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# HOMOGENIZATION-BASED ANALYSIS OF THE INFLUENCE OF MICROCRACKING ON THERMAL CONDUCTION AND THERMOELASTICITY

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**Summary** Microcracks are known to affect the behaviour of the materials and several modelling studies are devoted to finding their effective elasticity. This paper aims to extend such investigation to thermal properties, concerning both the steady-state conduction and the thermoelasticity. Accounting simultaneously for the anisotropy induced by microcracks orientation and for their unilateral effect (different behaviour whether they are open or closed) remains a difficult task. In this work, 3D homogenization-based approaches are developed to derive closed-form estimations of the effective thermal properties of a microcracked media with arbitrarily oriented microcracks, either open or closed.

## INTRODUCTION

Effects of microcracks on the elastic properties have been extensively studied through experiments and modelling approaches. In the latter case, the homogenization (up-scaling) technique appears as an effective tool to provide the overall properties from the microstructural features of the Representative Volume Element (RVE) [1]. Especially for microcracks, main difficulties arise from: (i) the anisotropy induced by the oriented nature of defects, (ii) their ability to be open or closed (according to tension or compression stresses) and to influence differently the overall response of the material. Regarding the thermal steady-state conduction and thermoelastic properties of microcracked media, very few investigations of these effects have been done in the literature, even on the experimental point of view. Some micro-macro modelling works have been proposed to derive effective properties of a given RVE but they consider only the open state of microcracks [2]. This work aims at exploring homogenization-based approaches to derive anisotropic thermal conductivity and thermoelasticity accounting for unilateral effect.

## MICROMECHANICAL FRAMEWORK

Assuming length scale separation, let us consider a 3D RVE composed of an isotropic matrix weakened by  $N$  number of randomly distributed families of parallel microcracks. Such media exhibits a matrix-inclusion topology in which each phase (matrix, microcracks) is homogeneous, allowing the use averaging techniques. Two conditions can be imposed on the outer boundary of the RVE, i.e. uniform macroscopic temperature gradient  $G$  (for the thermal conduction problem) and uniform macroscopic strain  $E$  with constant temperature difference  $\Delta T$  over the RVE (for the thermoelastic problem). Under equilibrium condition, the macroscopic quantities  $(G, E)$  correspond to the average value of their respective microscopic quantity  $(g, \epsilon)$  over the RVE, i.e.  $G = \langle g \rangle$ ,  $E = \langle \epsilon \rangle$ . The microscopic and macroscopic quantities can be linked linearly as  $g = A \cdot G$  and  $\epsilon = \mathbb{A} : E$ , where  $A$  and  $\mathbb{A}$  are the second-order gradient and fourth-order strain localization tensors, respectively. Regarding the steady-state heat conduction problem, the microscopic heat flux  $q$  is given by Fourier's law:

$$q = -\lambda \cdot g \quad (1)$$

where  $\lambda$  is the symmetric second-order thermal conductivity tensor. On the other hand, thermoelasticity is classically described by the following linear constitutive equation:

$$\sigma = \mathbb{C} : \epsilon - \kappa \Delta T \quad (2)$$

where  $\sigma$  is the microscopic stress tensor,  $\mathbb{C}$  is the fourth-order stiffness tensor and  $\kappa = \mathbb{C} : \alpha$  is the second-order thermoelasticity tensor with  $\alpha$  being the thermal expansion tensor.

Based on the boundary conditions, the overall heat flux  $Q = \langle q \rangle$  and stress  $\Sigma = \langle \sigma \rangle$  of the RVE can be expressed as:

$$Q = -\lambda_{hom} \cdot G \quad \text{and} \quad \Sigma = \mathbb{C}_{hom} : E - \kappa_{hom} \Delta T \quad (3)$$

where  $\lambda_{hom}$ ,  $\mathbb{C}_{hom}$ , and  $\kappa_{hom}$  are the effective tensors related to the thermal conductivity, stiffness and thermoelasticity of the microcracked media, respectively. Taking all simplifications into account, the effective thermal properties  $\lambda_{hom}$  and  $\kappa_{hom}$  are given as:

$$\lambda_{hom} = \lambda_m + \sum_{i=1}^N f_{c,i} (\lambda_{c,i} - \lambda_m) \cdot \langle A \rangle_{c,i} \quad \text{and} \quad \kappa_{hom} = \kappa_m + \sum_{i=1}^N f_{c,i} (\kappa_{c,i} - \kappa_m) : \langle \mathbb{A} \rangle_{c,i} \quad (4)$$

where  $\lambda_m$  (resp.  $\lambda_{c,i}$ ) and  $\kappa_m$  (resp.  $\kappa_{c,i}$ ) are the thermal conductivity and thermoelasticity tensors of the matrix (resp. of the  $i^{th}$  family of parallel cracks) and  $f_{c,i}$  is the cracks volume fraction for the  $i^{th}$  family.

Estimations of localization tensors  $A$  and  $\mathbb{A}$  can be obtained using the works of Eshelby on the single-inhomogeneity problem [3]. In line with the recent study of [4], each family of microcracks is modelled as a flat oblate ellipsoid with

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unit normal  $\mathbf{n}_i$ , crack density  $d_i$  and aspect ratio  $\omega_i \rightarrow 0$ . We assume both the matrix and cracks are isotropic and their properties can be given by  $\lambda_r = \lambda_r \mathbf{I}$ ,  $\kappa_r = \kappa_r \mathbf{I}$  and  $\mathbb{C}_r = 3k_r \mathbb{J} + 2\mu_r \mathbb{K}$  with  $r = \{m; c, i\}$  (where  $\lambda_r$  and  $\kappa_r$  are scalars while  $k_r$  and  $\mu_r$  are the bulk and shear moduli). The unilateral effect is taken into account via the  $\lambda_{c,i}$  and  $\kappa_{c,i}$ . For open cracks, it is assumed adiabatic (for thermal conduction) and stress-free (for thermoelasticity) conditions on the microcracks lips, so  $\lambda_{c,i} = 0$ ,  $\mathbb{C}_{c,i} = \mathbb{O}$  ( $k_{c,i} = 0, \mu_{c,i} = 0$ ) and  $\kappa_{c,i} = 0$ . Inspired by the works of Deudé *et al.* [5], the closed cracks (frictionless contact) are considered as a fictitious isotropic material such that  $\lambda_{c,i} \neq 0$ ,  $\kappa_{c,i} \neq 0$  and  $\mathbb{C}_{c,i} = 3k_m \mathbb{J}$  ( $k_{c,i} = k_m, \mu_{c,i} = 0$ ), which accounts for some heat (for conduction) and stress (for thermoelasticity) transfer continuity across the closed microcracks lips.

## RESULTS

From these assumptions, closed-form expressions of the whole thermal properties (Eq (4)) are derived for dilute and Mori-Tanaka (MT) schemes and Ponte Castañeda-Willis (PCW) bound. For a single family of open microcracks,  $\lambda_{hom}$  is function of  $d, \mathbf{n}$ ,  $\lambda_m$  and  $\kappa_{hom}$  is function of  $d, \mathbf{n}$ ,  $\kappa_m$  and matrix Poisson ratio  $\nu_m$ . Also the material shows a damage-induced anisotropy, irrespective of the estimation. To be precise, the effective thermal properties are transversely isotropic around the axis  $\mathbf{n}$  of the crack. As an illustration, Fig. 1 presents the generalized scalar thermal conductivity  $\lambda(\mathbf{v})$  and thermoelasticity  $\kappa(\mathbf{v})$  in any direction of unit vector  $\mathbf{v}$  defined by:  $\lambda(\mathbf{v}) = \mathbf{v} \cdot \lambda_{hom} \cdot \mathbf{v}$  and  $\kappa(\mathbf{v}) = \mathbf{v} \cdot \kappa_{hom} \cdot \mathbf{v}$  respectively. While both properties are mostly degraded along the direction  $\mathbf{n}$  normal to the crack surface, the cracks influence is stronger on thermoelastic properties for a given cracks density. We should also note the differences for both properties according to the estimations, related to the account (MT and PCW) or not (dilute) of interactions between microcracks and, in the former case, to the considered spatial distribution of microcracks (ellipsoidal for MT and spherical for PCW).

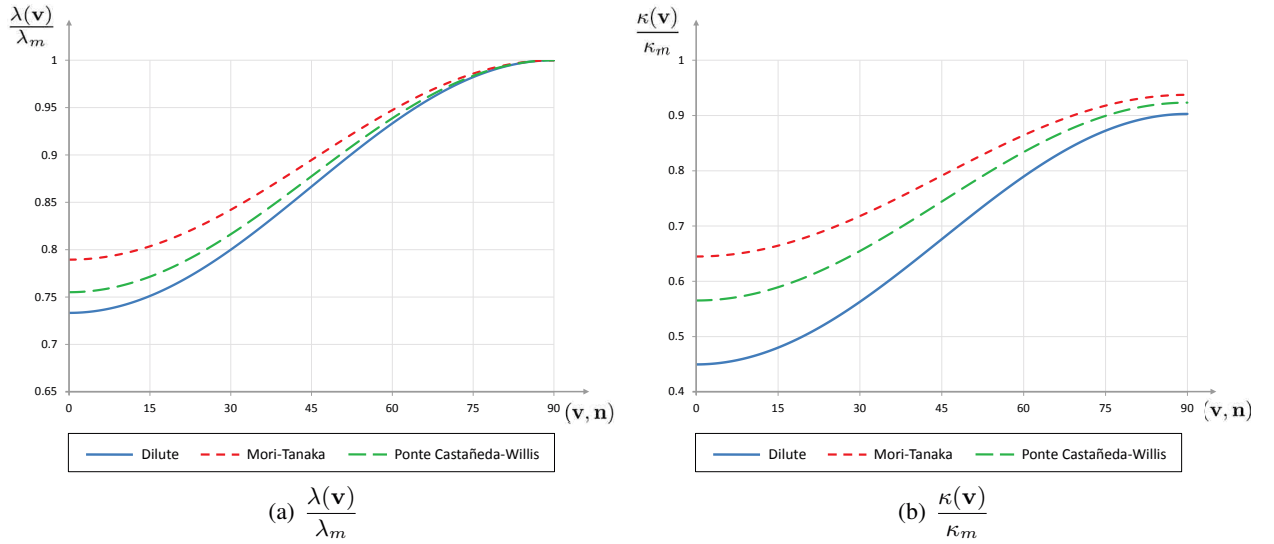


Figure 1: Generalized thermal conductivity  $\lambda(\mathbf{v})$  and coefficient  $\kappa(\mathbf{v})$  normalized by their initial values for a material weakened by a single family of parallel open microcracks of unit normal  $\mathbf{n}$  (cracks density  $d = 0.1$ , matrix Poisson ratio  $\nu_m = 0.15$ ).

On the other hand, it is demonstrated that closed cracks do not contribute to the degradation of the thermal conductivity or thermoelasticity. This result is irrespective of the estimation (dilute, MT, PCW), fictitious cracks properties or space direction, i.e. effective conductivity and thermoelasticity in any direction  $\mathbf{v}$  recover their respective values for the virgin material:  $\lambda(\mathbf{v}) = \lambda_m$  and  $\kappa(\mathbf{v}) = \kappa_m$ ,  $\forall \mathbf{v}$ . While elastic properties are only fully recovered in the direction  $\mathbf{n}$  normal to closed microcracks [6], we observe here a total deactivation of microcracking influence on thermal properties.

## CONCLUSIONS

Using Eshelby approach, we have derived in this work several homogenization-based expressions for effective thermal properties of a microcracked media, both regarding steady-state conduction and thermoelasticity. Special attention has been paid to the unilateral effect and its consequences on the overall behaviour. This study provides some relevant information to further develop a fully coupled thermo-mechanical model with evolving damage.

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