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What is the Shape of Geographical Time-Space? A three-dimensional Model made of Curves and Cones

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Abstract: *Geographical time-spaces* exhibit a series of properties, including space inversion, that turns any representation effort into a complex task. In order to improve the legibility of the representation and leveraging the advances of three-dimensional computer graphics, the aim of the study is to propose a new method extending time-space relief cartography introduced by Mathis and L'Hostis [1–4]. The novelty of the model resides in the use of cones to describing the terrestrial surface instead of graph faces, and in the use of curves instead of broken segments for edges. We implement the model on the Chinese space. The Chinese geographical time-space of the reference year 2006 is produced by the combination and the confrontation of the fast air transport system and of the 7.5 times slower road transport system. Slower, short range flights are represented as curved lines above the earth surface with longer length than the geodesic, in order to account for a slower speed. The very steep slope of cones expresses the relative difficulty of crossing terrestrial time-space, as well as the comparably extreme efficiency of long-range flights for moving between cities. Finally, the whole image proposes a coherent representation of the *geographical time-space* where fast city to city transport is combined with slow terrestrial systems that allow to reach any location.

Keywords: Geographical Time-Space; Transport; Cartography

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1. Introduction

Geographic space is known through experience and its narration, and through representation. Geographic distances are experienced through the use of means of transport, while representations usually take the form of maps. In the domain of the representation of geographic space, even considering the earliest maps [5], many efforts have focused on improving the coherence between the experience of space through movement and the representation itself. Most of these proposals introduced deformations of the conventional map [6–10] and belong to the family of anamorphic cartography. Other proposals focused on network representation [11–13] with springs more or less compressed to express the temporal length of routes. These representations have issues that we will expose below¹.

Time geography [14] and its developments in GIScience [15–17] indicate another stream of investigation of geographical time-space but more focused on understanding and modelling movement and its constraints than on producing a new representation of geographical time-space. In particular time geography contains no significant development regarding geographical distances. In a broader scope, cartographers have started importing methods developed in the field of scientific visualisation [18,19]. This stream of "visual analytics" [20,21] aims at implementing improved graphical outputs to make complex spatial data intelligible. While our datasets are comparably much more simple

¹ See section 4 [Cartographic Representations of Geographical Time-Space](#)

36 than what is generally used in visual analytics, we also make use of sophisticated visual
37 representation methods borrowed from scientific visualisation.

38 More generally, beyond these cartographic and visualisation works, in geography
39 and related fields, only a few efforts have been dedicated to the study of distances
40 [22–28]. This observation is also valid concerning time geography and visual analytics
41 that both dedicated most efforts on movement data rather than on distances.

42 We propose in this paper a geographical time-space model extending time-space
43 relief cartography introduced by Mathis and L’Hostis [1–4]. We will expose the principles
44 of the model and, in the annexes, detail the mathematics needed for the graphical
45 implementation.

46 Furthermore, we want to produce a cartographic model of current geographi-
47 cal time-space. In this aim we start with the study of the properties of geographical
48 time-space transport modes. After an empirical section, we draw a state of the art of
49 cartographic representation of geographical time-space. Finally, we introduce our model.

50 2. The Properties of Geographical Time-Space

51 We call geographical time-space a representation of the geographical space ex-
52 pressing the time spent to reach places. Representations of geographical time-space
53 usually come with a timescale. The study of geographical time-space reveals a series of
54 properties.

55 Firstly, geographers observed, at least since the antiquity [29, 19-3], an *acceleration* of
56 human movement, and expressed the idea of the shrinking of geographical time-space
57 [7,30].

58 Secondly, the crossing of geographical space is performed by a range of coexisting
59 *transport means*. In the current transport system, several transport means with distinct
60 characteristics and especially distinct speed levels, can be observed. The principle of
61 coexistence of several transport modes can also be traced back in the history of transport
62 [31]. Each transport system has its own domain of relevance, and the principle of their
63 co-existence is a key feature of geographical time-space. But this represents a critical
64 challenge for the representation of time-space and geographical distances due to the
65 coexistence of different speeds.

66 Thirdly, movements in the geographical space are mostly realised by means of
67 *transport networks*. This means that the description of geographical time-space should
68 exhibit network properties. Transport networks may be continuously or discontinu-
69 ously accessible. The usual road network or the urban public space are an example of
70 continuously accessible networks, as opposed to the cases of expressways, railways or
71 the airlines that are only accessible through access points. The other key properties of
72 networks relates to the dialectic of straight lines and detour [32]. Paths through networks
73 follow predefined lines and almost always escape the straight line between origin and
74 destination.

75 Finally, the most geometrically disturbing property of geographical time-spaces
76 derives from an exacerbation of network configurations exposed before. *Space inversion*
77 [33,34] occurs when the initial portion of a trip goes in the opposite direction to that of
78 the end destination. It is abundantly observed in geographical time-space and occurs in
79 the proximity of access-point of discontinuous networks such as expressway entrances,
80 railway stations and airports. In these cases the order of proximities in geographical
81 time-space is profoundly disturbed. As shown in figure 1, a place B located in-between
82 two other places A and C in *geographical space* may have a different position in *geographical*
83 *time-space*: C is closer to A (1 hour) than to B (1h20) in *time-space*.

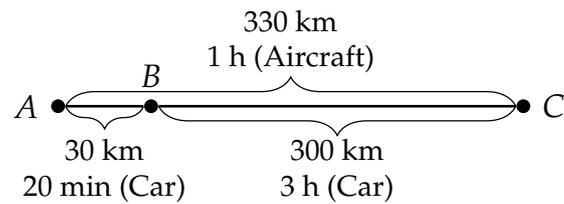


Figure 1. Space inversion between three places located in *geographical space* (kilometres) and in *geographical time-space* (duration)

84 It can be observed that these four properties of geographical time-spaces are tightly
 85 linked to each other. Thus, space inversion is generated by the design of transport
 86 networks, and implies the principle of coexistence of several transport means. After this
 87 first section on the properties of geographical time-space we will expose our empirical
 88 approach of the measurement of transport speed.

89 3. The Speed of Air and Road Transport Systems

90 The air transport system lies at the top of the current hierarchy in terms of speed
 91 level. The top speed of a typical jet aircraft is about 900 km/h which converts into a
 92 typical commercial speed of 750 km/h if we consider for example a current London to
 93 New York 7h45 flight.

94 We have collected data on a sample of air routes with great circle kilometre distance
 95 and duration from timetable². Long distance east-west flights exhibit significantly
 96 different schedules according to the direction of flight regarding trade wind. But the
 97 model needs symmetrical data for the network. In this case we have chosen as duration
 98 the greatest measure, generally from west to east.

² We extracted data from the website flightaware.com in January 2016.

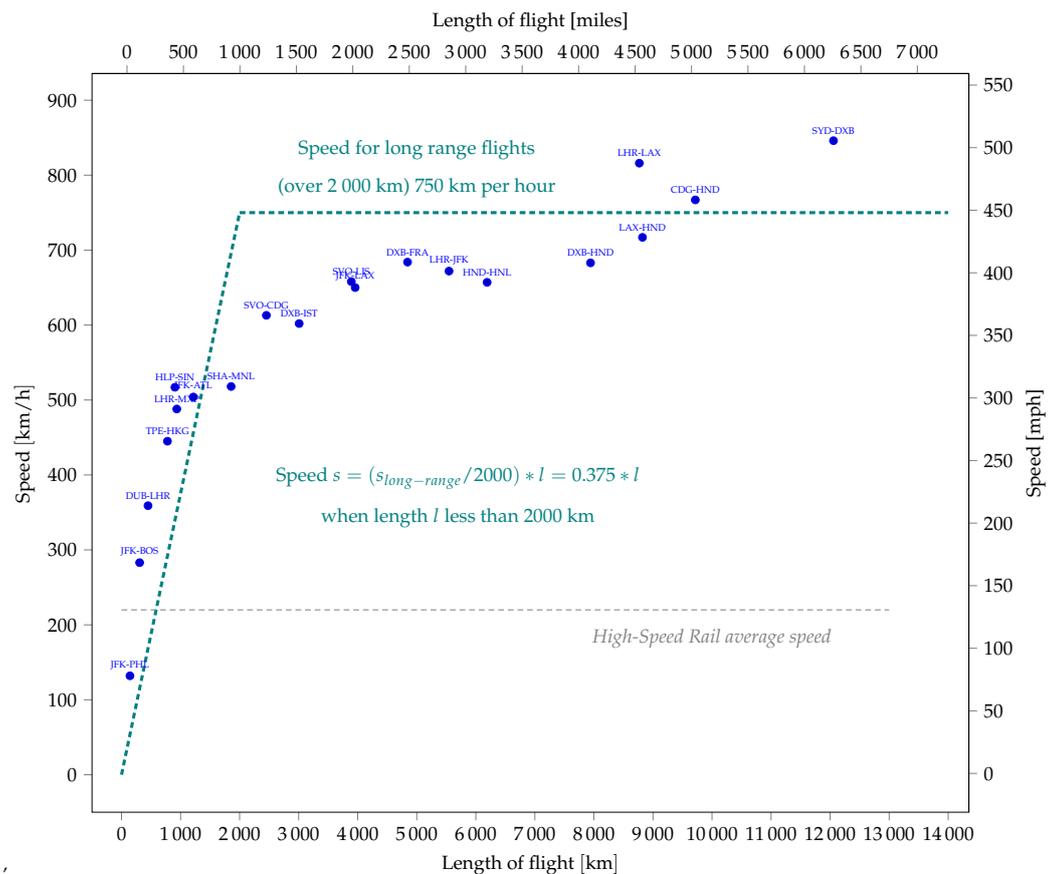


Figure 2. The commercial speed of aircraft services on a sample of origin destination pairs (data from flightglobal.com in 2016) and a linear approximation

99 The analysis of current timetables on a sample origin-destination pairs on figure
 100 2 shows that, in terms of speed, we can consider two situations. Below 2000 kilo-
 101 metres in length, the speed of services can be approximated with a linear function,
 102 $s = (s_{long-range}/2000) * l = 0.375 * l$, where l is the length of the service between origin
 103 and destination following the geodesic line. Beyond 2000 kilometres of service length
 104 we approximate speed with the value of 750 kilometres per hour³.

105 According to the principle of coexistence of transport modes, in complement to the
 106 air transport system, reaching destinations necessitates the use of terrestrial transport
 107 systems. The terrestrial transport system can be described with a unique speed of 100
 108 kilometres per hour [35]. This value represents an approximation of car speed over long
 109 distance on motorways, as well as train speed on classical rail systems.

110 Once the key parameters are fixed we can start considering the issue of the repre-
 111 sentation of geographical time-space.

112 4. Cartographic Representations of Geographical Time-Space

113 The improvement of transport means over time has had profound influence on our
 114 representation of geographical space. The more common sense idea, the first property
 115 of geographical time-spaces, states that the world is getting smaller. Nevertheless, the
 116 idea of a uniform shrinkage [36,37] has been challenged by geographers [9,11,30]. The
 117 main critic issued against the principle of uniform shrinkage is the idea that the entire
 118 intermediate and surrounding geographical space benefits from transport conditions
 119 improvements in a given area or between two locations. These representations do not
 120 consider the widely accepted fact that geographical time-space is not homogeneous [9].

³ Further developments could introduce an exponential function in order to better represent the empirical data.

121 More generally, the issue of the cartographic representation of time-space can be
122 seen as transforming a *kilometre-space* into a *time-space*. In the cartographic representation,
123 we will then replace the usual and conventional kilometre scale by a timescale.

124 The various methods for the production of cartographic time-space can be subdivi-
125 ded into two categories (38, 39, 90). Most methods found in the literature move the
126 position of places on the map in order to reflect time-space distances. Many represen-
127 tations of this type were introduced during the *plastic spaces* period in the history of
128 geography [39, 90], in reference to the expression proposed by Forer [8] to describe a
129 geography freed from cartographic conventions. Since this very active period, regularly
130 new implementations emerge [40–43]. Cartographic representations attached to this
131 movement are of the type of the anamorphic cartography [33,44]. In *plastic space* maps
132 the heterogeneous shrinkage of space is assumed. But these maps imply that all the
133 space located in-between places that are moved closer to each other benefit from the
134 time-space contraction. The fact that these maps do not account for the phenomenon of
135 *space inversion* (see figure 1), a key feature of geographical time-spaces, is the main critic
136 formulated against this type of representation [4].

137 The second category of geographical time-space representation consists in pre-
138 serving the location of places, and in drawing the edges in a way that expresses time-
139 distances. Janelle introduced curved lines in network style diagrams to account for
140 lower speeds of slow transport systems [11]. This approach produced a coherent repre-
141 sentation of time-distances with multiple speed. Tobler proposed sketch maps where
142 compressed springs express variations in the efficiency of edges in a network [13]. These
143 two first proposals are relevant as network representations, but fail at representing space
144 in a continuous way. Overcoming this limitation, and belonging also to this category,
145 time-space relief cartography were conceptualised by Mathis [1] and implemented by
146 L/Hostis [2].

147 The time-space relief cartography is based on three general principles:

- 148 • Cities, considered as nodes of the transport network, remain at their conventional
149 geographical location.
- 150 • Edges are drawn in the third dimension, proportionally to the travel time needed
151 between nodes.
- 152 • The geographic surface is attached to the slowest network, i.e. the road network.

153 These principles imply that the height of cities does not have the meaning it has in
154 classical thematic cartography. Proportional time-distances is the only quantity that the
155 map conveys, by design.

156 When drawing edges in the three-dimension time-space representation, one princi-
157 ple must be observed, but several rules can be chosen. The principle of the representation
158 implies that edges length must be proportional to their associated time-distance.

159 The shortest edges must be drawn as straight lines in the plane, or as geodesic
160 curves on the sphere. All the other edges, characterized by a slower speed than the
161 fastest existing speed in the considered space, must be drawn longer than the fastest
162 edges, proportionally to the longer time needed to join the two location. This type
163 of cartography permits, by design, the simultaneous representation of fast and slow
164 transport modes. The design of the model conveys a significant level of abstraction
165 regarding the actual path of connections or links between cities for all transport modes.
166 For instance while air connection rarely follow the straight line or the geodesic [45], our
167 model takes for reference the most direct route in order to allow for visually reading
168 lengths that converts in time.

169 Providing the principle of proportionality is respected, a degree of freedom is
170 allowed in drawing the edges between two nodes of the graph. The simplest way
171 consists in drawing two segments with a middle point located under the surface of
172 the earth. If the time-space structure is an obstacle to the drawing of slow edges, it is
173 possible to draw the edges above the surface of earth [46]. From the representation of
174 terrestrial transport modes edges it is possible to infer a transport surface.

175 Initially applied only to terrestrial transport modes, the model has been extended
 176 in the 2000' to the air transport system, on continental spaces [4]. The next step is the
 177 current proposal that develops a new design for the transport surface, and introduces
 178 curves for long edges. While in previous time-space cartographic designs the terrestrial
 179 transport surface was based on the faces of a graph, we introduce the use of cones.

180 5. The Principles of A Representation of Geographic Time-Space Based on 181 Three-Dimensional Cones

182 Unlike all previous time-space relief representation [2–4,47] where the geographical
 183 surface is based on the edges of a graph, we introduce conic shapes to represent terrestrial
 184 time-space. Each city and its surrounding space are then represented by a cone. The
 185 spiky edge of the cone ends at the usual geographic location of the city, and the cone
 186 geometry extends below the earth surface. The terrestrial time-space is formed by the
 187 three-dimensional surface resulting from the aggregation of all the cones together.

188 The use of cones allows to generalising the terrestrial transport surface, and to
 189 simplifying the principle of construction of the three-dimensional model. In addition,
 190 the design based on cones permits to avoid the random slopes generated by the three-
 191 dimensional facets and replaces them by a uniform, controlled slope. A uniform slope
 192 for the terrestrial transport surface is, from a theoretical point of view, more satisfactory
 193 than previous three-dimensional network based facets [4] where the slope of facets was
 194 not fully controlled, as it is more coherent with the basic principle of the representation
 195 which indicates a unique slope for a given transport mean.

196 The cone is the basic structure of this representation. Cones are characterized by
 197 a slope that follows the ratio between the speed of basic terrestrial networks, i.e. the
 198 road system, and the maximum available speed, which is attained, on the period that
 199 we consider, through the air transport system.

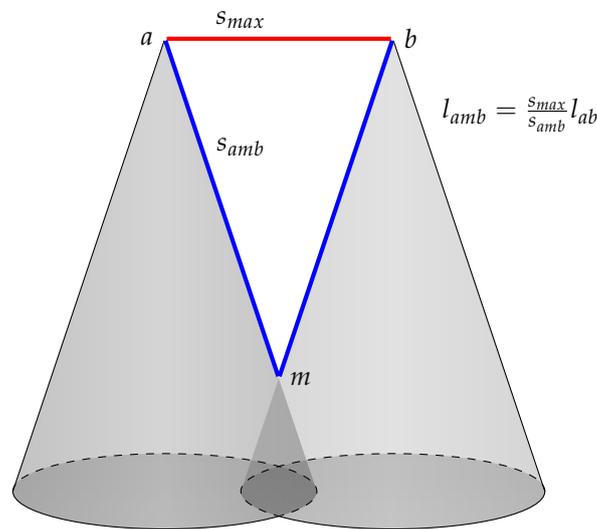


Figure 3. Cones and edges in three dimensions as a basic structure for time-space representation

200 On figure 3 two cities *a* and *b* are joined by a fast non-stop transport system (in
 201 red), and also joined by a slower terrestrial transport mode (in blue). The fast transport
 202 link is represented through a straight line of speed s_{max} , while the cones represent the
 203 surface supporting the slower terrestrial transport mode of speed s_{amb} , with *m* as the
 204 midpoint between the two cities and the intersecting point of the two cones. Assuming
 205 that terrestrial travel speed are similar in all the geographic space considered, the slope
 206 of the cones centred on the two cities is a coherent representation of time-space. On
 207 figure 3, the slope of the cones implies that the length of the straight red line, and the
 208 length of the blue line drawn on the cone and joining the two cities, are proportional
 209 to the respective travel times, according to a provided formula of a ratio of speed. This

210 three-dimensional geometry forms a time-space representation with different transport
211 speed.

212 The mathematical formulae of the representation are presented in the annexes of the
213 paper: the geometry of the cones (annex A), the geometry of edges in spherical geometry
214 (annex B) and the geometry of smoothed edges in projected geometry (annex C).

215 6. Theoretical Validation

216 We can test the ability of the representation to respect and render the four properties
217 introduced in section 2.

218 The property of acceleration may be rendered by producing side by side two
219 states of the representation at two different periods where transport speed differ. A
220 common timescale will help to express the transformations provoked by an acceleration
221 of transport speed. Hence, the representation is able to account for the acceleration of
222 human movement over historical time.

223 Secondly, the coexistence of transport means is introduced by design since the
224 representation is based on a graph of the transport network characterised by several
225 transport modes.

226 Thirdly, the combination of continuous and discontinuous access is provided by the
227 design of the three-dimensional structure: the cones represent the continuous geographic
228 surface, the lines represent the links of the transport network, and the summit of cones,
229 where the geographical surface and fast transport network meet, represent the cities. In
230 the representation, the two modes of representation of geographical time-space – namely
231 continuous and network based discontinuous – coexist and are attached to each other
232 through cities.

233 Finally, the space inversion is made intelligible and readable on the representation:
234 from a point located on the slope of a cone, close to the top, the fastest path to a distant
235 city implies going back *up* to the nearby city, and taking the fast transport mode.

236 Next steps of the research will involve user tests in order to provide a practical
237 validation. They extend beyond the scope of the current article.

238 7. Dataset

239 In order to build a map of the geographical time-space, we firstly need to establish a
240 list of cities. We used the 2014 revision of the World Urbanisation Prospect by the United
241 Nations [48] for the cities, being the most recent high quality urban dataset available at
242 the time of creation of the representation.

243 The air services network is derived from the data provided by openflights.org⁴. The
244 network of flights is from the year 2006, the most recent dataset available from this source.
245 In addition, the aerial transport system of 2006 can be considered as representative of
246 the 2014 condition. Links in the network correspond to air routes served daily by at
247 least three aircraft of a capacity of more than 70 passengers. We defined this threshold to
248 consider only origin-destinations pairs with a significant service available.

249 Tests⁵ shown that the shorter the list of cities, the easier it is to produce a readable
250 representation. In this view, we used two criteria to operate the selection of cities. Only
251 cities with more than a million inhabitants were considered. In addition, we chose only
252 those cities served by aerial services in 2006. This selection rules applied to the UN cities
253 database generated a list of 45 Chinese cities, including Taiwan.

⁴ <https://openflights.org/data.html>

⁵ The blog <https://timespace.hypotheses.org> shows several tests of representation.

254 8. Implementation of the Geographical Time-Space of China

255 The three-dimensional model on figure 4 was produced by the *Shriveling world*⁶
256 software developed by the authors and B Helali.

257 The Chinese geographical time-space in 2006 is characterised by two transport
258 modes. Road transport is modelled with a uniform speed of 100 kilometres per hour
259 which is consistent with the measurement of travel times by road between major Chi-
260 nese cities. The air transport system is divided in two sets of links. According to our
261 analysis (see section 3) long range flights, beyond 2000 kilometres, follow a speed of 750
262 kilometres per hour. Lower range flights have a decreasing speed following their length
263 in kilometres. The Chinese geographical time-space is generated by the confrontation
264 of long range flight speed (750 km/h) and road speed (100 km/h). The slope of cones,
265 where cars circulate, is the same for all the geographic surface⁷ and is computed (see
266 annex A) from the fundamental ratio of the maximum speed divided by road speed
267 giving the value of 7.5.

268 The implementation of the geographical time-space model to China has obeyed to
269 a series of choices and parameters that we will expose and justify now.

- 270 • On this representation, no geographical projection is used: cities are located on the
271 terrestrial sphere which explains that the fastest arcs, in red, are not straight lines
272 but curves: they follow the geodesic.
- 273 • An angle of 45 degrees between the camera axis and the tangent to the surface
274 of the earth, has been chosen as a compromise between the readability of the
275 final image, and the preservation of proportionality of north-south compared to
276 east-west lengths.
- 277 • Lights have been introduced in order to maximise the readability of the three-
278 dimensional structure: a directional light is completed by punctual lights between
279 the cones in order to improve contrasts.
- 280 • Cones are set in white colour with shadows to express their three-dimensional
281 nature.
- 282 • Aerial edges are coloured in two colours according to the categories of long and
283 short haul. Long-haul speed, i.e. the fastest observed speed on this geographical
284 space, determines the parameters of the geometry of all the representation: cones
285 slope, and the geometry of the other edges; given their importance we attributed
286 them a highly visible colour, red. Slower edges, on short haul routes, are coloured in
287 green to highlight their relatively weaker performance in geographical time-space.

288 This map is the first representation of the Chinese geographical time-space in relief,
289 and is also the first map ever with this new three-dimensional model with cones and
290 rounded edges.

291 On the representation, the speed of the road transport is expressed through the slope
292 of cones; the idea that the road allows to access all space is expressed by the fact that the
293 cone is a surface. The long range links of the air transport system are the fastest transport
294 mode available: they form direct geodesic lines between cities, shown in red. Slower,
295 shorter range flights have length proportional to the time needed to connect cities, and
296 expand above the geodesic line; they are represented in green. If the air transport system
297 is dramatically efficient for reducing time-space on long ranges, its efficiency drops
298 on shorter distances. To such extent that fast terrestrial modes can compete with air
299 transport on many city pairs, as shows the development of the high-speed rail in China
300 since the end of the first decade of the 21st century.

⁶ The *Shriveling world* application is available here: <https://bit.ly/shrivelApp>. Open source code can be accessed here: https://github.com/theworldisnotflat/shriveling_world. The map of this article was created by using the version v0.9-alpha of *Shriveling world*: <http://bit.ly/shrivelZen>, with post-treatments in Blender (<https://www.blender.org/>).

⁷ The road system being modelled as an anisotropic surface with uniform speed, physical geography constraints like mountains and rivers are completely ignored in the representation.

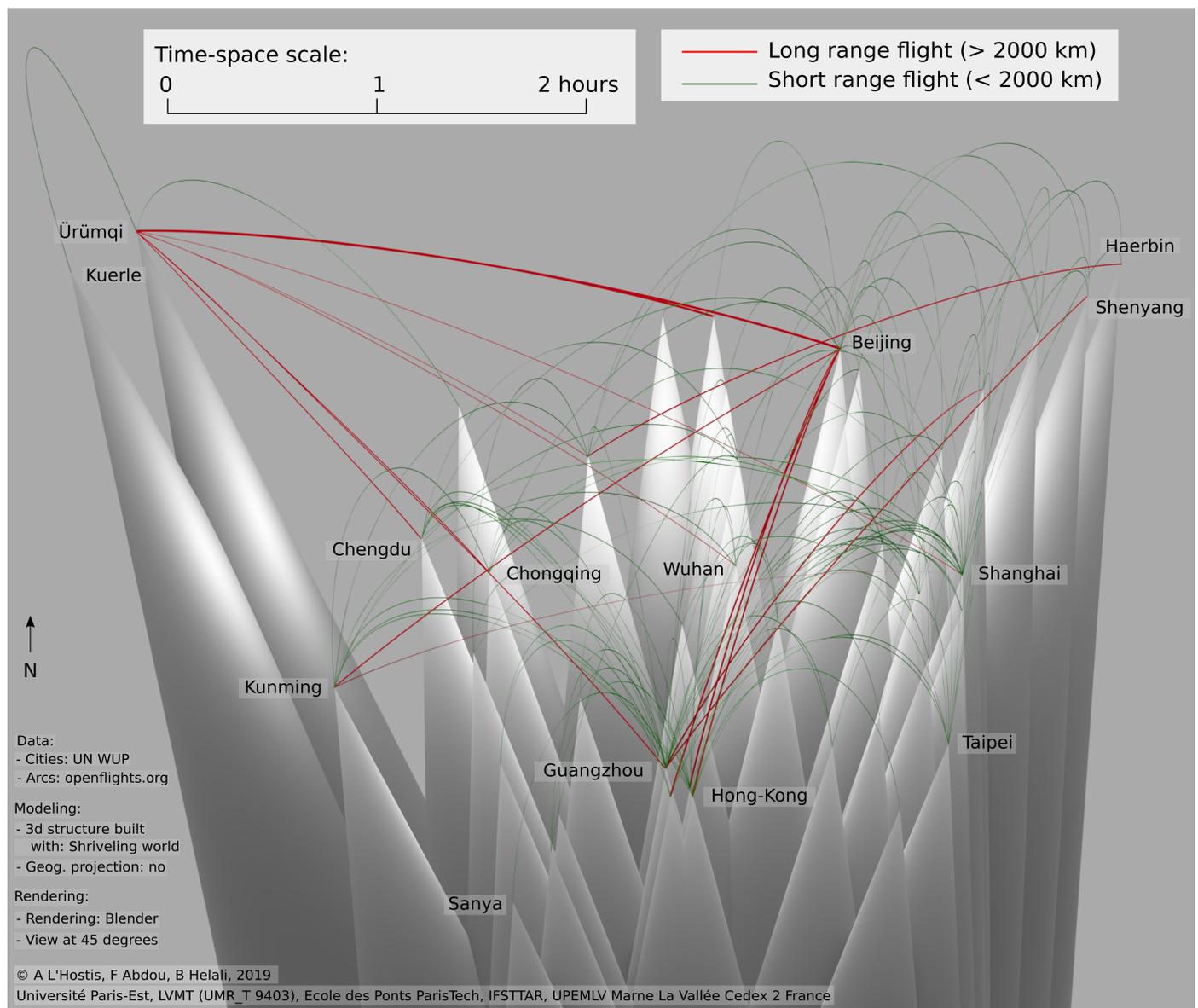


Figure 4. A representation of the Chinese geographical time-space in 2006. View of a model generated by the *shriveling world* software, unprojected Chinese cities, flight information from openflights.org, UN WUP cities data.

301 The representation opposes the efficiency of the air transport system in crossing
 302 time-space and the relative slowness of terrestrial modes. Considering that most move-
 303 ments at the scale of the country will involve combining flying and driving, the map
 304 proposes a coherent representation of time-space for such trips, where a time-scale for
 305 approximate measurement is available. The coherence between the measurements of
 306 time-space and the proposed graphic representation is not observed on the usual map or
 307 on other types of anamorphic cartography [49].

308 In exchange for the theoretical benefits it provides, from a general point of view,
 309 this implementation has engaged the need to address a whole new range of issues, some
 310 of them unprecedented in cartography. The main issues identified so far are listed here:

- 311 • the need to consider the phenomenon of occultation of three dimension shapes
 312 • the conflict between the desire to rotate the structure in all directions – a common
 313 practice in three-dimensional modelling – and the convention of cartography that
 314 invite to restrict it to rotate exclusively on an east-west axis

- 315 • the combination of geographical projection, from the geoid to a two-dimensional
316 flat surface, with other types of projection used to transform a three-dimensional
317 scene into a computer screen image
- 318 • the need for shadings to reveal three-dimensional shapes that may conflict with the
319 greying conventions of cartography used to express terrestrial relief, and conven-
320 tional colour choices in cartography [50]
- 321 • the need for a background which is not generally dealt with in cartography
- 322 • the need to adjust the graphical parameters of networks – width, colour, shape and
323 transparency – to allow for reading the three-dimensional surface situated below
324 the treatment of terrestrial and maritime borders

325 The model exposed in this paper is a proposition that aims at solving these issues.
326 But many alternatives can be imagined, and will be investigated in further work. This
327 could include projecting different GIS layers onto the three-dimensional surface in order
328 to facilitate the reading of the representation. In addition, the current implementation
329 covers a single large country; new issues may occur when considering other smaller or
330 larger geographical entities.

331 9. Conclusion

332 We introduce a new model of geographical time-space representation aiming at
333 improving and developing a model proposed in the 1990s. Our aim has been to ratio-
334 nalise the production of the maps, and to improve the readability. Compared to previous
335 representations of the same type, the novelty of the model resides in the use of cones to
336 describe the terrestrial surface instead of graph faces, and in the use of curves instead of
337 broken segments for edges.

338 The higher complexity of the three-dimensional model has necessitated to writing a
339 new code for generating the three-dimensional structure, and to using existing three-
340 dimensional rendering software.

341 The approach lies at the intersection of two domains involving graphic representa-
342 tion i.e. cartography and three-dimensional computer graphics, that, exception made
343 of the implementation of time-geography in GIS [17,51,52], have rarely been fully com-
344 bined: we intend to respect as far as possible the rules of cartography, but we also want
345 to take advantage of the potential of three-dimensional computer graphics to improve
346 the readability of the final image. Consequently, the representation produced should be
347 understood as borrowing to the two domains.

348 We implement the model on the Chinese space, for the first time. The Chinese
349 geographical time-space of the reference year 2006 is produced by the combination
350 and the confrontation of the fast air transport system and of the 7.5 times slower road
351 transport system. The very steep slope of cones expresses the relative difficulty of
352 travelling in the terrestrial time-space, as well as the comparably extreme efficiency of
353 long-range flights for movement between cities. The image also shows that the very high
354 efficiency of the air transport for crossing geographical time-space is not met on most
355 origin-destination pairs in the Chinese space: many flights follow a long curve above the
356 earth surface that account for their relative inefficiency, i.e. the relatively slow speed of
357 short haul flights, as is the case between Beijing and Shanghai⁸. This situation highlights
358 the relevance of the current development of high-speed rail in China. Finally, the whole
359 image proposes a coherent representation of the geographical time-space where fast
360 city to city transport is combined with slow terrestrial systems that allow to reach any
361 location.

362 The major challenge posed by geographical time-space three-dimensional repre-
363 sentation lies in the issue of readability. Proposing a coherent geographical time-space
364 representation that moves away from conventional cartography and that is fully based

⁸ Beijing (PEK) and Shanghai (SHA) airports are 1075 km (great circle distance) away and connected in 2h15 resulting in a commercial speed of 478 kph

365 on the three dimensions entails the introduction of an unprecedented set of cartographic
 366 and graphical issues. The developments exposed here – the cones and the curved edges
 367 – are two proposals in this direction of a more intelligible representation. Much remains
 368 to be done, in a domain where graphical solutions should meet cartographic constraints.
 369 Once a suitable level of legibility is obtained, the next steps should involve users testing
 370 in order to assess the improvement of the representation over previous designs, and
 371 validate it beyond the theoretical arguments provided here.

372 Appendix A Cones parameters in projected geometry

373 Considering the geometry of cones as shown in figure A1, we have the maximum
 374 speed as the ab straight line and the road speed along the cone in am . Cities a and b
 375 are linked by a fast transport mode and by a terrestrial transport mode characterized
 376 by a lower speed. By construction of the time-space map, the length of links ab by fast
 377 transport and amb by slow transport are proportional to the duration of the respective
 378 trips.

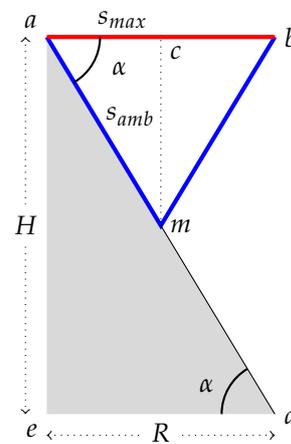


Figure A1. Cones and edges in projected geometry

379 The representation indicates the duration t of trips by measuring the length of
 380 segments. Hence, a fixed factor k corresponding to a time-space scale links duration
 381 and length so that $t_{ab} = k \cdot l_{ab}$ and $t_{am} = k \cdot l_{am}$. Time, length, and speed are linked by
 382 the simple formula of $t_{ab} = l_{ab}/s_{max}$, but on the segment am the duration of the trip
 383 corresponds to the length ac travelled at the speed on ab and hence $t_{am} = l_{ac}/s_{amb}$. From
 384 this we obtain a formula linking the length ac and am to a ratio of speed:

$$385 \quad t_{ac} = k \cdot l_{ac}$$

$$386 \quad l_{ac}/s_{max} = k \cdot l_{ac}$$

$$387 \quad k = 1/s_{max}$$

$$388 \quad t_{am} = l_{ac}/s_{amb} = k \cdot l_{am}$$

$$389 \quad l_{ac}/s_{amb} = l_{am}/s_{max}$$

$$390 \quad l_{am} = l_{ac} \frac{s_{max}}{s_{amb}}$$

391 The slope is then given by the ratio between speed s_{max} on ac and the speed s_{amb} on
 392 am .

Applying the trigonometric formula $1 + \tan(\alpha)^2 = \frac{1}{\cos(\alpha)^2}$ with $\tan(\alpha) = \frac{H}{R}$ and
 $\cos(\alpha) = \frac{s_{max}}{s_{amb}}$ gives $1 + \frac{H^2}{R^2} = \frac{1}{(\frac{s_{max}}{s_{amb}})^2}$ and finally:

$$R = \frac{H}{\sqrt{(\frac{s_{max}}{s_{amb}})^2 - 1}}$$

393 This final formula links the radius of the cone to its height and to the ratio between
 394 maximum speed and speed along the slopes of the cone. For a given radius, the height
 395 of the cone will follow this formula A1, according to the speed differential:

$$H = R \sqrt{\left(\frac{s_{max}}{s_{amb}}\right)^2 - 1} \quad (A1)$$

396 Appendix B Cones and Straight Edges in Spherical Geometry

397 In spherical geometry, the fastest edge is not a straight euclidean line, but rather a
 398 geodesic, or great circle edge between two cities, following the curvature of the earth.
 399 In the simplest variant of the model, a slower edge is drawn as two chained rectilinear
 400 segments of equal length, in the plane formed by the two cities orthogonal to the surface
 401 of the earth. The general principle of the model states that the length of the edge amb , in
 402 figure A2, is proportional to the length of the fastest speed edge g , hence to the ratio of
 403 speed.

404 The length of the arc g between a and b is given by the formula $g = r\theta$, and hence:

$$405 \quad am + mb = \frac{s_{max}}{s_{amb}} g = \frac{s_{max}}{s_{amb}} r\theta \quad am + mb = \frac{s_{max}r\theta}{s_{amb}} \quad am = \frac{s_{max}r\theta}{2s_{amb}}$$

We need to determine the coordinates of point m and the length of segment om .

$$om = r - (cd + dm)$$

In case the edges are drawn above the surface of the earth, then the length of segment om' is:

$$om' = om + 2dm = r - cd + dm$$

Considering the circle centred in a and having am' for radius we have:

$$x^2 + y^2 = r^2$$

The point m has for coordinate $x = ad = ab/2$ and $y = dm$ then:

$$(ab/2)^2 + dm^2 = am^2$$

$$dm = \sqrt{am^2 - (ab/2)^2}$$

$$dm = \sqrt{\left(\frac{s_{max}r\theta}{2s_{amb}}\right)^2 - (ab/2)^2}$$

Considering the triangle adO :

$$\sin \frac{\theta}{2} = ad/r = ab/2r$$

$$ab = 2r \sin \frac{\theta}{2}$$

We then update the previous equation as:

$$dm = \sqrt{\left(\frac{s_{max}r\theta}{2s_{amb}}\right)^2 - \left(r \sin \frac{\theta}{2}\right)^2}$$

$$dm = r \sqrt{\left(\frac{s_{max}\theta}{2s_{amb}}\right)^2 - \sin^2 \frac{\theta}{2}}$$

Considering the two points m and m' we will now focus on the general case of m' , knowing that $dm = -dm'$:

$$om = r - (cd + dm)$$

Considering the trigonometric formula of the height of the chord between a and b :

$$cd = r - \sqrt{r^2 - \frac{ab^2}{4}}$$

$$om' = r - \left(r - \sqrt{r^2 - \frac{ab^2}{4}} \right) + dm$$

$$\Rightarrow om' = \sqrt{r^2 - \frac{ab^2}{4}} + dm$$

406 Considering the formula of the length of the chord $ab = 2r \sin \frac{\theta}{2}$ we have:

$$om' = \sqrt{r^2 - r^2 \sin^2 \frac{\theta}{2}} + dm = r \sqrt{1 - \sin^2 \frac{\theta}{2}} + dm$$

$$om' = r \cos \frac{\theta}{2} + dm$$

$$om' = r \cos \frac{\theta}{2} + r \sqrt{\left(\frac{s_{max}\theta}{2s_{amb}} \right)^2 - \sin^2 \frac{\theta}{2}}$$

407 Here comes the formula A2 of the length of the segment om' , in the general case, as
408 a function of speed, r , and θ as:

$$om' = r \left(\cos \frac{\theta}{2} + \sqrt{\left(\frac{s_{max}\theta}{2s_{amb}} \right)^2 - \sin^2 \frac{\theta}{2}} \right) \quad (A2)$$

409 In order to account for the lower commercial speed of aircraft services on short
410 distances, as introduced before [4], we consider a cruise speed of 750 kilometres per hour
411 beyond 2000 kilometres of range, and the following formula below 2000 kilometres:

$$412 s_{amb} = (s_{long-range}/2000)g = 0.375g$$

413 with g as the length of the geodesic minimum path. Considering that $g = r\theta$, we
414 have:

$$415 s_{amb} = 0.375r\theta$$

$$416 om' = r \left(\cos \frac{\theta}{2} + \sqrt{\left(\frac{s_{max}\theta}{0.75r\theta} \right)^2 - \sin^2 \frac{\theta}{2}} \right)$$

417 In the case of aircraft links with length below 2000 kilometres the formula of the
418 length of the segment om' becomes the following :

$$om' = r \left(\cos \frac{\theta}{2} + \sqrt{\left(\frac{s_{max}}{0.75r} \right)^2 - \sin^2 \frac{\theta}{2}} \right) \quad (A3)$$

Finally we provide the formula of the midpoint between two given geographical loca-
tions with ϕ as latitude, λ as longitude, and $\Delta\lambda = (\lambda_2 - \lambda_1)$:

$$Bx = \cos \phi_2 \cos \Delta\lambda$$

$$By = \cos \phi_2 \sin \Delta\lambda$$

$$\phi_m = \arctan 2$$

$$\left(\sin \phi_1 + \sin \phi_2, \sqrt{(\cos \phi_1 + B_x)^2 + B_y^2} \right) \quad (A4)$$

$$\lambda_m = \lambda_1 + \arctan 2(B_y, \cos \phi_1 + B_x)$$

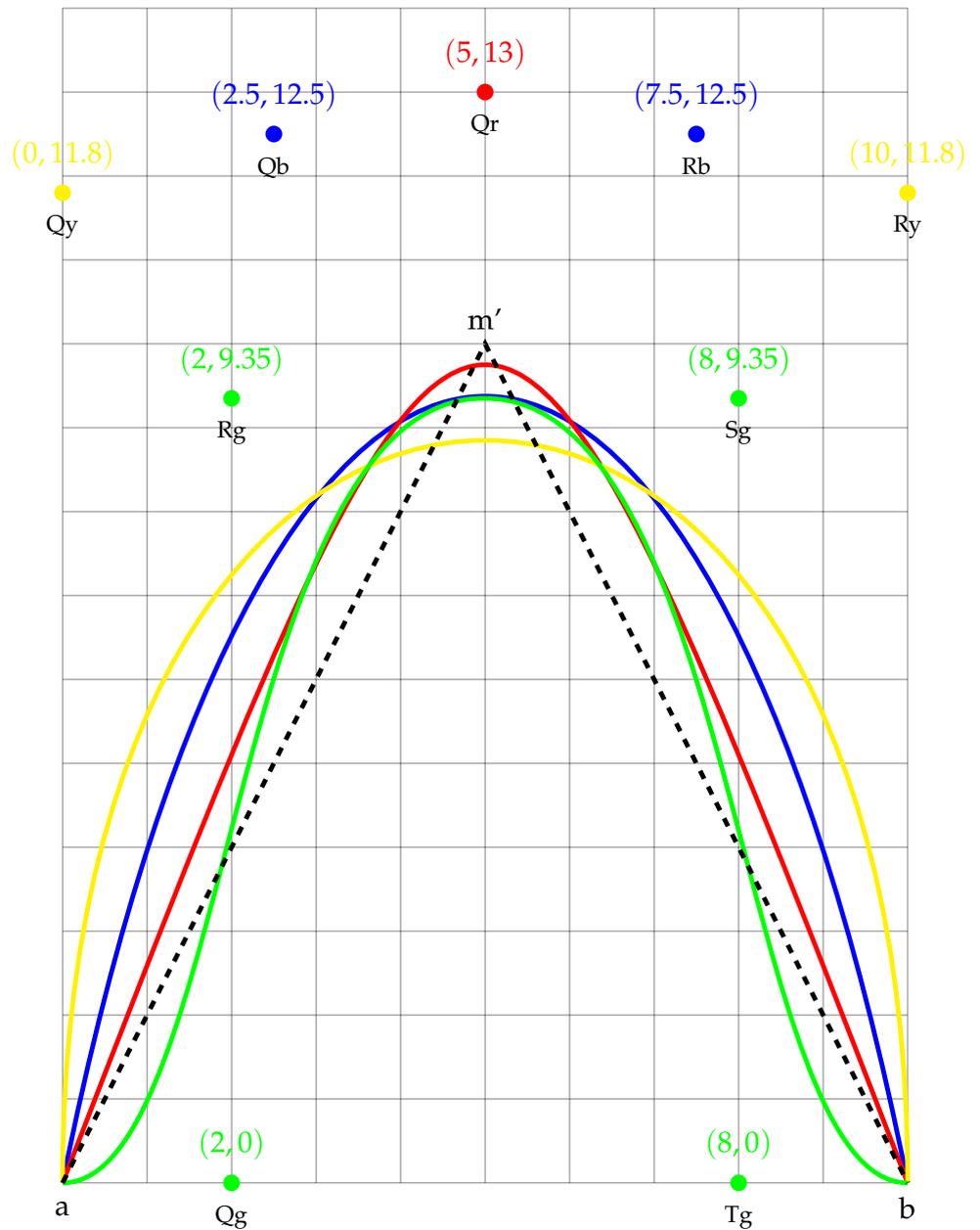


Figure A3. Several graphical solutions to the constraint of drawing an edge with a given length between cities a and b : two straight segments (dotted) and different Bézier curves (red with one control point Qr ; blue with two control points Qb and Rb ; yellow with two control points Qy and Ry ; green with four control points Qg , Rg , Sg and Tg). The position of control points was adjusted to constrain the curve length.

419 Appendix C Drawing Smoothed Edges in Projected Geometry

420 In the three-dimensional time-space, the length of edges is proportional to the time
421 needed to travel between cities.

422 On figure A3 we provide several graphical solutions to the constraint of drawing an
423 edge with given length between cities A and B . The straight segments solution geometry
424 is described in annex B and is shown here in dotted black line. Four Bézier curve
425 solutions are represented with their respective control points. The Bézier curve with
426 one single control point is shown in red. The blue and yellow Bézier curves with two
427 control points differ in their extremities. The yellow curve follows a vector orthogonal

428 to the earth surface in *A* and in *B*. Finally, a Bézier curve with four control points allows
429 a solution that tangents the earth surface in start and end points.

430 The proposed solutions are graphical variations respecting the general principle of
431 the representation. They allow to explore the graphical possibilities in terms of edges
432 layout. Smoothed curves avoid to graphically highlight the midpoint of the straight
433 segments, as in [4], which has no significant geographical meaning. Orthogonal or
434 tangent starting and ending curves propose different ways to graphically highlight cities
435 or transport nodes which bears true geographical meaning.

436 Geolocation information

437 The representation of the geographical time-space in relief covers:

- 438 • People's Republic of China
- 439 • Taiwan

440 **Author Contributions:** "Conceptualization, A.L.H.; methodology, A.L.H. and F.A.; software, F.A.;
441 data curation, A.L.H. and F.A.; writing—original draft preparation, A.L.H.; writing—review and
442 editing, A.L.H. and F.A.; visualization, A.L.H. and F.A.. All authors have read and agreed to the
443 published version of the manuscript."

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446 **Data Availability Statement:** Two sources of data are needed to produce the representation. We
447 used the 2014 revision of the World Urbanisation Prospect by the United Nations [48] for the cities.
448 The air services network is derived from the data provided by openflights.org representative of
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