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# Group belief and grounding in conversation

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## 1 Introduction

When we study dialogue between artificial agents, two traditions have emerged: a subjective one with for example [8, 31, 32, 33] and an objective one with [9, 48, 39, 15, 46, 4].

The former approach, often named mentalist approach, considers that dialogue is function of the agents' mental attitudes, usually formalized with BDI (Belief Desire Intention) modal logics. The speech acts that an agent can perform depend on his mental attitudes and are seen as modifying cognitive state of agents. Its most important application is the definition of FIPA-ACL (Agent Communication Language) semantics [13], the most used Agent Communicative Language.

This approach is also based on the recognition of mental attitudes (in particular intention). Linguistically, statement significance does not depend only on the statement but also on the speaker's intentions. That is what Grice names non-natural significance [21]. The example of indirect speech acts is the most obvious. But nothing proves that the hearer well recognizes the correct mental state and the speaker has no warranty to have been correctly understood (even if the communication is perfect).

The mentalist approach has a lot of predictive power (dialogue can be viewed as a driven by intentions and that can be planed [6, 1]). But in exchange, it needs very strong hypotheses on agents' behavior: e.g. sincerity, cooperation...

This approach has often been criticized (see e.g. [39, 15]) because hypotheses are too strong in open and heterogeneous multi-agents systems (MAS for short). Moreover no verification on the dialogue can be performed because it would need to access to private mental attitudes of agents, and such hypothesis is not realistic in open MAS.

To get round this problem objective approaches, also called conventional or structural approaches, take into account only what is public in the dialogue. The semantics of speech acts is described in terms of commitments, *i.e.* propositions that are registered in the commitment store, a kind of public black board for every agent taking part in the dialogue. This approach is much more descriptive in particular because commitments are not related to private mental attitudes. But it does not need hypotheses on agents anymore: agents can be insincere, uncooperative and based on approaches other than the BDI paradigm, in particular they may have no intentions at all (such as automata used in most nowadays “dialogue systems”).

The first aim of this paper is to bridge the gap between these two approaches by extending a BDI-like logical framework with an operator formalizing what is public in the dialogue.

We will use a small example to illustrate some properties of the notion we will study. Consider three rational agents in a company. The agent 0 thinks privately for some reasons that his boss (agent 2) is smart. But this idea is not widespread in his department: agent 0 meets agent 1, a very charismatic agent who often claims publicly that his boss (agent 2) is dumb. They discuss about their boss and agent 0 asserts that he is really a moron (for some social reasons) and of course agent 1 confirms. At this moment the boss comes and enters the conversation. Soon he oriented the discussion on himself and agent 0 congratulates him by asserting he is smart. And agent 2 and agent 1 (given the boss’ attendance) express their agreement.

It is interesting to see that agent 0 expressed fully different points of view depending on his hearers. As we are interested in what is public in a dialogue, we have to make precise which group of agents constitutes the public because a speaker’s behavior depends on who can hear what he says as illustrated above. Thus the second aim of this paper is to formalize such a dialogue with different groups of hearers.

Firstly we will philosophically ground this notion and define its major features (Section 2). Then we will formalize it by introducing it into a logic of belief, choice and action (Section 3). Afterwards we will show some applications of our new notion. We can apply it to formalize commitments and dialogue games *à la* Walton & Krabbe (Section 4). Finally we will show its link with group belief (Section 5).

The present paper is part of Benoit Gaudou's PhD thesis “*Formalizing social attitudes in modal logic*” (PhD thesis of the University of Toulouse, July 2008).

## 2 Notion of Grounding

We name our central notion *grounding*<sup>1</sup> in reference to works of Traum [41]. He defined it as “the process of adding to the common ground between conversational participants”. We use it in a more general sense: for us grounding refers to what is public in the dialogue. We give a more precise definition below.

### 2.1 Philosophical foundations

Our notion stems from speech act theory, where Searle’s *expression of an Intentional state* [36] concerns a psychological state related to the state of the world. Even if an utterance was insincere an Intentional state has been *expressed*, and that state corresponds to a particular belief of the speaker in some way.

Vanderveken [44, 45] has captured the subtle difference between *expressing* an Intentional state and *really being in* such a state by distinguishing *success conditions* from *non-defective performance conditions*, thus refining Searle’s felicity conditions [34, 35, 37]. According to Vanderveken, when we assert  $p$  we *express* that we believe  $p$  (success condition), while the speaker’s belief that  $p$  is a condition of non-defective performance.

The notion of groundedness is also behind Moore’s paradox, according to which one cannot successfully assert “ $p$  is true and I do not believe  $p$ ”. The paradox follows from the fact that: on the one hand, the assertion entails expression of the sincerity condition about  $p$  (the speaker believes  $p$ ); on the other hand, the assertion expresses the speaker believes he ignores that  $p$ . If we accept introspection then this expresses that the speaker does not believe  $p$ , and the assertion is contradictory (if we accept that beliefs are consistent).

### 2.2 Definition and features

We view grounded information as information that is *publicly expressed or accepted as being true by all the agents participating in a conversation*.

A piece of information might be grounded even when some agents privately disagree, as long as they do not publicly manifest their disagreement.

If we consider the above example, by asserting that his boss is a moron in front of agent 1, agent 0 has expressed that he believes that his boss is a moron (although 0 does not think so) and thus this proposition is grounded for agents 1 and 2. After acceptance by the second agent, it becomes grounded for the group of agents

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<sup>1</sup> This notion has nothing to do with Wooldridge’s computational grounding notion [49].

consisting of 0 and 1 that their boss is a moron. The contrary is grounded afterwards for the group of agents  $\{0,1,2\}$ .

Groundedness is an objective notion: it refers to what can be observed, and only to that. It is different from other objective notions such as that of social commitment of [38, 39, 15, 46]. In our approach we do not try to determine whether  $j$  *must* do such or such action or not: we just establish the facts, without any hypothesis on the agents' beliefs, goals, intentions... or commitments.

As we will show in Section 4, grounding and commitment paradigms are equivalent in the case of an assertion. To see the difference for requests consider the speech act where agent  $i$  asks agent  $j$  to pass him the salt. Thereafter it is established (if we assume that the speech act is well and completely understood) that  $i$  has the intention that  $j$  passes him the salt and nothing is grounded for  $j$ . In contrary, in a commitment-based approach this typically leads to a conditional commitment (or precommitment) of  $j$  to pass the salt, which becomes an unconditional commitment upon a positive reaction, whereas the requester is not committed to the fact that he performed a request.

We make also the hypothesis that grounding is a rational notion, *i.e.* a proposition and its contrary cannot be grounded in the same group. But this can be the case for two different groups, see the example.

In a previous paper [16], we presented a modal logic of belief and choice augmented by the modal operator  $\mathbf{G}$  to express the notion of grounding. But this operator was a bit too restricted:  $\mathbf{G} \varphi$  expresses that  $\varphi$  is publicly grounded, where “publicly” means for all agents. Thus in a given group of agents, we cannot distinguish a private dialogue between two agents from a public debate. In the former a piece of information could be grounded between only two agents and stay secret for the other agents of the group.

In the next section we will present the logical framework of the grounding operator: it is the one of [17] augmented with the mutual belief operator.

### 3 Logical Framework

In this section, we present a light version of the logic of belief, choice and action we developed in [24] which builds on the works of Cohen & Levesque [7] and Sadek [33], and augments it by a modal operator expressing groundedness in a group. We show that groundedness for the single-agent group  $\{i\}$  corresponds to belief of  $i$ . Thus a particular individual belief operator is superfluous. We neither develop here temporal aspects nor dynamics between action and mental attitudes.

### 3.1 Semantics

Let  $AGT = \{i, j, \dots\}$  be a finite set of agents. A *group of agents* (or a *group* for short) is a nonempty subset of  $AGT$ . We use  $I, J, K, \dots$  to denote groups. When  $I' \subseteq I$  we say that  $I'$  is a subgroup of  $I$ . Let  $ATM = \{p, q, \dots\}$  be the set of atomic formulas. Complex formulas are denoted by  $\varphi, \psi, \dots$

A model includes a set of possible worlds  $W$  and a mapping  $V: W \rightarrow (ATM \rightarrow \{0, 1\})$  associating a valuation  $V_w$  to every  $w \in W$ . Models moreover contain accessibility relations that will be detailed in the sequel.

**Grounding.** To each possible world  $w$  and each non-empty  $I \subseteq AGT$ , we associate the set of possible worlds that are consistent with all propositions grounded in world  $w$  for the group  $I$ . This set is characterized by the mapping:  $\mathbf{G}: 2^{AGT} \rightarrow (W \rightarrow 2^W)$  associating an accessibility relation to each non-empty subgroup of  $AGT$ .  $\mathbf{G}_I(w)$  contains those worlds where all grounded propositions hold.

$\mathbf{G}_I \varphi$  reads “it is publicly grounded for group  $I$  that  $\varphi$  is true” (or for short: “ $\varphi$  is grounded for  $I$ ”). When  $I$  is a singleton,  $\mathbf{G}_{\{i\}} \varphi$  reads “ $\varphi$  is grounded for (agent)  $i$ ” and  $\mathbf{G}_{\{i\}}$  is identified with the standard belief operator **Bel**, *à la* Hintikka [25]. We write  $\mathbf{G}_i \varphi$  for  $\mathbf{G}_{\{i\}} \varphi$ .

The truth condition for  $\mathbf{G}_I$  stipulates that  $\varphi$  is grounded in  $w$ , noted  $w \Vdash \mathbf{G}_I \varphi$ , if and only if  $\varphi$  holds in every world that is consistent with the set of grounded propositions:

$$w \Vdash \mathbf{G}_I \varphi \quad \text{iff} \quad w \Vdash \varphi \text{ for every } w \in \mathbf{G}_I(w).$$

We assume that:

$$[\text{SC1}] \quad \mathbf{G}_I \text{ is serial.}$$

Thus, groundedness is rational: at least one world that is consistent with the set of grounded propositions exists.

Furthermore we postulate the following constraints on accessibility relations, for groups  $I$  and  $I'$  such that  $I' \subseteq I$ :

$$[\text{SC2}] \quad \text{if } u \mathbf{G}_{I'} v \text{ and } v \mathbf{G}_I w \text{ then } u \mathbf{G}_I w;$$

$$[\text{SC3}] \quad \text{if } u \mathbf{G}_{I'} v \text{ and } u \mathbf{G}_I w \text{ then } v \mathbf{G}_I w;$$

$$[\text{SC4}] \quad \text{if } u \mathbf{G}_I v \text{ and } v \mathbf{G}_I w_1 \text{ then there is } w_2 \text{ such that } u \mathbf{G}_I w_2 \text{ and}$$

- $V(w_1) = V(w_2)$ ,
- $\mathbf{G}_K(w_1) = \mathbf{G}_K(w_2)$  for all  $K$  such that  $K \cap I = \emptyset$ ,
- $\mathbf{C}_K(w_1) = \mathbf{C}_K(w_2)$  for all  $k$  such that  $k \notin I$ , where  $C$  is the accessibility

relation for choice to be defined below;

$$[\text{SC5}] \quad \mathbf{G}_I \subseteq \bigcup_{i \in I} \mathbf{G}_i \circ \mathbf{G}_i.$$

Constraint [SC2] stipulates that agents of a subset  $I'$  of the set  $I$  are aware of what is grounded in the group  $I$ : whenever  $w$  is a world for which it is grounded for  $I'$  that all  $I$ -grounded propositions hold in  $w$ , then all  $I$ -grounded propositions indeed hold in  $w$ . This is a kind of *attention* property: each subgroup taking part in a conversation is aware of what is grounded in the group.

Similarly [SC3] expresses that subgroups are aware of what is ungrounded in the group, too.

[SC2] and [SC3] together make that if  $u \mathbf{G}_{I'} v$  then  $\mathbf{G}_I(u) = \mathbf{G}_I(v)$ , *i.e.* if  $u \mathbf{G}_{I'} v$  then what is grounded for  $I$  at  $u$  is the same as what is grounded for  $I$  at  $v$ . From [SC2] and [SC3] it also follows that  $\mathbf{G}_I$  is transitive and Euclidian.

[SC4] says that if an information “about something outside group  $I$ ” (see the definition in the following subsection) is grounded for  $I$  then it is grounded for  $I$  this information is grounded for every subgroup of  $I$ .

[SC5] says that if it is grounded for a set  $I$  that a proposition is established for every agent then it is grounded for  $I$ , too.

**Mutual belief.** From individual belief (*i.e.* grounding for singleton groups), we define the notion of mutual belief of a group of agents. Semantically we have the mapping  $\mathbf{MB}: 2^{AGT} \rightarrow (W \rightarrow 2^W)$  associating an accessibility relation  $\mathbf{MB}_I$  to each  $I \subseteq AGT$ .  $\mathbf{MB}_I(w)$  denotes the set of possible worlds compatible with mutual beliefs of the group  $I$ . For each group  $I$ ,  $\mathbf{MB}_I$  is defined as the transitive closure of the set of accessibility relations associated to the  $I$ 's members beliefs (*i.e.*  $\mathbf{G}_i$  for each  $i \in I$ ):

$$[\text{SC6}] \quad \mathbf{MB}_I = (\bigcup_{i \in I} \mathbf{G}_i)^+$$

$\mathbf{MBel}_I \varphi$  reads “it is mutual belief for the group  $I$  that  $\varphi$  is true”. It means that every member of the group believes individually that  $\varphi$  and that it is mutual belief for the group that  $\varphi$  is true<sup>2</sup>. (See [12] for more details about the logic of mutual belief.)

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<sup>2</sup> This definition is a recursive one. We can define the mutual belief with an infinite disjunction: every agent believes  $\varphi$ , that other agents believe  $\varphi$ , that other agents believe that every agent believe  $\varphi$ ...

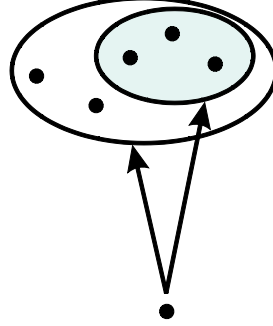


Figure 1. Grounding and choices

**Choice.** Among all the worlds in  $G_i(w)$  that are possible for agent  $i$ , there are some that  $i$  prefers. Semantically, these worlds are identified by yet another mapping  $C : AGT \rightarrow (W \rightarrow 2^W)$  associating an accessibility relation  $C_i$  to each  $i \in AGT$ .  $C_i(w)$  denote the set of worlds the agent  $i$  prefers.

$\mathbf{Ch}_i \varphi$  reads “agent  $i$  chooses that  $\varphi$ ”. Choice can be viewed as a preference operator and we sometimes also say that “ $i$  prefers that  $\varphi$ ”. Note that we only consider individual choices, group choices being beyond the scope of the present article.

The truth condition for  $\mathbf{Ch}_i$  stipulates that  $w \Vdash \mathbf{Ch}_i \varphi$  if  $\varphi$  holds in all chosen worlds:

$$w \Vdash \mathbf{Ch}_i \varphi \quad \text{iff} \quad w \Vdash \varphi \text{ for every } w \in C_i(w).$$

We assume that:

$$[\text{SC7}] \quad C_i \text{ is serial, transitive and Euclidian.}^3$$

(See [24] for more details about the logic of choice, and the definition of intention from choice.)

**Choice and grounding.** As said above, an agent only chooses worlds he considers possible (see Figure 1):

$$[\text{SC8}] \quad C_i(w) \subseteq G_i(w).$$

Hence what is grounded for an agent must be chosen by him, and choice is a mental attitude that is logically weaker than groundedness.

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<sup>3</sup> The choice operator used is not fully classical. In particular it refers to what Cohen and Levesque named “goal” with stronger properties (Cohen and Levesque only assumed seriality). A more detailed comparison with other choice operators is developed in [24].



We moreover require that worlds chosen by  $i$  are also chosen from  $i$ 's "grounded worlds", and *vice versa*.

$$[\text{SC9}] \quad \text{if } w \mathbf{G}_i w' \text{ then } \mathbf{C}_i(w) = \mathbf{C}_i(w').$$

This constraint means that agent  $i$  is aware of his choices.

**Action.** Let  $ACT = \{\alpha, \beta, \dots\}$  be the set of actions. Sometimes we write  $(i:\alpha)$  to denote that  $i$  is the author of (*i.e.* performs) the action  $\alpha$ .<sup>4</sup>

The model contains a mapping  $\mathbf{R}: ACT \rightarrow (W \rightarrow 2^W)$  associating an accessibility relation  $\mathbf{R}_\alpha$  to every  $\alpha \in ACT$ .  $\mathbf{R}_\alpha(w)$  is the set of worlds accessible from  $w$  through the execution of  $\alpha$ . Just as Cohen and Levesque we suppose here that there is at most one possible execution of  $\alpha$ . Hence  $\mathbf{R}_\alpha$  can also be viewed as a partial function on  $W$ .

The formula **After** $_\alpha \varphi$  reads: " $\varphi$  holds after every execution of  $\alpha$ ". As there is at most one possible execution of  $\alpha$ , the dual operator **Happens** $_\alpha \varphi \stackrel{\text{def}}{=} \neg \mathbf{After}_\alpha \neg \varphi$  reads: " $\alpha$  is happening and  $\varphi$  is true just afterwards".

Hence **After** $_\alpha \perp$  expresses that  $\alpha$  does not happen and **Happens** $_\alpha \top$  that  $\alpha$  happens. We often write **Happens** $(\alpha)$  for **Happens** $_\alpha \top$ .

The truth condition is:

$$w \Vdash \mathbf{After}_\alpha \varphi \quad \text{iff} \quad w' \Vdash \varphi \text{ for every } w' \in \mathbf{R}_\alpha(w)$$

The formula **Before** $_\alpha \varphi$  reads: " $\varphi$  holds before every execution of  $\alpha$ ". The dual **Done** $_\alpha \varphi \stackrel{\text{def}}{=} \neg \mathbf{Before}_\alpha \neg \varphi$  expresses that the action  $\alpha$  has been performed before which  $\varphi$  held. Hence **Done** $_\alpha \top$  reads: " $\alpha$  has just happened". We often write **Done** $(\alpha)$  for **Done** $_\alpha \top$ .

The accessibility relation for **Before** $_\alpha$  is the converse of the above relation  $\mathbf{R}_\alpha$ . The truth condition is:

$$w \Vdash \mathbf{Before}_\alpha \varphi \quad \text{iff} \quad w' \Vdash \varphi \text{ for every } w' \in \mathbf{R}_\alpha^{-1}(w).$$

As said above, we do not detail here the relationship between action and mental attitudes (belief and choice) and refer the reader to [24]. We only consider here the link between action and grounding.

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<sup>4</sup> In particular for a speech act  $\alpha$ , this notation allows to specify the author of  $\alpha$  without mentioning the addressee, the illocutionary force...

**Action and grounding.** As it is often the case in the structural approaches to dialogue, we consider in this paper that actions are public for attending agents, in the sense that they are completely and soundly perceived by them.

For example, when agent  $i$  performs an assertive speech act only towards agent  $j$  then  $j$  will perceive the assertion. If no other agent perceives this action then the attentive group is limited to  $K = \{i, j\}$ , and the action is public for exactly this group. But when agent  $i$  performs a speech act towards agent  $j$  in front of an assistance  $L$  then the attentive group is extended to  $K = \{i, j\} \cup L$ .

Let  $\alpha$  be an action performed by agent  $i$  in front of attentive group  $K$  (of which  $i$  is a member). The hypothesis of public actions (for group  $K$ ) corresponds to the constraint:

$$[\text{SC10}] \quad R_\alpha^{-1}(w) = \emptyset \quad \text{if and only if} \quad (\mathbf{G}_K \circ R_\alpha^{-1})(w) = \emptyset$$

### 3.2 Axiomatics

**Grounding.** The logic of the grounding operator is a normal modal logic of type KD:

$$(\text{D}_{\mathbf{G}_I}) \quad \mathbf{G}_I \varphi \rightarrow \neg \mathbf{G}_I \neg \varphi$$

$(\text{D}_{\mathbf{G}_I})$  expresses that grounded information in a group are consistent: it cannot be the case that both  $\varphi$  and  $\neg\varphi$  are simultaneously grounded.

In accordance with the preceding semantic conditions the following logical axioms respectively correspond to the semantic constraints [SC2] and [SC3]. Thus, for each  $I' \subseteq I$ :

$$(\text{SR}+) \quad \mathbf{G}_I \varphi \rightarrow \mathbf{G}_{I'} \mathbf{G}_I \varphi$$

$$(\text{SR}-) \quad \neg \mathbf{G}_I \varphi \rightarrow \mathbf{G}_{I'} \neg \mathbf{G}_I \varphi$$

The axioms of strong rationality (SR+) and (SR-) express that if a proposition  $\varphi$  is grounded (resp. ungrounded) for group  $I$  then it is grounded for each subgroup that  $\varphi$  is grounded (resp. ungrounded) for  $I$ . This is due to the public character of the grounding operator.<sup>5</sup>

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<sup>5</sup> In particular (SR+) and (SR-) axioms are a generalization to a group of the (positive and

The next axiom must be restricted to particular formulas, viz. objective formulas for a group, that we define as follows.

**Definition.** The set of formulas that are objective for a group  $I$  is defined inductively to be the smallest set such that:

- every atomic formula  $p$  is objective for  $I$ ;
- $\mathbf{G}_K \varphi$  is objective for  $I$  if  $K \cap I = \emptyset$ , for every formula  $\varphi$ ;
- $\mathbf{Ch}_j \varphi$  is objective for  $I$  if  $j \notin I$ , for every formula  $\varphi$ ;
- if  $\varphi$  and  $\varphi'$  are objective for  $I$  then  $\neg\varphi$ ,  $\varphi \wedge \varphi'$  are objective for  $I$ .

With respect to the semantic constraint [SC4], our third axiom, a weak rationality axiom, stipulates that if  $I$  is a subgroup of  $I$  and  $\varphi$  is objective for  $I$  then:

$$(WR) \quad \mathbf{G}_I \varphi \rightarrow \mathbf{G}_I \mathbf{G}_{I'} \varphi$$

(WR) expresses that if  $\varphi$  is objective for group  $I$  and grounded for  $I$  then it is necessarily grounded for  $I$  that for each subgroup  $I'$  the formula is grounded.

Note that this does not imply that for every subgroup  $\varphi$  is actually grounded, *i.e.* (WR) does not entail  $\mathbf{G}_I \varphi \rightarrow \mathbf{G}_{I'} \varphi$ . In particular, the fact that  $\varphi$  is grounded for group  $I$  does not imply that the members of  $I$  believe that  $\varphi$ .

It is very important to note that (WR) concerns only formulas  $\varphi$  that are objective for  $I$ . Indeed, if we applied (WR) to some mental states of an agent of the group, we would restrict the agents' autonomy.

For example, when an agent  $i$  asserts to another agent  $j$  that  $\varphi$  in presence of group  $I$ , he publicly expresses that he believes  $\varphi$  [34, 44] and thus he socially commits himself on the fact that he believes  $\varphi$ , as we will develop in Section 4. Thus his belief that  $\varphi$  is immediately and without discussion grounded for the group.

Now if agent  $i$  asserts that  $\mathbf{G}_j \varphi$  in presence of group  $I$ , then the formula  $\mathbf{G}_I \mathbf{G}_i \mathbf{G}_j \varphi$  holds afterwards, and if (WR) applied unrestrictedly then  $j$  could not express later that he ignores whether  $\varphi$ , or believes  $\neg\varphi$ . If he made this last speech act, the formulas  $\mathbf{G}_I \mathbf{G}_i \neg\varphi$  and, thanks to (WR),  $\mathbf{G}_I \mathbf{G}_i \mathbf{G}_j \neg\varphi$  would hold, which is

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negative) introspection axioms commonly accepted for mental attitudes (like belief, choice...): each agent  $i$  member of the group  $I$  if aware of what is grounded (resp. ungrounded) for the group  $I$ :  $\mathbf{G}_I \varphi \rightarrow \mathbf{G}_i \mathbf{G}_I \varphi$  and  $\neg \mathbf{G}_I \varphi \rightarrow \mathbf{G}_i \neg \mathbf{G}_I \varphi$ .

inconsistent with the above formula  $\mathbf{G}_I \mathbf{G}_i \mathbf{G}_j \varphi$  [16].

This restriction highlights that a formula  $\varphi$  can be grounded in two different manners: either  $\varphi$  is objective for group  $I$  and it must be discussed by all the agents of  $I$ , or  $\varphi$  is not and it is grounded directly by being expressed. We will discuss this distinction in the sequel (Section 5).

And finally, corresponding to the semantic constraint [SC5], we have the last axiom of common grounding:

$$(CG) \quad (\bigwedge_{i \in I} \mathbf{G}_I \mathbf{G}_i \varphi) \rightarrow \mathbf{G}_I \varphi$$

It expresses that if a proposition is established for every agent in  $I$ , then it is grounded for the group  $I$ . Together, (WR) and (CG) stipulate that for formulas  $\varphi$  that are objective for  $I$  we have:

$$(1) \quad (\bigwedge_{i \in I} \mathbf{G}_I \mathbf{G}_i \varphi) \leftrightarrow \mathbf{G}_I \varphi$$

From axioms (SR<sup>+</sup>) and (SR<sup>-</sup>), we can prove that we have the modal axioms (4) and (5) for  $\mathbf{G}_I$  operators as theorems of our logic:

$$(4_{Gi}) \quad \mathbf{G}_I \varphi \rightarrow \mathbf{G}_I \mathbf{G}_I \varphi$$

$$(5_{Gi}) \quad \neg \mathbf{G}_I \varphi \rightarrow \mathbf{G}_I \neg \mathbf{G}_I \varphi$$

Thus operator  $\mathbf{G}_I$  is in a normal modal logic of type KD45. Hence for individual groundedness we obtain the standard logic of belief KD45.

We can moreover show that if  $I' \subseteq I$  then:

$$(2) \quad \mathbf{G}_I \varphi \leftrightarrow \mathbf{G}_{I'} \mathbf{G}_I \varphi$$

$$(3) \quad \neg \mathbf{G}_I \varphi \leftrightarrow \mathbf{G}_{I'} \neg \mathbf{G}_I \varphi$$

These theorems express that subgroups of a group are aware of what is grounded (resp. ungrounded) in the group. The formula  $(\bigwedge_{i \in I'} \mathbf{G}_I \mathbf{G}_i \varphi) \rightarrow \mathbf{G}_I \varphi$  is provable from our axiom (CG). Moreover we can prove that:

$$(4) \quad \mathbf{G}_I \varphi \leftrightarrow \mathbf{G}_I \mathbf{G}_{I'} \mathbf{G}_I \varphi$$

$$(5) \quad \neg \mathbf{G}_I \varphi \leftrightarrow \mathbf{G}_I \mathbf{G}_{I'} \neg \mathbf{G}_I \varphi$$

These theorems say that if  $\varphi$  is (not) grounded for a group, it is grounded for this group that it is grounded for every subgroup of this group that  $\varphi$  is (not) grounded for the group.

Even if  $I'$  is a subgroup of  $I$  we do not necessarily have  $\mathbf{G}_I \varphi \rightarrow \mathbf{G}_{I'} \varphi$ . Such a principle would be too strong because it would restrict the autonomy of subgroups  $I'$  of  $I$ : a proposition can be grounded for a group  $I$  while there is a dissident subgroup  $I'$  of  $I$ , i.e. a group where the contrary is grounded:  $\mathbf{G}_I \varphi \wedge \neg \mathbf{G}_{I'} \varphi$  is consistent in our logic even if  $I \cap I' \neq \emptyset$ .

**Mutual belief.** Axiomatically mutual belief is defined by the Fixpoint Axiom, which specifies that a mutual belief about  $\varphi$  holds if and only if every agent believes that  $\varphi$  and that the mutual belief holds (see [12] for more details):

$$(\text{FP}_{\mathbf{MBel}_I}) \quad \mathbf{MBel}_I \varphi \leftrightarrow \bigwedge_{i \in I} \mathbf{G}_i (\varphi \wedge \mathbf{MBel}_I \varphi)$$

and the Least Fixpoint axiom, that will be used in the sequel:

$$(\text{LFP}_{\mathbf{MBel}_I}) \quad \bigwedge_{i \in I} \mathbf{G}_i \varphi \wedge \mathbf{MBel}_I (\varphi \rightarrow \bigwedge_{i \in I} \mathbf{G}_i \varphi) \rightarrow \mathbf{MBel}_I \varphi$$

From this axiomatics, we can deduce that  $\mathbf{MBel}_I$  is a normal modal operator of type KD4:

$$\mathbf{MBel}_I \varphi \rightarrow \neg \mathbf{MBel}_I \neg \varphi$$

$$\mathbf{MBel}_I \varphi \rightarrow \mathbf{MBel}_I \mathbf{MBel}_I \varphi$$

**Mutual belief and grounding.** By definition, it follows from the Fixpoint Axiom that when it is mutual belief for a group  $I$  that  $\varphi$  holds then necessarily every member of  $I$  believes individually that  $\varphi$ :

$$(6) \quad \mathbf{MBel}_I \varphi \rightarrow \bigwedge_{i \in I} \mathbf{G}_i \varphi$$

Now we will show the link between mutual belief of a group  $I$  and the grounding for the whole group  $I$ :

**Theorem 3.1** *We have the equivalence:*

$$(7) \quad \mathbf{G}_I \varphi \leftrightarrow \mathbf{MBel}_I \mathbf{G}_I \varphi$$

That means that a formula  $\varphi$  is grounded in a group  $I$  if and only if there is mutual belief in the group that  $\varphi$  is grounded. This property is mainly due to the public nature of the grounding operator.

**Proof.**

1.  $\vdash \mathbf{MBel}_I \mathbf{G}_I \varphi \rightarrow \mathbf{G}_i \mathbf{G}_I \varphi$ , by theorem (6)
2.  $\vdash \mathbf{MBel}_I \mathbf{G}_I \varphi \rightarrow \mathbf{G}_I \varphi$ , from 1. by theorem (2)
3.  $\vdash \mathbf{G}_I \varphi \rightarrow \mathbf{G}_i \mathbf{G}_I \varphi$ , by theorem (2), for every  $i \in I$
4.  $\vdash \mathbf{G}_I \varphi \rightarrow \bigwedge_{i \in I} \mathbf{G}_i \mathbf{G}_I \varphi$ , from 3. because it holds for every  $i \in I$
5.  $\vdash \mathbf{MBel}_I (\mathbf{G}_I \varphi \rightarrow \bigwedge_{i \in I} \mathbf{G}_i \mathbf{G}_I \varphi)$   
from 4. by the Rule of Necessitation for  $\mathbf{MBel}_I$
6.  $\vdash \mathbf{MBel}_I (\mathbf{G}_I \varphi \rightarrow \bigwedge_{i \in I} \mathbf{G}_i \mathbf{G}_I \varphi) \rightarrow (\bigwedge_{i \in I} \mathbf{G}_i \mathbf{G}_I \varphi \rightarrow \mathbf{MBel}_I \mathbf{G}_I \varphi)$ ,  
from Axiom (LFP<sub>MBel</sub>)
7.  $\vdash \bigwedge_{i \in I} \mathbf{G}_i \mathbf{G}_I \varphi \rightarrow \mathbf{MBel}_I \mathbf{G}_I \varphi$ , from 5. and 6. by Modus Ponens
8.  $\vdash \mathbf{G}_I \varphi \rightarrow \mathbf{MBel}_I \mathbf{G}_I \varphi$ , from 4. and 7.
9.  $\vdash \mathbf{G}_I \varphi \leftrightarrow \mathbf{MBel}_I \mathbf{G}_I \varphi$ , from 2. and 8. □

**Choice.** With respect to the semantic constraint [SC7], the choice operator is defined in a normal modal logic of type KD45 and we have the axioms (D<sub>Ch<sub>i</sub></sub>), (4<sub>Ch<sub>i</sub></sub>) and (5<sub>Ch<sub>i</sub></sub>):

- $$\begin{aligned} (\text{D}_{\text{Ch}_i}) \quad & \mathbf{Ch}_i \varphi \rightarrow \neg \mathbf{Ch}_i \neg \varphi \\ (4_{\text{Ch}_i}) \quad & \mathbf{Ch}_i \varphi \rightarrow \mathbf{Ch}_i \mathbf{Ch}_i \varphi \\ (5_{\text{Ch}_i}) \quad & \neg \mathbf{Ch}_i \varphi \rightarrow \mathbf{Ch}_i \neg \mathbf{Ch}_i \varphi \end{aligned}$$

**Choice and Grounding.** Due to the semantic constraint [SC8] we have the following axiom:

$$(8) \quad \mathbf{G}_i \varphi \rightarrow \mathbf{Ch}_i \varphi$$

which means that every formula grounded for agent  $i$  must necessarily be chosen by this agent.

Our semantics also validates the principles:

$$(9) \quad \mathbf{Ch}_i \varphi \leftrightarrow \mathbf{G}_i \mathbf{Ch}_i \varphi$$

$$(10) \quad \neg \mathbf{Ch}_i \varphi \leftrightarrow \mathbf{G}_i \neg \mathbf{Ch}_i \varphi$$

that correspond with constraint [SC9]. This expresses that agents are aware of their choices.

**Action.** With respect to the semantic constraints, the action operators  $\mathbf{After}_\alpha$  and its converse  $\mathbf{Before}_\alpha$  are defined in a  $K_i$  logic, *i.e.* a normal modal logic with following conversion axioms:

$$(\mathbf{I}_{\mathbf{After}_\alpha, \mathbf{Done}_\alpha}) \quad \varphi \rightarrow \mathbf{After}_\alpha \mathbf{Done}_\alpha \varphi$$

$$(\mathbf{I}_{\mathbf{Before}_\alpha, \mathbf{Happens}_\alpha}) \quad \varphi \rightarrow \mathbf{Before}_\alpha \mathbf{Happens}_\alpha \varphi$$

These axioms characterize the fact that the relation  $R_\alpha^{-1}$  is the converse of  $R_\alpha$ .

**Action and grounding.** As we have said above we only consider public actions and  $\alpha$  be an action performed by an agent  $i$  in front of attentive group  $K$  (of which  $i$  is member). Thus we have following axioms of public actions corresponding to the semantic constraint [10], for each group  $K$  observing an action  $\alpha$ :

$$(\mathbf{PA}_{K,\alpha}) \quad \mathbf{G}_K \mathbf{Done}_\alpha \top \leftrightarrow \mathbf{Done}_\alpha \top$$

$$(\mathbf{NA}_{K,\alpha}) \quad \mathbf{G}_K \neg \mathbf{Done}_\alpha \top \leftrightarrow \neg \mathbf{Done}_\alpha \top$$

To sum it up, an action has been (resp. has not been) performed by a member of group  $I$  if and only if it is grounded for the group that it has been (resp. has not been) performed.

### 3.3 Action laws

Action laws come in two kinds: *executability laws* describe the preconditions of the action, and *effect laws* describe the effects. The preconditions of an action are the conditions that must be fulfilled in order that the action be executable. The effects (or postconditions) are properties that hold after the action because of it. For example, to toss a coin, we need a coin (precondition) and after the toss action the coin is heads or tails (postcondition).

The set of all action laws is noted *LAWS*, and some examples are collected in Table 2. The general form of an executability law is

$$(\mathbf{Int}_{\mathbf{Ch}_i, i:\alpha}) \quad \mathbf{Ch}_i \mathbf{Happens}(i:\alpha) \wedge \text{precond}(i:\alpha) \leftrightarrow \mathbf{Happens}(i:\alpha)$$

This expresses a principle of intentional action: an action happens exactly when its preconditions hold and its author chooses it to happen [27]. The general form of an effect law is  $\varphi \rightarrow \mathbf{After}_a \text{postcond}(a)$ . In order to simplify our exposition we suppose that effect laws are unconditional and therefore the general form of an effect law is here:

$$\mathbf{After}_a \text{postcond}(a)$$

A way of capturing the conventional aspect of interaction is to suppose that these laws are common to all the agents. Formally they are thus global axioms to which the necessitation rule applies [14].

### 3.4 Example

To highlight our proposal for the semantics of grounding we will formalize the introduction example. Let consider the example where there are three agents  $AGT = \{0, 1, 2\}$ :

1. Agent 0 (privately) believes that 2 is smart, formally written  $\mathbf{G}_0 \text{smart}_2$ .
2. Now suppose that in private conversation agent 0 tells 1 that 2 is not smart. The illocutionary effect is  $\mathbf{G}_{\{0,1\}} \mathbf{G}_0 \neg \text{smart}_2$ .
3. If 1 publicly adopts  $\neg \text{smart}_2$  (e.g. by confirming publicly that  $\neg \text{smart}_2$ ) we moreover obtain  $\mathbf{G}_{\{0,1\}} \neg \text{smart}_2$ .
4. Then agent 2 joins in the conversation, and later on 0 informs 1 and 2 that 2 is smart: the illocutionary effect is  $\mathbf{G}_{\{0,1,2\}} \mathbf{G}_0 \text{smart}_2$ .
5. Then if both 1 and 2 publicly adopt  $\text{smart}_2$  we moreover obtain  $\mathbf{G}_{\{0,1,2\}} \text{smart}_2$ .

This illustrates that even for nested groups  $J_0 = \{0\} \subset J_1 = \{0, 1\} \subset J_2 = \{0, 1, 2\}$  we might have states of public groundedness for the different groups which are about propositions that are mutually inconsistent, viz. here:

$$\mathbf{G}_{J_0} \text{smart}_2$$

$$\mathbf{G}_{J_1} \neg \text{smart}_2$$

$$\mathbf{G}_{J_2} \text{smart}_2$$

## 4 Application to Walton & Krabbe's dialogue games

We now apply our formalism to a particular kind of dialogue, viz. persuasion dialogues defined by Walton&Krabbe (W&K for short). [48] presents a dialogue



type hierarchy based on the notion of conflict. Among them, a detailed study is made for persuasion dialogues, with quite precise descriptions of game rules and speech act semantics in term of commitments. These works mainly follow from Hamblin's works [23]. This section adapts the case study of [16] to the present logical framework.

A persuasion dialogue takes place when there is a conflict between two agents' belief. The goal of the dialogue is to resolve this situation: an agent can persuade the other party to concede his own thesis (in this case he wins the dialogue game) or concede the point of view of the other party (and thus lose the game).

W&K define two types of persuasion dialogue: the Permissive Persuasion Dialogue (PPD for short) and the Rigorous Persuasion Dialogue (RPD). RPD is asymmetric (participants have different roles viz. proponent and opponent), and is analytic (the initial proposition is decomposed during the dialogue), while PPD is symmetric and non analytic (allows to introduce new arguments).

We show, by characterizing  $PPD_0$ , how our formally well-grounded operator can be used to define speech acts semantics and game rules instead of the informal commitments *à la* W&K.

In order to simplify our exposition we suppose with W&K that there are only two agents (but the account can easily be generalized to  $n$  agents).

## 4.1 Strong and Weak commitments

W&K distinguish two kinds of commitment: those which must be defended by a proof or a justification when challenged, called *assertions*, and those which in contrarily does not need, called *concessions*. We formalize this distinction with the notions of strong commitment ( $SC_{i,K}$ ) and weak commitment ( $WC_{i,K}$ ). They are linked by the fact that a strong commitment to a proposition implies a weak commitment to it [48, p. 133]. We use the logical framework presented above to formalize these two notions, and apply it to  $PPD_0$ . In relation with this logical framework, we define:<sup>6</sup>

$$(Def_{SC_{i,K}}) \quad SC_{i,K} \varphi = G_K \mathbf{Bel}_i \varphi$$

$$(Def_{WC_{i,K}}) \quad WC_{i,K} \varphi = G_K \neg \mathbf{Bel}_i \neg \varphi$$

---

<sup>6</sup> This is an approximation of W&K's *assertion*, noted  $a$ . Indeed, our  $G_K G_i \varphi$  is "more logical" than W&K's  $a(\varphi)$ : W&K allow both  $a(\varphi)$  and  $a(\neg \varphi)$  to be the case simultaneously, while for us  $G_K G_i \varphi \wedge G_K G_i \neg \varphi$  is inconsistent. In the case of weak commitment, we agree with W&K's works: in our framework,  $WC_{i,K} \varphi \wedge WC_{i,K} \neg \varphi$  is consistent.

In terms of the preceding abbreviations we can prove:

$$(11) \quad \mathbf{SC}_{i,K} \varphi \rightarrow \neg \mathbf{SC}_{i,K} \neg \varphi$$

$$(12) \quad \mathbf{SC}_{i,K} \varphi \leftrightarrow \mathbf{SC}_{i,K} \mathbf{SC}_{i,K} \varphi$$

$$(13) \quad \neg \mathbf{SC}_{i,K} \varphi \leftrightarrow \mathbf{SC}_{i,K} \neg \mathbf{SC}_{i,K} \varphi$$

(11) shows the rationality of the agents: they cannot commit both on  $\varphi$  and  $\neg\varphi$ . (12) and (13) account for the public character of commitment. With these three theorems, we can show that  $\mathbf{SC}_{i,K}$  is an operator of a normal modal logic of type KD45, too.<sup>7</sup>

$$(14) \quad \mathbf{G}_K \varphi \leftrightarrow \mathbf{SC}_{i,K} \mathbf{G}_K \varphi$$

$$(15) \quad \neg \mathbf{G}_K \varphi \leftrightarrow \mathbf{SC}_{i,K} \neg \mathbf{G}_K \varphi$$

$$(16) \quad \mathbf{SC}_{i,K} \varphi \leftrightarrow \mathbf{SC}_{j,K} \mathbf{SC}_{i,K} \varphi$$

$$(17) \quad \neg \mathbf{SC}_{i,K} \varphi \leftrightarrow \mathbf{SC}_{j,K} \neg \mathbf{SC}_{i,K} \varphi$$

These theorems express the public character of the commitment. (14) and (15) entail that it is grounded that the agents are committed to the grounded (resp. ungrounded) propositions. (16) and (17) mean that each agent is committed to the other agents' commitments, and non-commitments.

The following theorems show links between strong and weak commitments.

$$(18) \quad \mathbf{SC}_{i,K} \varphi \rightarrow \mathbf{WC}_{i,K} \varphi$$

$$(19) \quad \mathbf{WC}_{i,K} \varphi \rightarrow \neg \mathbf{SC}_{i,K} \neg \varphi$$

(18) says that strong commitment implies weak commitment. (19) expresses that if agent  $i$  is weakly committed to  $\varphi$  then  $i$  is not strongly committed to  $\neg\varphi$ .

$$(20) \quad \mathbf{WC}_{i,K} \varphi \leftrightarrow \mathbf{SC}_{j,K} \mathbf{WC}_{i,K} \varphi$$

$$(21) \quad \neg \mathbf{WC}_{i,K} \varphi \leftrightarrow \mathbf{SC}_{j,K} \neg \mathbf{WC}_{i,K} \varphi$$

(20) expresses that weak commitment is public. (21) is similar for absence of weak commitment.

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<sup>7</sup> From the definition of  $\mathbf{G}_K$  we can prove that the modal property K is a theorem for  $\mathbf{SC}_{i,K}$  and that the necessitation rule can be applied to it.

## 4.2 Speech acts and grounding

In our framework, speech acts are just particular actions. In a general way [17], they are 5-tuples of the form  $\langle i, J, K, FORCE, \varphi \rangle$  where  $i \in AGT$  is the *author* of the speech act (*i.e.* the speaker),  $K \in AGT$  the *group* of agents attentive to the conversation,  $J \subseteq K \setminus \{i\}$  the set of its *addressees*, *FORCE* its illocutionary force, and  $\varphi$  a formula denoting its propositional content. As said above we consider dialogues between only two agents, thus the hearers group  $J$  is restricted to the singleton  $\{j\}$  and the attentive agents group  $K$  is limited only to  $\{i, j\}$ . In the sequel we will use a simplified notation for speech acts:  $\langle s, h, FORCE, \varphi \rangle$ , where  $s$  is the speaker and  $h$  the hearer and  $K$  will denote the set  $\{s, h\}$ .

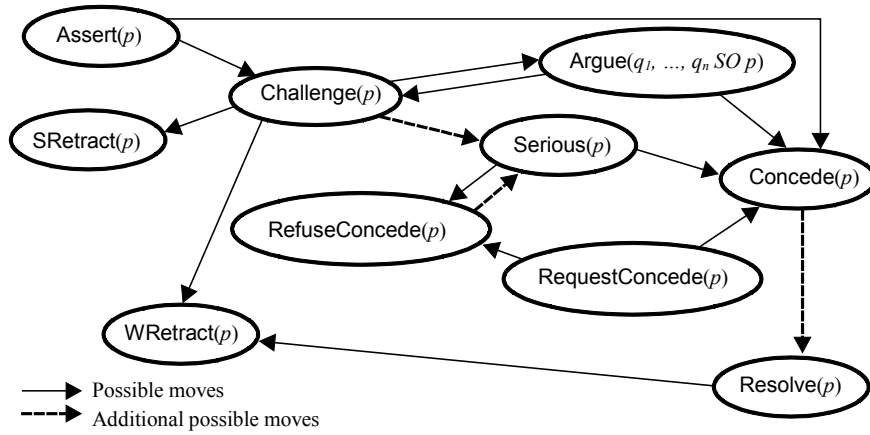


Figure 2. (Additional) possible moves after each act

The dialogues that we want to formalize (W&K-like dialogues) are controlled by some conventions: the rules of the game, which describe the allowed sequences of speech acts. The allowed sequences of acts are those of W&K's  $PPD_0$  (cf. [48, p. 150-151]). They are depicted in Figure 2 and will be discussed below. For example, after a speech act  $\langle s, h, \text{Assert}, p \rangle$ , the hearer can only challenge  $p$  or concede it. We formalize them in our logic by expressing that an act grounds that the hearer's choices are limited only to some acts. Speech acts have two different effects: one is on the commitment store in terms of weak and strong commitments (cf. Table 1) and the other one is the set of acts the hearer can perform in response (cf. Table 2).

We suppose that initially nothing is grounded, *i.e.* the belief base is  $\{\neg G_I \varphi : \text{for every } I \in 2^{AGT} \text{ such that } \text{Cardinality}(I) > 1 \text{ and where } \varphi \text{ is a formula such that } \not\models \varphi\}$ .<sup>8</sup> The **Assert** act on  $p$  can only be used by the two parties in some preliminary

<sup>8</sup> This is an infinite set. In practice one would resort to default reasoning here.

moves of the dialogue to state the theses of each participant. The effect of the act is that it is grounded that its content  $p$  holds for the speaker: he has expressed a kind of strong commitment (an *assertion* for W&K) on  $p$  in the sense that he must defend his commitment by an argument if it is challenged.

**Table 1. Preconditions and effects of speech acts (with commitments)**

Precond( $\alpha$ )	$A$	Effects( $\alpha$ )
$\neg \text{SC}_{s,K} p$	$\langle s, h, \text{Assert}, p \rangle$	$\text{SC}_{s,K} p$
$\text{SC}_{s,K} p$	$\langle s, h, \text{SRetract}, p \rangle$	$\neg \text{SC}_{s,K} p$
$\text{WC}_{s,K} p$	$\langle s, h, \text{WRetract}, p \rangle$	$\neg \text{WC}_{s,K} p$
$\text{SC}_{s,K} p \wedge \neg \text{WC}_{h,K} p$	$\langle s, h, \text{Argue}, (q_1, \dots, q_n \text{SO} p) \rangle$	$\bigwedge_{1 \leq i \leq n} \text{SC}_{s,K} q_i \wedge$ $\text{SC}_{s,K} (\bigwedge_{1 < i < n} q_i \rightarrow p)$
$\neg \text{WC}_{s,K} p$	$\langle s, h, \text{Concede}, p \rangle$	$\text{WC}_{s,K} p$
$\neg \text{WC}_{s,K} p$	$\langle s, h, \text{RefuseConcede}, p \rangle$	$\neg \text{WC}_{s,K} p$
$\text{SC}_{s,K} q \wedge \neg \text{WC}_{h,K} q \wedge$ $\neg \text{WC}_{h,K} p$	$\langle s, h, \text{RequestConcede}, p \rangle$	$\emptyset$
$\neg \text{WC}_{s,K} p \wedge \text{SC}_{h,K} p \wedge$ $\neg \mathbf{G}_k \text{Done}_{(s,h,\text{Challenge},p)} \top$	$\langle s, h, \text{Challenge}, p \rangle$	$\emptyset$
$\neg \text{WC}_{h,K} p$	$\langle s, h, \text{Serious}, p \rangle$	$\emptyset$
$\text{WC}_{h,K} p \wedge \text{WC}_{h,K} q \wedge$ $(p \leftrightarrow \neg q)$	$\langle s, h, \text{Resolve}, p \rangle$	$\emptyset$

To **Concede**  $p$  means to admit that  $p$  could hold, where  $p$  is a strong commitment of the other party (e.g.  $p$  has been asserted). The effect of this act is that it is grounded that the speaker has taken a kind of commitment on  $p$ . But the nature of this commitment seems weaker than the former one: this one has not to be defended when it is attacked. W&K call it *concession* and it corresponds to our notion of Weak Commitment.

The **Challenge** act on  $p$  forces the other participant to either put forward an argument for  $p$ , or to retract the assertion  $p$ . For a given propositional content this act can only be performed once.

To defend a challenged assertion  $p$ , an argument, expressed by **Argue**, must have  $p$  as conclusion and a set of propositions  $q_1 \dots q_n$  as premises. We write it as follows:

$$(\text{Def}_{\text{SO}}) \quad q_1 \dots q_n \text{SO} p \stackrel{\text{def}}{=} q_1 \wedge \dots \wedge q_n \wedge (q_1 \wedge \dots \wedge q_n \rightarrow p)$$

The effect of this act is that the speaker is strongly committed on all premises  $q_1, \dots, q_n$  and on the implicit implication  $q_1 \wedge \dots \wedge q_n \rightarrow p$ . It follows that the agent

who has been challenged must explicitly take position in the next move (challenge or concede) on each premise and on the implicit implication. To challenge one premise means that the argument cannot be applied, while to challenge the implicit implication means that the argument is incorrect. If he does not challenge a proposition, he (implicitly) concedes it. But as soon as he has conceded all the premises and the implication, he must also concede the conclusion. To avoid some digressions, W&K suppose that an unchallenged assertion cannot be defended by an argument. Moreover, we took over their form of the support of arguments, viz.  $\varphi \rightarrow \psi$ , although we are aware that more complex forms of reasoning occur in real world argumentation.

**Table 2. Additional effects of speech acts**

$\alpha$	Constraints on the possible actions following $\alpha$
$\langle s, h, \text{Assert}, p \rangle$	$\mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Challenge}, p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Concede}, p \rangle))$
$\langle s, h, \text{SRetract}, p \rangle$	$\emptyset$
$\langle s, h, \text{WRetract}, p \rangle$	$\emptyset$
$\langle s, h, \text{RequestConcede}, p \rangle$	$\mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{RefuseConcede}, p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Concede}, p \rangle))$
$\langle s, h, \text{Argue}, (q_1, \dots, q_n \text{ SO } p) \rangle$	$\bigwedge_{1 \leq i \leq n} \mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Challenge}, q_i \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Concede}, q_i \rangle)) \vee \mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Challenge}, q_1 \wedge \dots \wedge q_n \rightarrow p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Concede}, q_1 \wedge \dots \wedge q_n \rightarrow p \rangle))$
$\langle s, h, \text{Challenge}, p \rangle$	$\mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{SRetract}, p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{WRetract}, p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Argue}, (q_1, \dots, q_n \text{ SO } p) \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Serious}, p \rangle))$
$\langle s, h, \text{Concede}, p \rangle$	$\emptyset$
$\langle s, h, \text{RefuseConcede}, p \rangle$	$\emptyset$
$\langle s, h, \text{Serious}, p \rangle$	$\mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{RefuseConcede}, p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{Concede}, p \rangle))$
$\langle s, h, \text{Resolve}, p \rangle$	$\mathbf{G}_K (\mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{WRetract}, p \rangle) \vee \mathbf{Ch}_h \text{ Happens} (\langle h, s, \text{WRetract}, \neg p \rangle))$

At any time, the speaker may request more concessions (with a **RequestConcede** act) from the hearer, to use them as premises for arguments. The hearer can then accept or refuse to concede.

W&K use the same speech act type to retract a concession and to refuse to concede something (the act  $nc(p)$ ). But it seems to us that it is not the same kind of act, and we decided to create two different acts:  $\langle s, h, \text{WRetract}, p \rangle$  to retract his own weak commitment on  $p$ , and  $\langle s, h, \text{RefuseConcede}, p \rangle$  to decide not to concede  $p$ . A strong commitment can be retracted with a  $\langle s, h, \text{SRetract}, p \rangle$ . This act removes the strong commitment from the commitment store, but not the weak

commitment, whereas the  $\langle s, h, \text{WRetract}, p \rangle$  act removes the weak commitment and, if it exists, the strong commitment, too.

In our logic,  $\text{WC}_{i,K} \varphi \wedge \text{WC}_{i,K} \neg \varphi$  is satisfiable, but not  $\text{SC}_{i,K} \varphi \wedge \text{SC}_{i,K} \neg \varphi$ . Thus we are more restrictive than W&K: in the following, a contradiction in an agents' commitment store is only due to contradictory Weak Commitments.<sup>9</sup> When a party detects a contradiction in the other party's commitment store, it can ask him to resolve it (with the act  $\text{Resolve}(p,q)$  where “ $p$  and  $q$  are explicit contradictories” [48, p. 151]). The other party must retract one of the inconsistent propositions. W&K do not make any inference in the commitment store, so  $\text{Resolve}$  only applies to explicit inconsistency (that is:  $\text{Resolve}(p, \neg p)$ ). We will write  $\text{Resolve}(p)$  instead of  $\text{Resolve}(p,q)$  where  $q$  is  $\neg p$ .  $\text{Resolve}(p)$  and  $\text{Resolve}(\neg p)$  are thus equivalent. To perform the speech act  $\text{Resolve}(p)$ , we can show that it is necessary and sufficient that the propositions  $p$  and  $\neg p$  are weak commitments of the opponent. In our formalism, the act  $\text{Resolve}$  holds only to weak commitments. Moreover the two contradictory weak commitments cannot be derived from two inconsistent strong commitments (which W&K allow), because such are consistent in our logic.

When an agent chooses to challenge a proposition  $p$  or to refuse to concede it, his opponent can query him to reassess his position. Finally the speech act  $\text{Serious}(p)$  imposes that the agent must concede  $p$  or refuse to concede it.

Note that W&K define another commitment store that contains what they call *dark-side commitments*. Whereas assertions and concessions (light-side commitments) are public, no agent is necessary aware of these commitments. They characterize deep features of agents. If  $p$  is a dark-side commitment, it must be revealed after a  $\text{Serious}(p)$  and the agent must concede  $p$  and cannot retract it.<sup>10</sup>

We do not consider such commitments here because, we focus on what is observable and objective in the dialogue: so if an agent chooses to concede  $p$ , we do not know if it was a dark-side commitment or not, consequently the agent may, even if it had a dark-side commitment on  $p$  and contrary to W&K's theory, retract it in a subsequent dialogue move dialogue.

The action preconditions are not mutually exclusive. This gives the agents some freedom of choice. We do not describe here the subjective cognitive processes that lead an agent to a particular choice.

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<sup>9</sup> W&K allow the agents to have some contradictory concessions ( $\text{WC}_{i,K}$ ) and assertions ( $\text{SC}_{i,K}$ ) in their commitment store (*i.e.*  $\text{SC}_{i,K} \varphi$  and  $\text{SC}_{i,K} \neg \varphi$  or  $\text{WC}_{i,K} \varphi$  and  $\text{WC}_{i,K} \neg \varphi$  can hold simultaneously).

<sup>10</sup> Thus W&K consider agents can be publicly inconsistent but they cannot hide dark-side commitments if the opponent insists with a  $\text{Serious}$ .

### 4.3 Example

We recast an example of a persuasion dialogue given by W&K [48, p. 153] to illustrate the dialogue game  $PPD_0$  (see Figure 3): initially, agent  $i$  asserts  $p_1$  and agent  $j$  asserts  $p_2$ . Thus, the following preparatory moves have been performed:  $\langle i, j, \text{Assert}, p_1 \rangle$  and  $\langle j, i, \text{Assert}, p_2 \rangle$ . After each move, the agents' commitment stores are updated (see Table 3 next page). In his first move,  $j$  asks  $i$  to concede  $p_3$  and challenges  $p_1$ .  $i$  responds by conceding  $p_3$ , etc. In move (vii), agent  $j$  concedes  $p_1$  which is the thesis of his opponent. He thus loses the game in what concerns the thesis of  $i$  but in what concerns his own thesis, the game is not over yet.

<ol style="list-style-type: none"> <li>1. <math>\langle j, i, \text{RequestConcede}, p_3 \rangle,</math> <math>\langle j, i, \text{Challenge}, p_1 \rangle</math></li> <li>2. <math>\langle i, j, \text{Concede}, p_3 \rangle,</math> <math>\langle i, j, \text{Serious}, p_1 \rangle,</math> <math>\langle i, j, \text{Argue}, (p_3 \text{ SO } p_1) \rangle,</math> <math>\langle i, j, \text{Challenge}, p_2 \rangle</math></li> <li>3. <math>\langle j, i, \text{RefuseConcede}, p_1 \rangle,</math> <math>\langle j, i, \text{Concede}, p_3 \rightarrow p_1 \rangle,</math> <math>\langle j, i, \text{Argue}, (p_4, p_5 \text{ SO } p_2) \rangle,</math> <math>\langle j, i, \text{Challenge}, p_3 \rangle</math></li> <li>4. <math>\langle i, j, \text{Concede}, p_5 \rangle,</math> <math>\langle i, j, \text{Concede}, p_4 \wedge p_5 \rightarrow p_2 \rangle,</math> <math>\langle i, j, \text{Serious}, p_3 \rangle,</math> <math>\langle i, j, \text{Argue}, (\neg p_4, p_5 \text{ SO } p_3) \rangle,</math> <math>\langle i, j, \text{Challenge}, p_4 \rangle</math></li> <li>5. <math>\langle j, i, \text{WRetract}, p_3 \rightarrow p_1 \rangle,</math></li> </ol>	<ol style="list-style-type: none"> <li><math>\langle j, i, \text{Concede}, p_3 \rangle,</math> <math>\langle j, i, \text{Concede}, \neg p_4 \rangle,</math> <math>\langle j, i, \text{Concede}, \neg p_4 \wedge p_5 \rightarrow p_3 \rangle,</math> <math>\langle j, i, \text{Argue}, (p_3 \text{ SO } p_4) \rangle,</math> <math>\langle j, i, \text{Challenge}, p_3 \rightarrow p_1 \rangle</math></li> <li>6. <math>\langle i, j, \text{Resolve}, p_4 \rangle,</math> <math>\langle i, j, \text{Argue}, (\neg p_4 \text{ SO } p_3 \rightarrow p_1) \rangle,</math> <math>\langle i, j, \text{Challenge}, p_3 \rightarrow p_4 \rangle</math></li> <li>7. <math>\langle j, i, \text{WRetract}, p_4 \rangle,</math> <math>\langle j, i, \text{WRetract}, p_3 \rightarrow p_4 \rangle,</math> <math>\langle j, i, \text{SRetract}, p_5 \rangle,</math> <math>\langle j, i, \text{SRetract}, p_3 \rangle,</math> <math>\langle j, i, \text{WRetract}, p_4 \wedge p_5 \rightarrow p_2 \rangle,</math> <math>\langle j, i, \text{Concede}, \neg p_4 \rightarrow (p_3 \rightarrow p_1) \rangle,</math> <math>\langle j, i, \text{Concede}, p_3 \rightarrow p_1 \rangle,</math> <math>\langle j, i, \text{Concede}, p_1 \rangle,</math> <math>\langle j, i, \text{Argue}, (p_6 \text{ SO } p_2) \rangle</math></li> </ol>
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Figure 3. Example of dialogue (from [48, p. 153])

As we have said, in order to stay consistent with our logical framework, we have to add an effect to the W&K speech act of concession: when  $i$  concedes a proposition  $p$ , every strong commitment of  $i$  on  $\neg p$  is retracted. Agent  $i$  is then weakly committed on both  $p$  and  $\neg p$ . We thus weaken the paraconsistent aspects of W&K, viz. that an agent can have assertions or concessions that are jointly inconsistent, in order to keep in line with standard properties of the modal operator  $\mathbf{G}_K$ .

Now we can establish formally that our logic captures W&K's  $PPD_0$ -dialogues. For example we have:

**Theorem 4.1.**

$$LAWS \models \mathbf{After}_{\langle s, h, \text{Assert}, p \rangle} ( (\neg \mathbf{WC}_{h,K} p \wedge \neg \mathbf{Done}(\langle h, s, \text{Challenge}, p \rangle)) \rightarrow \mathbf{G}_K (\mathbf{Happens}(\langle h, s, \text{Challenge}, p \rangle) \vee \mathbf{Happens}(\langle h, s, \text{Concede}, p \rangle)) )$$

**Table 3. Commitment stores in the example dialogue**

Grounded propositions	SC <sub>i</sub>	WC <sub>i</sub>	SC <sub>j</sub>	WC <sub>j</sub>
∅	p <sub>1</sub>		p <sub>2</sub>	
WC <sub>i,K</sub> p <sub>3</sub> SC <sub>i,K</sub> p <sub>3</sub> , SC <sub>i,K</sub> p <sub>3</sub> → p <sub>1</sub>	p <sub>1</sub> , p <sub>3</sub> , p <sub>3</sub> → p <sub>1</sub>		p <sub>2</sub>	
WC <sub>j,K</sub> p <sub>3</sub> → p <sub>1</sub> , SC <sub>j,K</sub> p <sub>4</sub> , SC <sub>j,K</sub> p <sub>5</sub> , SC <sub>j,K</sub> p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub>			p <sub>2</sub> , p <sub>4</sub> , p <sub>5</sub> , p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub>	p <sub>3</sub> → p <sub>1</sub>
WC <sub>i,K</sub> p <sub>5</sub> , SC <sub>i,K</sub> ¬p <sub>4</sub> , SC <sub>i,K</sub> p <sub>5</sub> , SC <sub>i,K</sub> ¬p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>3</sub> , WC <sub>i,K</sub> p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub>	p <sub>1</sub> , p <sub>3</sub> → p <sub>1</sub> , p <sub>3</sub> , ¬p <sub>4</sub> , p <sub>5</sub> , ¬p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>3</sub>	p <sub>5</sub> , p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub>		
¬SC <sub>j,K</sub> p <sub>3</sub> → p <sub>1</sub> , WC <sub>j,K</sub> p <sub>3</sub> , WC <sub>j,K</sub> ¬p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>3</sub> , SC <sub>j,K</sub> p <sub>3</sub> , SC <sub>j,K</sub> p <sub>3</sub> → p <sub>4</sub> , WC <sub>j,K</sub> ¬p <sub>4</sub>			p <sub>2</sub> , p <sub>4</sub> , p <sub>5</sub> , p <sub>3</sub> , p <sub>3</sub> → p <sub>4</sub> , p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub>	¬p <sub>4</sub> , ¬p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>3</sub>
SC <sub>i,K</sub> ¬p <sub>4</sub> , SC <sub>i,K</sub> ¬p <sub>4</sub> → (p <sub>3</sub> → p <sub>1</sub> )	p <sub>3</sub> , p <sub>3</sub> → p <sub>1</sub> , p <sub>1</sub> , ¬p <sub>4</sub> , p <sub>5</sub> , ¬p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>3</sub> , ¬p <sub>4</sub> → (p <sub>3</sub> → p <sub>1</sub> )	p <sub>5</sub> , p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub>		
¬SC <sub>j,K</sub> p <sub>4</sub> , ¬WC <sub>j,K</sub> p <sub>4</sub> ¬WC <sub>j,K</sub> p <sub>3</sub> → p <sub>4</sub> , ¬SC <sub>j,K</sub> p <sub>3</sub> , ¬SC <sub>j,K</sub> p <sub>5</sub> , ¬SC <sub>j,K</sub> p <sub>3</sub> → p <sub>4</sub> , ¬WC <sub>j,K</sub> p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub> , ¬SC <sub>j,K</sub> p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>2</sub> WC <sub>j,K</sub> p <sub>3</sub> → p <sub>1</sub> , WC <sub>j,K</sub> p <sub>1</sub> , WC <sub>j,K</sub> ¬p <sub>4</sub> → (p <sub>3</sub> → p <sub>1</sub> ) SC <sub>i,K</sub> p <sub>6</sub> , SC <sub>j,K</sub> p <sub>6</sub> → p <sub>2</sub>			p <sub>2</sub> , p <sub>6</sub> , p <sub>6</sub> → p <sub>2</sub>	¬p <sub>4</sub> , ¬p <sub>4</sub> → (p <sub>3</sub> → p <sub>1</sub> ), p <sub>3</sub> , p <sub>5</sub> , p <sub>3</sub> → p <sub>1</sub> , p <sub>1</sub> , ¬p <sub>4</sub> ∧ p <sub>5</sub> → p <sub>3</sub>

Thus after an assertion of  $p$  the only possible reactions of the hearer are to either challenge or concede  $p$ , under the condition that he has not doubted that  $\neg p$ , and that he has not challenged  $p$  in the preceding move.

**Proof.** LAWS contains (see Table 2) the formula

$$\mathbf{After}_{\langle s, h, \text{Assert}, p \rangle} \mathbf{G}_K ( \mathbf{Ch}_h \mathbf{Happens}(\langle h, s, \text{Challenge}, p \rangle) \vee \mathbf{Ch}_h \mathbf{Happens}(\langle h, s, \text{Concede}, p \rangle) )$$



The precondition for  $\langle h, s, \text{Challenge}, p \rangle$  is

$$\neg \mathbf{WC}_{h,K} p \wedge \mathbf{SC}_{s,K} p \wedge \neg \mathbf{Done}(\langle h, s, \text{Challenge}, p \rangle)$$

Now the postcondition of  $\langle s, h, \text{Assert}, p \rangle$  is  $\mathbf{SC}_{s,K} p$ . Hence we have by the law of intentional action ( $\text{Int}_{\text{Chi},i,a}$ ):

$$\begin{aligned} LAWS \models \mathbf{After}_{\langle s, h, \text{Assert}, p \rangle} ( & \neg \mathbf{WC}_{h,K} p \wedge \neg \mathbf{Done}(\langle h, s, \text{Challenge}, p \rangle) \rightarrow \\ & (\mathbf{Ch}_h \mathbf{Happens}(\langle h, s, \text{Challenge}, p \rangle) \rightarrow \mathbf{Happens}(\langle h, s, \text{Challenge}, p \rangle)) ) \end{aligned}$$

Similarly, for concede we have:

$$\begin{aligned} LAWS \models \mathbf{After}_{\langle s, h, \text{Assert}, p \rangle} ( & \neg \mathbf{WC}_{h,K} p \rightarrow \\ & (\mathbf{Ch}_h \mathbf{Happens}(\langle h, s, \text{Concede}, p \rangle) \rightarrow \mathbf{Happens}(\langle h, s, \text{Concede}, p \rangle)) ) \end{aligned}$$

Combining these two with the law of intentional action for **Assert** we obtain our theorem.  $\square$

Similar results for the other speech acts can be stated. They formally express and thus make more precise further properties of W&K's dialogue games. For example, the above theorem illustrates something that remained implicit in W&K's  $\text{PPD}_0$  dialogues: the hearer of an assertion that  $p$  should not be committed that  $p$  himself because, if he were the dialogue would no more be a persuasion dialogue and no rule would apply.

Similarly, in a context where  $h$ 's commitment store contains  $\mathbf{SC}_{h,K} (p \vee q)$ ,  $\mathbf{SC}_{h,K} \neg p$ , and  $\mathbf{SC}_{h,K} \neg q$  (and is thus clearly inconsistent), W&K's dialogue rules do not allow  $s$  to execute  $\langle h, s, \text{Resolve}, p \vee q, \neg p \wedge \neg q \rangle$ . This seems nevertheless a natural move in this context. Our formalization allows for it, the formal reason being that our logic of  $\mathbf{G}_K$  is a normal modal logic, and thus validates  $(\mathbf{SC}_{i,K} p \wedge \mathbf{SC}_{i,K} q) \rightarrow \mathbf{SC}_{i,K} (p \wedge q)$ .

## 5 Formalization of group belief

As expressed above, our grounding operator formalizes what has been “publicly expressed and accepted as being true by all the agents participating in a conversation”. In the previous section, we developed the expression aspect of this definition, in particular by describing the link between our operator  $\mathbf{G}_I$  and speech acts. In the sequel we will investigate relations between  $\mathbf{G}_I$  and the notions of group acceptance and group belief, in particular with the notion of proper group belief as defined by

Gilbert [18] and Tuomela [42] opposed to the so-called we-belief or shared belief. The aim is to show that our grounding operator can also be viewed as a proper group belief operator.

In this paper, we will not consider refinements of the notion of belief (at individual and collective levels): in particular we ignore the distinction between belief, acceptance and holding true. These distinctions were the subject of many investigations, for example [5, 11, 43]. At the collective level, we will not take part either in the debate between the authors who attribute beliefs to groups (like [20, 40]) and the “rejectionnists” (e.g. [20, 40]) who consider that a group can only accept propositions. (See also [22] for a very interesting discussion about this distinction). We are aware that this is an important philosophical debate. However, from a logical point of view, these are quite subtle distinctions that we do not take account for the time being.

**Remark.** In our framework, we have assimilated the peculiar operators  $\mathbf{G}_i$  to private beliefs. To take into account the distinction between belief and acceptance in the individual layer, we could drop this hypothesis and study relations between  $\mathbf{G}_i$  and a belief operator *à la* Hintikka [25]. But this is out of the scope of this paper.

Thus we consider with Tuomela [42] that we can relate a belief to a whole group. And with Gilbert [18] we will use indifferently the words group belief and group acceptance. Our aim is not to formalize all the subtle distinctions of the domain but only to highlight links between our “grounding” operator and “proper group belief”. Firstly we will summarize the commonly admitted features of group beliefs and show that our operator verifies them, and secondly we will detail its links with two important group belief approaches.

## 5.1 Common features of proper group beliefs

We base our analysis on the works of Margaret Gilbert [18] and Raimo Tuomela [42]. These are the main authors considering group belief as a proper object studies (the term proper group belief is used to name group belief in this sense) and not only as an aggregation of individual beliefs. Indeed, before Gilbert’s papers, group belief was rather formalized in a “summative approach” [42] (e.g. Quinton [30]): a group believes that  $\varphi$  (or has the group belief that  $\varphi$ ) if every agent or at least the majority of the group members believes individually that  $\varphi$ . A more complex summative approach defines the group belief with the common/mutual belief [26]. These approaches are criticized because they relate too deeply individual and collective belief: there a collective belief cannot hold without private beliefs, which discards group belief as the result of a consensus. Thus this constraint is too restrictive and motivated other approaches to emerge. Before studying in details their specificities, we will stress their common features.

***Proper group belief is in no case related to individual beliefs.*** This property is likely the major criticism against the summative approaches. Already Durkheim in [10] expressed that any proper group belief must be “external to individual consciousness”. In fact a group belief can be the effect of a negotiation, a deliberation or a persuasion process and thus a consensus between two or more parts with very different viewpoints. It can even be the result of more or less ethical processes as propaganda or threat. Thus in borderline cases it can be the case that the group has a group belief while no group member individually has the corresponding belief. Tuomela gives the following example [42]: “The Communist Party of Ruritania believes that capitalist countries will soon perish (but none of its members really believes so).” He names these cases “spurious beliefs”.

As said above, our grounding operator  $\mathbf{G}_I$  is linked in no way to the private beliefs, and in particular for every agent  $i$ , member or not of the group  $I$ , neither  $\mathbf{G}_I \varphi \rightarrow \mathbf{G}_i \varphi$  nor  $\mathbf{G}_i \varphi \rightarrow \mathbf{G}_I \varphi$  is a theorem of our logic.<sup>11</sup>

***There is a kind of commitment on the proper group belief.*** As soon as the group belief has been established, even if some group members disagree with this belief, they must act in compliance with it, *i.e.* they are committed in some way to this belief. When they violate it, they are liable for sanctions, ranking from blames of the group [18] to the exclusion of the group [42].

This property is an axiom of our logic: it corresponds to Axioms (11) and (WR). If it is grounded for a group that  $\varphi$ , then no group member can assert that he believes  $\neg\varphi$  (in the case of  $\varphi$  objective, *i.e.* when the grounding arises from a discussion between every agent and not only from the assertion of one).<sup>12</sup>

***The group members share a mutual belief about the proper group beliefs.*** One of the major criticisms against the “simple summative approach” of group belief is that every group member can believe individually that  $\varphi$  without any collective belief on  $\varphi$  because agents are not aware of what other agents believe. A kind of mutual belief is thus necessary, but not about the content of the group belief (as in the “complex summative approach”), but rather on the group belief itself. Tuomela [42] defends this thesis arguing that group belief is grounded due to a joint and intentional group action.

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<sup>11</sup> Nevertheless  $\mathbf{G}_I \varphi \wedge \mathbf{G}_i \varphi$  is a consistent formula, *i.e.* proper group belief does not ban the existence of a mutual belief.

<sup>12</sup> Indeed, if it is grounded that  $\varphi$  for the group  $I$ , then due to (WR), in particular  $\mathbf{G}_I \mathbf{G}_i \varphi$  holds for every member  $i$  of the group  $I$  and with Axiom (11),  $\neg \mathbf{G}_I \mathbf{G}_i \neg\varphi$  holds too. No member of  $I$  can perform an assertive speech act with propositional content  $\neg\varphi$  [17].

This feature is a theorem of our logic: as proven above, the formula  $\mathbf{G}_I \varphi \rightarrow \mathbf{MBel}_I \mathbf{G}_I \varphi$  is a theorem. Our logic is stronger because we even have the equivalence.

After this first examination our grounding operator has good properties to represent a kind of group belief. We will examine more deeply its link with Gilbert's and Tuomela's approaches.

## 5.2 Details on Gilbert's and Tuomela's approaches

*The Plural Subject Account (Gilbert [18, 19]).* In her book [19], in opposition to "summative" approach, Gilbert gives the following characterization of proper group belief:

1. *A group  $G$  believes that  $p$*  if and only if the members of  $G$  jointly accept that  $p$ .
2. The members of  $G$  *jointly accept* that  $p$  if and only if it is common knowledge in  $G$  that the members of  $G$  individually have intentionally and openly expressed their willingness jointly to accept that  $p$  with the other members of  $G$ .

From this standpoint, group belief *à la* Gilbert seems equivalent to the common belief in a group on the public and intentional expression by each group member of his joint acceptance.

In the sequel, we will show how our grounding operator is close to this definition thank to Axioms (WR) and (CG). In particular in the case of objective formulas  $\varphi$ , we have the equivalence:

$$(22) \quad \mathbf{G}_I \varphi \leftrightarrow (\bigwedge_{i \in I} \mathbf{G}_I \mathbf{G}_i \varphi)$$

Due to the formula (7), we can deduce the equivalence:

$$(23) \quad \mathbf{G}_I \varphi \leftrightarrow \mathbf{MBel} (\bigwedge_{i \in I} \mathbf{G}_I \mathbf{G}_i \varphi)$$

This equivalence is very close to Gilbert's characterization of the group belief. In fact, formula  $\mathbf{G}_I \mathbf{G}_i \varphi$  expresses that agent  $i$  expressed in front of group  $I$  that he believes  $\varphi$  (*i.e.*  $i$  asserted  $\varphi$ ), or with the abbreviation introduced in the former section, that he involved a strong commitment face of  $I$  on  $\varphi$ . Thanks to Axiom (CG)  $((\bigwedge_{i \in I} \mathbf{G}_I \mathbf{G}_i \varphi) \rightarrow \mathbf{G}_I \varphi)$ , by asserting  $\varphi$ , agent  $i$  expresses also implicitly his

acceptance that  $\varphi$  to be grounded in the group.<sup>13</sup>

Thus formula (23) can be read: “an objective formula  $\varphi$  is grounded for group  $I$  if and only if it is mutual belief in group  $I$  that every group member publicly expressed that they believe the objective formula  $\varphi$ ”.<sup>14</sup>

It follows from this informal proof that for objective formulas our grounding operator matches group belief as defined by Gilbert. Thus our grounding operator is an admissible formalization of Gilbert’s account of group belief (for objective formulas).

***The Positional Account (Tuomela [42]).*** Tuomela in [42] discusses Gilbert’s group belief definition and shows in particular that in the cases of structured groups (for example with representatives) Gilbert’s approach [18] needs some updates. Tuomela takes the following example: “the United States believe that the [Soviet] invasion of Afghanistan was an unconscionable act” [19]. Not all Americans have accepted this utterance, but only a small subgroup called the government, that we could name also the leaders.<sup>15</sup> Tuomela gives the following analysis of the proper group belief, being inspired by his group intention formalization based on the distinction between “operative” and “non-operative” members (the ones who form the intention and the others who accept it tacitly) and definition of a “right social and normative circumstances” (a kind of institution composed by norms, roles, social rules and tasks...):

$G$  believes that  $p$  in the social circumstances  $C$  if and only if in  $C$  there are operative members  $A_1, \dots, A_m$  of  $G$  in respective positions  $P_1, \dots, P_m$  such that:

- (1) the agents  $A_1, \dots, A_m$ , when they are performing their social tasks in their positions  $P_1, \dots, P_m$  and due to exercising the relevant authority system of  $G$ , (intentionally) jointly accept that  $p$ , and because of this exercise of authority system, they ought to continue to accept and positionally believe it;
- (2) there is a mutual belief among the operative members  $A_1, \dots, A_m$  to the effect that (1);

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<sup>13</sup> In particular, because he is aware of this theorem (thanks to Rule of Necessitation of the grounding operator), he knows that by asserting  $\varphi$ ,  $G_I \varphi$  could hold if the other agents asserted their acceptance.

<sup>14</sup> which is equivalent in our framework to accept that  $\varphi$  could be grounded in the group.

<sup>15</sup> Tuomela’s purpose is not to give an example of a subgroup imposing its beliefs to the whole group. Every member of the government could believe the opposite of the grounded group belief but decide to accept it as the group belief. On the contrary, the other members of the group could be in accordance with it, and only have to tacitly accept it.

- (3) because of (1), the (full-fledged and adequately informed) non operative members of  $G$  tend tacitly to accept –or at least ought to accept–  $p$ , as members of  $G$ ;
- (4) there is a mutual belief in  $G$  to the effect that (3).

This characterization seems much more realistic and complex than the Gilbert's one. It is also more general: if we consider that every agent is an operative agent and we ignore the social and normative circumstances, Tuomela's approach is brought down to Gilbert's one. With our simple framework, we obviously cannot formalize the whole complexity of the Tuomela's definition. In particular, we introduce neither notions of roles, institution or norms, nor do we distinguish between operative/nonoperative agents. But following Tuomela, we will simplify a bit his formalism to show an interesting property: we will ignore the social and normative circumstances (assuming that they do not impose any restrictions).

As said above, the formula  $\mathbf{G}_I \mathbf{G}_i \varphi$  means: it is grounded for the group  $I$  that  $i$  believes  $\varphi$ . But another reading that we have previously given is: agent  $i$  has expressed in front of  $I$  that he believes  $\varphi$ . This formula is in fact the effect (in the sense of the action theory) of an assert speech act. Thus some non-objective formulas can be immediately grounded as soon as an agent performs the corresponding speech act. These formulas neither require discussion nor explicit acceptance by every agent of the group. Only a single agent is needed to ground such formula for the whole group. It appears clearly that such formulas do not correspond to group belief *à la* Gilbert. We will show that it corresponds in fact to a group belief in Tuomela's approach.

In our logic, an agent is operative (in Tuomela's sense) in what concerns his own beliefs:  $\mathbf{G}_I \mathbf{G}_i \varphi$  holds whenever  $i$  asserts  $\varphi$ . This implies in our logic that he ought to continue to accept publicly that he believes  $\varphi$ . By theorem (7), there is mutual belief that  $\mathbf{G}_I \mathbf{G}_i \varphi$ . The other agents (called non-operative in this case) ought to accept that it is grounded that agent  $i$  believes  $\varphi$ . Moreover there is mutual belief about this grounded formula. Due to what we have said above about Tuomela's group belief, we have that  $\mathbf{G}_I \mathbf{G}_i \varphi$  implies group belief that  $i$  believes  $\varphi$  (in Tuomela's sense).

We could be closer to Tuomela's approach by introducing the concept of *leaders* of a group about a proposition in our framework. The leaders would be a subgroup of the group of agents  $I$ , verifying properties such as:  $\mathbf{G}_I \mathbf{G}_{leaders(I, \varphi)} \varphi \leftrightarrow \mathbf{G}_I \varphi$ . This means that if it is grounded for the whole group  $I$  that it is grounded for leaders that  $\varphi$ , then  $\varphi$  is *de facto* grounded for the whole group (*i.e.* if leaders have jointly accepted  $\varphi$ , then other agents have to accept it tacitly and thus  $\varphi$  becomes a proper group belief *à la* Tuomela). The group of leaders could be for example the government for every decision concerning the whole nation (thus leaders get their power from citizen's votes) or a group of specialists of a domain for every fact

concerning their competence domain (they are leaders due to their knowledge and skills).

To summarize we have proven that, despite our simple logical framework without complex social constructions as norms, roles or institutions, our grounding operator is an operator of proper group belief. It is moreover more general than that of Gilbert’s approach but still less than that of Tuomela’s characterization, because of the specific process of grounding for **Bel**<sub>*i*</sub>  $\varphi$ -like formulas.

## 6 Conclusion

### 6.1 Related works

A way to bridge the gap between mentalist and structural approaches and to adapt BDI based agent to social speech act semantics has been recently the object of many works from a logical point of view<sup>16</sup>, among which Boella et al. [2], Pasquier et al. [29] and Nickles et al. [28].

Firstly Nickles et al. [28] developed a logic of ostensible mental attitudes (Ostensible Belief and Ostensible Intention) that represents the public counterpart of mental attitudes and shows links between private mental attitudes (beliefs and intentions) and ostensible mental attitudes. As we discussed in [17], our two approaches are from a semantic point of view more or less equivalent. Nickles et al.’s ostensible belief of *i* w.r.t. *j* that  $\varphi$  corresponds in our logic to  $\mathbf{G}_{\{i,j\}} \mathbf{G}_i \varphi$ . Our logic is completely formalized in particular with a semantics, whereas theirs one is not.

Pasquier et al. [29], following their works on dialogue games and commitments [4], brings to light links between private intention and commitment. Following the distinction of Bratman [3] between *intention-to-do* (about an action) and *intention-to-be* (about a proposition), they consider that commitments in action are the social counterpart of private intention-to-do whereas propositional commitments are the one of the intention-to-be. Their work is not yet logically well-founded and they do not consider any links between private beliefs and a public counterpart.

Boella et al. [2] use what they name the “role metaphor” to bridge the gap between mentalist and social approaches of the ACL speech acts. Every agent taking part in a dialogue plays a role and is assumed to act in accordance with his role. They consider that agents must be sincere and cooperative not as individuals but as playing a role. Mental attitudes involved in the dialogue are associated to roles but

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<sup>16</sup> from a more philosophical standpoint, we can cite without discussing them [3, 47].

not to agents. Thus, these attitudes are public and can be checked by every agent. With this concept they formalize both FIPA-ACL speech act semantics and commitments-based semantics.

## 6.2 Summary and perspectives

We have defined the modal operator of grounding to characterize what is public in the dialogue and more particularly what has been openly expressed or accepted by every agent. It can bridge the gap between the public and the private layer of the agent, because it is public, objective and linked to the belief. It is well grounded philosophically and formally.

Moreover we have shown how it can be used to formalize speech acts semantics and game rules, in particular in the case of Walton&Krabbe dialogue games.

In addition the grounding operator is very close of the notion of proper group belief although our logical framework does not contain any concept such as norm, role or institution.

Finally a very important improvement of the framework could be to add roles and institution to permit a much more precise analysis of the proper group belief. Moreover, one could extend the framework to take into account proper group intentions and obtain a unified framework for a group BDI logic.

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