

Appendix 1. Proof that the algorithmic dissimilarity D_{KG} is always not higher than the Rogers dissimilarity D_R .

Let p_i and q_i be the relative abundances of species i in plots P and Q , respectively, such that $0 \leq p_i \leq 1$, $0 \leq q_i \leq 1$ and $\sum_i^S p_i = \sum_i^S q_i = 1$, where S is the total number of species in both plots, and Δ be a symmetric $S \times S$ matrix of pairwise functional dissimilarities d_{ij} between species i and j ($i, j = 1, 2, \dots, S$) in the range $[0, 1]$ with $d_{ij} = d_{ji}$ and $d_{ii} = 0$.

We define a matching between plots P and Q as a matrix M with dimension $S \times S$ and coefficients in $[0, 1]$ such as:

$$\begin{aligned} \forall j \in \{1, \dots, S\}, \sum_{i=1}^S M_{ij} &= p_j \\ \forall i \in \{1, \dots, S\}, \sum_{j=1}^S M_{ij} &= q_i \end{aligned} \quad (1.1)$$

We define the dissimilarity between P and Q associated to the matching M and the dissimilarity matrix Δ as:

$$D(M, \Delta) = \sum_{i=1}^S \sum_{j=1}^S M_{ij} \Delta_{ij} \quad (1.2)$$

The set of possible matching pairs between P and Q is a closed and bounded subset of the $S \times S$ matrices. Consequently, the algorithmic dissimilarity D_{KG} between P and Q can be defined as $D_{KG}(\Delta) = \min_M D(M, \Delta)$ and there exists an admissible matching $M_{KG}(\Delta)$ such that $D_{KG}(\Delta) = D(M_{KG}(\Delta), \Delta)$

Denoting with Δ^* the dissimilarity matrix with $\Delta_{ii}^* = 0$ and $\Delta_{ij}^* = 1$ for $i \neq j$, we will prove that $D_{KG}(\Delta) \leq D_{KG}(\Delta^*)$ for all dissimilarity matrices Δ .

Proof

Let Δ be a dissimilarity matrix.

a) Because $d_{ij} \leq 1$ for all $i \in \{1, \dots, S\}$ and $j \in \{1, \dots, S\}$, it is straightforward that $D(M, \Delta) \leq D(M, \Delta^*)$ for all matchings M and all dissimilarity matrices Δ . This implies that $D(M_{KG}(\Delta^*), \Delta^*) \geq D(M_{KG}(\Delta^*), \Delta)$ and thus $D_{KG}(\Delta^*) \geq D(M_{KG}(\Delta^*), \Delta)$.

b) As $D_{KG}(\Delta) = \min_M D(M, \Delta)$, by definition $D_{KG}(\Delta) \leq D(M_{KG}(\Delta^*), \Delta)$.

a) and b) imply $D_{KG}(\Delta) \leq D_{KG}(\Delta^*)$.

End of proof

It has been shown by Kosman (2014) that $D_{KG}(\Delta^*)$ equals the Rogers dissimilarity between plots P and Q , denoted D_R . Thus we proved that for any dissimilarity matrix Δ , $D_{KG}(\Delta) \leq D_R$.

References

Kosman, E. (2014) Measuring diversity: from individuals to populations. *European Journal of Plant Pathology* 138: 467–486.