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Micrometer Positioning Accuracy with a Planar Parallel Continuum Robot

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2 ABSTRACT

3 Parallel Continuum Robots (PCR) have several advantages over classical articulated robots,
4 notably a large workspace, miniaturisation capabilities and safe human-robot interactions.
5 However, their low accuracy is still a serious drawback. Indeed, several conditions have to
6 be met for PCR to reach a high accuracy, namely: a repeatable mechanical structure, a correct
7 kinematic model, and a proper estimation of the model's parameters. In this article, we propose a
8 methodology that allows reaching a micrometer accuracy with a PCR. This approach emphasizes
9 the importance of using a repeatable continuum mechanism, identifying the most influential
10 parameters of an accurate kinematic model of the robot and precisely measuring them. The
11 experimental results show that the proposed approach allows to reach an accuracy of $3.3\ \mu\text{m}$
12 in position and $0.5\ \text{mrad}$ in orientation over a 10 mm long circular path. These results push
13 the current limits of PCR accuracy and make them good potential candidates for high accuracy
14 automatic positioning tasks.

15 **Keywords:** soft robots, continuum robots, Cosserat-rod, Kirchhoff-rod, parallel robots, micro-positioning, robot calibration, accuracy

1 INTRODUCTION

16 Parallel Continuum Robots (PCR) are a recent and rising type of robots introduced in (Bryson and Rucker,
17 2014). They are composed of flexible slender elements arranged in parallel and linked to a platform.
18 Resulting robots present a higher rigidity than soft robots while keeping most of their advantages. This
19 structure provides higher safety than common industrial robots thanks to the flexibility of the PCR's limbs.
20 For instance, Campa *et al.* proposed a planar PCR to perform safer collaborative robot applications (Campa
21 *et al.*, 2019). The flexibility of the limbs provides large continuous deformations that allow for a large
22 workspace. Yang *et al.* proposed a continuum version of the Delta robot whose continuous joints allow
23 larger rotations than usual joints (Yang *et al.*, 2018). Flexible elements allow to reduce the number of joints
24 and eliminate the mechanical plays (Black *et al.*, 2018). Reducing the number of joints and mechanical
25 parts also facilitates the miniaturization of the designed robot. For example, the continuum Steward-Gough
26 platform introduced in (Bryson and Rucker, 2014) had its dimensions reduced in order to create a surgical
27 PCR whose platform (a gripper) is around 10 mm diameter (Orekhov *et al.*, 2017). This small prototype
28 illustrates the interest and the capability of PCR to perform applications inside confined space (like the
29 human body) where instrumentation and sensor-based controls are difficult to implement. To execute those
30 applications, an accurate PCR could be a solution. However, the reachable accuracy of those structures

31 is still an open question which needs more investigations. The objective of this paper is to propose a
32 methodology to reach a high accuracy with PCR by measuring, identifying and understanding the influence
33 of the different parameters, and have an estimate of a typical level of accuracy that a PCR can reach.

34 The accuracy corresponds to the difference between commanded poses and the barycenter of attained
35 ones (ISO 9283, 1998) in position and orientation. A high accuracy enables to position the end-effector
36 of the PCR thanks to its model and allows to successfully perform automated tasks. The accuracy of a
37 robot depends on three key elements: its repeatability, the correctness of its model and the knowledge
38 of the model's parameters. The repeatability corresponds to the deviations of the reached poses for a
39 same command, and mainly depends on robot's design, the quality of its mechanical structure and its
40 actuation system. A repeatable robot is essential to have a predictable behavior and thus to model it. The
41 modeling of continuum robots appears especially challenging because of their virtually infinite number
42 of Degrees-of-Freedom (DoF) and their highly nonlinear behavior due to the large deformations. Two
43 modeling approaches of parallel continuum robotics are widespread in the literature: physical-based models
44 and black-box models such as neural networks. Using an artificial neural network, Wu *et al.* reached a
45 positioning accuracy of 5 mm. One of the drawbacks in using black-box models is that they could not
46 identify why the positioning accuracy was low (Wu *et al.*, 2017). This is one of the reasons why most of
47 the studies use physical-based models.

48 We can distinguish two categories of physical-based models: high fidelity distributed parameter models
49 and low fidelity parameter models (Rone and Ben-Tzvi, 2014). The low fidelity parameter models use
50 strong hypothesis to reduce the number of parameters and to facilitate the modeling and the identification of
51 its parameters. For instance, constant curvature models represent continuum deformations by considering
52 that flexible element shapes are circle arcs (Lilge *et al.*, 2021; Nuelle *et al.*, 2020). Those approaches
53 are interesting for their simplicity but seem not suitable for high positioning accuracy. For this reason,
54 physical-based models with high fidelity distributed parameters and variable curvature assumption are
55 preferred. Depending on the structure, Cosserat-rod-based models (Trivedi *et al.*, 2008; Orekhov *et al.*,
56 2017; Till and Rucker, 2017; Black *et al.*, 2018) or Kirchhoff-rod-based ones (Altuzarra *et al.*, 2019;
57 Altuzarra and Merlet, 2019; Takano *et al.*, 2017; Campa *et al.*, 2019) gained consensus because of their
58 ability to predict the shape and forces of the rods. In the case of tendon-actuated continuum structures, Rao
59 *et al.* proposed guidelines to choose a model depending on the targeted application (Rao *et al.*, 2021). Such
60 guidelines do not currently exist for parallel continuum structures.

61 In addition to the modeling, the measurement and the identification of the model's parameters are also
62 important. Indeed, to get an accurate prediction of the robot's behavior, the model's parameters need to
63 be close to their real values. To address this issue, robot calibration is usually performed. However, as
64 Wu *et al.* pointed out, the calibration of PCR is more complex than for regular parallel robots (Wu and Shi,
65 2019). One example is the absence of an analytical model. This complexity generally conducts to choose
66 a simpler model to facilitate the identification step (Dehghani and Moosavian, 2013) or to measure the
67 parameters rather than identifying them (Orekhov *et al.*, 2016). The chosen model, the measurement and
68 the identification of its parameters have a strong and deeply intertwined impact on the robot's accuracy.

69 Recent studies quantified the accuracy of several PCR that are gathered in Table 1. Even if it is difficult to
70 compare them because of their different designs, those values give an overview of the expected reachable
71 accuracy of current PCR using different models. To be more representative to the PCR's design, the
72 positioning accuracy is generally expressed in percent of the nominal or the mean dimension of the
73 continuum flexible segment. For example, Orekhov *et al.* obtained a mean positioning accuracy of 2.8%

74 (1.19 mm) and a mean orientation accuracy of 3.81° after identifying extrinsic parameters of a 6-Degrees-
75 of-Freedom (DoF) robot (Orekhov et al., 2016). The accuracy is interesting for a spatial PCR even if it
76 is limited by the actuation system whose positioning accuracy is about 0.1 mm. The actuation system is
77 also one of the main uncertainty sources for the positioning accuracy calculated from the data presented
78 in (Yang et al., 2018). In the studies of Wu *et al.* (Wu and Shi, 2019; Wu et al., 2017), several reasons were
79 pointed out like the small construction and assembly tolerances or the friction forces that were not took
80 into consideration. Nuelle *et al.* proposed a study of a tendon-actuated planar PCR and reached an accuracy
81 of 1.4% (1.8 mm) after identifying all model's parameters thanks to a calibration process (Nuelle et al.,
82 2020). The shown accuracy was limited by the constant curvature approach and by the actuation and the
83 robot design, which suffer from gear backlash and static-friction.

84 To address the challenge of getting accurate PCR, the proposed approach consists in five key elements.
85 The first one consists in using a repeatable PCR. The robot recently introduced in (Mauzé et al., 2020) was
86 able to provide an outstanding repeatability of 9.13 nm in position and $1.2 \mu\text{rad}$ in orientation. Choosing
87 a planar architecture allows using a very resolute and long range multidimensional measurement system
88 which facilitates the understating of the proposed methodology. Thanks to its mechanical structure, this
89 robot seems suitable to study the accuracy reachable by PCR. This 3 DoF $XY\Theta$ planar parallel continuum
90 is illustrated in Fig. 1. The second point is the use of a correct mechanical model. The measurement of the
91 model's parameters is facilitated by the PCR design which also anticipates the calibration process. That is
92 why fiducial markers, third element of the proposed methodology, are introduced. Those fiducial markers,
93 placed at specific locations of the PCR, enable to efficiently measure the nominal values and to estimate
94 the uncertainties of the different models' parameters. The fourth element is a sensitivity analysis which
95 coupled with the uncertainties allows to determine the most influential parameters. The last key point is the
96 calibration process of the robot.

97 The next section presents the model of the PCR and the sensitivity analysis. This analysis enables to
98 understand the role of each parameter and to identify the most influential ones. Those parameters require
99 a special attention during the design and the calibration processes of the robot. The robot prototype and
100 the experimental setup are described in Section 3. The models' parameters measurement step with an
101 uncertainty analysis and the calibration process are respectively presented in Section 4 and 5. The last
102 section demonstrates the capability of the robot to perform desired trajectories and quantifies the accuracy
103 using the identified parameters.

2 3-DOF PLANAR PCR MODEL

104 To study Parallel Continuum Robots' (PCR) accuracy, we considered a high-grade repeatable robot
105 illustrated in figure 1. This robot is composed of three planar kinematic chains that are linked together to a
106 rigid moving platform. Contrary to classical 3-PRR mechanisms which inspired this design, each chain
107 is composed of a prismatic actuator and a flexible rod that deforms continuously. The three flexible rods
108 transmit the actuators' forces to the platform inducing its motions. Using the traditional naming convention,
109 the flexible rod is denoted using the letter 'F' and the resulting PCR design is then a 3-PF robot.

110 Fig. 2 illustrates the kinematic diagram of the 3-PF robot. The actuation stages, through the prismatic
111 joint values q_1, q_2, q_3 , push and pull the rods to move the mobile platform. The rods are slender beams
112 capable of continuous and large deformations. The movement of this mobile platform is restricted to planar
113 displacements (x, y translations and θ rotation).

114 A global work frame ($W, \mathbf{x}_W, \mathbf{y}_W, \mathbf{z}_W$) is defined with the \mathbf{z}_W axis perpendicular to the robot's base.
115 A frame ($P, \mathbf{x}_P, \mathbf{y}_P, \mathbf{z}_P$) is attached to the mobile platform. The three rods are clamped to this platform at

116 their distal ends $(B_i, \mathbf{x}_{B_i}, \mathbf{y}_{B_i}, \mathbf{z}_{B_i})$. The proximal ends of the rods clamped to the actuator are defined by
 117 the frames $(A_i, \mathbf{x}_{A_i}, \mathbf{y}_{A_i}, \mathbf{z}_{A_i})$.

118 In the following, we describe the model used to simulate the quasi-static behavior of the robot. First, the
 119 model of a single rod is detailed. Then, the three models of rods are linked together thanks to the static
 120 equilibrium of the platform. All those elements allow to implement the forward and the inverse kinematic
 121 models of the robot.

122 2.1 Rod Model

123 The PCR's model depends mainly on the modeling of the rods and its correctness, that is why a Kirchhoff-
 124 rod-based model is used. This model supposed that shear and extension can be neglected. It is possible as
 125 the used slender elements have their cross-section more than a hundred times smaller than their lengths.
 126 The proposed structure remains in a plane so, only the planar case of this model is considered without
 127 losing the generality of the proposed approach which can also be adapted for a spatial robot.

128 The curvilinear abscissa is represented by the scalar parameter $s \in [0, l]$ where l is the stress-free length
 129 of a rod. Along its arc length, $\mathbf{p}(s) = [x(s), y(s)]^T$ and $\theta(s)$ respectively define the cross-section centroid
 130 position and orientation in the frame attached to the proximal end of the rod. Fig. 3 shows the entire model
 131 of a limb.

132 All involved differential equations can be gathered into the following system:

$$\begin{pmatrix} \frac{dx(s)}{ds} \\ \frac{dy(s)}{ds} \\ \frac{d\theta(s)}{ds} \\ \frac{dn_x(s)}{ds} \\ \frac{dn_y(s)}{ds} \\ \frac{dm(s)}{ds} \end{pmatrix} = \begin{pmatrix} \cos(\theta(s)) \\ \sin(\theta(s)) \\ \frac{m(s)}{EI} \\ 0 \\ 0 \\ n_x(s) \sin(\theta(s)) - n_y(s) \cos(\theta(s)) \end{pmatrix} \quad (1)$$

133 Where $n_x(s), n_y(s)$ are respectively the X and Y components of the internal force $\mathbf{n}(s)$, $m(s)$ is the
 134 internal moment, E is the Young modulus of the rod material, and I is the second area moment of the rod
 135 cross-section which depends on the rod's diameter d .

136 2.2 Forward and Inverse Kinematic Models

137 To model the quasi-static behavior of the PCR, a forward and inverse kinematic models are created. Both
 138 result from a numerical resolution of the previous equations considering the rod's boundary conditions and
 139 the static equilibrium of the platform.

140 Those boundary conditions describe how the rods link the actuators to the mobile platform. They yield:

$${}^W\mathbf{T}_P = {}^W\mathbf{T}_{A_i} \cdot {}^{A_i}\mathbf{T}_{B_i} \cdot {}^{B_i}\mathbf{T}_P \quad (2)$$

141 where ${}^W\mathbf{T}_{A_i}$ depends on the joints coordinates q_i , ${}^{A_i}\mathbf{T}_{B_i}$ corresponds to the transformation resulting from
 142 the integration of the rod's equations ${}^{B_i}\mathbf{T}_P$ is constant reflecting the rigid-body conditions between the
 143 distal ends of the rods and the mobile platform, and ${}^W\mathbf{T}_P$ depends on the desired position such as:

$${}^W\mathbf{T}_P = \begin{bmatrix} \cos \theta & -\sin \theta & x \\ \sin \theta & \cos \theta & y \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

To know the pose of the platform, the static equilibrium conditions are considered:

$$\begin{cases} \sum_{i=1}^3 [\mathbf{n}_i(l_i)] - \mathbf{f}_P = 0 \\ \sum_{i=1}^3 [\mathbf{p}_{B_i} \times \mathbf{n}_i(l_i) + m_i] - \mathbf{p}_P \times \mathbf{f}_P - m_P = 0 \end{cases} \quad (4)$$

144 where \mathbf{f}_P , m_P are the external force and moment applied on the platform in the work frame, where \mathbf{n}_i , m_i
 145 are the rods' forces and moments applied on the platform in the work frame, l_i is the length of the rod i and
 146 \mathbf{p}_{B_i} and \mathbf{p}_P are the positions of B_i and P in the work frame.

147 Due to the coupling between the rods, there is no analytical solution of those equations considering the
 148 previous boundary and equilibrium conditions, thus, a numeral resolution is performed using a shooting
 149 method. This shooting method is based on a optimisation problem where the Eq. (2) and Eq. (4) constitute
 150 its residual vector (Till and Rucker, 2017; Mauzé et al., 2020).

151 2.3 Sensitivity Analysis of the models' parameters

152 The PCR accuracy depends on the values of the model's parameters. To estimate the influence of the
 153 different parameters, a sensitivity analysis is performed. It also allows for identifying which parameters
 154 required more consideration. There are three kinds of parameters: the intrinsic rod parameters (the Young
 155 modulus E of its material, its diameter d and its length l), the ones representing the poses of the rods'
 156 proximal and distal ends (respectively A_{x_i} , A_{y_i} , α_i and B_{x_i} , B_{y_i} , β_i), and the parameters γ_i which represent
 157 the misalignments between the directions of translations and the rods' orientation at the proximal ends.

158 The principle of the proposed sensitivity analysis is the following. An arbitrary set of 26 joints
 159 configurations is chosen in the center of the workspace. For each configuration, partial derivatives of the
 160 platform position considering the different parameters are numerically computed using a finite difference
 161 method. The maximal values of the partial derivatives among the configurations and the different rods are
 162 gathered in Table 2. All parameters are expressed in the SI base unit.

163 From the table results, three groups of parameters can be defined considering their influence on the model.
 164 The first group is composed of the Young modulus and the diameters of the rod. The Young modulus has
 165 no influence on the pose of the platform. It is the same observation for the diameter if the three parameters
 166 are equal. This assumption can be considered as true if the different rods are created from the same element.
 167 If one diameter is different than the other, it introduces an asymmetry and thus a small variation of the
 168 modeling. This difference of behavior is only visible for the diameters. Indeed, if all lengths (for example)
 169 are changed in the same time, the consequences on the model will be more important than if only one
 170 length was changed.

171 The second group is composed of the angular parameters α_i , β_i and γ_i . The influence of the parameter
 172 α_i is more important than the one of the other angular parameters. It can be explained by its role in the
 173 transmission of the actuator's movement. To understand the influence in terms of orientation, considering a
 174 1 mrad uncertainty on a α_i and the value of the table, the resulting positioning error for the model is about
 175 1.3 μm . If this value is compared with a case-study of an uncertainty of 1 mrad made on the orientation of a
 176 rigid bar of 30 mm, the estimation of the position is about 30 μm at the bar's end. So, it can be said that the
 177 influences of those angular parameters are relatively small. The elasticity, the deformation of the rod and
 178 the parallel structure reduced their influence.

179 The last group is composed of the parameters that are the most influential: the length of the rods l_i and
180 their end positions $A_{x_i}, A_{y_i}, B_{x_i}, B_{y_i}$. The differences between the direction X and Y are essentially due to
181 the asymmetry of the robot's structure which is induced by the initial orientation of the rods.

182 This sensitivity analysis give information about the influence of the different parameters but also indicate
183 the threshold of uncertainty where their influence can not be neglected anymore. To give an example of
184 comparison between the parameters' influence, considering the values of the table, an uncertainty of 1 mrad
185 observed on a proximal end's orientation α_i has the same influence of an uncertainty of 1 μm observed on
186 the estimation of the X coordinate of the proximal end position A_{x_i} .

187 In order to get an accurate robot, the parameters whose influences are the most important on the model
188 need to be estimated as precisely as possible. The next section will present the PCR design which takes
189 into account the presented results by introducing fiducial markers. The objective of those markers is to
190 reduce the measurement uncertainty of the most influential parameters.

3 ROBOT DESIGN AND MEASUREMENT SYSTEM

191 To validate the modeling of the robot, a prototype has been built by taking into account the previous
192 sensitivity analysis. To perform this comparison between the prototype's experimental behavior and its
193 model, the PCR poses and its models' parameters have been measured by two complementary vision
194 measurement systems. This section introduces the robot design and the associated measurement system.

3.1 Robot Design

196 The robot is composed of a mobile platform coupled to three actuated continuum limbs.

197 Each limb is assembled on a stack of three manual precision stages, two translation stages (Newport
198 SDS-40) and a rotation stage (Newport M-RS40). These manual stages are used to adjust the position and
199 the orientation of each actuator fixed above them.

200 The actuators are SmarAct (SLC-1730-S-HV) positioning stages. Their repeatability is below 30 nm and
201 their range is 21 mm. Those actuators possess accurate optical sensors and a repeatable behavior that are
202 necessary to get accurate inputs for the forward kinematic modeling of the PCR.

203 The rods are optical fibers stripped of their plastic part. Those fibers are in fused silica whose Young
204 Modulus E is about 69 GPa. They are able to do large deformations without viscous effect thanks to
205 their elasticity. They are 125 μm in diameter and about 30 mm in length. Those rods are connected to the
206 actuators and to the platform by the mean of rod mounts.

207 The rod mounts are the key elements to define precisely the position of the rods' ends and thus their
208 length. Each rod mount is drilled on one side to clamp the rods. On the top face of the mount, a fiducial
209 marker (QR-Code-like pattern) is engraved. Thanks to its manufacturing, the transformation between the
210 center of the fiducial marker and the hole where the rod are inserted is known with a small uncertainty.
211 Thus, knowing the pose of the fiducial marker will enable to know the relative pose of the proximal and the
212 distal ends of the three rods (more details will be given in the next section). The proximal rod mounts are
213 fixed on the actuators and the distal rod mounts are glued on the mobile platform.

214 The platform is a 100 mm silicon wafer lifted by a 50 mm diameter air bearing (S205001) from the IBS
215 company. This air bearing avoids friction and is preloaded to maintain a stable elevation of the platform
216 ($\pm 5 \mu\text{m}$). A manual linear stage (Newport M-DS25-Z) allows to adjust the level of the air bearing and to
217 get the required planarity of the entire PCR.

218 3.2 Vision Setups

219 The knowledge of the initial configuration of the robot is mandatory for an accurate simulation and
220 position control of the robot. For this purpose, two vision measurement systems have been set up as shown
221 in Fig. 4.

222 The first one relies on the localization of fiducial markers embedded on the rod mounts. The position
223 of these fiducial markers are tracked over a large field with a camera (IDS UI3008CP-3) mounted with a
224 50 mm lens. This vision setup is used to get accurate measurements of the different models' parameters of
225 the robot. More details will be given in the next section.

226 The second vision measurement system is dedicated to the measurement of the Cartesian pose of the
227 platform with a very high resolution. This system is composed of a camera with a microscope tube, a x20
228 lens from Mitutoyo and a pseudo-periodic pattern glued on the mobile platform of the PCR. This system is
229 able to measure the x , y position and the θ angle of the platform with a sub-nanometer resolution (Andre
230 et al., 2020). Both cameras are supported by a robust gantry in order to minimize mechanical noise.

4 PARAMETERS MEASUREMENT

231 After the PCR's model is created, the PCR accuracy will depend on the value of the model's parameters.
232 The closer the value of the parameters will be, the higher will be the PCR's accuracy. The measurement of
233 those parameters is an essential step. Specially those whose influence, resulting from the sensitivity analysis,
234 is important. This section explains how nominal values and measurement uncertainties are obtained in the
235 aim to have an accurate control of the robot.

236 The different parameters are related to the rods, the position of the rods' ends and the actuators' direction
237 of displacement. The quantification of the uncertainty will define intervals which will give more information
238 about the potential modeling errors, and help to identify the parameters during the calibration process.

239 4.1 Rod parameters

240 The diameter of the rods is measured thanks to a calliper which has a measurement uncertainty of 20 μm .
241 The length of the rods is measured from images taken with the first vision setup. The measurement
242 uncertainty is estimated to 3 pixels. Considering the pixel/metre ratio, this uncertainty is 126 μm .
243 Considering the uncertainties values and the results of the sensitivity analysis, the rod diameters influence
244 can be neglected while the rod lengths need to be identified carefully.

245 4.2 Rod end positions

246 The sensitivity analysis points out that the rod end positions are influential parameters. Those positions
247 depend on the rod mounts position and the clamping conditions. All rod mounts positions can be directly
248 measured with the first vision setup and a specific fiducial marker detection process. This algorithm
249 gives the Cartesian coordinates of all fiducial markers in the camera frame. The six rod mounts can be
250 differentiated thanks to missing squares in each fiducial markers (coding principle). The resolution of the
251 measurement is less than 0.25 μm for X and Y translations and 0.5 mrad for rotation.

252 As the transformation between the center of the fiducial marker and the clamping hole is known, the
253 Cartesian position of the proximal end A and the distal end B of each rod can be deduced from these
254 markers poses. By construction, the transformations between the center of the fiducial marker and the
255 clamping point of the rod are defined by two translations of 14 mm and 250 μm in the X and Y directions.
256 The machining accuracy is about 6% of this transformation leading to uncertainties of respectively 840 μm
257 and 15 μm . By acquiring an image close to the rod mount and the clamping area and considering the
258 resulting pixel/metre ratio, the uncertainty in the X direction is reduced to 160 μm . Due to the value of

259 the uncertainties and the influence of those parameters, they will need a special cares during the PCR
260 calibration process.

261 4.3 Actuator's direction of displacement

262 The last parameters that need to be measured are assembling defaults between the actuators and the rod
263 mounts (Fig. 5). The angle of the stage axis in the work frame, γ_i , is measured using 1 mm displacement
264 of the stage with a step size of 125 μm . Each pose of the pattern is recorded during this displacement. The
265 angle of the regression line through these points gives a precise measurement of γ_i . Then, the difference
266 between this angle γ_i and the pattern pose provides the misalignment angle α_i . The uncertainty for both
267 angles is estimated to 5 mrad. Considering their small uncertainties, the parameters β_i, γ_i will be neglected
268 and only the parameters α_i will be considered.

269 Thanks to this measurement process, nominal values of the different parameters and their uncertainties
270 are known. This knowledge is useful to prepare the calibration of the PCR.

5 ROBOT CALIBRATION

271 Even with an accurate measurement process, the addition of small uncertainties reduces the PCR's accuracy.
272 To reduce those errors and obtain a better fitting between the experimental behavior and the model
273 a calibration process has been implemented. In this article, three identification steps are used in this
274 calibration process described in Fig. 6. The principle of one identification is illustrated in Fig. 7. The
275 new values of the parameters are usually obtained thanks to a minimization of an objective function. This
276 section details the different considered parameters, the definition of this objective function, the identification
277 strategy, the experimental results and the obtained position and orientation errors.

278 5.1 Extrinsic and intrinsic parameters

279 There are two kinds of parameters considered for the identification step of the calibration process: the
280 intrinsic parameters π_r (defined in Table 2) which correspond to the model's parameters, and the external
281 parameters which depend on the measurement system (whose uncertainties are at the nanometer level)
282 used to get the pose of the PCR.

283 The experimental poses are acquired thanks to the second vision setup as shown in Fig. 4. They correspond
284 to the 2D poses of a frame attached to the pseudo-periodic pattern with regards to the camera frame ${}^{Se}\mathbf{T}_M$.
285 These measured poses are used to get the experimental Cartesian pose of the platform in the work frame
286 ${}^W\mathbf{T}_P$ as follows:

$${}^W\mathbf{T}_P = {}^W\mathbf{T}_{Se} \cdot {}^{Se}\mathbf{T}_M \cdot {}^M\mathbf{T}_P \quad (5)$$

287 The transformations ${}^W\mathbf{T}_{Se}$ and ${}^M\mathbf{T}_P$ respectively correspond to the transformation between the work
288 frame and the camera frame, and to the transformation between the pseudo-periodic pattern frame and the
289 platform frame. ${}^W\mathbf{T}_{Se}$ and ${}^M\mathbf{T}_P$ are constant but cannot be measured directly and must be identified. The
290 six corresponding extrinsic parameters are gathered in the vector π_{ext} . The experimental pose ${}^W\mathbf{T}_P$ defined
291 by $\mathbf{X} = [x, y, \theta]^T$ has to be compared with those resulting from the simulation defined by $\hat{\mathbf{X}} = [\hat{x}, \hat{y}, \hat{\theta}]^T$.

292 5.2 Objective Function

293 The parameters identification is based on a minimization of an objective function using a least squares
294 algorithm. This algorithm is defined by the sum of the squares of the differences between the experimental
295 Cartesian poses of the robot and the simulated ones using the forward kinematic model. Both experimental
296 and simulation poses depend on a set of n joint coordinates $\mathbf{Q} = [\mathbf{q}^1 \dots \mathbf{q}^n]$ where $\mathbf{q}^j = [q_1^j, q_2^j, q_3^j]^T$ is
297 the j -th joint vector in the set. Considering the measured and simulated poses which depend on \mathbf{q}^j and

298 $\boldsymbol{\pi} = [\boldsymbol{\pi}_r, \boldsymbol{\pi}_{ext}]^T$, the vector of residuals is defined by $\mathfrak{F} = [\mathfrak{F}^1 \dots \mathfrak{F}^n]^T$ where \mathfrak{F}^j is:

$$\mathfrak{F}^j(\boldsymbol{\pi}, \mathbf{q}^j) = \begin{bmatrix} \alpha * (\hat{\theta}^j - \theta^j) & (\hat{x}^j - x^j) & (\hat{y}^j - y^j) \end{bmatrix} \quad (6)$$

299 where α is a weighting factor. This factor has been chosen such as the residuals in orientation and position
300 were proportional to the repeatability of the robot.

301 Using this function, the identification problem can be formalized as:

$$\begin{aligned} & \underset{\boldsymbol{\pi}}{\text{minimize}} \quad \mathfrak{F}^T \mathfrak{F} \\ & \text{subject to} \quad \boldsymbol{\pi} \in [l_b, u_b] \end{aligned} \quad (7)$$

302 where, l_b and u_b are the lower and upper bound values of the parameters, provided by the sensitivity
303 analysis.

304 The minimisation of this optimization problem is performed using the built-in-function `lsqnonlin`
305 from Matlab software.

306 5.3 Calibration Process

307 To increase the PCR's accuracy, the parameters can be identified after their measurements. This initial
308 measurement of the parameters enables the reduction of the parameters' uncertainty intervals. Small
309 intervals help to avoid some local optima and ensure consistent parameters during the minimisation.
310 Moreover, the sensitivity analysis helps to distinguish the most influential parameters that should be
311 identified in priority. Indeed, the parameters could not be identified at the same time if their values are too
312 far from their true values. For this reason, the identification is carried out in three steps using the same
313 experimental data.

314 The parameters that need to be identified first are the extrinsic parameters $\boldsymbol{\pi}_{ext}$. Indeed, those parameters
315 cannot be measured and have only been coarsely estimated. Then, after setting the extrinsic parameters, the
316 most influential robot's parameters can be identified in a second step. From the analysis, those parameters
317 are the length of the rods, the orientation of the translation direction of the actuator and the position of their
318 ends. The two first steps aim at reducing substantially the errors but the values of the parameters may not
319 be optimal yet, because some parameters could compensate for the uncertainties of the others. In the last
320 step, all parameters are re-identified together, after the identification of the most influential parameters.
321 This final optimization begins with an initial set of parameters that assumed to be relatively closed to their
322 true values.

323 5.4 Calibration Results

324 As previously explained, the joint coordinates have been chosen inside the robot's workspace in which it
325 has a nanometer repeatability. These joint coordinates are sent to the robot and the Cartesian coordinates of
326 the platform are recorded. To ensure a good distribution in the available workspace, the joint coordinates
327 have been randomly chosen using a 3-dimensional Poisson-disc sampling. A set of 99 joints coordinates,
328 more than three times the number of considered parameters, is created to perform their identification.
329 56 joints coordinates are selected to validate the identified parameters. Fig. 8 presents the result of the
330 calibration process. With the warm-up cycle of the camera and the actuators, the experimental measurement
331 of the calibration process lasts around 6 hours. It shows the errors between the platform positions and
332 orientations simulated with the nominal model (model before calibration) and the model after calibration
333 using the experimental data.

334 The position error of the pose j is defined as the root mean square of the difference between the simulated
 335 position (\hat{x}^j, \hat{y}^j) and the experimental one (x^j, y^j) such as:

$$e_p^j = \sqrt{(\hat{x}^j - x^j)^2 + (\hat{y}^j - y^j)^2} \quad (8)$$

336 Similarly, the orientation error is:

$$e_\theta^j = \sqrt{(\hat{\theta}^j - \theta^j)^2} = |\hat{\theta}^j - \theta^j| \quad (9)$$

337 Table 3 reports the results of model errors before and after calibration. After calibration, position error
 338 is included between 0.66 μm and 15.26 μm while the absolute orientation error is included between
 339 4×10^{-3} mrad and 5.61 mrad. In order to better study the influence of the error made on the values of the
 340 intrinsic parameters π_r , the extrinsic parameters π_{ext} have been identified even for the nominal model
 341 (first step). It shows that the calibration process allows to reduce the position error by a factor of 10 and the
 342 orientation error by a factor of 13.

343 5.5 Forward Kinematic Model Validation

344 To check the validity of the identified forward kinematic model, simulations are performed with a
 345 validation set of joints coordinates. Fig. 9 shows the obtained results. Position and absolute orientation
 346 errors are respectively included between 1.5 μm to 13.88 μm and between 0.1 mrad to 5.57 mrad. Those
 347 results, reported in Table 3, have the same orders of magnitude as those for the calibration set. The small
 348 reduction of the position errors is explained by a more compact distribution of poses close to the center of
 349 the workspace. The sources of these errors will be discussed in the last section.

350 For both sets, the maximal resulting position modeling error is 15.26 μm corresponding to 0.05% of the
 351 30 mm length of the flexible rods.

6 ROBOT ACCURACY

352 This section presents the results of the proposed methodology on the PCR's accuracy. To quantify this
 353 accuracy, the robot is controlled in the Cartesian space. This control depends on the inverse kinematic
 354 model whose parameters resulted from the calibration process.

355 6.1 Robot Control

356 The forward kinematic model enables the prediction of the pose of the robot considering the joint
 357 coordinates as inputs. Conversely, the inverse kinematic model allows to calculate the joint coordinates
 358 corresponding to a given pose in the Cartesian space. A regular scheme for controlling the position of the
 359 robot in the Cartesian space using the inverse kinematic model is implemented as illustrated in Fig. 10. The
 360 parameters of the inverse kinematic model are those that have been identified for the forward kinematic
 361 model.

362 To validate the capability of the robot to reach commanded poses in the Cartesian space, two trajectories
 363 have been considered: a 2 mm side square and a 3 mm diameter circle with a constant null orientation of
 364 the mobile platform. Those trajectories are used as an input for the inverse kinematic model which returns
 365 the corresponding joints coordinates \mathbf{Q}_c . Those joints coordinates are sent to the robot and its poses are
 366 recorded by the second vision setup (the one composed by a microscope).

367 Fig. 11(A) and Fig. 12(A) show the desired trajectories (red points) and the experimental trajectories
 368 (blue points). Both positions and orientations obtained experimentally are closed to the desired ones. The

369 differences between them are too small to be seen at the same time than the performed trajectory. Those
 370 results show that the robot is capable of following a defined trajectory.

371 6.2 Evaluation of the robot accuracy

372 The robot accuracy evaluates the closeness of agreement between the pose attained by the robot and its
 373 commanded pose. The accuracy of a robot is defined by the standard ISO 9283:1998. The positioning
 374 accuracy is the difference between the commanded position and the barycenter of the reached positions:

$$AP_P = \sqrt{(\bar{x} - x_c)^2 + (\bar{y} - y_c)^2} \quad (10)$$

375 The orientation accuracy is the difference between the commanded angle and the average of the reached
 376 orientations:

$$AP_\theta = (\bar{\theta} - \theta_c) \quad (11)$$

377 The bar operator ($\bar{\cdot}$) is the barycenter (or the average) of the reached positions (or orientations) after
 378 repeating the same pose at least 30 times.

379 The accuracy of the robot has been evaluated for the two trajectories as shown in Fig. 11 (B-C) and
 380 Fig. 12 (B-C). For both cases, the means of positioning accuracy are below 2 μm (1.27 μm and 1.95 μm)
 381 and the worst positioning accuracies are respectively 2.60 μm and 3.28 μm for the square and circular
 382 trajectories. In orientation, the worst orientation accuracy is inferior to 0.532 mrad. Table 4 reports all
 383 those results. The resulting mean positioning accuracy is $10.9 \times 10^{-3}\%$ of the nominal length of a flexible
 384 continuum rod (around 30 mm). Considering the performances of the other PCR gathered in Table 1, the
 385 presented positioning accuracy is 10 times better than the current relative positioning accuracy.

386 6.3 Discussion

387 For both trajectories (square and circle), the pose accuracies depend on the pose of the platform. For
 388 instance, in the square trajectory, the orientation accuracy is worst in the corners. A deeper analysis on the
 389 experimental data of the calibration process shows that the angular errors are linked, with a correlation
 390 ratio of nearly one, to the differences between the initial angle and the current angle. Larger the orientation
 391 of the platform, higher is the angular error. Correlations between the position and the angular error are
 392 less straightforward to establish with typical ratios between 0.52 and 0.74. With local studies on different
 393 areas of the workspace, there are some locations near the workspace borders where the position errors are
 394 increased. In conclusion, the model is more accurate in the middle of its workspace in both orientation and
 395 position.

7 CONCLUSION

396 In this article, a methodology to reach micrometer positioning accuracy is proposed.

397 This micrometer positioning accuracy was reached thanks to the proposed methodology which consists
 398 in five key elements. The first one is the use of a repeatable PCR structure to be able to predict the robot's
 399 quasi-static behavior. The second element is to use a correct mechanical model. The third one is the use of
 400 fiducial markers in the design of the robot. Those markers allow to efficiently measure the nominal values
 401 of the model's parameters and to estimate their uncertainties. The fourth element is to conduct a sensitivity
 402 analysis to quantify the influence of the different parameters and find the most influential ones. This crucial
 403 information is considered in the robot design to be able to efficiently measure those parameters thanks to
 404 fiducial markers. The last step is to calibrate the whole robot by identifying all the parameters through three
 405 optimization steps. Using this approach on a $XY\Theta$ planar Parallel Continuum Robot (PCR), the maximal

406 reached positioning accuracy is 3.3 μm in position and 0.5 mrad in orientation over a 10 mm-long circular
407 trajectory.

408 In addition to the already recognized advantages of PCR (miniaturisation capabilities, lightweight, etc.),
409 the obtained results make also PCR worth to consider for high precision positioning tasks.

CONFLICT OF INTEREST STATEMENT

410 The authors declare that the research was conducted in the absence of any commercial or financial
411 relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS

412 All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it
413 for publication.

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REFERENCES

- 417 Altuzarra, O., Caballero, D., Campa, F. J., and Pinto, C. (2019). Position analysis in planar parallel
418 continuum mechanisms. *Mechanism and Machine Theory* 132, 13–29
- 419 Altuzarra, O. and Merlet, J.-P. (2019). Certified kinematics solution of 2-dof planar parallel continuum
420 mechanisms. *Advances in Mechanism and Machine Science* 73. doi:[https://doi.org/10.1007/](https://doi.org/10.1007/978-3-030-20131-9_20)
421 978-3-030-20131-9_20
- 422 Andre, A. N., Sandoz, P., Mauze, B., Jacquot, M., and Laurent, G. J. (2020). Sensing one nanometer
423 over ten centimeters: A micro-encoded target for visual in-plane position measurement. *IEEE/ASME*
424 *Transactions on Mechatronics* , 1193–1201doi:10.1109/TMECH.2020.2965211
- 425 Black, C. B., Till, J., and Rucker, D. C. (2018). Parallel Continuum Robots: Modeling, Analysis, and
426 Actuation-Based Force Sensing. *IEEE Transactions on Robotics* 34, 29–47. doi:10.1109/TRO.2017.
427 2753829
- 428 Bryson, C. E. and Rucker, D. C. (2014). Toward parallel continuum manipulators. In *IEEE International*
429 *Conference on Robotics and Automation* (Hong Kong, China), 778–785. doi:10.1109/ICRA.2014.
430 6906943
- 431 Campa, F. J., Diez, M., Diaz-Caneja, D., and Altuzarra, O. (2019). A 2 Dof Continuum Parallel Robot for
432 Pick & Place Collaborative Tasks. In *Advances in Mechanism and Machine Science*, ed. T. Uhl (Cham:
433 Springer International Publishing), vol. 73. 1979–1988. doi:10.1007/978-3-030-20131-9_196
- 434 Dehghani, M. and Moosavian, S. A. A. (2013). Characteristics identification of continuum robots for exact
435 modeling. In *IEEE RSI/ISM International Conference on Robotics and Mechatronics* (Tehran), 26–31.
436 doi:10.1109/ICRoM.2013.6510076
- 437 ISO 9283 (1998). *Manipulating Industrial Robots – Performance criteria and related test methods*.
438 Standard ISO 9283:1998, International Organization for Standardization
- 439 Lilge, S., Nuelle, K., Boettcher, G., Spindeldreier, S., and Burgner-Kahrs, J. (2021). Continuous Structures
440 in Planar Parallel Robots: A Theoretical Analysis. *Journal of Mechanisms and Robotics* , 32

- 441 Mauzé, B., Dahmouche, R., Laurent, G. J., André, A. N., Rougeot, P., Sandoz, P., et al. (2020). Nanometer
442 Precision with a Planar Parallel Continuum Robot. *IEEE Robotics and Automation Letters* 5, 3806–3813.
443 doi:10.1109/LRA.2020.2982360
- 444 Nuelle, K., Sterneck, T., Lilge, S., Xiong, D., Burgner-Kahrs, J., and Ortmaier, T. (2020). Modeling,
445 calibration, and evaluation of a tendon-actuated planar parallel continuum robot. *IEEE Robotics and*
446 *Automation Letters* 5, 5811–5818. doi:10.1109/LRA.2020.3010213
- 447 Orekhov, A. L., Aloï, V. A., and Rucker, D. C. (2017). Modeling parallel continuum robots with general
448 intermediate constraints. In *IEEE International Conference on Robotics and Automation* (Singapore),
449 6142–6149. doi:10.1109/ICRA.2017.7989728
- 450 Orekhov, A. L., Black, C. B., Till, J., Chung, S., and Rucker, D. C. (2016). Analysis and Validation of
451 a Teleoperated Surgical Parallel Continuum Manipulator. *IEEE Robotics and Automation Letters* 1,
452 828–835. doi:10.1109/LRA.2016.2525720
- 453 Rao, P., Peyron, Q., Lilge, S., and Burgner-Kahrs, J. (2021). How to model tendon-driven continuum
454 robots and benchmark modelling performance. *Front. Robot. AI* , 22doi:10.3389/frobt.2020.630245
- 455 Rone, W. S. and Ben-Tzvi, P. (2014). Mechanics Modeling of Multisegment Rod-Driven Continuum
456 Robots. *Journal of Mechanisms and Robotics* 6, 041006. doi:10.1115/1.4027235
- 457 Takano, R., Mochiyama, H., and Takesue, N. (2017). Real-time shape estimation of Kirchhoff elastic rod
458 based on force/torque sensor. In *IEEE International Conference on Robotics and Automation* (Singapore,
459 Singapore), 2508–2515. doi:10.1109/ICRA.2017.7989292
- 460 Till, J. and Rucker, D. C. (2017). Elastic Stability of Cosserat Rods and Parallel Continuum Robots. *IEEE*
461 *Transactions on Robotics* 33, 718–733. doi:10.1109/TRO.2017.2664879
- 462 Trivedi, D., Lotfi, A., and Rahn, C. (2008). Geometrically Exact Models for Soft Robotic Manipulators.
463 *IEEE Transactions on Robotics* 24, 773–780. doi:10.1109/TRO.2008.924923
- 464 Wu, G. and Shi, G. (2019). Experimental statics calibration of a multi-constraint parallel continuum robot.
465 *Mechanism and Machine Theory* 136, 72–85. doi:10.1016/j.mechmachtheory.2019.02.013
- 466 Wu, G., Shi, G., and Shi, Y. (2017). Modeling and analysis of a parallel continuum robot using artificial
467 neural network. In *IEEE International Conference on Mechatronics* (Churchill, VIC), 153–158. doi:10.
468 1109/ICMECH.2017.7921096
- 469 Yang, Z., Zhu, X., and Xu, K. (2018). Continuum Delta Robot: A Novel Translational Parallel Robot
470 with Continuum Joints. In *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*
471 (Auckland), 748–755. doi:10.1109/AIM.2018.8452695

FIGURE CAPTIONS

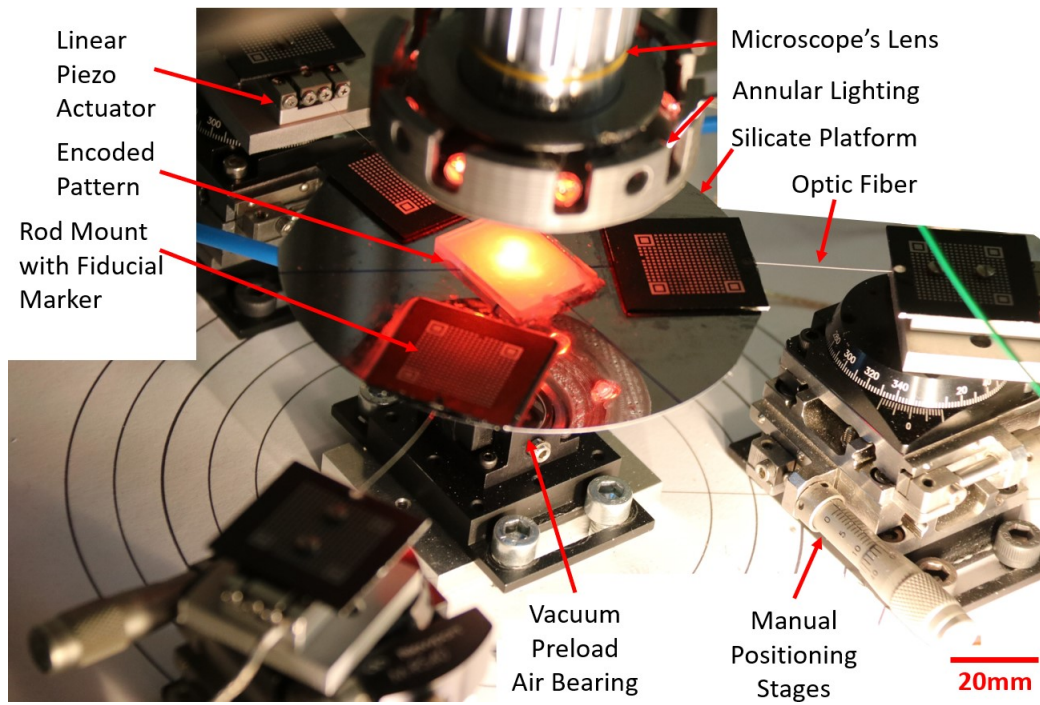


Figure 1. Picture of the $XY\theta$ parallel continuum robot

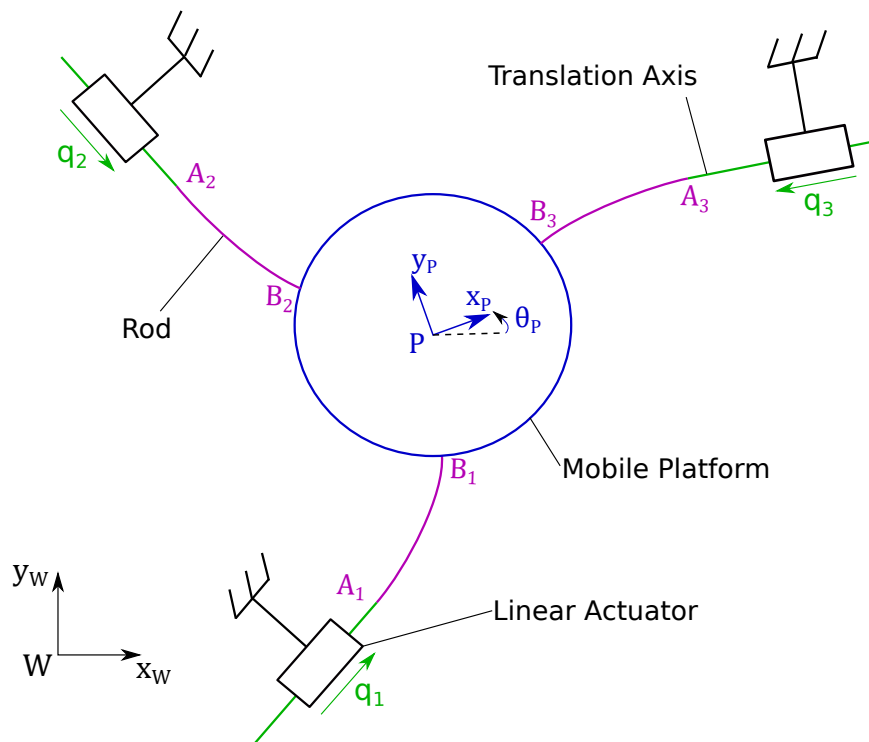


Figure 2. Kinematic diagram of the 3-PR planar Parallel Continuum Robot

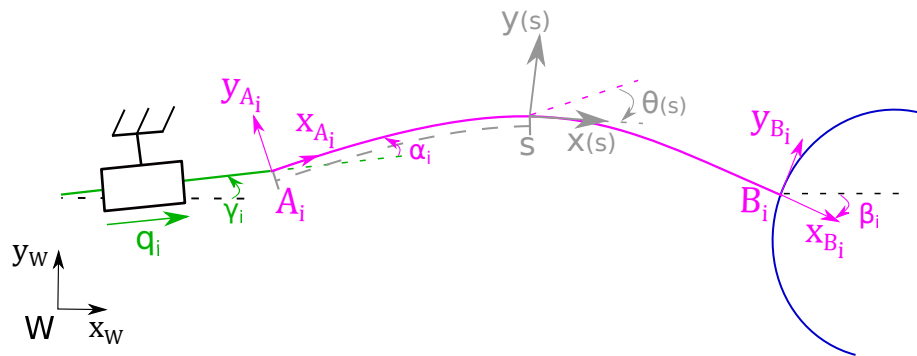


Figure 3. Detailed scheme of the i -th limb with frames and parameters.

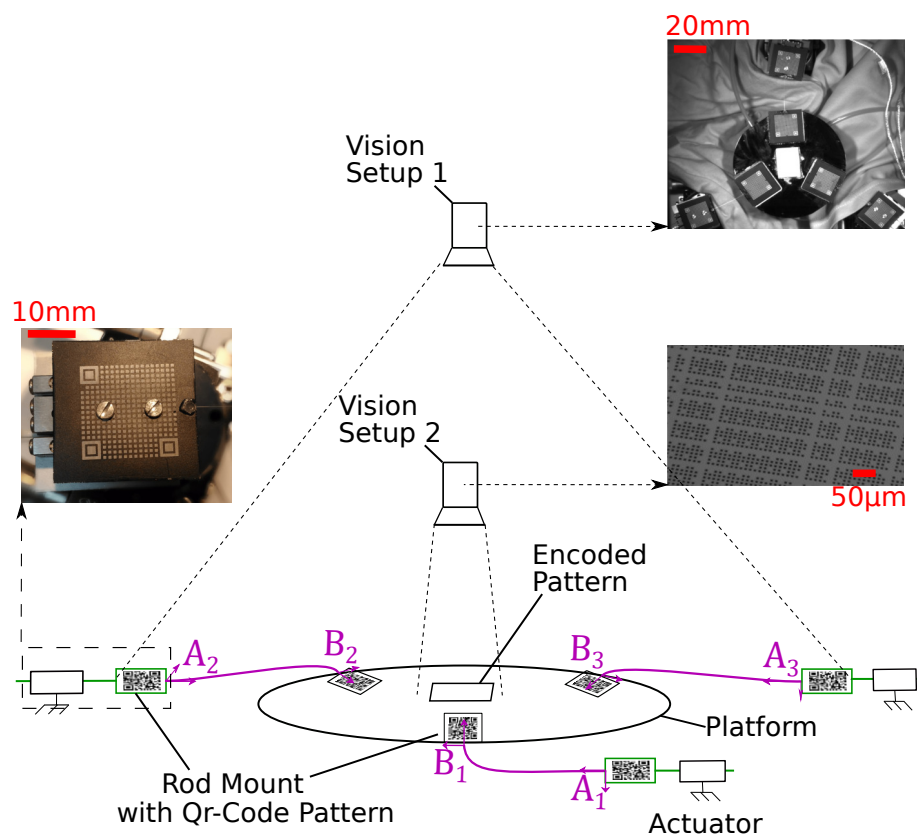


Figure 4. Scheme of the experimental setup with the several measurement vision systems.

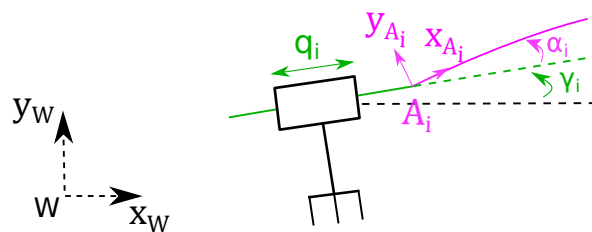


Figure 5. Detailed scheme of the misalignment between the rod mount and the translation axis of the piezo stage.

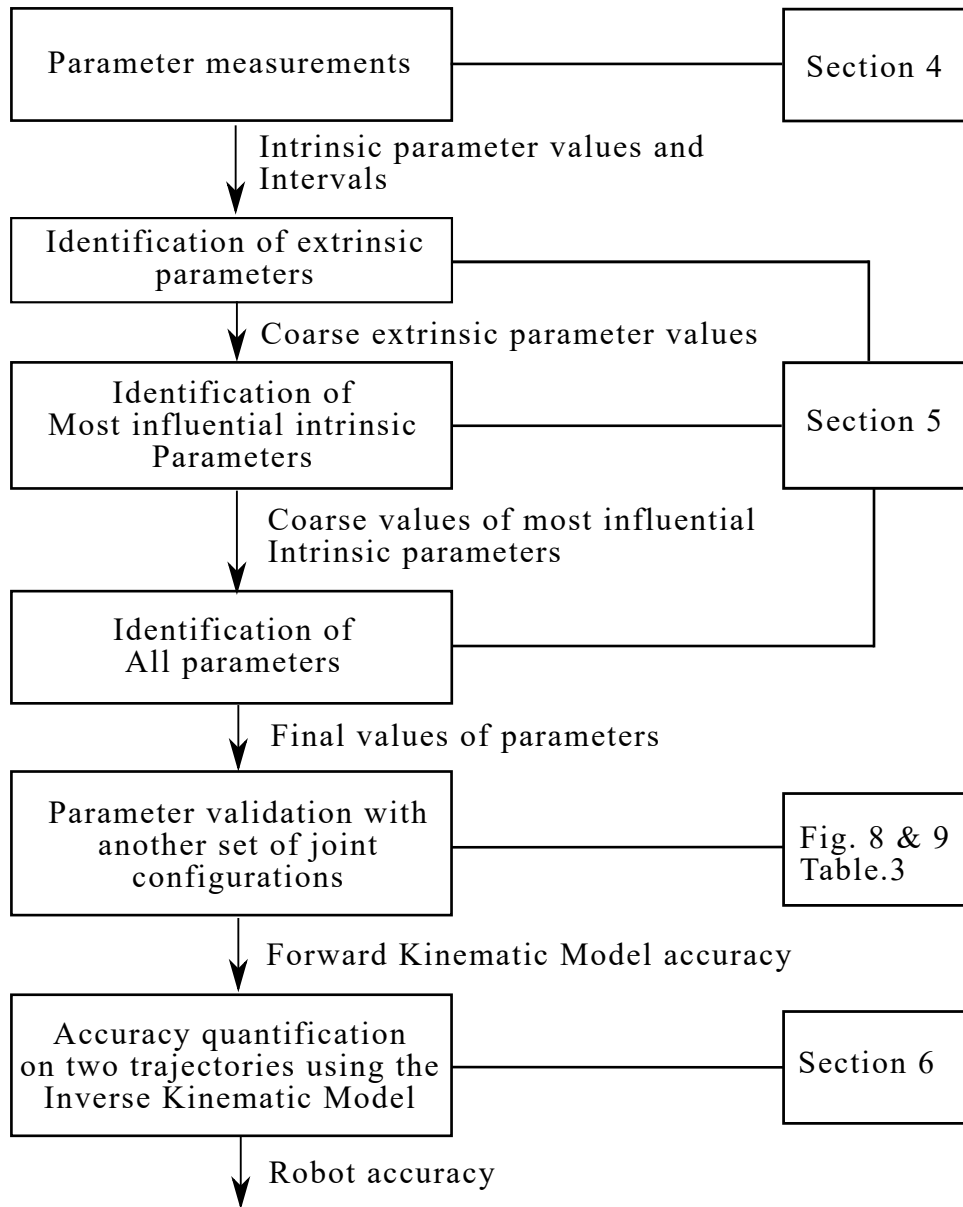


Figure 6. Flowchart of the parameter identification strategy.

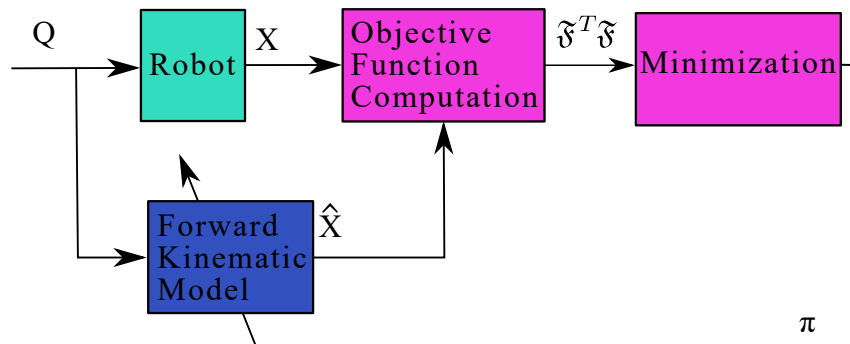


Figure 7. Block diagram of the identification principle using the Forward Kinematic Model. Considering a set of joints configuration Q , the parameters π of the model are changed such as the residual of the objective function $\tilde{\mathfrak{F}}$ is minimized.

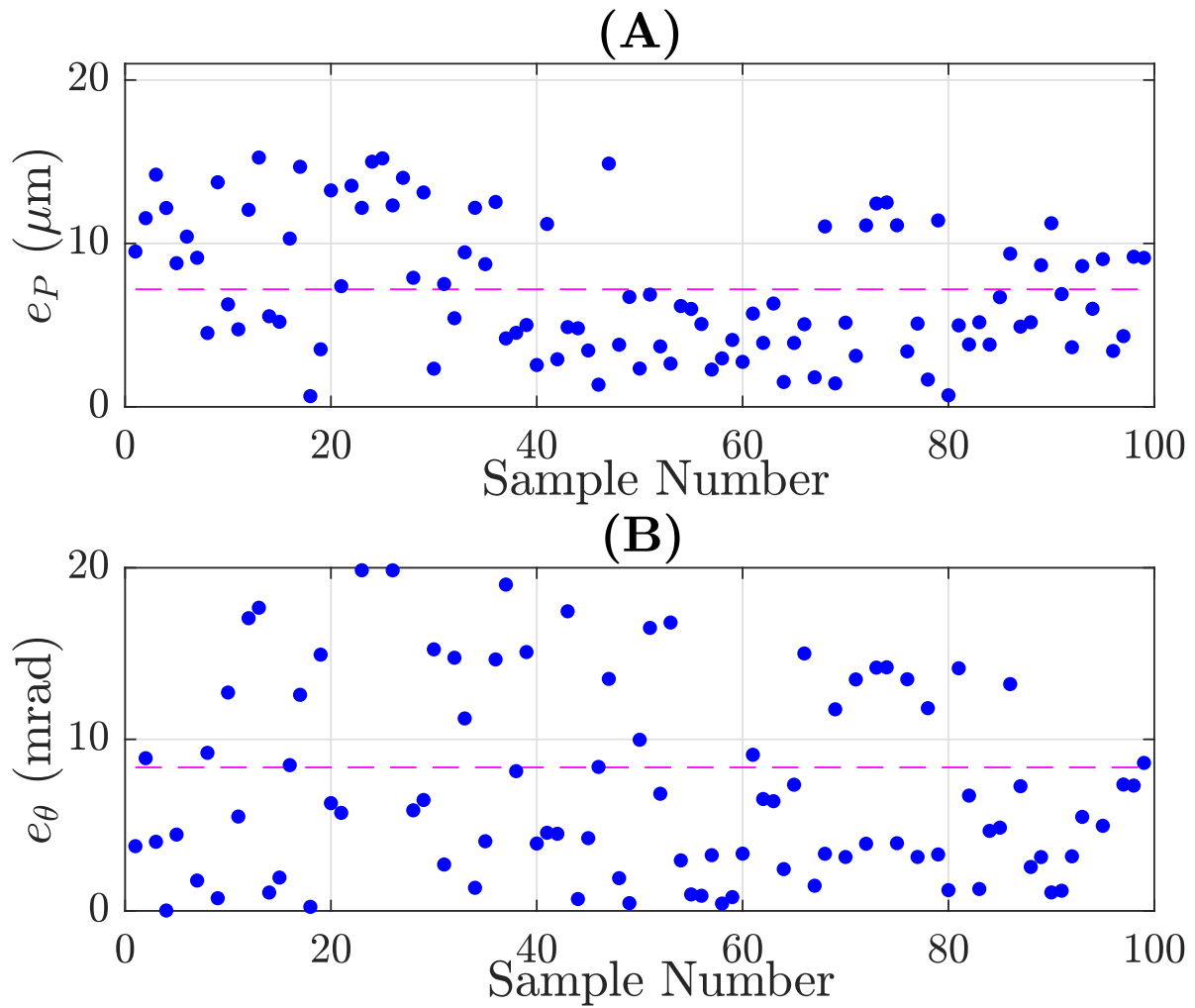


Figure 8. Positioning (A) and orientation (B) errors of the model (respectively e_P and e_θ) for the different samples of the calibration set of joints coordinates. The dashed-lines are the mean values of positioning and orientation errors and respectively correspond to 7.19 μm and 1.7 mrad.

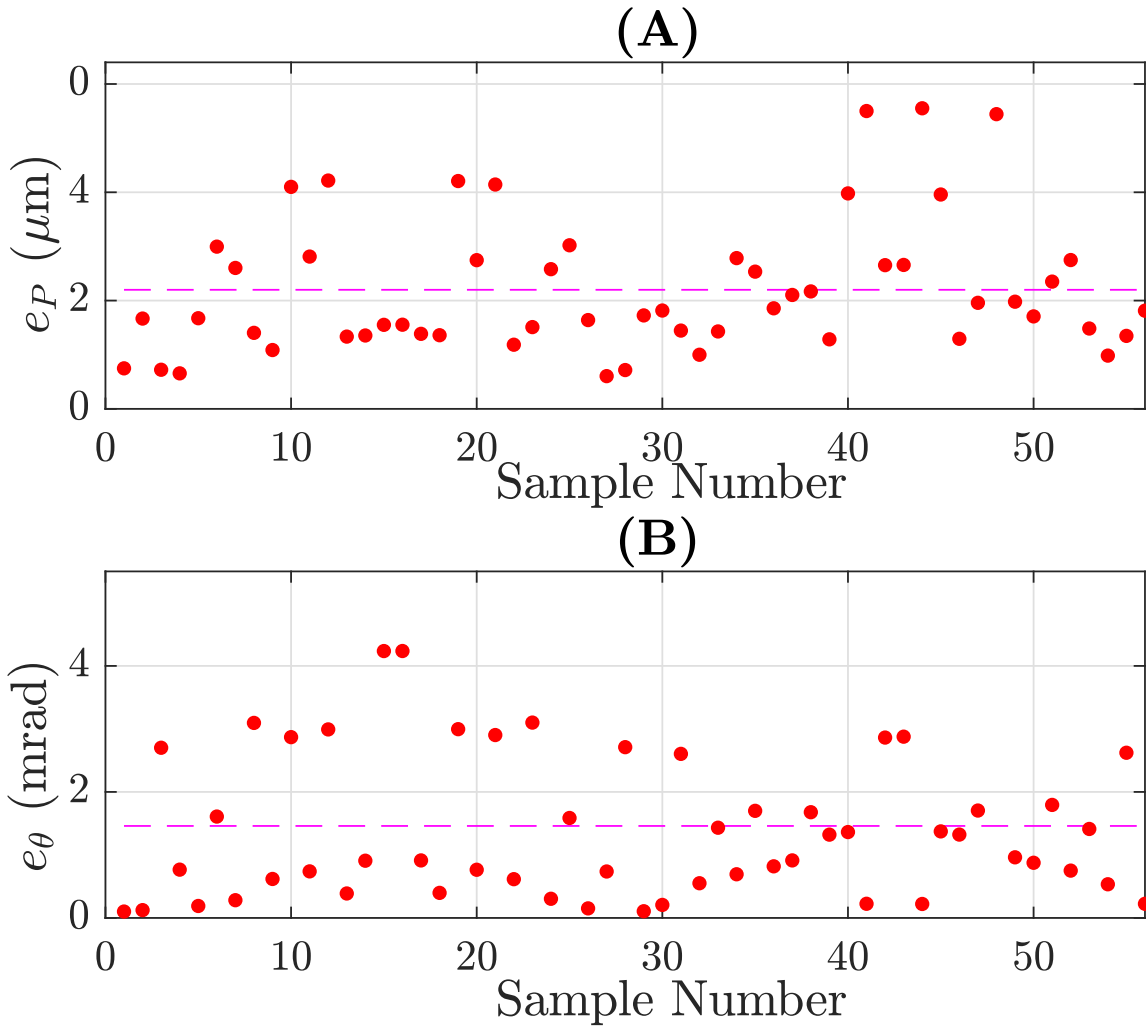


Figure 9. Positioning (A) and orientation (B) errors of the model (respectively e_P and e_θ) for the different samples the validation set of joints coordinates. The dashed-lines are the mean values of positioning and orientation accuracy and respectively correspond to 5.5 μm and 1.5 mrad .

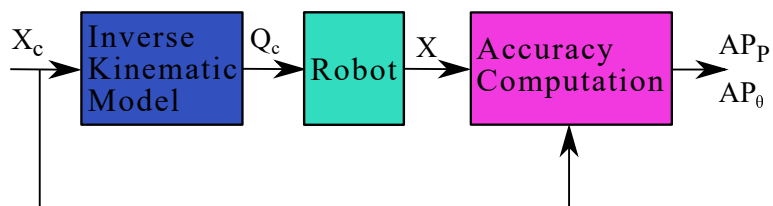


Figure 10. Block diagram of the position control of the robot using the Inverse Kinematic Model.

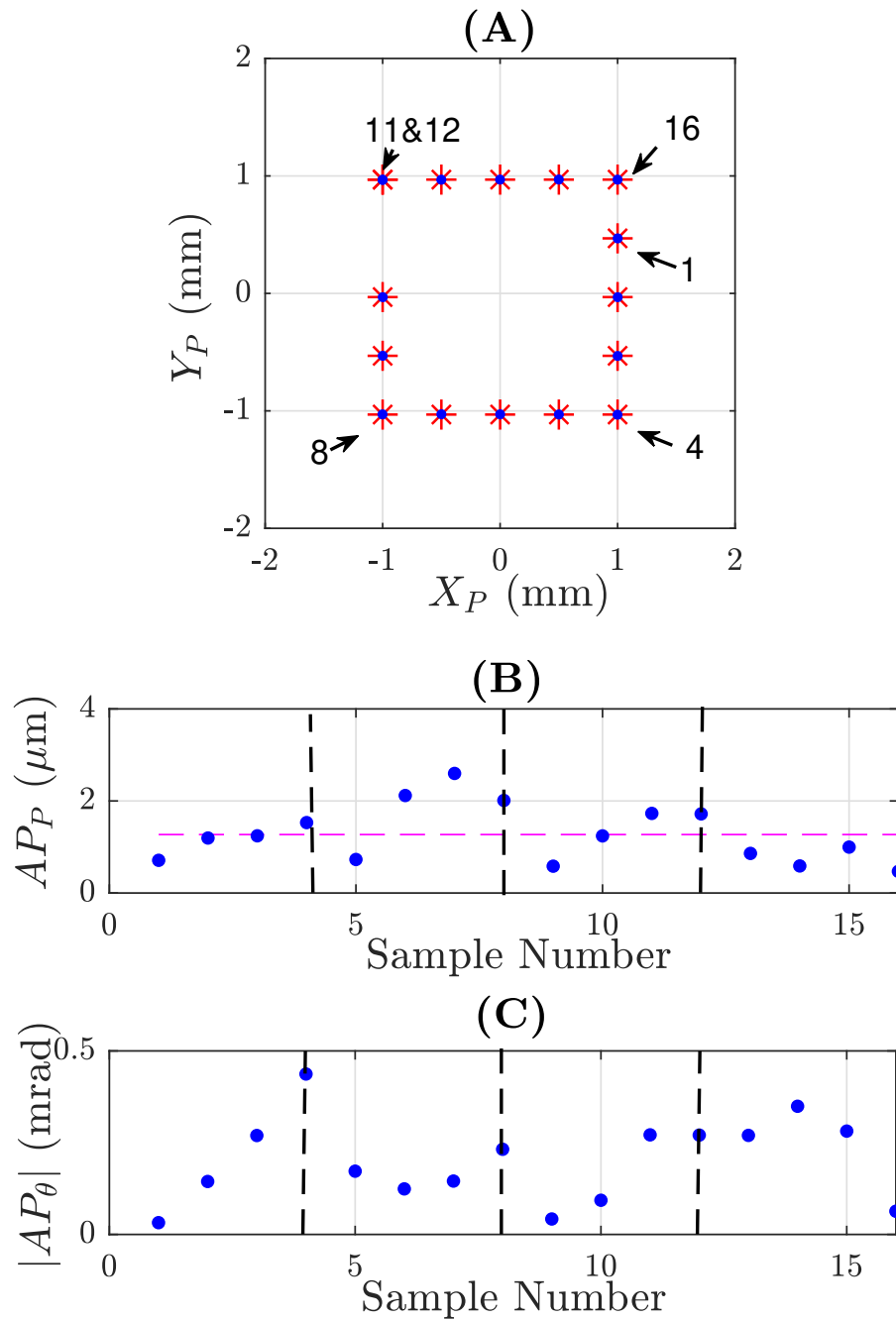


Figure 11. Result of the robot position control for a square trajectory: **(A)** commanded poses (in red) and attained by the robot (in blue), **(B)** position and **(C)** orientation accuracies obtained for the different poses. The horizontal dashed-line is the mean value of the positioning accuracy which is 1.27 μm . The black dashed-lines are characteristic points (numbered points) on the trajectories (corners of the square) which correspond to points where the AP_P or AP_θ monotony change.

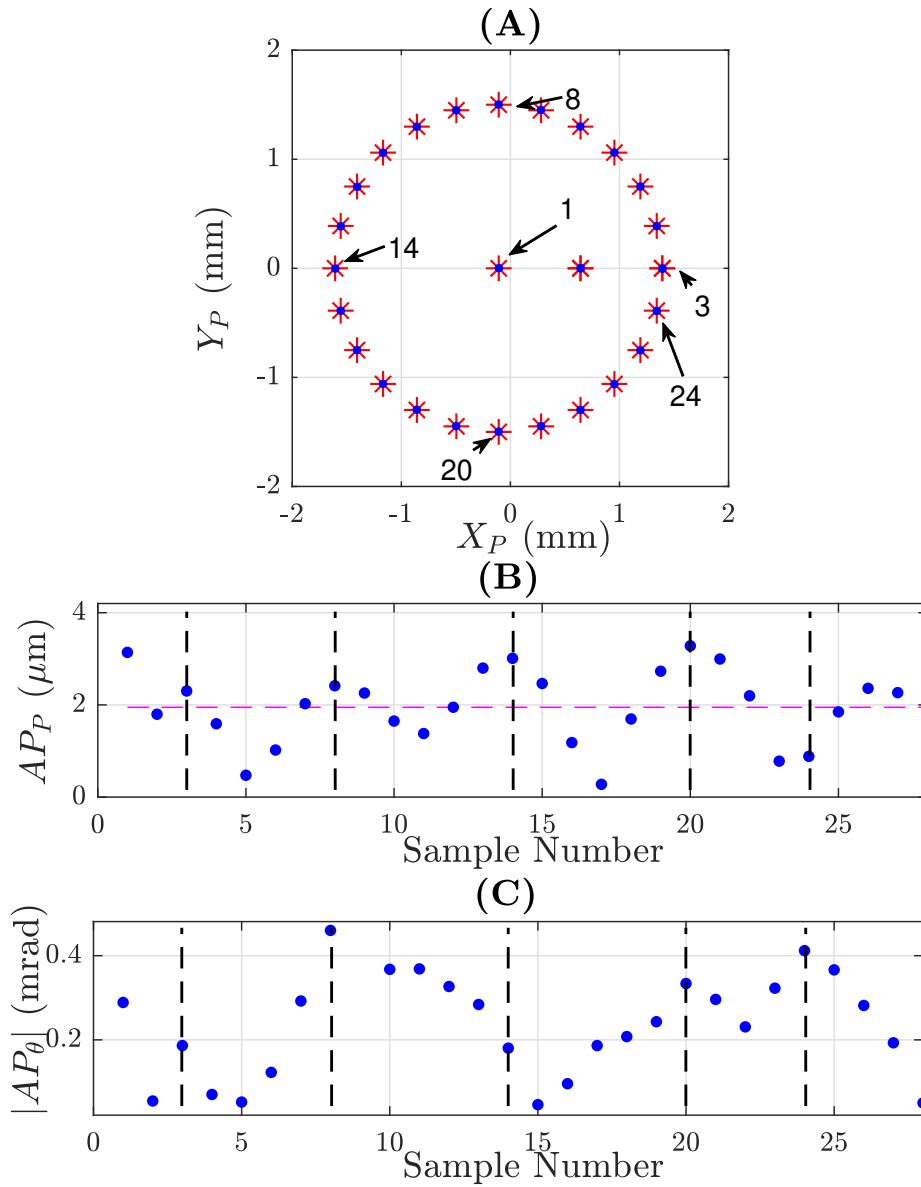


Figure 12. Result of the robot position control for a circular trajectory: (A) commanded poses (in red) and attained with the prototype (in blue), (B) position and (C) orientation accuracies obtained for the different poses. The horizontal dashed-line is the mean value of the positioning accuracy which is $1.95 \mu\text{m}$. The black dashed-lines are characteristic points (numbered points) on the trajectories where the AP_P or AP_θ monotony change.

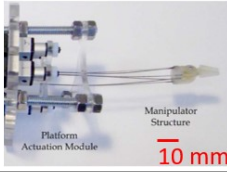
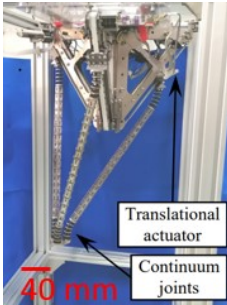


Design	Image	DoF	Structure	Models	Calibration	AP_P (%)	AP_θ (°)
Orekhov et al. (2016)		6	6PF	Cosserat	no	2.8	3.81
Yang et al. (2018)		3	3PFF	Cosserat	no	0.1	-
Wu et al. (2017)		3	Multi	Neural Network	no	0.43	-
Wu and Shi (2019)			Constraint	Cosserat	yes	0.8	-
Nuelle et al. (2020)		3	3PFR	Constant Curvature	yes	1.4	1.1

Table 1. Mean orientation AP_θ and positioning accuracy AP_P (in percent of the nominal or mean dimension dimension of the continuum flexible segment) for different PCR designs.

TABLES CAPTIONS

Table 2. Influences of the different models’ parameters (expressed in SI base unit) computed as the maximal finite difference for the different configurations and rods.

Parameters	$max_i(\frac{\partial x}{\partial param})$	$max_i(\frac{\partial y}{\partial param})$	$max_i(\frac{\partial \theta}{\partial param})$
E	0	0	0
d	0	0	0
d_i	0.2	0.2	0.05×10^3
α_i	1.3×10^{-3}	1.3×10^{-3}	0.4
β_i	0.3×10^{-3}	0.4×10^{-3}	0.2
γ_i	0.3×10^{-3}	0.3×10^{-3}	0.1
l_i	1.5	1.3	0.4×10^3
A_{x_i}	1.3	1.2	0.4×10^3
A_{y_i}	1.0	0.8	0.4×10^3
B_{x_i}	1.3	1.2	0.4×10^3
B_{y_i}	1.0	0.8	0.4×10^3

	e_P (μm)			e_θ (mrad)		
	max	mean	min	max	mean	min
Nominal model with calibration set	157.8	70.6	10.1	12.65	4.5	0.058
Identified model with calibration set	15.26	7.19	0.66	5.61	1.7	0.004
Identified model with validation set	13.88	5.5	1.5	5.57	1.5	0.10

Table 3. Positioning and orientation errors (respectively e_P and e_θ) with the nominal or the identified parameters and for the calibration or the validation configuration set.

	AP_P (μm)			$ AP_\theta $ (mrad)		
	max	mean	min	max	mean	min
Square	2.60	1.27	0.47	0.437	0.20	0.032
Circle	3.28	1.95	0.28	0.532	0.24	0.046

Table 4. Positioning and orientation accuracies (respectively AP_P and AP_θ) of the robot for two different trajectories