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To cite this version:

Margot Morlans, Jean Guerard, Jerome Juillard. Characterization of electrical and mechanical coupling in MEMS gyroscopes with electrical measurements. 2021 Symposium on Design, Test, Integration & Packaging of MEMS and MOEMS (DTIP), Aug 2021, Paris, France. 10.1109/DTIP54218.2021.9568500 . hal-03454425

HAL Id: hal-03454425

https://hal.science/hal-03454425v1

Submitted on 29 Nov 2021

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Characterization of electrical and mechanical coupling in MEMS gyroscopes with electrical measurements

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Abstract — We propose an intuitive way to characterize the parasitic couplings in a weakly-coupled resonator. Here the study is focused on MEMS gyroscopes, one example of coupled resonator. Knowing the value of electrical and mechanical coupling will then allow us to electronically cancel them, and thus to reduce the bias instability of the gyroscope measures.

Keywords — MEMS, gyroscope, mechanical coupling, sensor characterization, weakly-coupled resonators, quadrature error

I. Introduction

In MEMS gyroscopes, one seeks to measure the coupling induced by the Coriolis force between two mechanical modes. Some other phenomena, such as mechanical coupling or electrical one, tend to overshadow Coriolis coupling. In order to optimize the detection of the Coriolis coupling, some solutions have been proposed [1]. Here, we show how a Nyquist diagram can be used as an intuitive graphical method to assess at first glance the amount of parasitic coupling in a given MEMS gyroscope, from a frequency sweep. Indeed, when representing the results with a classical frequency response, it is hard to differentiate the amount of electrical coupling from the amount of mechanical coupling, which is problematic, as these two phenomena do not call for the same correction. The Nyquist diagram provides a way to do this in a single look. Furthermore, this representation is amenable to straightforward circle-fitting methods, which allows for a quantitative determination of such resonator parameters as natural frequency and quality factor, and of parasitic coupling.

Note that MEMS sensors based on weakly-coupled resonators, which rely on electromechanical coupling for detection of quantities as acceleration or electrical charge [2, 3], could also benefit from this approach in practice.

This paper consists in a brief overview of the interpretation that can be made of Nyquist diagrams applied to coupled resonators with large quality factors. The use of these diagrams for characterization is illustrated with both simulated (section II) and experimental (section III) data, obtained from measurements performed on ONERA's VIG gyroscope.

II. Quantitative and Qualitative Characterization of Coupling

MEMS gyroscopes rely on the coupling -through the Coriolis force- of two mechanical modes for measuring angular velocity. Besides the resonance frequency and quality factor of each mode, the calibration of a gyroscope requires the characterization of several "coupling" parameters, responsible for measurement errors.

A. Modeling

The first step to characterization is to model mathematically the gyroscope, accounting for its non-idealities. Supposing the angular velocity is zero and neglecting viscous coupling between the two modes, one may model a gyroscope being harmonically driven at angular frequency ω as

$$\left(\omega_D^2 - \omega^2 + j\frac{\omega\omega_D}{Q_D}\right)X_D + \kappa(X_D - X_S) = F \qquad (1)$$

$$\left(\omega_S^2 - \omega^2 + j\frac{\omega\omega_S}{Q_S}\right)X_S + \kappa(X_S - X_D) = 0 \qquad (2)$$

where ω_D (resp. ω_S) is the angular frequency of the drive (resp. sense) mode, Q_D (resp. Q_S) the quality factor of the drive (resp. sense) mode, κ represents elastic coupling, X_D and X_S are the complex amplitudes of the drive and sense mode, and F is the forcing input. X_D and X_S are transformed into voltages V_D and V_S through a set of transducers and analog front-ends. For the sake of this paper, we suppose

$$V_D = Z_D(X_D + \gamma_D V_{in}) \qquad (3)$$

$$V_S = Z_S(X_S + \gamma_S V_{in}) \qquad (4)$$

where Z_D (resp. Z_S) represents the complex value of the front-end and transducer transfer function at angular frequency ω and γ_D (resp. γ_S) represents the amount of parasitic feedthrough between the actuation electrode of the drive mode at voltage V_{in} and the detection electrode of the drive (resp. sense) mode.

Mechanical non-idealities (coupling κ) and electrical non-idealities (feedthrough) are responsible for

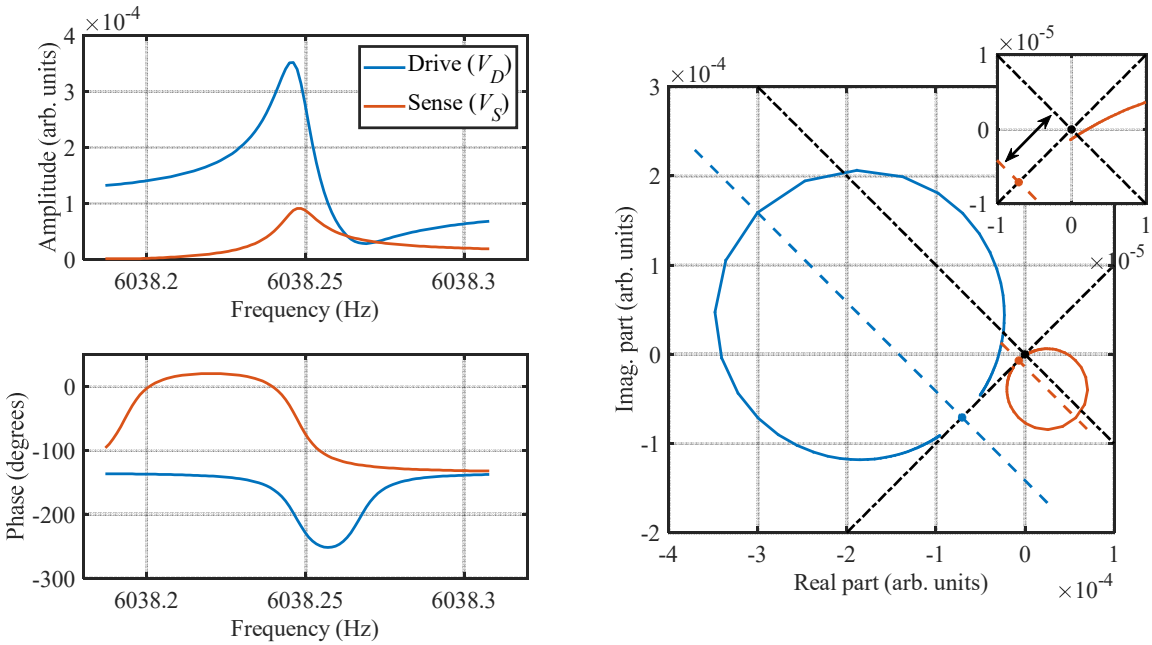

Fig. 1. Simulated response of a gyroscope with separate modes. Amplitude and phase frequency responses (left) vs. Nyquist diagram representation (right).

static (bias) errors when estimating angular velocity Ω from V_D and V_S. We will denote the mechanical error with M and the electrical error with E (both in $°/s$).

B. Parameter estimation

Estimating the transfer functions from V_{in} to V_D and from V_{in} to V_s can be performed using a variety of numerical methods, such as circle fitting methods applied to the Nyquist diagram of V_D and V_S. Furthermore, the representation of V_D and V_S in the complex plane lends itself very well to qualitative interpretation.

In our current setup, the Kasa and Coope method [4] is used to perform circle-fitting. This method relies on the solution of a linear least-squares problem which consists in determining the three optimal parameters which minimize $\sum_j d_j^2$, where

$$d_j^2 = r^2 - (x_C - x_j)^2 - (y_C - y_j)^2 \qquad (5)$$

is a measure of (the square of) the distance between a data point with coordinates (x_j, y_j) and a circle with center $(x_C, y_C) \in \mathbb{R}^2$ and radius $r \in \mathbb{R}^+$. The fact that the problem is linear guarantees a unique solution, with an analytical expression.

Furthermore, qualitative insight into the amount of electrical coupling can be gained by looking at the relative position and at the orientation of the "axes" of the circles (the circle's "axis" being defined as the straight line going through its centre and its limit far away from resonance). In particular, the angle between the circle axes and the real axis is equal to the phase-shift provided by Z_D and Z_S. The amount of feedthrough, and parameters γ_D and γ_S, can be qualitatively and quantitatively deduced from the distance between the limit of each circle and the origin of the complex plane, following a similar approach to [5]. Finally, the ratio ρ of the radii of the two circles is proportional to the mechanical coupling coefficient κ. In the specific case of a gyroscope with separated modes swept close its drive frequency (i.e. $\omega \approx \omega_D$), and assuming $|Z_D| = |Z_S|$, we have

$$\kappa \approx \rho \times (\omega_S^2 - \omega_D^2) \qquad (6)$$

A similar relation is in fact often used for resonant sensing in the field of "weakly-coupled resonators" [2], where (electro-)mechanical coupling between two resonators is in fact engineered to cause a phenomenon known as mode-localization. In this kind of application, the ratio of the oscillation amplitudes is often used as an output metric for measuring the stiffness mismatch of the two resonators.

In order to illustrate this interpretation of the Nyquist diagram, we have represented in Fig. 1 a simulated Nyquist diagram and the corresponding frequency responses for the following set of parameters:

$$\omega_D = 2\pi \times 6000\text{Hz}$$

$$\omega_S = 2\pi \times 5900\text{Hz}$$

$$Q_D = Q_S = 5 \times 10^5$$

$$\kappa = 5 \times 10^3 \times \omega_D^2 / Q_D \approx 1.42 \times 10^7 \ (\text{rad/s})^2$$

$$|Z_D| = |Z_S| = 1 \ \text{V/m}$$

$$\arg Z_D = \arg Z_S = \frac{5\pi}{4}$$

$$\gamma_D = 10^{-4} \ \text{m/V} = 10 \times \gamma_S$$

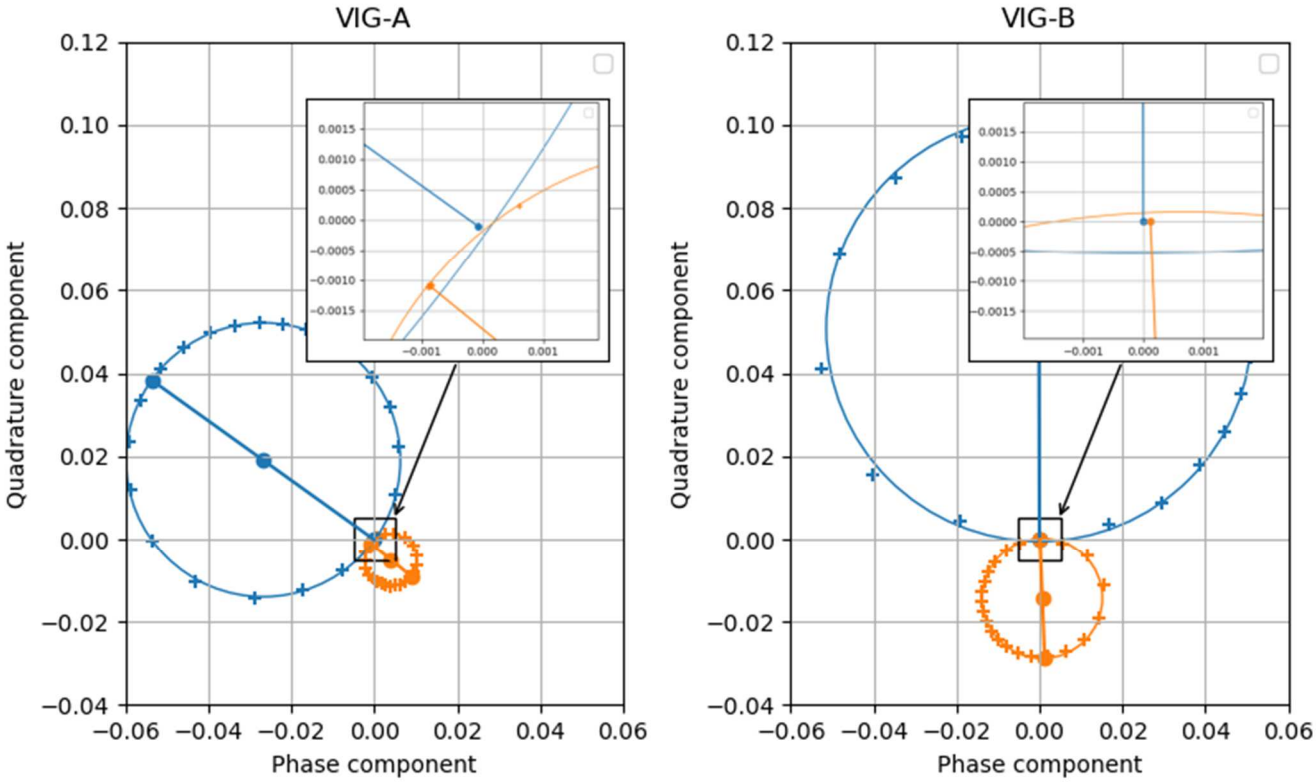

Fig. 2. Nyquist diagram for VIG-A and VIG-B.

The circles' axes are represented with dashed lines. The angles between these axes and the real axis yields $\arg Z_{S,D}$. The distance between each circle's axis and the real axis after a rotation of $\arg Z_{S,D}$ (as shown in the inset) yields the amount of electrical feedthrough. One appreciates at a glance that, in this example, there is 10 times as much electrical feedthrough on the drive mode than there is on the sense mode. The amount of mechanical coupling is deduced from the relative radius of the two circles.

III. Results

In this section, we report some experimental characterization results. The setup is based on a HF2LI lock-in amplifier: the drive mode of the gyroscope is driven with a harmonic signal, no rotation being applied to the device. The sense mode is left without any activation. The frequency of the drive signal is swept close to the resonance frequency of the drive mode. The lock-in amplifier is used to pick the signals output by the readouts of both the (separated) drive and sense modes. Both signals may either be represented in terms of their amplitude and phase frequency responses, or in the complex plane, as Nyquist diagrams.

The circle-fitting approach is demonstrated at constant temperature with two different VIG gyroscopes (denoted VIG-A and VIG-B in Table I), whose parameters are very well-known from extensive simulation and separate characterization experiments. The proposed approach gives results that are in excellent agreement with the expected values of the different system parameters. Table I reports relevant parameters estimated from our measurements. In particular, we report the input-referred mechanical and electrical errors, M and E, of these two devices, thus highlighting the need to compensate for these errors in order to achieve accurate angular velocity measurements.

TABLE I. Experimental characterization results

	VIG-A	VIG-B
ω_D (Hz)	5943.887	5888.496
Q_D (no dim.)	286056	448960
M (°/s)	252	300
E (°/s)	58.6	1.25

The Nyquist diagrams corresponding to the two devices are represented in Fig. 2, along with the circles fitted with the Kasa and Coope method. This representation shows the effectiveness of the method to determine the quality factor or the couplings. As presented here, it is also interesting to quickly compare two different gyroscopes.

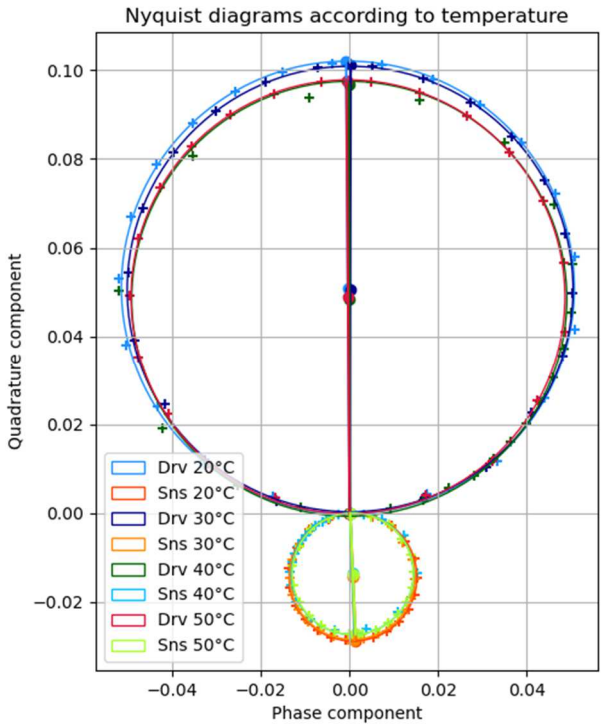

Fig. 3. Evolution of Nyquist diagrams with temperature.

In Fig. 3, we also represent the evolution of the Nyquist diagram of VIG-B according to temperature. It is of course possible to qualitatively appreciate the (well-known) variation of resonance frequency and quality factor with temperature. Perhaps more surprisingly, this curve also shows that the mechanical coupling coefficient is also quite strongly temperature-dependent, with mechanical coupling being a decreasing function of temperature.

IV. Conclusion and Future Work

The Nyquist diagram, whose use is classically limited to graphically determining the stability of feedback-loop controllers, is in fact a very powerful tool for qualitative insight and quantitative characterization of electrical coupling and mechanical coupling in MEMS gyroscopes. Moreover, this representation can be used in the characterization of several other parameters, such as amplifier nonlinearity for example. While we have so far applied this approach only to separate-mode gyroscopes, it may also be used for characterizing degenerate-mode gyroscopes. One should however be reminded that such an expression as (6) is only valid provided the modes are separate. In fact, part of our ongoing work is dedicated to extending this characterization framework for use with ONERA's next generation gyroscope cells.

The main drawback of this method is the time it takes to establish a proper Nyquist diagram, since all measurements must be made at a steady-state operation point. An extension for this work is then to investigate ways of accelerating the characterization process, even if it means losing the intuitive interpretation that can be made of Nyquist diagrams.

References

[1] M. Hodjat-Shamami, F. Ayazi, Eigenmode operation of piezoelectric resonant gyroscopes, *Microsyst Nanoeng* **6**, 108 (*2020*).

[2] C. Zhao, M. H.Montaseri, G. S.Wood, S. Hui Pu, A. A.Seshia, M. Kraft, A review on coupled MEMS resonators for sensing applications utilizing mode localization, *Sensors and Actuators A: Physical,* **249**, 93-111 *(2016).*

[3] J. Guérard, L. Delahaye, R. Levy, Digital electronics for inertial MEMS and space applications, *Symposium on Design, Test, Integration & Packaging of MEMS and MOEMS (DTIP 2018).*

[4] L.D. Coope, Circle fitting by linear and non-linear least squares, *Journal of Optimization Theory and Applications (1992).*

[5] J. E.-Y. Lee, A. A. Seshia, Direct parameter extraction in feedthrough-embedded capacitive MEMS resonators, *Sensors and Actuators A: Physical,* **167**, 237-244 *(2011).*