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Balanced electric car sharing charging station optimization

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Abstract

This work focuses on finding optimal locations for charging stations for one-way electric car sharing programs. The relocation of vehicles by a service staff is generally required in vehicle sharing programs in order to correct imbalances in the network. We seek to limit the need for vehicle relocation by strategic location of charging stations given estimates of traffic flow. A mixed-integer linear programming formulation is presented with an exponential number of potential charging station locations. A column generation approach is used which finds the optimal set of locations for the continuous relaxation for our problem. Numerical experiments using real world traffic and GIS location data which shows the benefit of our method.

1 Introduction

Electric car sharing programs are a method for urban centres to combat traffic congestion and pollution (??) as well as to promote the use of green technologies. In one-way car sharing programs such as in Paris, France (?), users are able to use and return vehicles to any charging station. Consequently, we generally see large imbalances within the supply of vehicles and parking spaces in the network, requiring a service staff to continuously transport vehicles to satisfy demand.

In ? the problem of determining optimal locations for charging stations for electric car-sharing systems under stochastic demand is addressed. The number of stations and vehicles, and their optimal placement is determined in order to maximize profit. One assumption

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is that decision makers do not consider operational activities of the service staff, such as vehicle relocation. Given a set of charging stations, a number of researchers have developed methodologies for the vehicle relocation problem. ? consider the use of folding bicycles by workers, which fit into the trunks of cars. ? propose the use of towtrucks which are capable of moving a number of vehicles at a time. In ?, not only is the scheduling considered, but also the number of vehicles and workers to use each day. An added complexity with moving electric vehicles by driving them is that there may not be enough battery power at a given time. ? consider this, as well as the optimal number of workers and the relocation of workers themselves across the network assuming the use of bicycles.

The idea of having to relocate vehicles runs counter to the objective of decreasing traffic congestion, and will cut into profit and system efficiency. In this work we consider a set of nodes, with an estimated expected traffic flow between each per time period. We place charging stations strategically, so as to limit supply imbalances in the network by matching demand for vehicles and parking spots at each charging station.

2 Balanced electric car sharing optimization model

Let N be the set of trip nodes and let $T = \{1, 2, \dots, M_T\}$ be a set of time intervals over a 24 hour cycle with lengths L_t . For each $t \in T$ we have an origin-destination matrix $OD^t \in \mathbb{Z}^{|N| \times |N|}$, indicating for each pair of nodes $\{n, n'\} \in N$, an estimate of the number of trips from n to n' . For each node $n \in N$, its outward flow over t is $f_n^{t-} = \sum_{n' \in N} OD_{n,n'}^t$, requiring an electric vehicle near n , and its inward flow over t is $f_n^{t+} = \sum_{n' \in N} OD_{n',n}^t$, requiring a parking space near n .

Let S be the set of potential charging stations. We assume that people are willing to walk up to $w = 0.5$ km to or from an electric vehicle station as used in (??). For each $s \in S$ we define its neighbourhood as $\mathcal{N}(s) = \{n \in N : d(n, s) \leq w\}$, where $d(n, s)$ is the distance in km between n and s . Likewise, we define the neighbourhood of each $n \in N$ as $\mathcal{N}(n) = \{s \in S : d(s, n) \leq w\}$. We have to assign the flow from node n to its neighbouring stations. Let $F^{t-} \in \mathbb{R}_+^{|S| \times |N|}$ be a matrix of the number of trips from each node n assigned to leave from each station s over t , and likewise let $F^{t+} \in \mathbb{R}_+^{|S| \times |N|}$ be a matrix of the number of trips to each node n assigned to arrive at each station s over t . Let $E^t \in \mathbb{R}_+^{|N| \times |N|}$ be the traffic flow in OD^t which we do not satisfy, and e be a vector of ones. We then assign the outward flow of n to its neighbouring stations, $f_n^{t-} = \sum_{s \in \mathcal{N}(n)} F_{s,n}^{t-} + E_{n,\cdot}^t \cdot e$, as well as its inward flow, $f_n^{t+} = \sum_{s \in \mathcal{N}(n)} F_{s,n}^{t+} + (E_{\cdot,n}^t)^T \cdot e$. The net flow over t for station s is $\sum_{n \in \mathcal{N}(s)} F_{s,n}^{t+} - F_{s,n}^{t-}$, which we want to be zero to minimize network imbalances.

We must decide how many parking spaces at each station. We always choose an even number of spots, to have half the number of cars as parking spots at each station, given that we want the demand for both to always be equal. Assuming that OD contains an estimate of the total traffic flow within the city, we assume that $k = 0.5\%$ of flow will eventually be fulfilled using our service as in (?).

In order to determine the usage capacity of a pair of parking spots, we need an estimate of how long it will take for someone to park and plug in a vehicle, and to register a vehicle and leave the station. We assume that both tasks will take p hours. We also need an estimate of the average trip length, from which we can estimate the average charging time required after each trip.

For each time period t , we calculate the average trip length over the network,

$$l_t = \frac{\sum_{n \in N} \sum_{n' \in N} OD_{n,n'}^t d(n, n')}{\sum_{n \in N} \sum_{n' \in N} OD_{n,n'}^t}.$$

Given an estimate of the electric vehicle's charging time per kilometre driven, u in hours/km, we can estimate the average charging time required after each trip, $h_t = ul_t$. The amount of time on average required between trips for a car to be dropped off and recharged, or picked up is then $p + \frac{h_t}{2}$, and the maximum amount of flow a pair of parking spots per time period $t \in T$ can service is

$$v_t = \left\lfloor \frac{2 * L_t}{k(p + \frac{h_t}{2})} \right\rfloor.$$

Whereas the objective of a company running a combustion engine car sharing program is likely to maximize profit, in this work we assume that the electric car sharing program is funded or heavily subsidized by a government in order to reduce pollution and road congestion, and to promote green technologies. For this reason, the objective in our optimization problem (??) is to maximize the number of electric vehicle trips. z_s is the decision variable for the number of pairs of parking spaces to install at station s , m_s is the maximum number of pairs of parking spaces that can be put at station s , and c_s is the cost of constructing a pair of parking spaces at station s . We assume all costs, including the electric vehicles themselves are embedded into the price per pair of parking spaces. b is the budget the government has allocated for the construction of the car sharing network.

The constraint set (4) enforces that the net flow over each station is zero, so as to minimize the need for vehicle relocation, as the expected number of cars parking and leaving over each time period are equal. We have also indicated the corresponding dual variables of each constraint set for reference.

$$\begin{aligned}
\min \quad & \sum_{t \in T} \sum_{n, n' \in N} E_{n, n'}^t & (OP) \\
\text{s.t.} \quad & \sum_{n \in \mathcal{N}(s)} F_{s, n}^{t+} + F_{s, n}^{t-} \leq v_t z_s \quad \forall t \in T, s \in S \quad U_{t, s} \geq 0 & (1) \\
& \sum_{s \in \mathcal{N}(n)} F_{s, n}^{t-} + E_{n, \cdot}^t e = f_n^{t-} \quad \forall t \in T, n \in N \quad G_{t, n} & (2) \\
& \sum_{s \in \mathcal{N}(n)} F_{s, n}^{t+} + (E_{\cdot, n}^t)^T e = f_n^{t+} \quad \forall t \in T, n \in N \quad P_{t, n} & (3) \\
& \sum_{n \in \mathcal{N}(s)} F_{s, n}^{t+} - F_{s, n}^{t-} = 0 \quad \forall t \in T, s \in S \quad R_{t, s} & (4) \\
& \sum_{s \in S} c_s z_s \leq b \quad q \geq 0 & (5) \\
& z_s \leq m_s \quad \forall s \in S \quad h_s \geq 0 & (6) \\
& F^{t+} \in \mathbb{R}_+^{|S| \times |N|}, \quad F^{t-} \in \mathbb{R}_+^{|S| \times |N|}, \quad E \in \mathbb{R}_+^{|N| \times |N|} \quad z \in \mathbb{Z}_+^{|S|} & (7)
\end{aligned}$$

2.1 Determining S using column generation

The most challenging aspect of (??) is determining the optimal set of charging station locations, as there are up to $2^{|N|} - 1$ stations if we consider all non-empty sets of nodes. We begin with some initial set of stations S' and iteratively add stations using a column generation technique until we have found the optimal set of stations for the continuous relaxation of (??) and use this set to find a solution to (??). The dual program of the continuous relaxation of (??) follows which will be used in the solution technique.

$$\begin{aligned}
\max \quad & -bq - \sum_{s \in S} m_s h_s - \sum_{t \in T} \sum_{n \in N} (f_n^{t+} P_{t, n} + f_n^{t-} G_{t, n}) & (D) \\
\text{s.t.} \quad & 1 + G_{t, n} + P_{t, n'} \geq 0 \quad \forall t \in T, n, n' \in N & (1) \\
& U_{t, s} + P_{t, n} + R_{t, s} \geq 0 \quad \forall t \in T, s \in S, n \in \mathcal{N}(s) & (2) \\
& U_{t, s} + G_{t, n} - R_{t, s} \geq 0 \quad \forall t \in T, s \in S, n \in \mathcal{N}(s) & (3) \\
& h_s + qc_s - \sum_{t \in T} v_t U_{t, s} \geq 0 \quad \forall s \in S & (4) \\
& U \in \mathbb{R}_+^{M_T \times |S|}, \quad h \in \mathbb{R}_+^{|S|}, \quad q \geq 0 & (5)
\end{aligned}$$

In order to find a new station s' to add to S' , let $B \in \{0, 1\}^{|N| \times |S'|}$ be a binary matrix with $B_{\cdot, s}$ representing $\mathcal{N}(s)$. Each node n has coordinates x_n and y_n , stored in vectors $x \in \mathbb{R}^{|N|}$ and $y \in \mathbb{R}^{|N|}$. Let s'_x and s'_y be the coordinates of s' , which we write as convex combinations of the coordinates of the nodes, $s'_x = \alpha^T x$ and $s'_y = \alpha^T y$. We define d_n as the maximum feasible distance of any s' from n , which is $d_n = \max_{n' \in N} d(n, n')$. Each node n has a pre-estimated cost c_n^N for placing a pair of parking spaces at n , as well as a

maximum number of possible pairs of parking spaces m_n^N . A feasible value of $B_{\cdot,s'}$ will satisfy the following constraints in (??). We ensure the uniqueness of $B_{\cdot,s'}$ with the constraints $B_{\cdot,s}^T B_{\cdot,s'} + (1 - B_{\cdot,s})^T (1 - B_{\cdot,s'}) \leq |N| - 1$. We also include the constraints $\frac{B_{\cdot,s'}}{|N|} \leq \alpha$ which ensure that the location of each station is unique by not allowing a node to be assigned to a station while not contributing a positive weighting in α . In addition, we have estimated the price $c_{s'}$ and the maximum pairs of parking spots $m_{s'}$ as the weighted averages of the nodes' values within $\mathcal{N}(s')$.

$$\begin{aligned}
d(s', n) &\leq w + (1 - B_{n,s'}) (d_n - w) \quad \text{for } n \in N & (8) \\
s'_x &= \alpha^T x, \quad s'_y = \alpha^T y \\
\alpha^T e &= 1, \quad \frac{B_{\cdot,s'}}{|N|} \leq \alpha \leq B_{\cdot,s'} \\
B_{\cdot,s}^T B_{\cdot,s'} + (1 - B_{\cdot,s})^T (1 - B_{\cdot,s'}) &\leq |N| - 1 \quad \text{for } s \in S' \\
c_{s'} &= \alpha^T c^N, \quad m_{s'} = \alpha^T m^N \\
\alpha &\in \mathbb{R}_+^{|N|}, \quad B_{n,s'} \in \{0, 1\}^{|N|}
\end{aligned}$$

After solving the dual program (??) for some set S' , we seek to find a feasible solution to (??) which would result in a decrease in its objective. The dual objective will decrease if $h_{s'} > 0$, which will occur for sufficiently large $\sum_{t \in T} v_t U_{t,s'}$. From the second and third constraint sets of (??), $U_{t,s'} \geq -\frac{(P_{t,n} + G_{t,n'})}{2}$ for all $n, n' \in \mathcal{N}(s')$ and will be tight for some particular $n, n' \in \mathcal{N}(s')$ or equal to 0. Let us build the matrices $PG^t \in \mathbb{R}_+^{|N| \times |N|}$, where $PG_{n,n'}^t = \max\left(-\frac{(P_{t,n} + G_{t,n'})}{2}, 0\right)$. The optimal value for $U_{t,s'}$ will be the maximum value of $PG^t \in \mathbb{R}_+^{|N| \times |N|}$. We can determine $U_{t,s'}$ with the following program, where $A \circ B$ is the Hadamard product.

$$\begin{aligned}
\max & \sum_{t \in T} v_t U_{t,s'} & (9) \\
\text{s.t.} & U_{t,s'} = \sum_{n \in N} \sum_{n' \in N} (PG^t \circ D^t)_{n,n'} \quad \text{for } t \in T \\
& e^T D^t e = 1 \quad \text{for } t \in T \\
& D_{n,n'}^t \leq B_{n,s'} \quad \text{for } t \in T, n, n' \in N \\
& D_{n,n'}^t \leq B_{n',s'} \quad \text{for } t \in T, n, n' \in N \\
& D \in \mathbb{R}_+^{|N| \times |N| \times M_T}
\end{aligned}$$

Putting things all together, we determine the next charging station s' to include in (??) by solving the following optimization problem.

$$\begin{aligned}
\max h_{s'} &= \sum_{t \in T} v_t U_{t,s'} - q c_{s'} & (\text{NS}) \\
\text{s.t. } U_{t,s'} &= \sum_{n \in N} \sum_{n' \in N} (PG^t \circ D^t)_{n,n'} \quad \text{for } t \in T \\
e^T D^t e &= 1 \quad \text{for } t \in T \\
D_{n,n'}^t &\leq B_{n,s'} \quad \text{for } t \in T, n, n' \in N \\
D_{n,n'}^t &\leq B_{n',s'} \quad \text{for } t \in T, n, n' \in N \\
d(s', n) &\leq w + (1 - B_{n,s'}) (d_n - w) \quad \text{for } n \in N \\
s'_x &= \alpha^T x, \quad s'_y = \alpha^T y \\
\alpha^T e &= 1, \quad \frac{B_{\cdot,s'}}{|N|} \leq \alpha \leq B_{\cdot,s'} \\
B_{\cdot,s}^T B_{\cdot,s'} + (1 - B_{\cdot,s})^T (1 - B_{\cdot,s'}) &\leq N - 1 \quad \text{for } s \in S' \\
c_{s'} &= \alpha^T c^N, \quad m_{s'} = \alpha^T m^N \\
\alpha &\in \mathbb{R}_+^{|N|}, \quad B_{\cdot,s'} \in \{0, 1\}^{|N|}, \quad D \in \mathbb{R}_+^{|N| \times |N| \times M_T}
\end{aligned}$$

If $h_{s'} > 0$ we add the station to S' and resolve the dual program, where we round $m_{s'}$ to the nearest integer. Else, we have found our set of stations, and proceed to solve the primal problem with our final set $S = S'$.

2.2 Valid inequalities for (??)

Many instances of (??) must be solved, which can be challenging for larger problem instances given the binary variables $B_{\cdot,s'}$. A set of valid inequalities were added to (??) which were found to reduce the computation time by an order of magnitude on average. The general idea is that if we are given a subset of nodes N' where $d(n, n') > 2w$ for all $n, n' \in N'$, then $\sum_{n \in N'} B_{n,s'} \leq 1$. The algorithm for adding these constraints is found in Algorithm ?? . In line 8 by unique we mean a constraint has not already been added for subset V .

Algorithm 1 Valid inequalities for (??)

```

1: for  $n \in N$  do
2:    $V = \{n\}$ 
3:   for  $n' \in N$  do
4:     if  $d(y, n') > 2w \forall y \in V$  then
5:        $V = V + \{n'\}$ 
6:     end if
7:   end for
8:   if  $|V| > 1$  and unique then
9:     add  $\sum_{y \in V} B_{y,s'} \leq 1$  to (??)
10:  end if
11: end for

```

2.3 Solution algorithm

We now present the solution algorithm for finding a balanced electric car sharing charging station network. In general we can expect there to be many "island" nodes, for which there does not exist another node within distance $2w$ from it. For this reason we start with $S' = N$ to prevent solving many instances of (??) to regenerate stations for which we already have all information for.

Algorithm 2 Balanced charging station algorithm (BCSA)

```
1: Initialize  $S' = N$ 
2: Solve (??)
3: Solve (??)
4: if  $h_{s'} > 0$  then
5:   go to 2
6: else
7:   Solve (??) with  $S = S'$ 
8: end if
```

3 Numerical Experiments

3.1 Traffic & GIS data

We test our methodology using trip data from the Transportation Tomorrow Survey (TTS) (??) covering the Greater Golden Horseshoe area of Ontario, Canada. We focused on trips made by car in Toronto. The city of Toronto comprises 16 planning districts (PD) containing 625 traffic zones. We focus on the first 6 planning zones which roughly form a square containing the downtown core and surrounding area. We further reduced the sparsity of the OD matrices by focusing on traffic zones which had trips to and from many other traffic zones. Given a number m , we found the subset of nodes N' such that for each $n \in N'$ there would be at least m other nodes in N' having trips to or from n . The maximum m was then found for which the resulting set N' was nonempty. With $m = 56$, our final dataset contained 149 traffic zones and 468,094 trips over a 24 hour period, broken down into 5 time periods: 6:00-9:00, 9:00-15:00, 15:00-19:00, 19:00-24:00, and 24:00-6:00. TTS Esri shapefiles were used to determine node locations as the centroid of each traffic zone.

3.2 Estimating v_t

The average trip lengths for our data set is $l = [5.1760, 4.4471, 4.7140, 4.7435, 5.9087]$ and p was chosen as 10 minutes. We calculated u based on information acquired about the bluecar used by Autolib' in Paris, which can travel up to 250 km with a recharge time of approximately 4 hours (?). The capacity of a pair of parking spots was then estimated as $v = [5767, 11867, 7828, 9774, 11218]$.

3.3 Parking spaces, station cost, and budget

The cost of installing a pair of parking spaces and the number of pairs of parking spots were set to vary between [1,2] and [1,3], respectively. PD1 contains the waterfront and downtown of the city. We took the centroid of all nodes within PD1 and considered this point P to be the most expensive and dense part of the city. A node's distance from P determined its cost and capacity, with the closest node having a cost of 3 and the furthest having a price of 1, with prices descending linearly with distance. All nodes within $\frac{1}{3}$ of the largest distance from P had a capacity of 1 pair, within $\frac{2}{3}$ had a capacity of 2 pairs and the remaining nodes had a capacity of 3 pairs of parking spaces. All distances in this work were calculated using the 1-norm. We then set our budget $b = \alpha c^T m$ for $\alpha = [0.1, 0.2, 0.3]$. Given an average cost of 2 to build a pair of parking spots, $\alpha = 0.1$ allowed 25 average priced pairs and $\alpha = 0.2$ allowed 51 average priced pairs of parking spots.

4 Results

All experiments were done on a Windows 10 Pro 64-bit, Intel Core i7-7820HQ 2.9GHz processor with 8 GB of RAM computer using Gurobi 8.01. Table ?? presents the results of comparing our method (BCSA) with the solution given by solving (??) with $S = N$. We can see that on average, $(S = N)$ does not satisfy 28% more trips than (BCSA). $|\mathcal{N}(s)|$ reports the number of stations with a neighbourhood of size [1, 2, 3, ...]. In the appendix we display figures of the set of nodes and the optimal stations found for each test problem. Table ?? presents the improved computation time of (??) using Algorithm ?. These statistics were calculated based on the first 50 calls to (??) with and without the use of the valid inequalities.

Table 1: Experimental results: (BCSA) vs. ($S = N$)

	(BCSA)	($S = N$)
$\mathbf{b} = 0.1\mathbf{c}^T\mathbf{m}$		
obj value	286731.4	325189
comp time (s)	7366.00	29.77
# (NS) calls	140	N/A
mean (NS) comp time (s)	31.42	N/A
$ \mathcal{N}(s) $	[16,10,3]	[32]
$\mathbf{b} = 0.2\mathbf{c}^T\mathbf{m}$		
obj value	198996.7	244815
comp time (s)	4515.50	29.18
# (NS) calls	76	N/A
mean (NS) comp time (s)	43.14	N/A
$ \mathcal{N}(s) $	[30,19,4,0,1]	[58]
$\mathbf{b} = 0.3\mathbf{c}^T\mathbf{m}$		
obj value	127549.7	188284
comp time (s)	3097.33	27.42
# (NS) calls	78	N/A
mean (NS) comp time (s)	25.06	N/A
$ \mathcal{N}(s) $	[45,29,2,1,0,1]	[81]

Table 2: Experimental results: benefit of valid inequalities

	with	without
min (NS) comp time (s)	19.17	48.87
max (NS) comp time (s)	36.15	614.12
mean (NS) comp time (s)	26.43	259.42

5 Conclusion

We have developed a novel approach to designing one-way car sharing networks, determining the locations of charging stations so as to maximize the balanced usage of the car sharing network. Though our method is computationally intensive requiring the use of an MILP subproblem, we were able to solve large scale problems and present its ability to increase network flow. Though we were interested in creating a self-regulating car sharing network without the need for vehicle relocation, there is nothing preventing the use of both methodologies. This could be an interesting extension of this work, which could be modeled as a

two-stage stochastic optimization problem, where we make the initial decision of charging stations' locations, and then given the random daily network usage, determine the number of required employees and their deployment. We also see our methodology to be useful in other programs, such a one-way bicycle sharing, which suffer from the same network imbalance problem and can be found in major cities around the world.

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6 Appendix

?? (BCSA) $b = 0.1c^T m$?? ($S = N$) $b = 0.1c^T m$?? (BCSA) $b = 0.2c^T m$
?? ($S = N$) $b = 0.2c^T m$?? (BCSA) $b = 0.3c^T m$?? ($S = N$) $b = 0.3c^T m$?? N

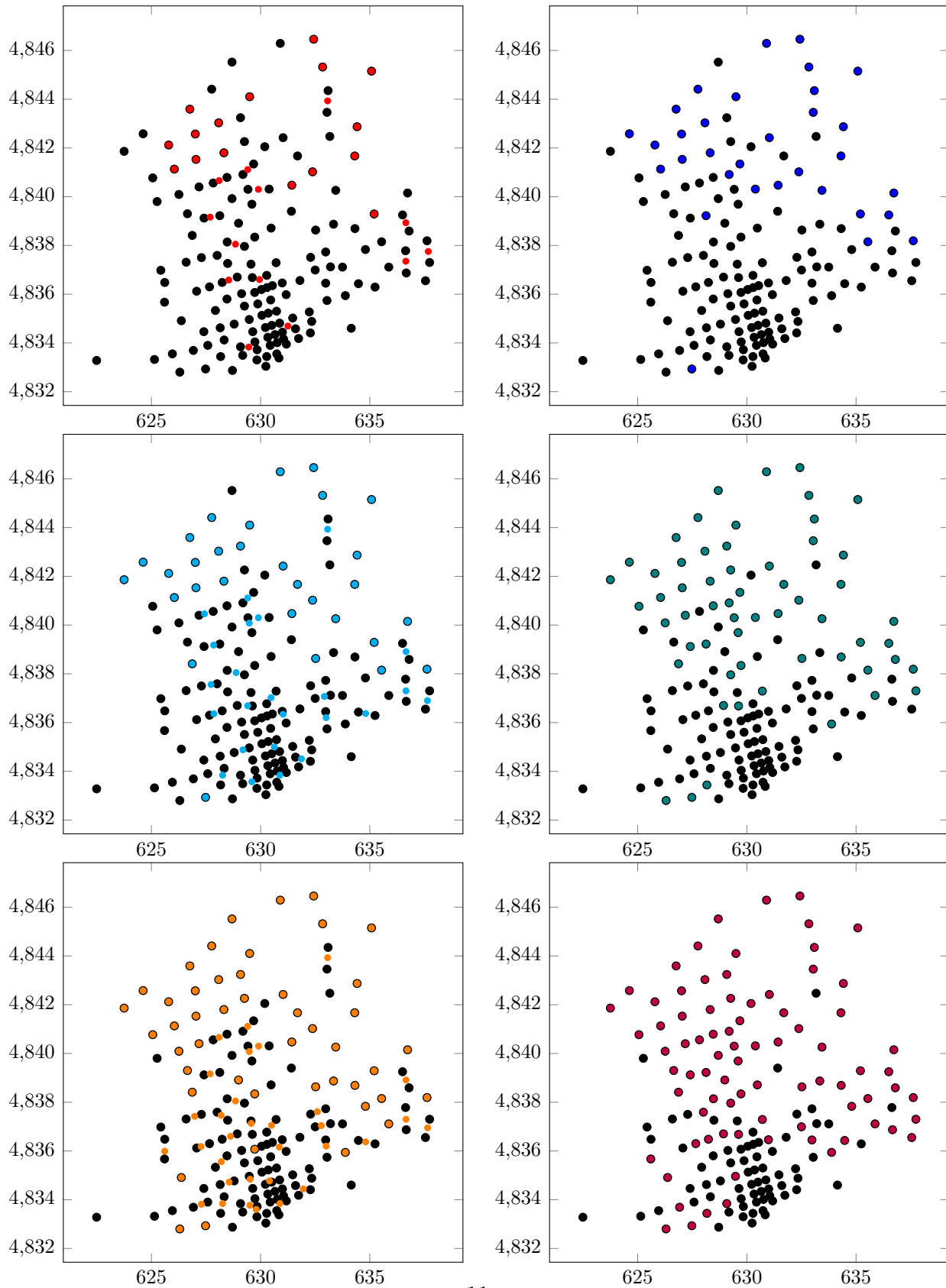


Figure 1: Traffic nodes and stations