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# A modal analysis oriented reduction methodology using component eigenmodes and Arnoldi enrichment

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## Résumé :

*L'utilisation de méthodes de réduction de modèle est aujourd'hui répandue dans le monde de l'industrie, ce qui permet entre autre d'écouter les phases de conception d'un produit, notamment lors de son analyse modale. Cependant, l'intérêt d'une telle approche est parfois détériorée par des interfaces de contact conséquentes entre les composants et par un manque de réutilisabilité des bases de réduction construites.*

*Dans cet article nous nous concentrons sur le développement d'une méthodologie pour la construction de modèles réduits destinés à l'analyse modale, projetés dans des bases de partiellement actualisables et dont les tailles ne sont pas liées au raffinement des interfaces entre les composants. En raison du contexte de conception dans lequel est construit le modèle réduit, un intérêt particulier est porté sur l'utilisation des modes propres libres des composantes dans les bases de réductions générées. Finalement un algorithme d'enrichissement des bases de réduction à l'aide de la méthode d'Arnoldi est réalisé en vue de permettre un certain contrôle de la qualité des modèles réduits produits.*

*Dans la dernière partie de cet article, la méthodologie de réduction proposée est confrontée à la méthode de Craig et Bampton. Cette comparaison met en avant un gain en termes de réutilisabilité, compacité et de qualité des modèles réduits avant enrichissement par rapport à la méthode de Craig et Bampton.*

## Abstract :

*In order to perform faster simulations, the model reduction is nowadays used in industrial contexts to solve large and complex problems. However, the efficiency of such an approach is sometimes cut by the interface size of the reduced model and its reusability.*

*In this article, we focus on the development of a reduction methodology for the build of modal analysis oriented and updatable reduced order model whose size is not linked to their contacting interface. In order to allow latter model readjusting, we impose the use of eigenmodes in the reduction basis. Eventually, the method introduced is coupled to an Arnoldi based enrichment algorithm in order to improve the accuracy of the reduced model produced.*

*In the last section the proposed methodology is discussed and compared to the Craig and Bampton reduction method. During this comparison we observed that even when not enriched, our work enables us to recover the Craig and Bampton accuracy with partially updatable and smaller reduced order model.*

## **Mots clefs : reduction basis update, interface reduction, Arnoldi enrichment**

### **1 Introduction**

The iterative design of a system involves several validation computation such as modal analysis that may slow its conception process. Moreover, in such a context, the required accuracy and detail level is expected to provide a model whose significant size leads to time consuming simulations.

Nowadays substructuring and reduction methods like the superelements or the Craig and Bampton method as instances are industrially used to solve large and complex structural dynamic problems and to cut simulation time.

In a design context, the Mac Neal and Rubin reduction methods are particularly interesting since they use substructures eigenmodes. Indeed, the use of free-free component modes enables to evaluate the contribution of each part into the global dynamics of a system. This may contain interesting information for the design of a system in terms of dynamic behavior. Furthermore, using eigenmodes enables to build updatable models that do not have to be entirely rebuilt when the relative position of a component is changed. Eventually, the use of these modes also allows to perform model readjusting.

Despite their numerous qualities, the Mac Neal and Rubin methods such as Craig and Bampton are facing limitations since they lead to reduced models whose size depends on the substructures contacting interfaces. As a results, this can lower the interest of a reduction approach when working on heavy models.

Several researches have been led to overcome these weaknesses. As an example, methodologies to improve the reusability of the Craig and Bampton basis by enrichment were performed [1], [2]. Rixen proposed a dual Craig and Bampton method using free-free eigenmodes [3] and an additional contribution that aimed at reducing the number of interface mode [5]. Eventually, interface reduction have already been investigated, Tran gives a short review of their applications in the CMS methods [9].

We suggest here an alternative methodology using interface reduction. The research of reusability for the reduced order model and the use of free-free eigenmodes in the reduction basis are conjointly performed. In a last stage, an Arnoldi based enrichment algorithm for a selective improvement of the model solutions is used.

### **2 Component coupling and reduction basis topology**

Let us consider a system  $\Sigma$  made of two components  $\Sigma_1$  and  $\Sigma_2$  tied to each other on their contacting interfaces  $\Gamma_1$  and  $\Gamma_2$  of the two components. For the sake of simplicity the two meshes of the contacting

interfaces are taken as coherent and the interfaces DoF  $j_1$  and  $j_2$  are sorted so that they exactly match in both FE characteristic matrix. The interior DoF of the components are noted  $i_1$  and  $i_2$ .

In order to allow evaluating the contribution of each component modal behavior in the modal behavior of the assemble structure the reduction basis is build as follow:

$$T = \begin{bmatrix} \phi_{1f} & 0 \\ 0 & \phi_{2f} \end{bmatrix} = \phi_f$$

Where  $\phi_{1f}$  and  $\phi_{2f}$  are truncated free modal basis of each component. However using such a topology lead to non-continuous reduction vector. Thus, using CMS method is not possible in this case and both component  $\Sigma_1$  and  $\Sigma_2$  have to be directly coupled.

In order to get coherent displacement fields on the contacting interface the first step of our work consists in primarily assemble both components so that the deformation continuity is imposed in a strong manner. Indeed, dual assemblies rely on the force continuity that may cause displacement jumps on the contacting interfaces. The dynamic stiffness of the assembled system  $Z(\omega)$  can be split into an elastic dynamic stiffness  $Z_E(\omega)$  that is block diagonal and an interaction stiffness matrix  $Z_I(\omega)$  that is mostly sparse so that:

$$Z(\omega) = Z_E + Z_I$$

$$Z(\omega) = \begin{bmatrix} Z_{1,ii} & Z_{1,ij} & 0 & 0 \\ Z_{1,ji} & Z_{1,jj} & 0 & 0 \\ 0 & 0 & Z_{2,ii} & Z_{2,ij} \\ 0 & 0 & Z_{2,ji} & Z_{2,jj} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & Z_{2,jj} & Z_{2,ji} & 0 \\ 0 & 0 & 0 & 0 \\ Z_{1,ji} & 0 & 0 & Z_{1,jj} \end{bmatrix}$$

The reduction of such a model with a basis  $\phi_f$  only made of free-free eigenmodes  $\phi_1$  and  $\phi_2$  lead to reduced model exclusively working for very low frequencies [6]. Thus, additional reduction vectors  $\phi_c$  called coupling vectors have to be taken into account to enable recovering the system behavior on a more spread frequency bandwidth. From now, the hunted reduction basis  $T$  have the following structure:

$$T = [\phi_f \quad \phi_c]$$

In order to increase the numerical conditioning of the reduced order model the coupling deformations  $\phi_c$  are orthogonalized to the free-free eigenmodes  $\phi_f$  using a Gram-Schmidt algorithm.

### 3 Coupling deformation and interface reduction

In the previous section the use free-free eigenmodes  $\phi_f$  to reduce the model was not satisfying, indeed this involves no global deformation of  $\Sigma_1$  and  $\Sigma_2$  simultaneously. Moreover, this projection of the coupled components on an uncoupled deformation basis may lead to interface discontinuities.

Another approach for the interface reduction consists in building a common interface displacement space  $T_j$  for both components and simultaneously imposing these deformations to both components  $\Theta_{T_j}$ . We suggest here the use of the singular value decomposition (SVD) for the build of the interface displacement basis:

$$T_j = \text{SVD}(\phi_{1,j}, \phi_{2,j})$$

Like in the previous method, the coupling deformations are computed for different circular frequencies using the analog formula:

$$\begin{cases} \Theta_{T_j,1} = -Z_{1,ii}^{-1} * Z_{1,ij} * T_j \\ \Theta_{T_j,2} = -Z_{2,ii}^{-1} * Z_{2,ij} * T_j \end{cases}$$

Once again, it is interesting to compute these deformations for several pulsations. Here the selection of the coupling deformations is performed according to the singular value associated to the interface deformations computed with the SVD.

Indeed the higher singular values are associated to the deformations that best engender the interface deformations observed in the eigenmodes.

## 4 Arnoldi enrichment procedure

### 4.1 Estimators for the quality of reduce model solutions

During the construction of the reduced order model the use of the MAC is not possible the high fidelity model (HFM) solutions are researched and the not exactly known. Over the years, several indicators were developed for an *a priori* accuracy evaluation of a model solution, some of them are presented in the following works [7], [8].

At first sight an interesting way to estimate the validity of a solution is to compute the energy ratio of the residual displacement obtained when replacing the approximated solution couple  $(\tilde{\omega}_i^2, \tilde{\Phi}_i)$  in the non-reduced problem operator. This lead to compute the residual force  $R_f(\tilde{\Phi}_i)$ :

$$R_f(\tilde{\Phi}_i) = (-\tilde{\omega}_i^2 M + K)\tilde{\Phi}_i$$

And then to compute the associated residual displacement  $R_\phi(\tilde{\Phi}_i)$  defined as follow:

$$R_\phi(\tilde{\Phi}_i) = \hat{K}R_f(\tilde{\Phi}_i)$$

Where the operator  $\hat{K}$  is the inverse operator of the dynamic stiffness and can be computed individually for each solution couple as  $(-\tilde{\omega}_i^2 M + K)^{-1}$ . This operator can also be computed only one time and used for all the approximated solution as the alpha-shifted pseudo inverse  $(-\alpha M + K)^{-1}$ . Here the choice of the mass shift  $\alpha$  is strategic and enable to cut the computation time of the pseudo inverse  $\hat{K}$ .

The drawback of such method is that the computation of the pseudo-inverse  $\hat{K}$  can be time consuming even when using shifted mass. Thus in our case we decided to use a force based analog indicator that is far cheaper to compute:

$$\varepsilon_i = \frac{\|R_f(\tilde{\Phi}_i)\|_K^2}{\|K\tilde{\Phi}_i\|_K^2} \text{ using the scalar product } \|g\|_K = g^T K g$$

Each solution  $\tilde{\Phi}_i$  whose indicators reach a given tolerance  $\varepsilon_{tol}$  is considered as poor and then enriched by an Arnoldi based algorithm that leads to construction of the new basis T. Once enriched, the solutions of the newly reduced model are computed and their quality indicator are evaluated once again.

In practice, each poor solution is enriched using  $n$  Arnoldi vectors that may fast lead to consequent reduction basis. Thus, the evaluations of the model solutions are expected to slow since the size of the reduce models tend to increase. To avoid this we use a restart step that consists in using the model solution in addition to the Arnoldi vectors to reduce the model Figure. 1.

Eventually, the coupling vectors computed in the loop are orthogonalized to the free-free eigenmodes and added to them that gives the reduction basis  $T$ .

### 4.3 Enrichment algorithm

Once identified, the badly approximated solution of the reduced order model are enriched using Arnoldi vectors. The solutions of the enriched model are then re-computed, their validity is re-evaluated and the reduced model is enriched again if needed until all the approximated solution reach a chosen quality threshold  $\varepsilon_i$ .

At each iteration, the computed solution are orthogonalized to the component eigenmodes and used as the coupling deformation, this step is called restart. The restart is useful to keep a compact reduction basis, somehow the reduced model accumulates more and more information at each enrichment iteration.

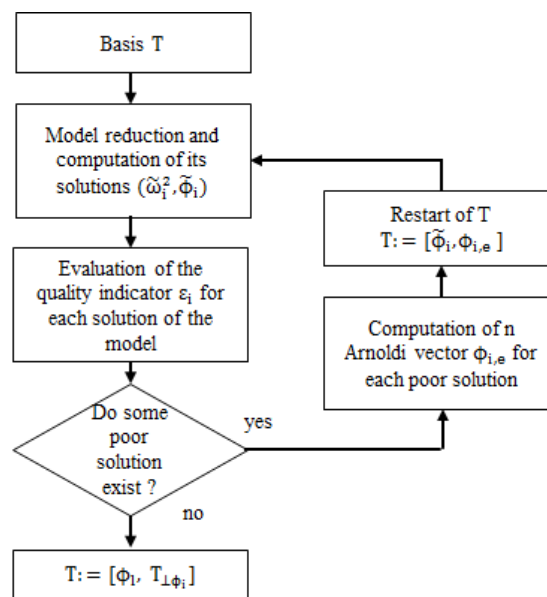


Figure 1.: Enrichment algorithm implementation

## 5 Reduction basis update

Unlike the Craig and Bampton method, the modification of the relative position of the contacting surface has a limited impact on the reduction basis. Indeed, this would only affect the coupling deformations. Thus, it can be interesting to also build a database containing both coupling and free-free deformations  $\phi_1$ ,  $\theta_{k/p,j}$  and/or  $\theta_{T_j}$  of the components in order to faster the rebuild of the initial reduction basis  $T$  and then enrich it.

It has however to be noticed that in the case of a structural modification of a component, the proposed methodology do not offer any advantage compared to the Craig and Bampton method.

## 6 Application of the proposed methodology

In order to support our purpose we apply the methodology previously described to a study case made of two simple parts. The parts of the model are linear and considers small deformations, their properties can be associated to those of a steel. The two parts are embedded and studied on a bandwidth  $\Delta f$  between 0 and 3000 Hertz that contains 41 eigenmodes.

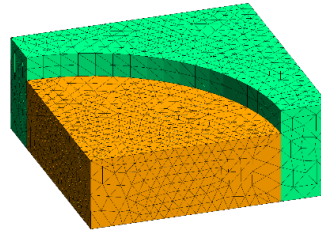


Figure 2.: Case study used for the test of the proposed methodology. the contacting interface of the component is curved and own 109 nodes (321 DoF)

The two interface reduction methods are compared themselves and with the Craig and Bampton method since it is a common method often considered as a reference in substructuring and that is already implemented in industrial codes like NASTRAN or CODE\_ASTER.

### 6.1 Application of the Craig and Bampton method

Here, the tied interface eigenmodes used are taken in the frequency range between [0; 4000] Hertz (54 modes) while the interface deformations are computed for a null pulsation (321 vectors).

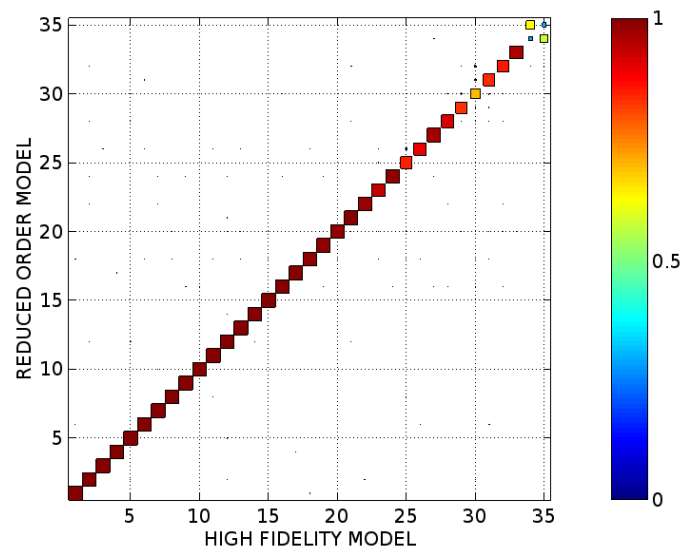


Figure 4.a.: MAC comparison of the flexible model solutions of the HFM and Craig and Bampton reduced model. Number of DoF: 375, mac average: 93.98%. Reduction basis conditioning: 133.65.

## 6.2 Application of the interface reduction methods

For the interface reduction methods, the free-free eigenmodes considered are taken below 3000 Hertz (47 modes). In order to get a rich model we computed the coupling deformations at the start, middle and end of the bandwidth (0, 1500 and 3000 Hertz).

The coupling deformations obtained using the SVD are selected by associated singular value. On Figure 3, the plot of the singular values of the interface deformation led us to only consider the singular values equal or higher than 0.2, this eventually lead to 126 coupling deformations.

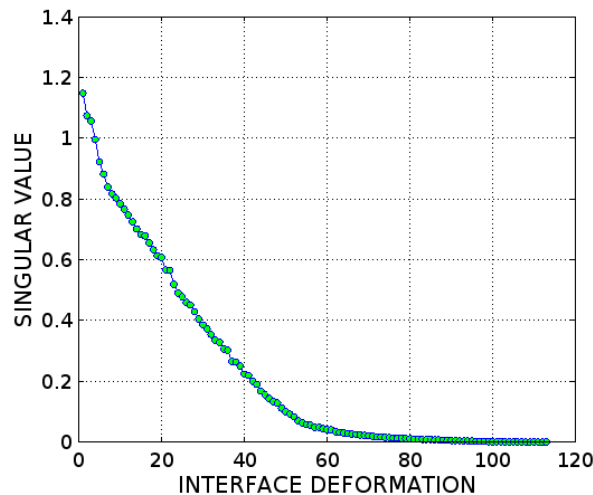


Figure 3.: Decrease of the singular values obtained by the SVD of the interface deformation observed in the eigenmodes.

We can observe on Figure 4.c that the method using the singular decomposition of the interface deformations provides very good results in terms of flexible MAC average and compactness. Indeed, compared to the Craig and Bampton method the compactness of the reduction basis is increase by 2 while the model accuracy is improved by 2.44% of MAC average. It can however be noticed that the conditioning of the reduction basis is higher (9069.67) than the one of the Craig and Bampton method (133.65). This loss is mostly brought by the computation of the coupling deformations for too closed frequencies (0, 1500 and 3000 Hz).

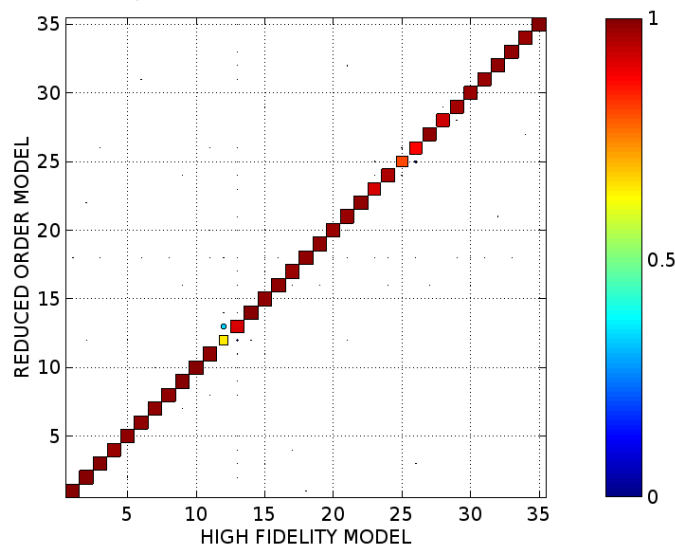




Figure 4.c. MAC comparison of the flexible model solutions of the HFM and reduced model using  $\theta_{t_j}$  as coupling deformations. Number of DoF: 173, mac average: 96.42%. Reduction basis conditioning: 9069.67.

## 7 Conclusion

The proposed method enable us to get accurate reduction basis for the Arnoldi enrichment. Thus, we expect this enrichment to be quick. The reduced model we build is more compact than those obtained with Craig and Bampton that was one of our first goal.

Although the reduced order model provided by the proposed reduction model is efficient for modal analysis it has to be noticed that its spatial convergence is not guaranteed for forced response simulations, unlike the Craig and Bampton method.

Eventually, the topology of the reduction basis we created is favorable to update if the position of a component is changed.

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