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# Compensation of loudspeaker's nonlinearities based on flatness and port-Hamiltonian approach

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## Abstract :

*The electrodynamics of a loudspeaker is affected by several nonlinear phenomena, mainly due to mechanical suspensions, coil with ferromagnetic core and temperature variations. In this paper we address the numerical correction of distortions in the high level response of these systems. The proposed method is based on one hand on the port-Hamiltonian approach for passive system modeling (including the nonlinear case), on the other hand on the flatness approach for exact trajectory planning (here the displacement of the loudspeaker's diaphragm). The model includes a simple linear acoustical load and a phenomenological position saturating nonlinear spring effect due to the suspension. The feed-forward controller is built and simulation results are shown. The method is applied here on a simulated system, with the perspective to be applied on a real device.*

**Mots clefs : Nonlinear control, Electroacoustic transducers, Port-Hamiltonian systems, Differential flatness.**

## 1 Introduction

The electrodynamic loudspeaker is a nonlinear electroacoustic transducer that includes several nonlinear phenomena, mainly due to mechanical suspensions, coil with ferromagnetic core and temperature variations (see *e.g.* [16]). In this paper we address the numerical correction of distortions in the high level response of these systems. Several signal processing or physically inspired approach have been considered (see *e.g.* [4],[3] and references therein). Such approaches do not allow straightforward inference of physical knowledge in the model and its parameters nor in the design process of the controller. Moreover, such system identification techniques can lead to non-physically unstable behaviors.

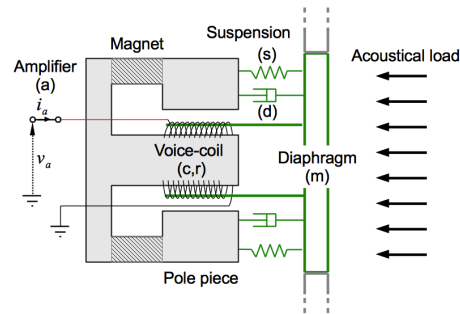


Figure 1: Schematic view of the electrodynamic loudspeaker.

We propose a method in the port-Hamiltonian (pH) formalism, which provides a continuous-time state-space model structured according to energy flows. This guarantees the passivity of the model, including the nonlinear case. In this preliminary work, magnetic and thermal effects are neglected, and we consider a simple low frequency approximation of the acoustic load. This leads to a simple open non-linear system which allows for an easy derivation of the controller. The proposed method is based on the differential flatness of the resulting pH system, which allows for exact trajectory planning.

The port-Hamiltonian modeling of the loudspeaker is given in section 2. A remainder on differential flatness theory is given in section 3. Then we build the feed-forward controller for the loudspeaker. Finally, section 4 is devoted to the application of the method in discrete-time. First, we present the numerical method which preserves the passivity property. Second, simulation results for a FANE Sovereign 12-500LF are shown.

## 2 The electrodynamic loudspeaker

This section details the physical model of the loudspeaker considered in the remainder. We firstly extend the usual linear modeling (namely, the *Thiele and Small* modeling) to account for the nonlinearity that first appears in practice. Secondly, we recast this model in the port-Hamiltonian framework. That proves the passivity of the model.

### 2.1 Physical description and model

The electrodynamic loudspeaker is a transducer which is composed of an electromagnetic and a mechanical parts, a simplified lumped parameter model of which has been proposed by Thiele and Small (see figure 2). This modeling includes a simple acoustical load as additional spring effect due to air volume in the cabinet, and additional air mass and dissipation due to acoustic radiation, see [13, 14, 15].

The electromagnetic part involves:

- a**: voltage source (amplifier)
- r**: wire resistance,
- c**: coil inductance,
- $\mathcal{L}$** : back electromotive force.

The mechanical part involves:

- m**: moving mass (diaphragm, coil and air)
- s**: spring effects (spider, dust cap and air)
- d**: damping effects (idem)
- $\mathcal{L}$** : Lorentz force.

The corresponding set of ordinary differential equations are derived by applying Kirchoff's laws to the electrical part and Newton's second law to the mechanical part:

$$v_a(t) = R_r \cdot i_a(t) + L \cdot \partial_t i_a(t) + Bl \cdot \partial_t q(t), \quad (1)$$

$$m \cdot \partial_t^2 q(t) = Bl \cdot i_a(t) - R_d \cdot \partial_t q(t) - F_s(q). \quad (2)$$

$v_a$  and  $i_a$  are respectively the input tension and current (in receiver convention with received power  $P_a = v_a \cdot i_a$ ),  $L$  is the inductance of the voice-coil (linear approximation) and  $R_r$  is the electrical resistance of the wires of the coil.  $q$  is the diaphragm's displacement (distance from equilibrium),  $m$  is the total mass of the moving part (including the diaphragm, voice coil and the equivalent mass due to acoustical load), and  $R_d$  is the linear approximation of the mechanical losses (including frictions and acoustical power radiation). The electro-mechanical coupling terms are the back e.m.f  $v_{\mathcal{L}} = Bl \cdot \partial_t q$ , and the Lorentz force  $F_{\mathcal{L}} = Bl \cdot i_a$ , with  $Bl$  the product of the (constant) magnetic induction field's magnitude with the length of the coil's wire in that field.

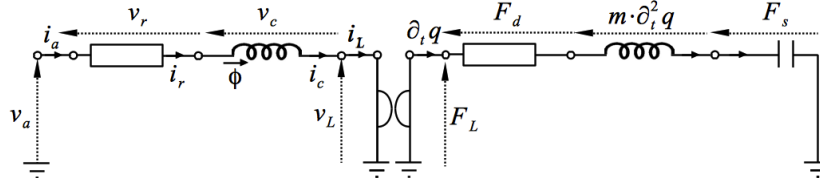


Figure 2: Thiele and Small equivalent electro-mechanic modeling of loudspeakers.

According to the literature [17, 16], the main nonlinearity is the spring effect  $F_s(q)$ . In the original model of Thiele and Small, it is supposed to be linear with stiffness  $k_0$ :

$$F_s(q) \equiv F_{lin}(q) = k_0 \cdot q. \quad (3)$$

Here, we consider a phenomenological position saturating nonlinear spring given in equation (4) and figure 3, with  $q_{sat}$  the saturating position. The nonlinear term  $F_{sat}$  does not contribute around the origin  $q = 0$ .

$$F_s(q) = F_{lin}(q) + F_{sat}(q) \quad (4)$$

$$F_{sat}(q) = \frac{4k_s}{4-\pi} \left( \tan\left(\frac{\pi \cdot q}{2 \cdot q_{sat}}\right) - \frac{\pi \cdot q}{2 \cdot q_{sat}} \right)$$

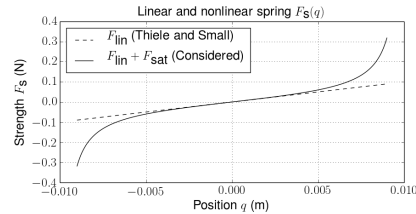


Figure 3: Linear and nonlinear spring.

## 2.2 Port-Hamiltonian formulation

**Port-Hamiltonian systems** The port-Hamiltonian (pH) formalism have been introduced in the 90's. It provides a framework for the passive guaranteed modeling of open dynamical system through a state space representation structured according to energy flows with state  $\mathbf{x} \in \mathbb{R}^{n_x}$ , input  $\mathbf{u} \in \mathbb{R}^{n_u}$  and storage function  $H(\mathbf{x}) \in \mathbb{R}_+$  (see [6, 5, 2] for details). Here, we consider the same algebraic-differential formulation as in [18], which includes a resistive variable  $\mathbf{w} \in \mathbb{R}^{n_w}$  and a dissipative relation  $\mathbf{z}(\mathbf{w}) \in \mathbb{R}^{n_w}$ :

$$\begin{pmatrix} \partial_t \mathbf{x} \\ \mathbf{w} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_x & -\mathbf{K} & \mathbf{G}_x \\ \mathbf{K}^\top & \mathbf{J}_w & \mathbf{G}_w \\ -\mathbf{G}_x^\top & -\mathbf{G}_w^\top & \mathbf{J}_y \end{pmatrix} \cdot \begin{pmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}, \quad (5)$$

where the output  $\mathbf{y} \in \mathbb{R}^{n_u}$ , and  $\mathbf{J}_x$ ,  $\mathbf{J}_w$  and  $\mathbf{J}_y$  are skew-symmetric matrices. Matrices  $\mathbf{K} \in \mathbb{R}^{n_x \times n_w}$ ,  $\mathbf{G}_x \in \mathbb{R}^{n_x \times \nu}$  and  $\mathbf{G}_w \in \mathbb{R}^{n_w \times \nu}$ . The incoming power is  $S = \mathbf{u}^\top \cdot \mathbf{y}$ , and function  $\mathbf{z}$  restores the (possibly nonlinear) dissipated power  $D(\mathbf{w}) = \mathbf{z}(\mathbf{w})^\top \cdot \mathbf{w} \geq 0$  with  $\mathbf{z}(0) = 0$  and positive definite Jacobian matrix  $[\frac{\partial \mathbf{z}(\mathbf{w})}{\partial \mathbf{w}}]_{m,n} = \frac{\partial z_m(\mathbf{w})}{\partial w_n}$ , so that the following power balance holds:

$$\begin{pmatrix} \nabla H(\mathbf{x}) \\ \mathbf{z}(\mathbf{w}) \\ \mathbf{u} \end{pmatrix}^\top \cdot \begin{pmatrix} \partial_t \mathbf{x} \\ \mathbf{w} \\ \mathbf{y} \end{pmatrix} = \partial_t H(\mathbf{x}) - D(\mathbf{w}) + S = 0 \quad (6)$$

**Port-Hamiltonian modeling of the loudspeaker** The pH system that corresponds to the loudspeaker's modeling (1), (2) and (4) is composed of  $n_x = 3$  storage components (coil, moving mass and spring),  $n_w = 2$  dissipative components (electric resistance and mechanical damping) and  $n_u = 1$  source (input tension). For storage components, the state is  $\mathbf{x} = [x_c, x_m, x_s]^\top$  with  $x_c = \phi$  the magnetic flux in the coil,  $x_m = m \cdot \partial_t q$  the momentum of the mass and  $x_s = q$  the elongation of the spring. The Hamiltonian is  $H(\mathbf{x}) = H_c(x_c) + H_m(x_m) + H_s(x_s)$  with quadratic storage functions  $H_c$  and  $H_m$  defined in table 1, and nonlinear potential energy  $H_s(x_s) = \frac{1}{2} k_0 \cdot x_s^2 + H_{sat}(x_s)$ ,

$$H_{sat}(x_s) = k_s \frac{8 q_{sat}}{\pi(\pi - 4)} \left( \ln \left( \cos \left( \frac{\pi \cdot x_s}{2 \cdot q_{sat}} \right) \right) + \frac{1}{2} \left( \frac{\pi \cdot x_s}{2 \cdot q_{sat}} \right)^2 \right).$$

The dissipative variables are  $\mathbf{w} = [w_r, w_d]^\top$ , with  $w_r = i_r$  the current in the resistance and  $w_d = \partial_t q$  the velocity associated to the damping, and associated function  $\mathbf{z}(\mathbf{w}) = [z_r(w_r), z_d(w_d)]^\top = [R_r \cdot w_r, R_d \cdot w_d]^\top$ . The input is the tension from the amplifier  $u_a = v_a$ , and the output is the current  $y_a = i_a$ .

The structure matrices in (5) are obtained applying conservation laws (namely, Kirchhoff and Newton's laws) to the following set of power variables (see table 1):

$$\begin{aligned} \nabla H(\mathbf{x}) &= [i_c, \partial_t q, F_s]^\top, & \partial_t \mathbf{x} &= [v_c, m \partial_t^2 q, \partial_t q]^\top \\ \mathbf{w} &= [i_r, \partial_t q]^\top, & \mathbf{z} &= [v_r, F_d]^\top \\ \mathbf{y} &= i_a, & \mathbf{u} &= v_a \end{aligned} \quad (7)$$

Label	Electric			Flux $\equiv i$ (A)	Effort $\equiv v$ (V)
(c)	coil inductance	$x_{\mathbf{c}} = \phi$	$\mathbf{H}_{\mathbf{c}}(x_{\mathbf{c}}) = \frac{x_{\mathbf{c}}^2}{2 \cdot L}$	$i_{\mathbf{c}} = \frac{\partial \mathbf{H}_{\mathbf{c}}}{\partial x_{\mathbf{c}}}$	$v_{\mathbf{c}} = \partial_t x_{\mathbf{c}}$
(r)	resistance	$w_{\mathbf{r}} = i_{\mathbf{r}}$	$z_{\mathbf{r}}(w_{\mathbf{r}}) = R_{\mathbf{r}} \cdot w_{\mathbf{r}}$	$i_{\mathbf{r}} = w_{\mathbf{r}}$	$v_{\mathbf{r}} = z_{\mathbf{r}}(w_{\mathbf{r}})$
(a)	external port			$i_{\mathbf{a}} = y_{\mathbf{a}}$	$v_{\mathbf{a}} = u_{\mathbf{a}}$
Label	Mechanic			Flux $\equiv \partial_t q$ (m.s <sup>-1</sup> )	Effort $\equiv F$ (N)
(m)	Total mass	$x_{\mathbf{m}} = m \cdot \partial_t q$	$\mathbf{H}_{\mathbf{m}}(x_{\mathbf{m}}) = \frac{x_{\mathbf{m}}^2}{2 \cdot m}$	$\partial_t q = \frac{\partial \mathbf{H}_{\mathbf{m}}}{\partial x_{\mathbf{m}}}$	$F_{\mathbf{m}} = \partial_t x_{\mathbf{m}}$
(s)	Nonlinear spider	$x_{\mathbf{s}} = q$	$\mathbf{H}_{\mathbf{s}}(x_{\mathbf{s}}) = \frac{k_0}{2} x_{\mathbf{s}}^2 + \mathbf{H}_{\text{sat}}(x_{\mathbf{s}})$	$\partial_t q = \partial_t x_{\mathbf{s}}$	$F_{\mathbf{s}} = \frac{\partial \mathbf{H}_{\mathbf{s}}}{\partial x_{\mathbf{s}}}$
(d)	Mechanical damping	$w_{\mathbf{d}} = \partial_t q$	$z_{\mathbf{d}}(w_{\mathbf{d}}) = R_{\mathbf{d}} \cdot w_{\mathbf{d}}$	$\partial_t q = w_{\mathbf{d}}$	$F_{\mathbf{d}} = z_{\mathbf{d}}(w_{\mathbf{d}})$

Table 1: Elementary components in the nonlinear pH modeling of the loudspeaker.

The nonlinear port-Hamiltonian system corresponding to the loudspeaker modeling (1), (2) and (4) is given by (5), with (7) and the following structure matrices:

$$\mathbf{J}_{\mathbf{x}} = \begin{pmatrix} 0 & -Bl & 0 \\ +Bl & 0 & -1 \\ 0 & +1 & 0 \end{pmatrix}, \quad \mathbf{K} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{G}_{\mathbf{x}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}^{\top} \quad (8)$$

$$\mathbf{J}_{\mathbf{w}} = 0, \quad \mathbf{G}_{\mathbf{w}} = 0, \quad \mathbf{J}_{\mathbf{y}} = 0.$$

**Remark 2.1 (Perspectives)** *The pH systems framework allows easy inclusion of modeling refinement, such as the dynamic behavior of the magnetic circuit (magnet, iron and air-gap) [19], or the fractional order dynamic [24] due to the losses by magnetic hysteresis and eddy-currents [22]. This will be studied in further works.*

### 3 Flatness and trajectory planning

The flatness-based feed-forward controller for the system (5) is derived in this section. After a short reminder on differential flatness, this method is used to derive a closed-form expression of the input that ensures the system's state follows a given trajectory. Then, the pH structure is exploited to derive an equivalent formal expression.

#### 3.1 Recalls

Here we recall the notion of differential flatness for the full linearization of nonlinear systems via dynamic feedback (see [7, 8, 9] for details). A classic dynamical system given by  $\partial_t \mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ , with state  $\mathbf{x} \in \mathbb{R}^{n_{\mathbf{x}}}$ , input  $\mathbf{u} \in \mathbb{R}^{n_{\mathbf{u}}}$  and nonlinear dynamic  $\mathbf{f} : \mathbb{R}^{n_{\mathbf{x}}} \times \mathbb{R}^{n_{\mathbf{u}}} \rightarrow \mathbb{R}^{n_{\mathbf{x}}}$  is *differentially flat* if both the following conditions are satisfied.

- (I) There exists a finite set  $\boldsymbol{\mu} \in \mathbb{R}^{n_{\boldsymbol{\mu}}}$  of variables which are differentially independent and such that the  $\mu_i$ 's are functions of the state  $\mathbf{x}$ , and the input  $\mathbf{u}$  with a finite number of its derivatives  $\boldsymbol{\mu} = \varphi_{\boldsymbol{\mu}}(\mathbf{x}, \mathbf{u}, \partial_t \mathbf{u}, \dots, \partial_t^{(\alpha_{\boldsymbol{\mu}})} \mathbf{u})$ ,  $\boldsymbol{\mu} \in \mathbb{R}^{n_{\boldsymbol{\mu}}}$ .

- (II) Any system variable is a function of  $\boldsymbol{\mu}$  and of a finite number of its derivatives:  
 $\mathbf{x} = \varphi_{\mathbf{x}}(\boldsymbol{\mu}, \partial_t \boldsymbol{\mu}, \dots, \partial_t^{(\alpha_{\mathbf{x}})} \boldsymbol{\mu})$  and  $\mathbf{u} = \varphi_{\mathbf{u}}(\boldsymbol{\mu}, \partial_t \boldsymbol{\mu}, \dots, \partial_t^{(\alpha_{\mathbf{x}}+1)} \boldsymbol{\mu})$ .

The set  $\boldsymbol{\mu}$  is the *linearizing* (or *flat*) output of the system, with  $\dim(\boldsymbol{\mu}) = \dim(\mathbf{u}) = n_{\boldsymbol{\mu}}$ .

### 3.2 Flat output and input design

The loudspeaker model (5) proves differentially flat: basic manipulations of equations (1), (2) and (4) show that the position is the flat output  $\mu = q$ , with  $\alpha_{\mu} = 0$ ,  $\alpha_{\mathbf{x}} = 2$  and

$$\begin{aligned} \mu &= \varphi_{\boldsymbol{\mu}}(x_3), \\ \mathbf{x} &= \varphi_{\mathbf{x}}(\mu, \partial_t \mu, \partial_t^2 \mu) \\ &= \begin{pmatrix} \frac{L}{Bl} (m \cdot \partial_t^2 \mu + R_d \cdot \partial_t \mu + F_s(\mu)) \\ m \cdot \partial_t \mu \\ \mu \end{pmatrix}. \end{aligned}$$

This leads to the following closed-form expression of the input  $v_a = \mathbf{u}$ :

$$\begin{aligned} \mathbf{u} &= \varphi_{\mathbf{u}}(\mu, \partial_t \mu, \partial_t^2 \mu, \partial_t^3 \mu) & (9) \\ &= \frac{m \cdot L}{Bl} \cdot \partial_t^3 \mu + \frac{R_d \cdot L + Re \cdot m}{Bl} \cdot \partial_t^2 \mu \\ &\quad + \left( \partial_x F_s(\mu) + \frac{L + Re \cdot R_d}{Bl} + Bl \right) \cdot \partial_t \mu & (10) \\ &\quad + \frac{R_r}{Bl} \cdot F_s(\mu). \end{aligned}$$

Then, given any admissible nominal trajectory of the flat output  $\boldsymbol{\mu}^*$ , the nominal input trajectory  $\mathbf{u}^*$  of the input  $\mathbf{u}$  is obtained from (9), replacing all occurrences of  $\boldsymbol{\mu}$  by  $\boldsymbol{\mu}^*$ , that is  $\mathbf{u}^* = \varphi_{\mathbf{u}}(\boldsymbol{\mu}^*, \partial_t \boldsymbol{\mu}^*, \dots, \partial_t^{(\alpha_{\mathbf{x}}+1)} \boldsymbol{\mu}^*)$  (see [10]). Here, the target is the diaphragm's displacement and the feed-forward controller is given by equation (10), replacing  $\mu$  with  $q^*$ .

### 3.3 Port-Hamiltonian formalization

The same basic manipulations on equations (5), (7) and (8) rather than on (1), (2) and (4) yields  $\mathbf{x} = \varphi_{\mathbf{x}}(\mu, \partial_t \mu, \partial_t^2 \mu)$ , with

$$\varphi_{\mathbf{x}}(\mu, \partial_t \mu, \partial_t^2 \mu) = \begin{pmatrix} (\partial_x \mathbb{H}_{\mathbf{C}})^{-1} \left( \frac{\partial_t (\partial_x \mathbb{H}_{\mathbf{M}})^{-1} (\partial_t \mu) + z_d (\partial_t \mu) + \partial_x \mathbb{H}_{\mathbf{S}}(\mu)}{Bl} \right) \\ (\partial_x \mathbb{H}_{\mathbf{M}})^{-1} (\partial_t \mu) \\ \mu \end{pmatrix}.$$

This formal expression combined with Kirchhoff's voltage law encoded in the pH structure (5) makes the input appear to be composed of three terms  $v_a = v_{\mathbf{C}} + v_{\mathbf{r}} + v_{\mathcal{L}}$  that find straightforward individual interpretations. The input of the mechanical subsystem (m, s, d) is the Lorentz force  $F_{\mathcal{L}} = Bl \cdot i_{\mathcal{L}}$ , which is designed to compensate the mechanical dynamic and loss  $i_{\mathcal{L}} = \frac{1}{Bl} [\partial_t (\partial_x \mathbb{H}_{\mathbf{M}})^{-1} (\partial_t \mu) + z_d (\partial_t \mu) + \partial_x \mathbb{H}_{\mathbf{S}}(\mu)]$ .

Then, the input  $\mathbf{u} = v_a$  for the electrical part (c, r) is designed to compensate the electrical dynamic and loss so that  $i_{\mathcal{L}} = i_{\mathcal{L}}^*$ . Finally, each term in the controller  $v_a$  corresponds to a chain of conversion from  $\mu$  to a tension:

$$\begin{array}{ll} \text{Coil} & v_{\mathcal{C}} \xleftarrow{\partial_t} \phi \xleftarrow{\partial_x \mathbf{H}^{-1}} i_{\mathcal{C}} = i_{\mathcal{L}} \\ \text{Wire} & v_{\mathcal{R}} \xleftarrow{z_{\mathcal{R}}} i_{\mathcal{R}} = i_{\mathcal{L}} \\ \text{back e.m.f} & v_{\mathcal{L}} = Bl \cdot \partial_t \mu. \end{array}$$

## 4 Simulation and results

This section is concerned with the discrete-time simulation of the algebro-differential system (5), that is, the computation of  $\mathbf{x}(k) \equiv \mathbf{x}(k \cdot T_s)$  from  $\mathbf{u}(k) \equiv \mathbf{u}(k \cdot T_s)$ , with  $k \in \mathbb{N}$ , for the constant sampling frequency  $f_s = 1/T_s$ , in such a way that a discrete power balance is satisfied. First, we present a structure-preserving numerical scheme and show direct simulation for a given loudspeaker. Second, results for the flatness-based controller are shown.

### 4.1 Passive-guaranteed numerical scheme and simulation

Here, the objective is to define the discrete time derivative of the state  $\delta_t \mathbf{x}(k)$  and the discrete gradient of the Hamiltonian  $\delta_x \mathbf{H}(k)$  so as to recover the chain rule in discrete time:  $\delta_t \mathbf{E}(k) = \delta_x \mathbf{H}(k)^{\top} \cdot \delta_t \mathbf{x}(k)$ . The level of energy is the one associated to the state  $\mathbf{x}(k)$  through storage function  $\mathbf{H}$ . Additionally, we choose a first order finite difference scheme:  $\delta_t \mathbf{x}(k) = \frac{\mathbf{x}(k+1) - \mathbf{x}(k)}{T_s}$  to approximate the variation of the state. Then, for a pH system composed of a collection of mono-variant energy storing components, the Hamiltonian is  $\mathbf{H}(\mathbf{x}) = \sum_{n=1}^{n_x} \mathbf{H}_n(x_n)$  and the solution is

$$[\delta_x \mathbf{H}(k)]_n = \frac{\mathbf{H}_n(x_n(k+1)) - \mathbf{H}_n(x_n(k))}{x_n(k+1) - x_n(k)}. \quad (11)$$

This numerical scheme restores the midpoint rule for linear systems, with its natural extension to the nonlinear cases. Applying this numerical method to the pH system (5) leads to an implicit system of algebraic equations, which is then solved by Newton-Raphson algorithm. The device used for simulation is a FANE Sovereign 12-500LF, with physical parameters in table 2. Simulation results are given in figure 4.

$m$	0.075 (kg)	$Bl$	16.37 (T·m)
$L$	2.36 (mH)	$k_0$	7.14 (N·m <sup>-1</sup> )
$R_{\mathcal{R}}$	5.9 ( $\Omega$ )	$k_s$	100 (N·m <sup>-1</sup> )
$R_{\mathcal{d}}$	3 (N·s·m <sup>-1</sup> )	$q_{sat}$	5.17 (mm)

Table 2: Physical parameters for the simulated FANE Sovereign 12-500LF loudspeaker.

### 4.2 Results

The feed-forward controller (10) is applied to the same loudspeaker device as in section 4.1, and numerical simulations are performed. We assign a sinusoidal trajectory to



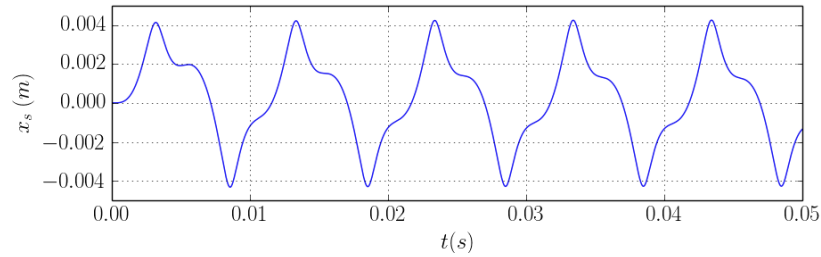


Figure 4: Simulated displacement for a FANE Sovereign 12-500LF excited with a sinusoidal tension (amplitude 100V and frequency 100Hz).

the target flat output:  $\mu^* = A \cdot \sin(2\pi \cdot f_0 \cdot t)$ , with amplitude  $A = 0.7 \cdot q_{sat}(m)$  and frequency  $f_0 = 100(\text{Hz})$ . Since the system is supposed initially at rest, initial values for the flat output and all the needed derivatives have to be zeros for the trajectory to be admissible [9]. Here, the set  $(\mu^*, \partial_t \mu^*, \partial_t^2 \mu^*, \partial_t^3 \mu^*)$  is smoothed between  $t = 0$  and  $t = 1/f_0$  with third order polynomial. Results for the target input  $v_a^*$  is given in figure 5. The comparison between the target  $\mu^*$  and the simulated flat output  $\mu$  for the input  $v_a^*$  are given in figure 5.

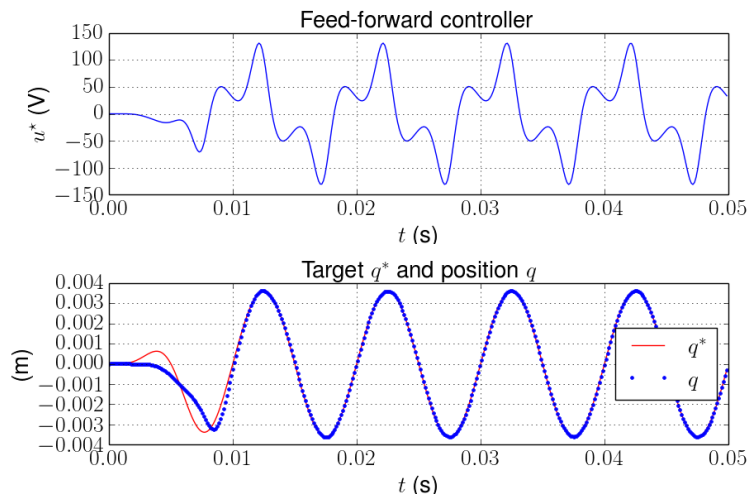


Figure 5: Upper: Input tension  $v_a^*$  (upper) for a sinusoidal target trajectory (smoothed at the origin) with amplitude 3.62mm and frequency  $f_0 = 100 \text{ Hz}$  ( $f_s = 48\text{kHz}$ ). Lower: Target and system's flat output.

## 5 Conclusion

Firstly, we recast the usual Thiele-Small modeling of loudspeaker in port-Hamiltonian framework. The model has been slightly refined to include the main nonlinearity according to the literature, preserving passivity due to pH structure. Secondly, we designed an numerically efficient flatness-based feed-forward controller. We have shown that port-Hamiltonian framework permits a physical interpretation of the generation of the controller. The main perspective is the application of this method to a real device. We shall include several refinement of the physical model (magnetic, thermic and acoustic) in future work, benefiting from the modular construction of pH systems. The change of variable from the pH state to the Brunovsky coordinate provided by the flatness property of the system shall be used to build an additional feedback controller to account for model or measurement errors in practice. Another perspective is to study the possible automated derivation of the flat output based on a port-Hamiltonian system and bond-graph formalism.

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