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Identification of a time-varying mechanical system using the Akaikeinformation criterion

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Résumé :

Cet article démontre l'intérêt du critère d'information d'Akaike pour les méthodes d'identification contenant une étape de projection de signaux de mesure sur des bases spécifiques. On s'intéresse particulièrement au problème d'identification de paramètres mécaniques variant dans le temps.La méthode est testée expérimentalement avec succès sur un système à un degré de liberté, constitué d'une inertie et d'une poutre à longueur variable travaillant en torsion. Les variations de cette longueur rendent les paramètres du système dynamique changeant. Une approche par moindre carré combinée à une projection des signaux mesuré sur une base polynomiale permet de reconstruire aisément les fluctuations des paramètres mécaniques. Cependant le choix de l'ordre de troncature de la base de projection est toujours un problème clef dans ce genre de méthode inverse et reste bien souvent une question ouverte. On propose ici d'utiliser le critère d'information Akaikepermettant ainsi une sélection robuste et automatique de cet ordre de troncature. L'étude présentée ici ainsi que l'expérimentation montrent la pertinence et l'intérêt de ce type de critère associé aux méthodes d'identification.

Abstract:

This article deals with the use of Akaike Information Criterion in the case of an identification problem of a time varying mechanical system.. This system studied to illustrate the approach is physically realized by a prismatic beam with a disk excited in torsion. The boundary conditions of the beam can change in a controlled way during the experiment. Therefore, our system can be considered as a 1 d.o.f. system with time-varying parameters. A method based on least-square estimates is used for the identification of the parameters. However, the major contribution of this article is the use of the Akaike information criterion in order to choose automatically the order of polynomial basis used during the estimationstep of the derivatives of the measured angle signal. It is shown through an experimental validation that the AIC criterion is robust and automatic tool for numerical estimation of signal derivatives.

Key words:Information criteria, inverse problems, identification

1 Introduction

In many engineering problems we are interested in identifying some real physical system. In other words, we want to identify a mathematical model which would describe the physical phenomenon, which we observe. In this article, we deal with the identification of a simple mechanical system with one degree of liberty and one time-varying parameter.

The systems with time-varying parameters can be found in many physical problems. For example, the mechanics of biological and human tissues and joints can be described by time-varying models [1]. In the same works, the robotics and joystick command systems are also considered as time-varying mechanical systems. The identification of the time-varying systems has been studied by different approaches: by the ensemble average method [2], by a parallel-cascade algorithm [3], or by a wavelet method [4]. Numerous techniques use the projection of the time-varying parameters on an orthogonal basis of functions [5, 6]. The choice of this basis can be a bit arbitrary, there were methods using Legendre polynomials [7, 8], block-pulse functions [9], Fourier series [10], Laguerre polynomials [11],Chebyshev polynomials [12-17] or classical polynomial basis [18, 19]. In principle, all of these methods work quite well. However, their weakness is the choice of the size of the functional basis. Indeed, we have no clue how large the basis of polynomial functions should be. Most often, one must use some iterative scheme and see how the results look like. Then, some human judgement is necessary to choose a good size of the basis of polynomial functions. This human factor makes these methods a bit cumbersome and time-costly.

In this article, we propose to apply a simple statistical tool called Akaike information criterion (AIC) to choose a suitable polynomial basis automatically. We deal with the same experimental data as described in [17]. The AIC criterion was developed by H.Akaike [20] in 1974. Its goal is to choose the optimal model from a set of candidate models and some observation. It was widely used in statistics and applied in many real-life problems, mostly in biology [21-24]. So far, its use for the mechanical problems was very limited. Its use for a choice of vibration models was proposed in [25].

This article is organized as follows. Section 2 describes the experimental set-up and measured data. Section 3 describes the technique of identification of the model and presents the results. Section 4 concludes the article and Appendix describes the AIC criterion.

2 Experimental set-up

To build an accurate and easy to used mechanical system with time varying parameters is not so easy. The experimental set-up can be seen in Fig.1. It consists of a triangular prism mounted vertically. At the lower end of the bar, there is a disk which serves as an inertia. At the upper end of the prism, there is a slider. This slider serves as a clamped boundary condition, but it can move freely up and down during the experiment. This motion changes the boundary condition and, consequently, changes also the torsional stiffness of the prismatic beam. If we know the position of the slider in time, we are able to calculate the theoretical torsional stiffness of the beam in time. This is very useful, because we can compare this time-varying stiffness to the stiffness identified by our method.

Finally, the excitation is achieved via an electric motor which is connected to the disk through the torque cell and rotation accelerometer. During the experiment, we are able to measure simultaneously the driving torque, the rotation angle and the position of the slider. A typical measure is shown in Fig.2. We note that the torque signal is by far the noisiest one. This is probably due to some micro-gaps in the screwing mechanism and maybe due to the influence of the flexural vibration of the beam. On the other hand, the rotation angle signal is very neat, which is necessary because we will be dealing

with estimation derivatives of this signal. The sampling frequency of the measure was 1000Hz. More information about this set up can be found in [17].

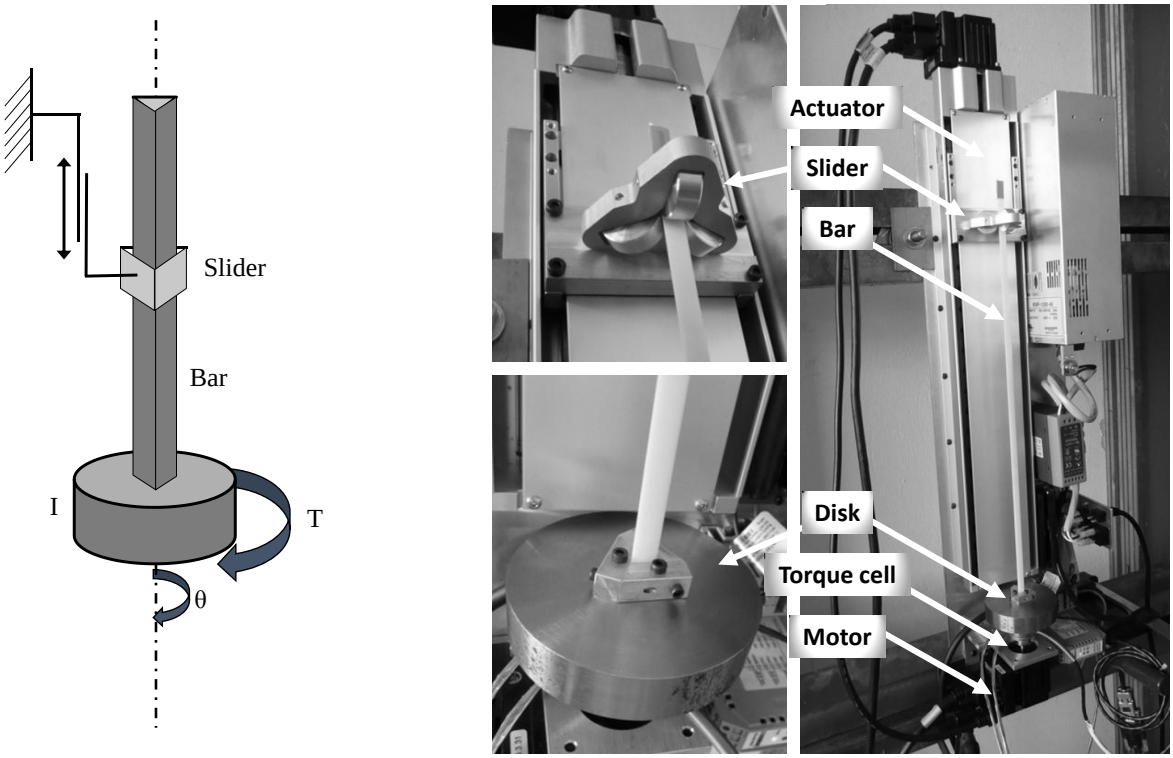

Fig.1: Experimental mounting of the prism beam in torsion and its representative scheme on left.

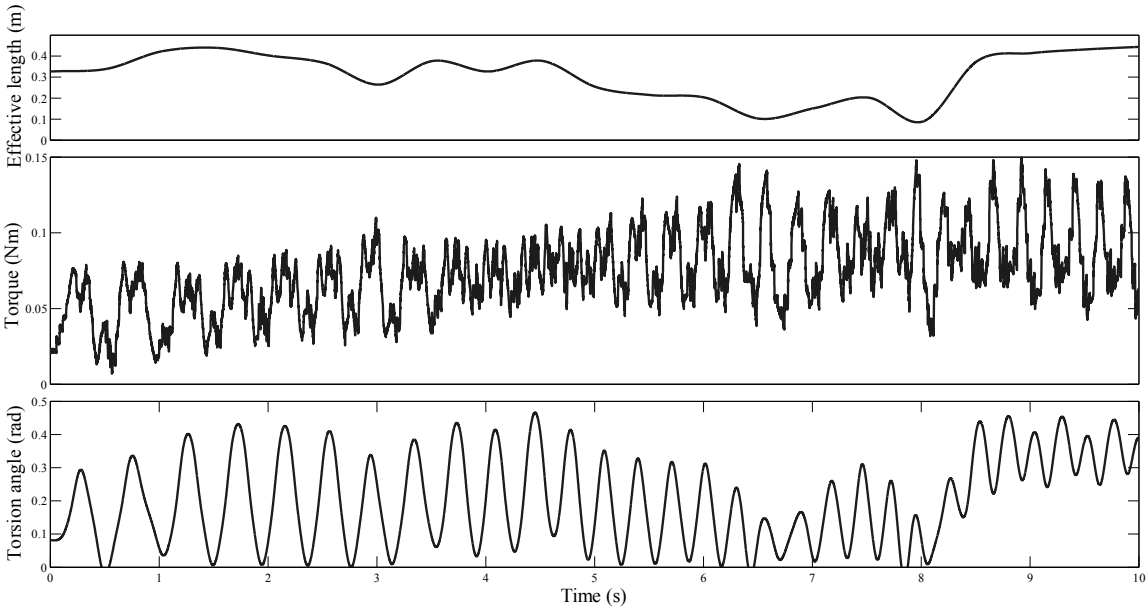

Fig.2: A typical measure obtained from our experimental setup: above – the slider position, in the middle – the torque applied, below – rotation angle.

3 Identification of the system

The system described in Section 2 can be approximately described as a system with one (rotational) degree of freedom. Its equation of motion can be written as

$$I\ddot{\theta} + C\dot{\theta} + K\theta = T \qquad (1)$$

Where *I* is the global inertia of the beam and the disk, *C* is the damping coefficient, *K* is the rotational stiffness, *θ(t)* is the rotation angle and *T(t)* is the driving torque.In our case, we want to identify the parameters *I*, *C* and *K* while we measure *θ(t)* and *T(t)*. In previous studies [17-19], the torsional inertia and damping were considered constant in time, but the torsional stiffness is necessarily a function of time *K=K(t)*. In this paper, the three mechanical parameters are considered as time-varying.

The process of identification can be separated into several parts as it can be seen in Fig.3. First, the first and second derivatives of the angle θ should be obtained. This is done with the help of the polynomial fitting and the AIC criterion. Second, once all the derivatives are estimated, we identify the optimal parameters I,C and K to satisfy the equation of motion given by (1). Both of these procedures are detailed in the following.

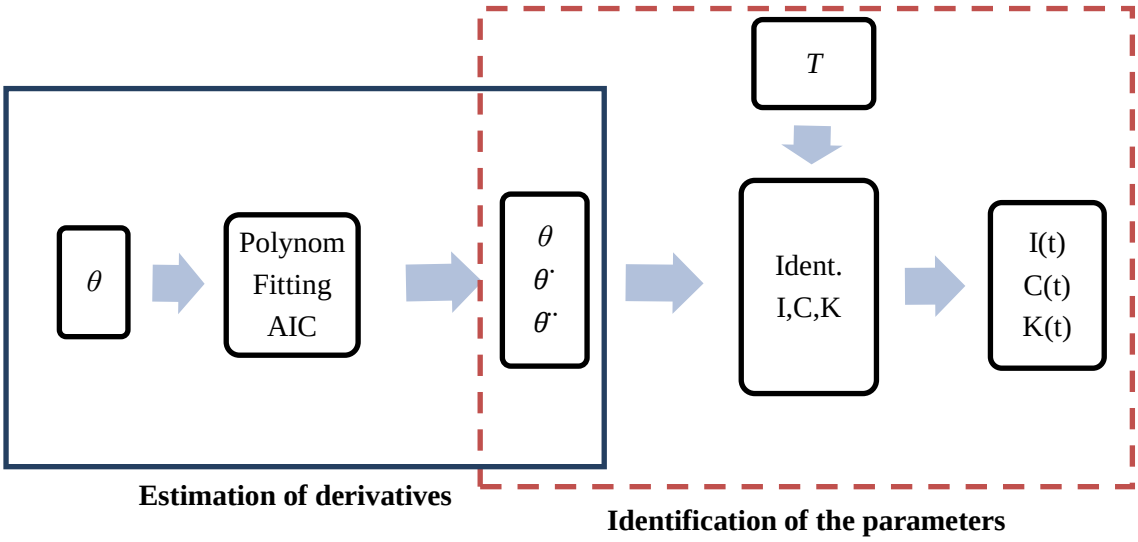

Fig.3: The steps of the identification method

3.1 Estimation of derivatives

The heart of the problem of identifying the parameters of the equation (1) lies in the fact that we only measure the angle θ. The time derivatives of this angle are unknown and we need to estimate them from θ. There are many methods for the estimation of derivatives. Here, we opt for the polynomial regression optimized by the AIC criterion.

Let us look the derivative of θ at some time t_i. We suppose that we dispose the measurement of θ for the neighboring times to t_i. In a small interval of interest we dispose 2N+1 measures of θ. We can put them in the vector $\boldsymbol{\theta} = [\theta(t_{i-N}),\ldots,\theta(t_i),\ldots,\theta(t_{i+N})]^T$.

In this small interval around the time t_i the function θ(t) can be developed in the Taylor series as

$$\theta(t) = \sum_{j=0}^{n} \alpha_j (t - t_i)^j \tag{2}$$

The coefficients α of the expansion given by (2) can be easily obtained by the least-square linear fit of the measured vector **θ** by the Taylor series decomposition. However, how should we choose the number of polynomials*n* in the Taylor series decomposition? This is the main problem of many identification methods using an expansion process.

The AIC criterion can answer this question (definition of the AIC criterion can be found in the appendix). Let us consider a small interval of time and the corresponding measure of angle θ. This measure can be seen in Fig.4 in the upper-left corner. Now, we can fit this function by Taylor series with different maximal polynomial order *n*. The optimal fit should be the one which minimizes the value of the AIC function. To illustrate the approach, let's consider the measured angle on a short time window (Fig.4(a)). According to the Fig.4(b), the Taylor series with n=13 is the optimal fit for the measured function. However, to highlight the main interest of this kind of criterion, if we add artificially some uncorrelated noise as it is shown in Fig.5(a), than the AIC criterion would choose the optimal model among the Taylor series the one having n=7(Fig.5(b)).Practically, that means that polynomial function of higher order would be used to fit the noise.

The strength of this approach is its *robustness* and *automatism*. A simple program can be constructed to automatically choose the model (Taylor series) having the lowest value of AIC among all the other models. Whatever is the shape of the function, we are dealing with, and whatever is the noise, AIC criterion always chooses the optimal fit for the measurement in question.

Once the optimal model (Taylor series) is identified via AIC criterion, it can be easily derived to obtain the first and second derivatives. In a certain point of view, the AIC avoids an unwanted expansion of the noise on the polynomial basis. These expansions usually disturb drastically the derivation step.

It should be noted that the maximal number of polynomials (so-called *candidate models*) to be used is a bit arbitrary. Here, we use the maximal polynomial degree 15 for all the measured intervals. It is recommended that the optimal model should not be the first neither the last model among the candidate models.

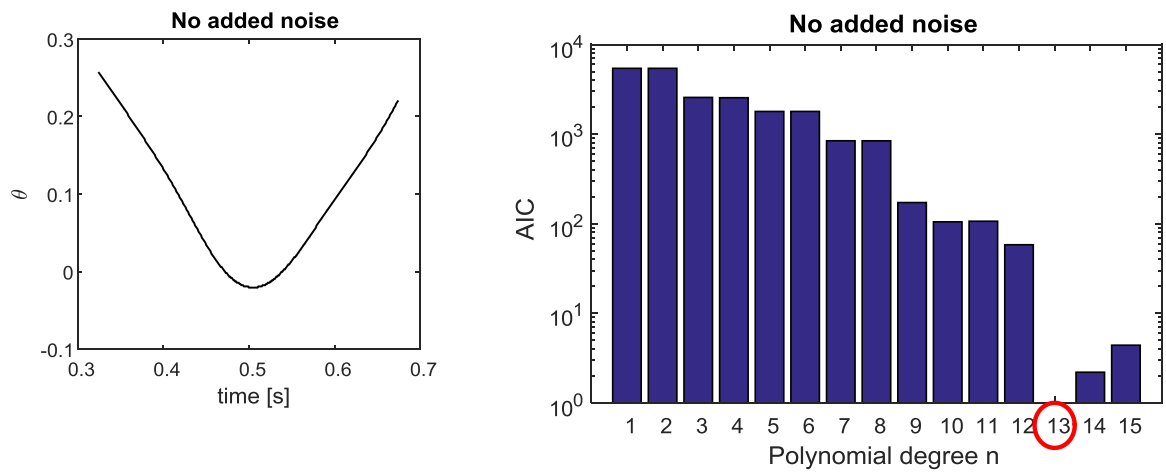

Fig.4 Example of the use of AIC to fit measured data. (a) By fitting an experimental curve with low noise, (b) AIC favors high polynomial fit.

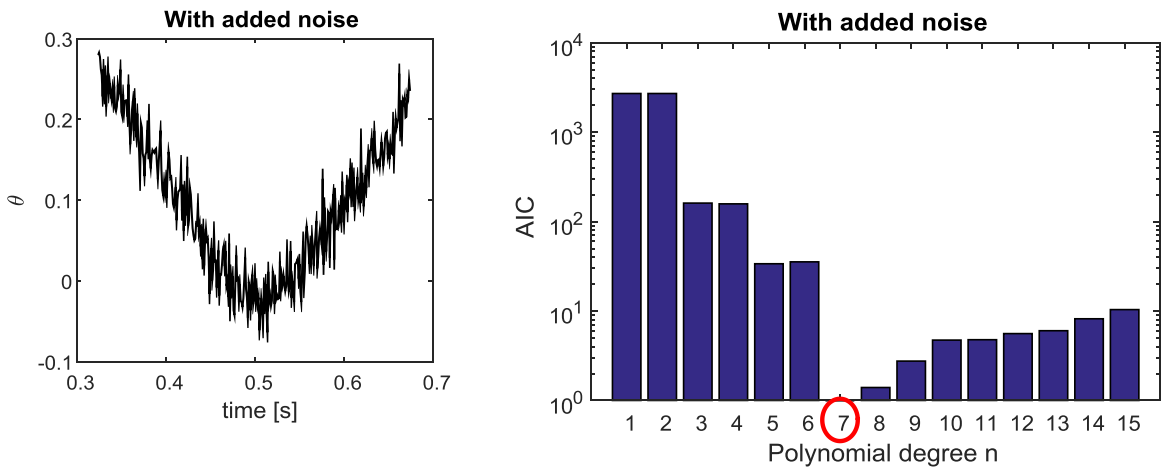

Fig.5 Example of the use of AIC to fit a noisy curve..(a) By fitting the same curve with artificially added noise, (b) AIC favors lower degree polynomials

3.2 Identification of parameters

Once the derivatives of θ are identified for all measured points, we can identify the optimal parameters of the equation of motion (1). For the *i-th* measured time t_i, we can write the equation (1) as

$$I\ddot{\theta}(t_i) + C\dot{\theta}(t_i) + K\theta(t_i) = T(t_i) \tag{3}$$

This equation cannot be solved for I, C and K, because we have more unknowns than equations. On the other hand, we can assume that the variation of these parameters is much slower than the sampling frequency of the time. Under these assumptions, we can consider the values of I, C and K constant over some short interval of time. According to the method used in Section 3.1, we consider that we dispose 2N+1 values of θ, $\dot{\theta}$, $\ddot{\theta}$ in the vicinity of time t_i. Then, we can write a system of linear equations for the parameters I, C and K at the time t_i

$$\begin{bmatrix} \ddot{\theta}(t_{i-N}) & \dot{\theta}(t_{i-N}) & \theta(t_{i-N}) \\ \vdots & \vdots & \vdots \\ \ddot{\theta}(t_i) & \dot{\theta}(t_i) & \theta(t_i) \\ \vdots & \vdots & \vdots \\ \ddot{\theta}(t_{i+N}) & \dot{\theta}(t_{i+N}) & \theta(t_{i+N}) \end{bmatrix} * \begin{bmatrix} I \\ C \\ K \end{bmatrix} = \begin{bmatrix} T(t_{i-N}) \\ \vdots \\ T(t_i) \\ \vdots \\ T(t_{i+N}) \end{bmatrix} \tag{4}$$

Where the parameters I, C and K correspond to the time point t_i. Parameters I, C and K can be easily obtained from Eq. (4) by pseudo-inversing the matrix of the system. The number of equations to take into account into the system (4) is a bit arbitrary. More equations we take into account, smoother is the solution, but this filtering property can badly reconstruct fast variations of the identified parameters.. It is possible to use some iterative technique to find a suitable number of equations to use. However, the influence of the number of equations in (4) is not so important in the studied case.

In Fig.6, there are identified parameters I, C and K obtained by the method described above. The number of equations in the system (4) was 200, which means physically that these parameters were considered constant over 0.2s (the sampling frequency was 1000Hz). We note that the identified

inertia parameter is noisy,which is probably due to its link with the second derivative, which ismuch more sensitive to noise than the others. On the other hand, the stiffness corresponds well to the theoretical stiffness parameter obtained from the measured position of slider (see Fig.2 above). Inertia and stiffness was the two main parameter of this study. They are well reconstructed, specially the variation of the stiffness. The damping of the real system is very low. In previous studies [17-19] it has been set to zero without disturbing drastically the identification of the time varying stiffness. The identified damping coefficient here is very small, but it cannot be compared to other independent measure. Never the less the mean value of this coefficient over the time domain is positive, which is in concordance with the physics of the system. The identification is generally bad at the beginning and the end of the measure. This is due to erroneous derivative estimation at the borders.

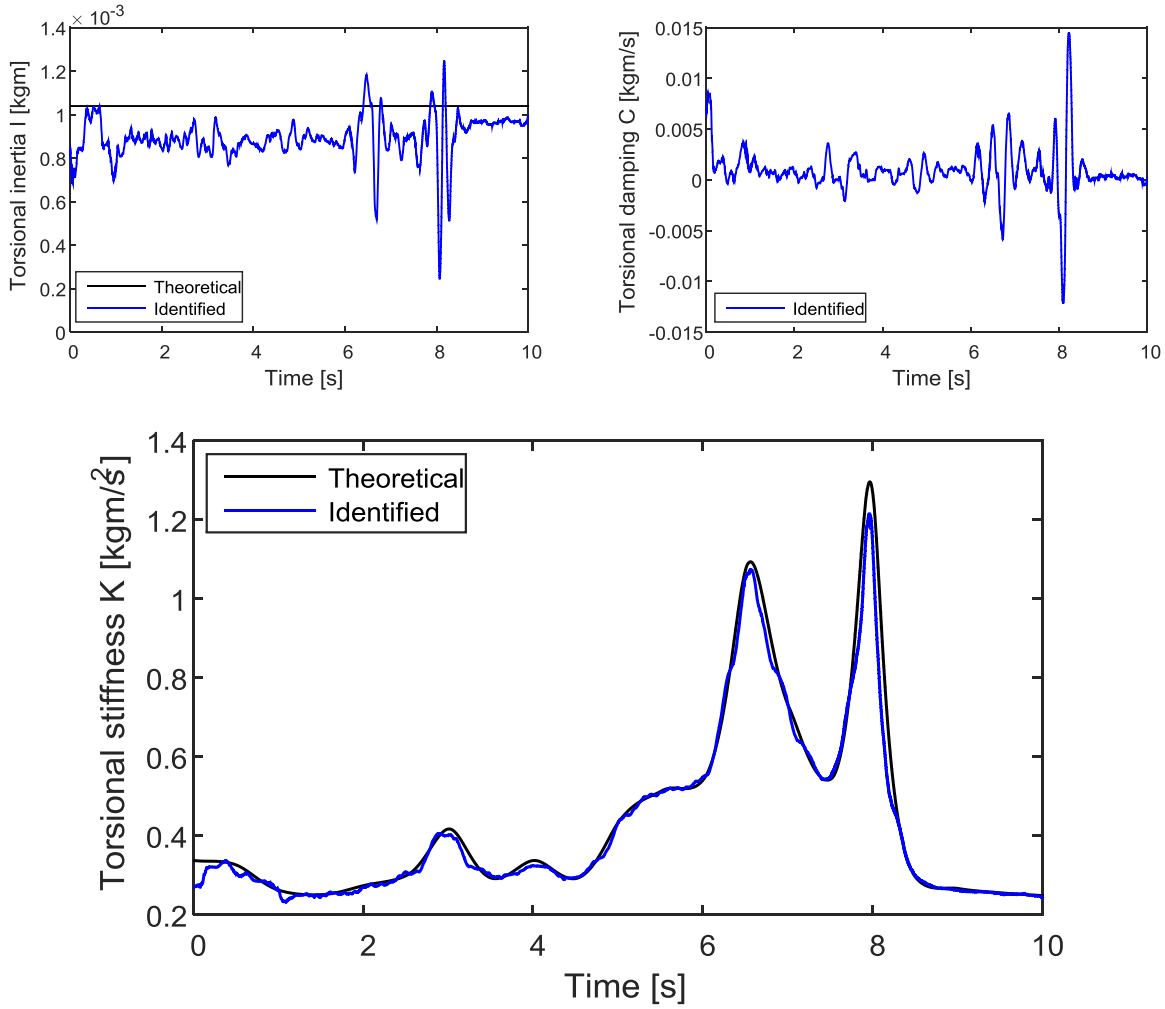

Fig.6: Parameters of the time-varying system identified by the inverse method. Above on the left: the identified torsional inertia (blue) compared to the theoretical inertia (black). Above on the right: the identified torsional damping coefficient. Below: the identified torsional stiffness (blue) compared to the stiffness obtained from the slider position (black).

4 Conclusion

In this article, an inverse method using on the Akaike information criterion is presented. This method enables us to estimate the temporal derivatives of a measured signal and consequently to identify the

parameters of a mechanical system. It has been illustrated with a single degree of freedom system. The AIC criterion is used to choose automatically the optimal polynomial for fitting locally a measured signal which is of great interest for this kind of identification problem. The advantage of this approach is the robustness of this method. It can deal with different level of noise and automatically choose the optimal polynomial model, contrary to previous developed methods [17-19]

Three parameters of the single degree of freedom system were determined: inertia, damping and stiffness coefficients. While the identified damping parameter cannot be verified independently, the identified inertia and stiffness coefficients correspond wellto other independent measures (see [17]).

The proposed approach could be applied to other identification problems. One perspective is the identification of the systems with a few degrees of freedom. Another perspective would be to increase the accuracy of the identificationby making constant (but still unknown) some of the non-varying parameters (I and C in this case).

Appendix: Akaike information criterion

The Akaike information criterion was introduced by Akaike in [20]. A more recent critical review is proposed by Burnham and Anderson in [21]. In a simplified version the problem solved by the AIC criterion is as follows. Let us have a system of N (generally nonlinear) equations

$$f_i\left(M_j\left(p_j\right)\right) = d_i,\ i=1..N \tag{5}$$

wherethe coefficient i represents different independent measures d_i. The *i-th* value of the model M_j is described by the function $f_i(M_j(p_j))$. This function depends on some vector of parameters p_j, which is to be determined. This parameter vector can be determined by the nonlinear least squares, in other words by minimizing the sum of residuals$R_j = \sum_i \varepsilon_{ij}^2$, where

$$\varepsilon_{ij}\left(p_j\right) = f_i\left(M_j\left(p_j\right)\right) - d_i \tag{6}$$

The optimal least-square estimate of the parameter $\hat{p}_j$ for the model j is obtained as

$$\hat{p}_j = \underset{p_j}{\operatorname{argmin}}\, R_j\left(p_j\right) \tag{7}$$

The AIC function for a model j is defined as follows

$$AIC_j = N \ln \sigma_j^2 + 2K_j + \frac{2K_j\left(K_j+1\right)}{N-K_j-1} \tag{8}$$

Where K_j is the number of elements in the vector of parameters p_j, and σ_j is the estimate of the standard deviance of the measure using the model j:

$$\sigma_j^2 = \frac{R_j\left(\hat{p}_j\right)}{N-1} \tag{9}$$

Under the assumptions that the measures d_i are independent identically normally distributed random variables, the AIC function is an unbiased estimate of the Kullback-Leibler distance between the measure and the model. In practice, we want this distance to be minimal; we want the model to be as close to the reality as possible. This means, that we search for the model with the *lowest* value of AIC. In order to compare different models, we must choose some candidate models at first. In our case,

these candidate models are simple polynomials of the Taylor series, but these models can be represented by any function. Then, the optimal parameters $\hat{p}_j$ of the models are determined by the least-squares and the minimal residuals (9) are evaluated. The model chosen by the AIC criterion is not simply the model having the lowest residuals. It is a model which has a low residual but a low number of parameters as well. AIC criterion selects a model which is optimal in a sense of the prediction. It means that the selected model would give the best prediction of unmeasured data.

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