



**HAL**  
open science

# Determination of the fine-structure constant with a 81 parts-per-trillion accuracy

Pierre Cladé, Léo Morel, Zhibin Yao, Saïda Guellati-Khélifa

► **To cite this version:**

Pierre Cladé, Léo Morel, Zhibin Yao, Saïda Guellati-Khélifa. Determination of the fine-structure constant with a 81 parts-per-trillion accuracy. 55th Rencontre Moriond - Electroweak interactions and Unified Theories, Mar 2021, Moriond (online), France. hal-03443551

**HAL Id: hal-03443551**

**<https://hal.science/hal-03443551v1>**

Submitted on 27 Nov 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# DETERMINATION OF THE FINE-STRUCTURE CONSTANT WITH A 81 PARTS-PER-TRILLION ACCURACY

P. Cladé<sup>1</sup>, L. Morel<sup>1</sup>, Z. Yao<sup>1</sup>, S. Guellati-Khélifa<sup>1,2</sup>

<sup>1</sup> *Laboratoire Kastler Brossel, Sorbonne Université, CNRS, ENS-PSL, Collège de France, 4 place Jussieu, 75005 Paris, France*

<sup>2</sup> *Conservatoire National des Arts et Métiers, 292 rue Saint Martin, 75003 Paris, France*

The fine-structure constant  $\alpha$  is a crucial parameter of the quantum electrodynamics theory especially for evaluating the magnetic moment of electron or muon predicted by the standard model. By using a matter-wave interferometer to measure the recoil velocity, we obtained recently a new value of the fine-structure constant with an accuracy of 81 parts per trillion. Using this value of the fine-structure constant, we obtained a prediction of the anomalous magnetic moment of the electron with a relative uncertainty below 0.1 ppt. This is the most accurate prediction of the standard model. This result is an important step in the way of testing, in the electron sector, the discrepancy observed in the magnetic moment anomaly of the muon.

## 1 Introduction

The fine-structure constant was introduced in 1916 by A. Sommerfeld to include relativistic correction to the Bohr model of the hydrogen atom. In this model, the velocity of the electron in the first orbit is given by  $\alpha c$  and the ionisation energy of the atom is given by  $hcR_\infty = \frac{1}{2}m_e c^2 \alpha^2$ , where  $R_\infty$  is the Rydberg constant and  $m_e$  the mass of the electron. The first quantum description of the hydrogen atom with relativistic correction was obtained using the Dirac equation in the 1930's. Thanks to this equation, the fine-structure of the atom was precisely calculated. This equation also predicts that the g-factor of the electron was exactly 2.

However, it turns out that those predictions were incomplete. In the late 1940's, experiments exhibit discrepancy with the Dirac theory in the hydrogen spectra<sup>1</sup> and the g-factor of the electron<sup>2</sup>. Both results were explained using the Quantum Electrodynamics theory developed the following years. In this theory, the fine-structure constant appears as the coupling constant between elementary charged particles and photons.

The electron g-factor can be theoretically calculated. The correction to the Dirac equation is called the anomalous magnetic moment:  $a_e = \frac{g_e - 2}{2}$ . It can be decomposed in terms from the QED, weak interactions and hadronic corrections.

$$a_e(\text{theo}) = a_e(\text{QED}) + a_e(\text{Weak}) + a_e(\text{Hadron}) \quad (1)$$

where  $a_e(\text{QED})$  depends on the fine-structure constant:

$$a_e(\text{QED}) = \sum_{n=1}^{\infty} A^{(2n)} \left(\frac{\alpha}{2\pi}\right)^n + \sum_{n=1}^{\infty} A_{\mu,\tau}^{(2n)} \left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \left(\frac{\alpha}{2\pi}\right)^n$$

The graph on Fig. 1 represents the order of magnitude of each correction.

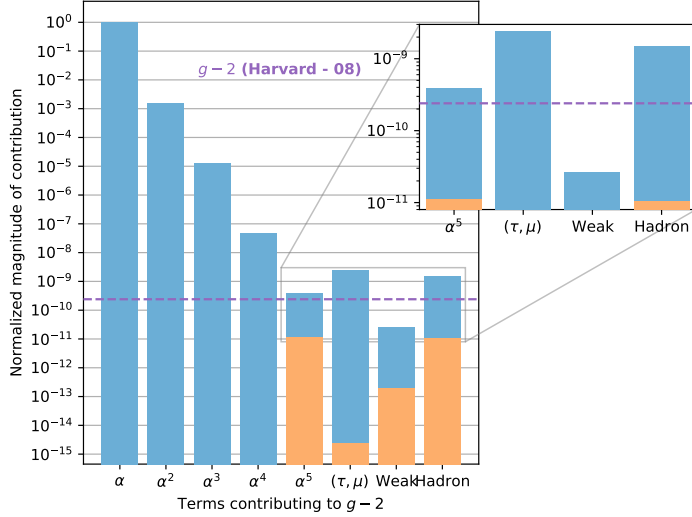


Figure 1 – Comparison of the order of magnitude of each contribution to the anomalous magnetic moment of the electron. In orange, are the uncertainty of those contributions. The dashed line is the uncertainty of the Harvard measurement.

The anomalous magnetic moment of the electron was measured in 2008 at Harvard<sup>3</sup>. The precision of this measurement (dashed line in Fig. 1) is smaller than the tenth order correction from QED and the hadronic corrections (but not the weak correction). However, in order to compare the experiment with the theory, one need to evaluate the QED correction with sufficient accuracy, which require a precise determination of  $\alpha$ .

## 2 Measuring the fine-structure constant with atom interferometry

Our determination of the fine-structure constant relies on the Rydberg constant. Indeed, it can be obtained using :

$$\alpha^2 = \frac{2R_\infty}{c} \frac{h}{m_e} = \frac{2R_\infty}{c} \frac{A_r(X)}{A_r(e)} \frac{h}{m_X} \quad (2)$$

where we have introduced in the equation the relative atomic masses  $A_r(X)$  of an atom X. All terms are known with relative accuracy below  $10^{-11}$  except the ratio  $\frac{h}{m_X}$  (or the mass  $m_X$ , since the Planck constant has a fixed value in the current SI). This is this last term whose precision has been improved.

The principle of the experiment relies on the measurement of the recoil velocity  $v_r$  of an atom that absorbs a photon of wavevector  $k$  :  $v_r = \frac{\hbar k}{m_X}$ . This velocity for a rubidium atom is of the order of 6 mm/s.

The experimental setup is depicted on Fig. 2. A cloud of rubidium atoms is prepared and cool down to  $4\mu\text{K}$  using standard laser cooling techniques. Atoms are then launched in a magnetically shielded tube where the atom interferometer is performed. The principle of the measurement as been described in several references and relies on combination of Bloch oscillations with a Ramsey-Bordé interferometer<sup>4,5</sup>. The technique of Bloch oscillations<sup>6</sup> allows to transfer an large number of recoil to the atoms (typically 1000) and the atom interferometer is used to measure the change of velocity induced by the recoil : thanks to Raman transitions, the atomic wave packets are split and recombined leading to interference fringes. The phase difference is scanned by changing the frequency of the last two Raman pulses. The central fringe is observed when the frequency compensate the Doppler effect induced by the Bloch oscillations.

Figure 3a shows an example of the spectrum obtained. The Doppler effect associated with the 1000 recoils is of the order of 15 MHz. The interfringe is 50 Hz and it is possible to measure the position of the central fringe with an uncertainty of about 50 mHz by fitting 50 points (about one minute of measurement). The relative uncertainty that we obtain on the determination of the recoil velocity is therefore  $6 \times 10^{-10}$  for one hour of integration, as confirmed by the Allan deviation of the measurements (Figure 3b).

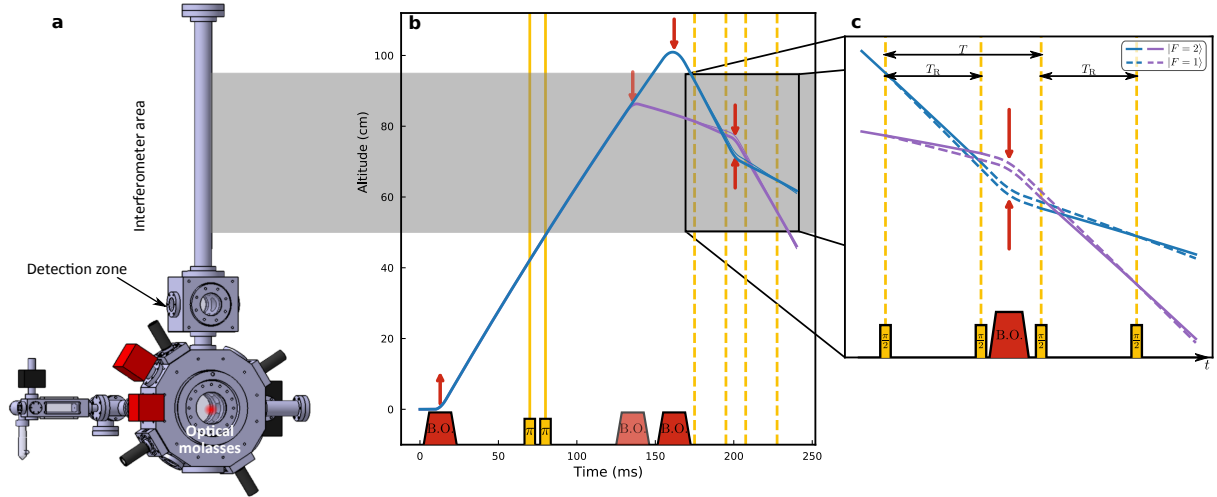


Figure 2 – Overview of the experimental setup (a) and of the trajectories of atoms (b and c). Atoms are prepared using an optical molasses. The time sequence is the following : atoms are launched upward using Bloch oscillations (B.O. in red). During their climb, two Raman  $\pi$  pulses are used to select the magnetically insensitive internal state. Once atoms have reached to top of the chamber, Bloch oscillations are used to decelerated them. The sequence of the interferometer starts then. It consists of four  $\pi/2$  Raman pulses and one Bloch acceleration (c).

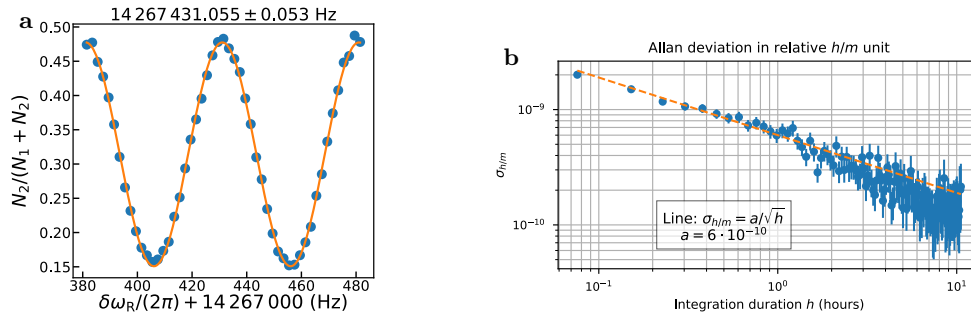


Figure 3 – a : example of spectrum : probability to be in the one output of the interferometer as a function of the Raman frequency. Each point corresponds to one realisation and takes about 1 second to acquire. b : Allan deviation of measurements taken over 48 continuous hours.

The statistics of our experiment allow us to have a relative uncertainty of  $8.5 \times 10^{-11}$  on the mass of the atom, or  $4.3 \times 10^{-11}$  on the fine-structure constant in 48 hours of integration. This good uncertainty represents an important asset of our experiment because it is significantly better compared to previous experiments<sup>7,5</sup>. It was thus possible to make measurements under different experimental conditions and thus to study systematic effects.

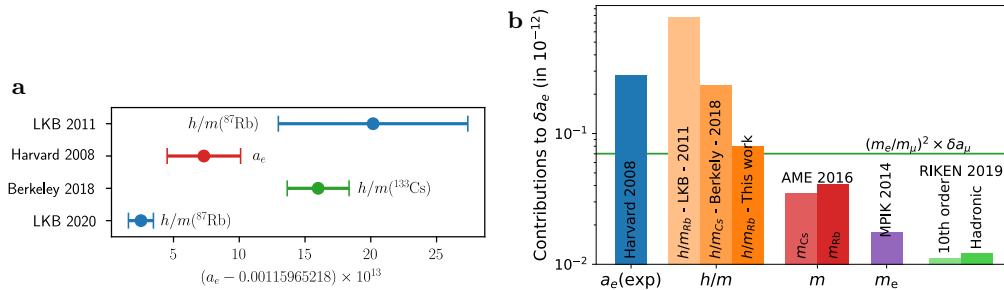
### 3 Systematic effects

The main limitation of the experiment comes from systematic effects and not statistics. The table 1 shows the relevant systematic effects evaluated in the experiment. A detailed description of each of these effects can be found in the reference<sup>8</sup> (and in particular in the method supplement).

We will focus here only on the largest correction, the Gouy phase. In order to extract the mass of the atom from the recoil velocity, one needs to know precisely the norm of the wavevector  $\mathbf{k}$ . In the case of a plane wave, the wavevector is proportional to the angular frequency of the laser  $\omega$  - quantity which can precisely measured :  $k = \frac{\omega}{c}$ . In a more general way, the wavevector is given by the phase gradient of the laser field. For a pure Gaussian beam, the phase can be

Source	Correction [ $10^{-11}$ ]	Relative uncertainty [ $10^{-11}$ ]
Gravity gradient	-0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave front curvature	0.6	0.3
Wave front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman phase shift	2.3	2.3
Index of refraction	0	< 0.1
Internal interaction	0	< 0.1
Light shift (two-photon transition)	-11.0	2.3
Second order Zeeman effect		0.1
Phase shifts in Raman phase lock loop	-39.8	0.6
Global systematic effects	64.2	6.8
Statistical uncertainty		2.4
Relative mass of $^{87}\text{Rb}$ : 86.909 180 531 0(60)		3.5
Relative mass of the electron: 5.485 799 090 65(16) $\cdot 10^{-4}$		1.5
Rydberg constant: 10 973 731.568 160(21) $\text{m}^{-1}$		0.1
Total: $\alpha^{-1} = 137.035 999 206(11)$		8.1

Table 1: Error budget of the experiment (corrections and their relative uncertainty). Value of the fine-structure constant obtained using eq. 2.



analytically calculated and, on the axis of the laser beam, there is a correction to the plane wave given by :

$$k = \frac{\omega}{c} - \frac{2}{kw^2}, \quad (3)$$

where  $w$  is the waist of the laser at the position of the measurement. This effect has been identified a long time ago<sup>9</sup>. It is related to the Gouy phase shift which states that there is a  $\pi$  phase shift at the waist of a Gaussian beam compared to a plane wave. Over the past years, we have studied more carefully this effect. We have obtained a general formula to calculate the wave vector in the paraxial approximation and shown that distortion on the wavefront due to imperfections in the optics can change significantly the wave vector and induce systematic effects<sup>10,11</sup>. A Monte-Carlo simulation of the trajectories of atoms was performed in order to calculate this effect which depends strongly on the position of atoms in the beam.

## 4 Results

The value of the fine-structure constant was used to compute a new theoretical value of the anomalous magnetic moment of the electron. The value obtained is displayed on the figure ??a and is compared to the direct experimental measurement (red) and to the other determinations from previous measurement of atom recoil (green and blue). This new value is in relatively good agreement with the direct measurement. However, it differs significantly with the Cs experiment at Berkeley ( $5.4\sigma$ ). This difference remains yet unexplained.

Figure ??b shows the origin of the uncertainties on the evaluation of the difference  $\delta a_e$  between the theoretical value and the experimental value of  $a_e$ . The largest contribution comes from the experimental measurement of  $a_e$ . Thanks to our result, the contribution of the  $h/m$  ratio measurement has decreased and is now close to the uncertainties coming from the mass ratios (see eq. 2).

The comparison between the experimental value and the theoretical value of  $a_e$  constitutes one of the most accurate tests of the standard model. It can be linked to the similar comparison performed on the muon. From a theoretical point of view, the limitation for the muon does not come from the fine-structure constant but from the hadronic contributions. This theoretical value has recently been re-evaluated<sup>17</sup>. Concerning the experiment, a recent measurement obtained at Fermilab<sup>18</sup> confirms the Brookhaven measurement<sup>19</sup>. The conclusion is that there is a discrepancy of  $2.51(59) \times 10^{-9}$  between the experiment and the theory for the muon.

The uncertainty and the discrepancy associated with the muon are much larger than that obtained for the electron, however the muon is much heavier than the electron and therefore more sensitive to potential new physics. A naive scaling implies that the ratio of  $\delta a_e/\delta a_\mu$  should scale as  $(m_e/m_\mu)^{2,13}$ . The green line on ??b shows the discrepancy of the muon scaled to the electron. Uncertainties are now too large to tell anything, however we can clearly see that with some reasonable improvement on experiment involved in this comparison, it should be possible to test the discrepancy observed for the muon is also present for the electron.

## 5 Conclusion

Using an atom interferometer to measure the recoil velocity of an atom that absorbs a photon, we have been able to measure precisely the mass of this atom. Thanks to a dedicated experimental setup, we achieved an unprecedented statistical uncertainty which allowed us to investigate many systematic effects. This measurement allow us to present a determination of the fine-structure constant  $\alpha$  with an unprecedented relative uncertainty of 81 ppt. This measurement differs significantly for the measurement made at Berkeley. This unexplained results need to be clarified.

In order to improve this measurement, several path are investigated in our group : a new measurement with rubidium 85 isotope could be performed in order to consolidate the measurement made with the 87 isotope. Several systematic effects depends on the temperature of the rubidium cloud. Using evaporative cooling, it is possible to reduce the temperature of the cloud and produce a Bose-Einstein condensate. Comparison of the measurement at different temperature with the model, will then be used to consolidate and improve the error budget. Finally, a new experiment setup with a longer interaction zone should be build in order to further increase the sensitivity.

In the next years, improvement of one order of magnitude is expected for the measurement of  $a_e$ <sup>20</sup>. Recently, measurement of relative atomic masses in a Penning with uncertainties of  $8 \times 10^{-12}$  have been demonstrated<sup>21</sup> and could be applied to rubidium atoms. It will then be possible to probe physics beyond the standard model with comparable information from both the electron and muon.

## References

1. W. E. Lamb and R. C. Retherford. “Fine Structure of the Hydrogen Atom by a Microwave Method”. *Physical Review* 72.3 (Aug. 1947), pp. 241–243. DOI: [10.1103/PhysRev.72.241](https://doi.org/10.1103/PhysRev.72.241).
2. P. Kusch and H. M. Foley. “The Magnetic Moment of the Electron”. *Physical Review* 74.3 (Aug. 1948), pp. 250–263. DOI: [10.1103/PhysRev.74.250](https://doi.org/10.1103/PhysRev.74.250).
3. D. Hanneke, S. Fogwell, and G. Gabrielse. “New Measurement of the Electron Magnetic Moment and the Fine Structure Constant”. *Physical Review Letters* 100.12 (Mar. 2008), p. 120801. DOI: [10.1103/PhysRevLett.100.120801](https://doi.org/10.1103/PhysRevLett.100.120801).
4. M. Cadoret et al. “Combination of Bloch Oscillations with a Ramsey-Bordé Interferometer: new determination of the Fine Structure Constant”. *Physical Review Letters* 101.23 (2008), p. 230801. DOI: [10.1103/PhysRevLett.101.230801](https://doi.org/10.1103/PhysRevLett.101.230801).

5. R. Bouchendira et al. “New Determination of the Fine Structure Constant and Test of the Quantum Electrodynamics”. *Physical Review Letters* 106.8 (Feb. 2011), p. 080801. DOI: [10.1103/PhysRevLett.106.080801](https://doi.org/10.1103/PhysRevLett.106.080801).
6. M. B. Dahan et al. “Bloch Oscillations of Atoms in an Optical Potential”. *Physical Review Letters* 76 (1996), p. 4508. DOI: [10.1103/PhysRevLett.76.4508](https://doi.org/10.1103/PhysRevLett.76.4508).
7. R. H. Parker et al. “Measurement of the Fine-Structure Constant as a Test of the —Standard Model”. *Science* 360.6385 (2018), pp. 191–195. DOI: [10.1126/science.aap7706](https://doi.org/10.1126/science.aap7706).
8. L. Morel et al. “Determination of the Fine-Structure Constant with an Accuracy of 81 Parts per Trillion”. *Nature* 588.7836 (Dec. 3, 2020), pp. 61–65. DOI: [10.1038/s41586-020-2964-7](https://doi.org/10.1038/s41586-020-2964-7).
9. D. S. Weiss, B. C. Young, and S. Chu. “Precision Measurement of the Photon Recoil of an Atom Using Atomic Interferometry”. *Physical Review Letters* 70.18 (May 1993), pp. 2706–2709. DOI: [10.1103/PhysRevLett.70.2706](https://doi.org/10.1103/PhysRevLett.70.2706).
10. S. Bade et al. “Observation of —Extra Photon Recoil in a —Distorted Optical Field”. *Physical Review Letters* 121.7 (Aug. 16, 2018), p. 073603. DOI: [10.1103/PhysRevLett.121.073603](https://doi.org/10.1103/PhysRevLett.121.073603).
11. P. Cladé et al. “State of the Art in the Determination of the Fine-Structure Constant and the Ratio  $h/m_u$ ”. *Comptes Rendus Physique* 20.1-2 (Jan. 2019), pp. 77–91. DOI: [10.1016/j.crhy.2018.12.003](https://doi.org/10.1016/j.crhy.2018.12.003).
12. G. F. Giudice, P. Paradisi, and M. Passera. “Testing New Physics with the Electron  $g - 2$ ”. *Journal of High Energy Physics* 2012.11 (Nov. 2012), p. 113. DOI: [10.1007/JHEP11\(2012\)113](https://doi.org/10.1007/JHEP11(2012)113).
13. F. Terranova and G. M. Tino. “Testing the  $a_\mu$  Anomaly in the Electron Sector through a Precise Measurement of  $h/m$ ”. *Physical Review A* 89.5 (May 19, 2014), p. 052118. DOI: [10.1103/PhysRevA.89.052118](https://doi.org/10.1103/PhysRevA.89.052118).
14. W. Huang et al. “The AME2016 Atomic Mass Evaluation : evaluation of Input Data; and Adjustment Procedures”. *Chinese Physics C* 41.3 (Mar. 2017), p. 030002. DOI: [10.1088/1674-1137/41/3/030002](https://doi.org/10.1088/1674-1137/41/3/030002).
15. S. Sturm et al. “High-Precision Measurement of the Atomic Mass of the Electron”. *Nature* 506.7489 (2014), pp. 467–470. DOI: [10.1038/nature13026](https://doi.org/10.1038/nature13026).
16. T. Aoyama, T. Kinoshita, and M. Nio. “Theory of the Anomalous Magnetic Moment of the Electron”. *Atoms* 7.1 (Feb. 22, 2019), p. 28. DOI: [10.3390/atoms7010028](https://doi.org/10.3390/atoms7010028).
17. T. Aoyama et al. “The Anomalous Magnetic Moment of the Muon in the Standard Model”. *Physics Reports* 887 (Dec. 2020), pp. 1–166. DOI: [10.1016/j.physrep.2020.07.006](https://doi.org/10.1016/j.physrep.2020.07.006).
18. B. Abi et al. “Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm”. *Physical Review Letters* 126.14 (Apr. 7, 2021), p. 141801. DOI: [10.1103/PhysRevLett.126.141801](https://doi.org/10.1103/PhysRevLett.126.141801).
19. G. W. Bennett et al. “Final Report of the E821 Muon Anomalous Magnetic Moment Measurement at BNL”. *Physical Review D* 73.7 (Apr. 2006), p. 072003. DOI: [10.1103/PhysRevD.73.072003](https://doi.org/10.1103/PhysRevD.73.072003).
20. G. Gabrielse et al. “Towards an Improved Test of the Standard Model’s Most Precise Prediction”. *Atoms* 7.2 (2019). DOI: [10.3390/atoms7020045](https://doi.org/10.3390/atoms7020045).
21. S. Rau et al. “Penning Trap Mass Measurements of the Deuteron and the HD+ Molecular Ion”. *Nature* 585.7823 (Sept. 3, 2020), pp. 43–47. DOI: [10.1038/s41586-020-2628-7](https://doi.org/10.1038/s41586-020-2628-7).