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# Non-reflective inlet conditions for the calculation of unsteady turbulent compressible flows at low Mach number

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## Résumé :

*Pour le calcul instationnaire d'écoulements turbulents compressibles subsoniques, en particulier en LES ou DNS, l'obtention de conditions d'entrée satisfaisantes pose problème. En effet, générer l'écoulement en entrée via l'imposition du champ de vitesse peut conduire à une réflexion non physique des ondes acoustiques qui remontent l'écoulement. Dans la présente contribution, une méthode permettant à la fois de filtrer ces ondes et d'imposer les variables requises en entrée est proposée. Cette méthode repose sur l'identification des rôles des variations d'amplitude des ondes acoustiques et convectives en entrée. Elle est testée numériquement pour la simulation d'un écoulement instationnaire à bas nombre de Mach avec un signal d'entrée turbulent.*

## Abstract :

*For the unsteady simulation of compressible subsonic flows (LES or DNS), the proper handling of the inlet boundary is a challenging task. Indeed, the inflow generation through the imposition of the velocity may lead to a non-physical reflection of the upstream acoustic waves. In the present contribution, a method that allows both to filter these waves and to impose the required variables is proposed. This method is based on the proper identification of the roles of the temporal rate of change of wave amplitudes at the inlet. The formulation obtained is tested numerically on unsteady flow at low Mach number. A test-case with an unsteady inlet signal that mimicks turbulence at the inlet is considered.*

**Mots clefs :** Inlet conditions; Turbulence; Low Mach number

## 1 Introduction

With compressible subsonic flows, difficulties may arise from boundary conditions. The imposition of variables such as velocity or pressure may lead to non-physical reflection of the outgoing acoustic waves on the boundary of the computational domain. The problem is particularly severe for inlet conditions when the inflow is turbulent, since then turbulence generation is carried out through the imposition of time varying inlet variables.

Linear relaxation methods, introduced by Rudy and Strikwerda [7], represent a trade-off between the imposition of variables and the partial reflection of acoustic waves. Following this approach, the linear relaxation must be applied to the velocity at the inlet, which becomes a low-pass filter for the reflected waves. Selle *et al.* [8] established that the relaxation coefficient is proportional to the highest frequency of the acoustic waves sent back into the computational domain. Therefore, the smaller the gap between the current and the imposed velocities is, the more the band of the reflected acoustic waves at the inlet is large. In short, velocity imposition and non-reflection at the inlet are incompatible if a basic linear relaxation approach is used.

As a cure to the problem of reflection, Polifke *et al.* [6] proposed to detect outgoing acoustic fluctuations and to eliminate them. For the inlet, the term that carries out this filtering is introduced in the temporal rate of change

of the ingoing wave amplitude, written under a linear relaxation form with the velocity variable. However, (1) the detection of the outgoing acoustic waves requires an additional computational cost, (2) the detection of the outgoing acoustic fluctuations among the turbulent fluctuations is satisfactory only for plane acoustic waves, and (3) the relaxation coefficient must be adjusted for each flow problem considered.

Following considerations of Prosser [4], Guézennec and Poinso [2] proposed inlet conditions based on the interpretation of low Mach number asymptotic expansions of the temporal rate of change of the ingoing acoustic wave amplitude, under the frozen turbulence hypothesis. An advantage of this approach is avoiding the linear relaxation, and thus the issue of the relaxation coefficient tuning. Another advantage is avoiding the detection of the upstream acoustic waves to be filtered. We will see however that the formulation of [2] leads to a non-satisfactory calculation of the acoustic energy level.

We propose here a formulation of the inlet conditions that preserves the advantages previously mentioned of the method of [2], and that allows to properly calculate the acoustic field in the computational domain. The proposed method is based on the following simple physical principle: At the inlet, the possible gap between the target value to be imposed on the velocity and its current value is due to the upstream waves, which are necessarily acoustic. At the inlet, this principle can be applied to density and temperature as well.

The presented numerical experiments are focused on low Mach number flows. The used numerical method is of pressure correction form, with a SIMPLE-type pressure-velocity coupling in combination with a momentum interpolation method (see [3] for a detailed description). We will show that small time-steps must be used to allow for the proper handling of the inflow turbulent fluctuations. The often mentioned problems of momentum interpolation methods when a small time-step is used for unsteady calculation (see *e.g.* [9]) are avoided thanks to the proper definition of the transporting velocity presented in [3].

## 2 Formulation of the inlet conditions

Consider a one-dimensional flow of air. The temporal rate of change of the wave amplitudes (or, in short, the wave amplitudes) are

$$\begin{aligned}\mathcal{L}_1 &= (v - c)\left(\frac{1}{\rho c}\partial_x p - \partial_x v\right), \\ \mathcal{L}_2 &= v\left(\partial_x \varrho - \frac{1}{c^2}\partial_x p\right), \\ \mathcal{L}_3 &= (v + c)\left(\frac{1}{\rho c}\partial_x p + \partial_x v\right).\end{aligned}$$

$\mathcal{L}_1$  and  $\mathcal{L}_3$  are the upstream and downstream acoustic wave amplitudes, respectively, and  $\mathcal{L}_2$  is the entropic wave amplitude [5]. These wave amplitudes satisfy the LODI<sup>1</sup> equations,

$$\partial_t \varrho + \frac{\rho}{2c}(\mathcal{L}_1 + \mathcal{L}_3) + \mathcal{L}_2 = 0, \quad (1a)$$

$$\partial_t v + \frac{1}{2}(\mathcal{L}_3 - \mathcal{L}_1) = 0, \quad (1b)$$

$$\partial_t p + \frac{\rho c}{2}(\mathcal{L}_1 + \mathcal{L}_3) = 0. \quad (1c)$$

The principle adopted for the inlet conditions is to consider that the possible gap between the target velocity  $v^\dagger$  and the current value of the velocity variable  $v$  is due to the upstream acoustic waves only, which correspond to  $\mathcal{L}_1$ . Then, as the downstream acoustic waves are not involved,  $\mathcal{L}_3 = 0$  is set in Eq. (1b), which results in  $\partial_t(v - v^\dagger) + \frac{1}{2}(0 - \mathcal{L}_1) = 0$ , or equivalently,

$$\mathcal{L}_3 = -2\partial_t v^\dagger. \quad (2)$$

Since  $\hat{\mathcal{L}}_3(\omega) = 2i\omega\hat{v}^\dagger(\omega)$ , where  $\hat{\cdot}$  designates the Fourier transform in time, the inlet reflection coefficient  $\hat{\mathcal{L}}_3/\hat{\mathcal{L}}_1$  is zero<sup>2</sup>. It is worth noticing that this result holds independently of the frequency of the outgoing acoustic signal. Thus, from Eq. (2), the suitable non-reflective properties of the inlet are immediately satisfied.

<sup>1</sup>For Locally One-Dimensional Inviscid [5].

<sup>2</sup>To define the inlet reflection coefficient, the target velocity  $v^\dagger$  is considered as constant, since the acoustic waves generated by the fluctuations of  $v^\dagger$  are not *reflected* waves.

An equation similar to Eq. (2) was proposed by Polifke *et al.* [6] if external acoustic signals to the computational domain were to be imposed. However, the acoustic filtering properties just mentioned which arise from Eq. (2) are not used in the method proposed in this reference, unlike the approach we suggest here. To obtain a non-reflective inlet treatment that allows to control the inlet velocity field and to generate vortices and turbulence at the inflow, Guézennec and Poinso [2] proposed to adopt the equations  $\mathcal{L}_3 = -\partial_t v^\dagger$  and  $\mathcal{L}_2 = 0$ . The expression of  $\mathcal{L}_3$  of this reference and our proposed expression in (2) differ by the factor 2. This difference arises from the interpretation of the low Mach number asymptotic expansions of the  $\mathcal{L}_i$ 's. Namely, it is established by Prosser [4] that  $\partial_t p = 0$  at the leading order of the low Mach number expansions, if Taylor's hypothesis [10] is adopted. In [2], the relation  $\mathcal{L}_3 = -[\partial_t p / (\rho c) + \partial_t v^\dagger] = -\partial_t v^\dagger$ , which therefore holds at the leading order, is used to introduce the target velocity into the expressions of the wave amplitudes. Indeed, at the leading order,  $\partial_t p = d_t p^{(0)}$ , where  $p^{(0)}$  is the thermodynamic part of the pressure (see [4]). Therefore, setting  $\partial_t p = 0$  for the total pressure  $p$  has as a consequence to consider the acoustic and the hydrodynamic parts of the pressure<sup>3</sup> as constant in time, which is not correct. Results obtained with the approach proposed in [2] are presented in the following section, and compared to those obtained with Eq. (2). Notice that, with the choice proposed in [2], the non-reflective property of the inlet is satisfied as for the method proposed here.

An argument similar to the one applied to the velocity LODI equation (1b) to obtain expression (2) of  $\mathcal{L}_3$  can be used for the density LODI equation (1a). When the density is imposed at the inlet in addition to the velocity, the entropic wave amplitude  $\mathcal{L}_2$  can be expressed in terms of  $\rho^\dagger$  and  $v^\dagger$  as follows. As previously done for the velocity, we consider that the gap between the target density  $\rho^\dagger$  and the current density  $\rho$  is due to the presence of the upstream acoustic waves. This leads to  $\partial_t(\rho - \rho^\dagger) + \frac{\rho}{2c}\mathcal{L}_1 = 0$ , and then, with (1a) and (2), to

$$\mathcal{L}_2 = \frac{\rho}{c}\partial_t v^\dagger - \partial_t \rho^\dagger.$$

If the temperature is imposed, using the temperature LODI equation,

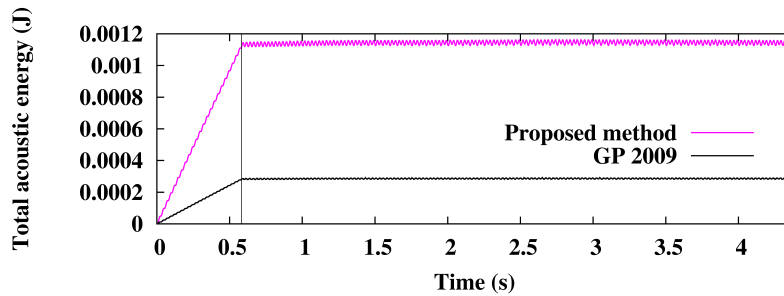
$$\partial_t T + \frac{T}{\rho}[(\gamma - 1)\frac{\rho}{2c}(\mathcal{L}_1 + \mathcal{L}_3) - \mathcal{L}_2] = 0,$$

the same argument results in

$$\mathcal{L}_2 = \frac{\rho}{T}\partial_t T^\dagger - (\gamma - 1)\frac{\rho}{c}\partial_t v^\dagger.$$

### 3 Test with harmonic inlet velocity

As a first test, we consider a flow of air in a pipe of length  $L = 100$  m divided into 5 000 cells of equal length. The inlet velocity is harmonic,  $v^\dagger(t) = \langle v \rangle [1 + A \sin(2\pi f t)]$ , with  $A = 10^{-2}$ ,  $f = 20$  Hz and  $\langle v \rangle = 0.30886$  m/s. Initially, there is no flow in the pipe. At the outlet, we write  $\mathcal{L}_1 = K_p(p - p^\dagger)$  with  $p^\dagger = 101\,300$  Pa. The acoustic waves are reflected at the outlet. In practice this is achieved by taking  $K_p = 10^3$ . The Mach number of the mean flow is approximately  $10^{-3}$ .

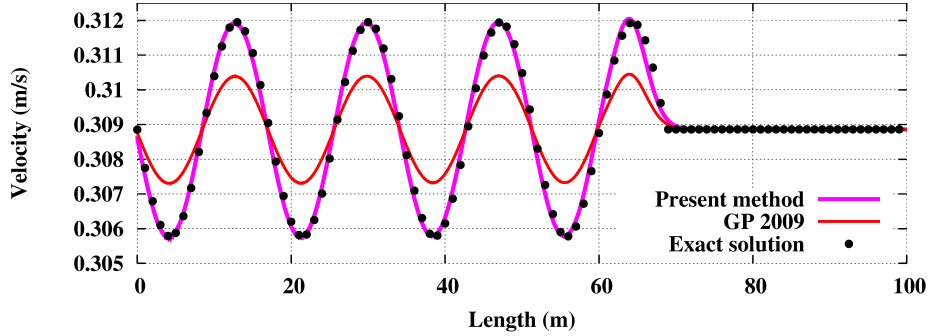


**Figure 1:** Test of Sec. 3. Total acoustic energy versus time. 'GP 2009' refers to the method proposed in [2].

The non-reflective inlet property is checked by calculating the total acoustic energy in the pipe,  $\frac{1}{2} \int_0^L \{ \rho_0 (\delta v)^2 + \frac{(\delta p)^2}{\rho_0 c_0^2} \}$  (see Figure 1). Here  $\rho_0$  and  $c_0$  designate the density and the speed of sound in the mean flow, and we set

<sup>3</sup>See *e.g.* [4].

$\delta p = p - p_0$  and  $\delta v = v - v_0$ , where  $p_0$  and  $v_0$  are the pressure and velocity in the mean flow. The simulation length is 15 times the duration necessary for an acoustic wave to cross the pipe. In Figure 1, the vertical line in  $t = 0,58$  s indicates the time necessary for forward and backward travel of the acoustic wave front. We observe that both methods lead to a non-reflective inlet. The acoustic energy level reached in the steady phase is significantly higher with Eq. (2) than with the method proposed in [2], however. The reason of the difference is evidenced in Figure 2. The amplitude of the imposed velocity is correct only when Eq. (2) is used. The amplitude obtained with the method proposed in [2] is twice smaller than the correct one.



**Figure 2:** Test of Sec. 3. Velocity field at time  $t = 0.2$  s. 'GP 2009' refers to the method proposed in [2].

#### 4 Turbulent velocity generation

We follow here Biferale *et al.* [1], where a method based on the solution of Langevin stochastic differential equations is proposed. The inlet velocity is written as  $v^\dagger(t) = \langle v \rangle + v'(t)$ , with

$$v'(t) = \sum_{k=1}^K g_k(t) \chi_1(t) \chi_2(t) \dots \chi_k(t). \quad (3)$$

In Eq. (3), each  $g_k(t)$  is solution of the linear Langevin equation

$$dX_t = -\frac{1}{\Lambda_k} X_t dt + \sigma_k \sqrt{\frac{2}{\Lambda_k}} dW_t, \quad (4)$$

and each  $\chi_j(t)$  is solution of the non-linear Langevin equation

$$dX_t = \frac{2}{\Lambda_j} \frac{dV}{dX_t} dt + \sqrt{\frac{2}{\Lambda_j}} dW_t. \quad (5)$$

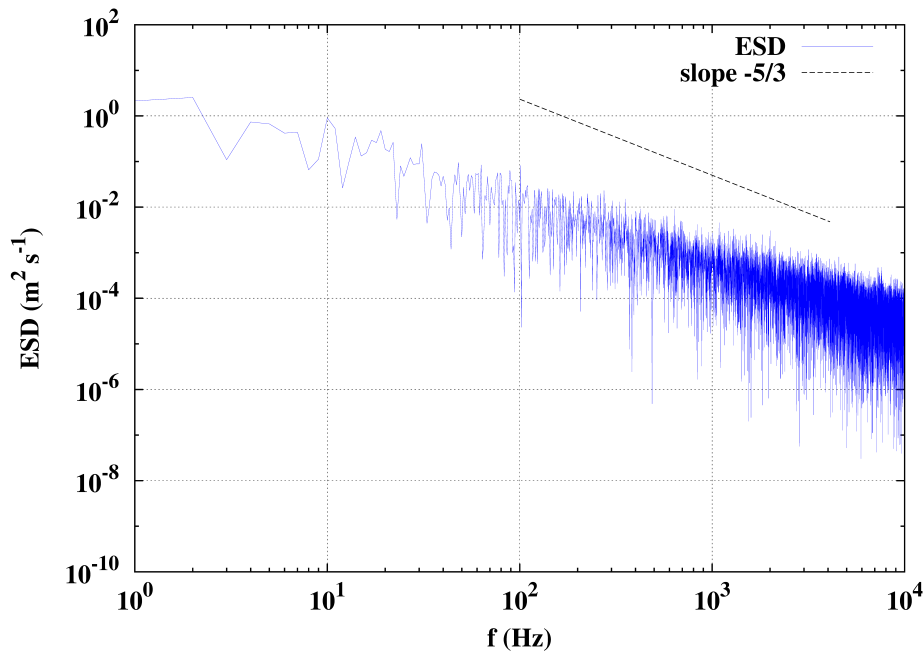
$W_t$  designates the Wiener process and we set  $\Lambda_k = 2^{-k}$  and  $\sigma_k = (\langle v \rangle \Lambda_k)^{1/3}$  for  $k = 1, 2, \dots, K$ . Further, setting  $0 < b < 1$ :

$$V(x) = \begin{cases} -2 \ln x & \text{if } (1-b)^{1/3} < x < (1+b)^{1/3}, \\ \infty & \text{else.} \end{cases}$$

The structure functions of the generated field, written in time through the Taylor hypothesis,  $S_q(\tau) = \langle |\delta v(t + \tau) - \delta v(t)|^q \rangle$ , are of order  $\tau^{\zeta_q}$  with  $\zeta_q = hq - \log_2 \langle x^q \rangle$ , which characterises the suitable multifractal structure of the signal (see [1]). In particular, the slope of the energy spectrum density of  $v'$  obtained matches the theoretical slope of  $-5/3$  (see Figure 3).

#### 5 Test with harmonic inlet velocity and superimposed turbulence

In this section, the aim is to illustrate on a toy problem how to impose a turbulent velocity signal at the inlet of the computational domain, while maintaining the non-reflective property already checked in Section 3 for

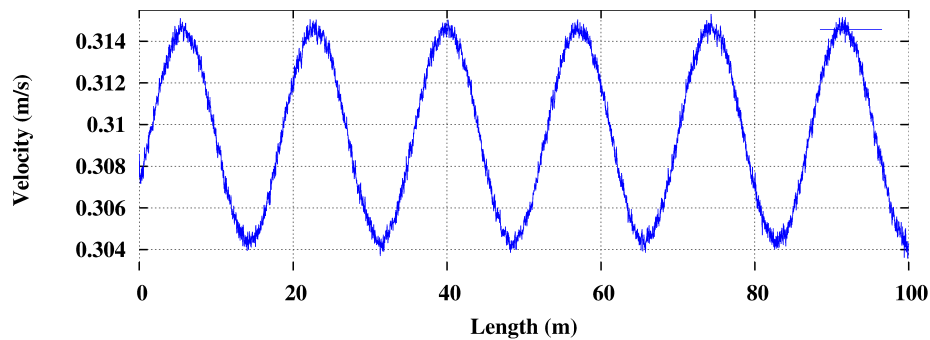


**Figure 3:** Energy spectrum density of  $v'$ , cf. Eq. (3).  $K = 15$ ,  $b = 0.9$ ,  $\langle v \rangle = 1$  m/s. Simulation length: 1 s.

a harmonic inlet velocity. The computational domain is the one of Section 3, with the same initial and outlet conditions. At the inlet, the turbulent velocity terms are generated with the method of Section 4 and introduced as perturbations of the target velocity adopted in Section 3, as

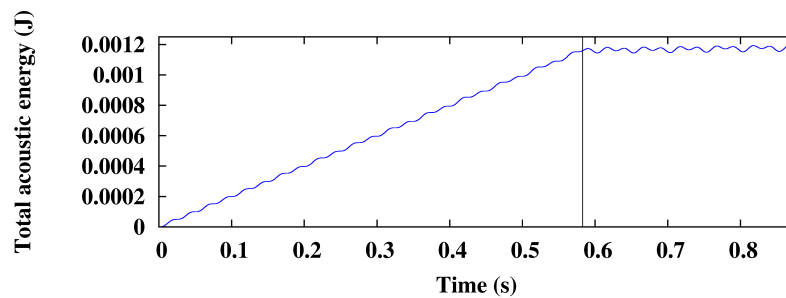
$$v^\dagger(t) = \langle v \rangle [1 + A \sin(2\pi ft)] + v'(t).$$

The values of  $\langle v \rangle$ ,  $A$  and  $f$  are the same as in Section 3. Eqs. (4) and (5) are solved with  $K = 15$  octaves. Practically the time-step  $\Delta t$  must be about  $2^{-K}/10$  (see [11]), so that the value of the convective CFL number is  $\text{CFL}_v = v\Delta t/\Delta x \simeq 0.30886 \times 3 \times 10^{-6}/(2 \times 10^{-2}) \simeq 5 \times 10^{-5}$ , and the acoustic CFL number is  $\text{CFL}_{v+c} = (1 + M^{-1})\text{CFL}_v \simeq 5 \times 10^{-2}$ . For such small time-steps, the convenience of the numerical method used in the present paper relies on the time consistency property of the transporting velocity, as evidenced in [3].



**Figure 4:** Test of Sec. 5. Velocity field at time  $t = 0.874$  s. Turbulence intensity: 1 %.

The velocity field obtained with  $K = 15$ ,  $b = 0.9$  and  $\text{CFL}_{v+c} = 5 \times 10^{-2}$ , is shown in Figure 4 at time  $t = 0.874$  s. Although this simulation length corresponds to three times the duration necessary for an acoustic wave to cross the pipe, no reflection is visible at the inlet that is the left side of the computational domain. The non-reflective behaviour of the inlet is confirmed by Figure 5.



**Figure 5:** Test of Sec. 5. Total acoustic energy versus time.

## 6 Conclusion

In this paper, we emphasized the importance of the proper identification of the role of the wave amplitudes to obtain non-reflective boundaries in compressible subsonic simulations. With the LODI equations solved at the inlet, a simple and effective way to impose a time varying velocity – and others quantities, if needed, such as density or temperature –, while satisfying non-reflective properties of the inlet, was proposed and tested on simple problems. It was shown that the inclusion of turbulent velocity fluctuations in the velocity imposed at the inlet does not deteriorate the non-reflective behaviour of the inlet.

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