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Experimental study of the Rayleigh-Bénard convection in a yield stress fluid

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Résumé :

Les travaux portent sur l'étude expérimentale de la convection de Rayleigh-Bénard dans le cas d'un fluide à seuil (Carbopol 980). Pour différentes puissances de chauffe P , en combinant les mesures, des différences de température entre les deux plans horizontaux parallèles, et la mesure locale du champ de vitesse au sein du fluide, deux régimes distincts ont été observés. Pour des puissances de chauffe inférieures à une valeur critique P_c , un régime purement conductif est observé. Une augmentation progressive de la puissance de chauffe au-delà de ce seuil révèle l'apparition d'un régime convectif qui se manifeste par une dépendance non linéaire, avec la puissance de chauffe, de la différence de température entre les plaques. En parallèle de cette observation, les mesures locales du champ de vitesse montrent une augmentation non linéaire de l'amplitude des cellules. Indépendamment de la concentration en Carbopol, la convection de Rayleigh-Bénard dans le gel de Carbopol apparaît comme une bifurcation imparfaite qui peut être modélisée par la théorie de Landau-Ginsburg des transitions de phase. Les résultats obtenus dans cette étude sont au final comparés à ceux existants dans la littérature.

Abstract :

An experimental study of the Rayleigh-Bénard convection in a yield stress fluid (Carbopol 980) is presented. By combined integral measurements of the temperature difference between two parallel plates and local flow velocity in a wide range of heating powers P , two distinct regimes are observed. For heating powers smaller than a critical value P_c a purely conductive regime is observed. A gradual increase of the heating power beyond this onset reveals a convective regime manifested through a nonlinear dependence of the temperature difference between plates on the heating power. Local measurements of the flow fields reveal a nonlinear increase of the roll pattern amplitude. Regardless the Carbopol concentration, the Rayleigh-Bénard convection in the Carbopol gel is found to appear as an imperfect bifurcation that can be correctly modelled by the Landau theory of phase transitions. The paper closes with a comparison of our findings with existing previous works.

Mots clefs : Rayleigh-Bénard convection ; yield stress ; continuous bifurcation

1 Introduction

During the past several decades, the yield stress fluids have found an increasing number of practical applications in various industrial settings related to the cosmetic, food and pharmaceutical industry. From a fundamental perspective, such fluids continue triggering debates and posing various challenges. Probably the best known debate concerning the yield stress fluids is related to the very existence of the yield stress behaviour defined as the onset of a flow when the applied stresses exceed a critical value, [1]. The classical theoretical frameworks of yielding as proposed by Bingham and Herschel-Bulkley predict a direct transition from a solid behaviour to a fluid one.

However, recent hydrodynamic studies, have demonstrated that in spite of its apparent simplicity, the solid-fluid transition in yield stress materials remains elusive and can not always be fully captured by the above mentioned models. Among such studies we mention the sedimentation of spherical objects in a yield stress fluid which have revealed a negative wake phenomenon, [2]. The experimentally observed flow patterns could not be explained within the classical Bingham and/or Herschel-Bulkley frameworks and a new model accounting for the role of elastic effects around the yielding transition and the irreversibility of deformation states has been recently proposed, [3].

Another classical hydrodynamic phenomenon little understood in the context of yield stress fluids is the Rayleigh-Bénard instability where a fluid contained in between two parallel horizontal plates which are differentially heated/cooled. For the yield stress fluid case, understanding the transition to convective states might shed light onto various geophysical problems involving non-isothermal flows of magma. The start up of the Rayleigh-Bénard instability corresponds to the point when the temperature difference between plates exceeds a critical value ΔT_c and the buoyancy stresses overcome the viscous one. The balance between these two contributions is quantified by the Rayleigh number, Ra . Beyond a critical value Ra_c , the balance is broken and thermo-convective rolls appear. For the case of yield stress fluids, the instability can only triggered if buoyancy force is large enough to overcome both the yield stress effect and the viscous ones. The stability analysis performed by Zhang et al [4] indicates that in the presence of yield stress the system is linearly stable. Rust and Balmforth [5] have found experimentally that the presence of the yield stress inhibits the Rayleigh Bénard convection but a sufficiently strong perturbation (whose magnitude increases with the yield stress) may, however, initiate it. Thus, the central prediction of Rust and Balmforth is that the system is linearly unstable. In addition, for large enough values of the perturbation, the transition to convective states occurs via a sub-critical bifurcation.

The global aim of the present work is first to probe the existence of the Rayleigh Bénard instability in a yield stress fluid (Carbopol 980) and to characterise its main features by both integral and local measurements with a particular focus on the nature of the bifurcation.

2 Experimental setup and methods

The Rayleigh-Bénard set-up is schematically illustrated in Fig. 1. The convection cell consists of four vertical plexiglass walls bonded with silicone, (L=38 cm, W=18cm, H=2cm). The top/bottom plates are made of polycarbonate and are 1.6 cm thick. The bottom plate is heated electrically and the heating power is controlled by controlling the intensity I of the current generated by the current supply **CS**, $P = R I^2$ (R stands for the Ohmic resistance of the heating circuit).

The temperature of the top plate T_t was maintained constant through our experiments by means of a cold water circuit embedded within the plate. We illustrate in Fig.2 the method by which the temperature gradients are measured. The steady state values of the temperature of the top/bottom plates T_t, T_b are obtained by linear extrapolation of the temperature readings of 6 thermocouples embedded in the top/bottom plates. The main advantage of this technique is that it is entirely non-intrusive. The accuracy of the temperature measurements is insured by passing the thermocouple's signals to a reference box **RB** via an analogue to digital (**A2D**) multi-channel acquisition block.

The measurements of the temperature gradient between the top/bottom plates allows one to detect the onset of Rayleigh-Bénard instability (the procedure will be later detailed). Simultaneously with

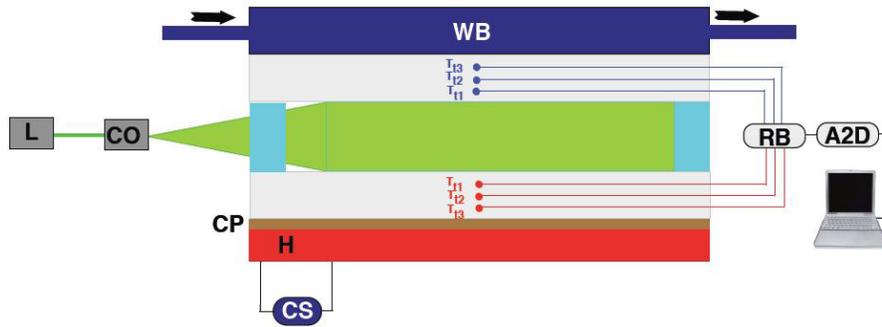


FIGURE 1 – Rayleigh Bénard experimental setup. The symbols are : L : laser, CO : cylindrical optic, CP : copper plate, WB : water bath, A2D : analogical to digital treatment, RB : reference box.

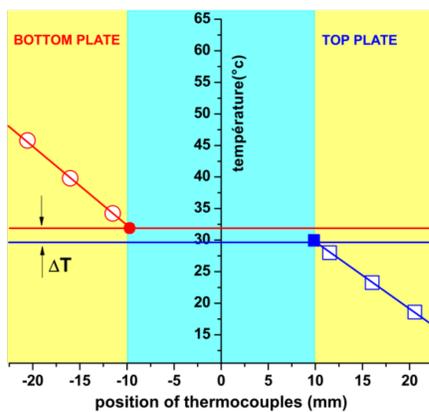


FIGURE 2 – Example of temperature measurements. The circles/squares refer to the thermocouples of the bottom/top plates, respectively. The dotted lines are linear fitting functions. The full symbols are the data linearly extrapolated at the contact between the fluid and the top/bottom plates.

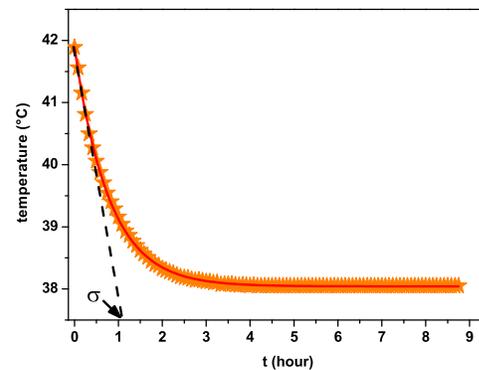


FIGURE 3 – Example of transient temperature measurement. The full line is an exponential fit.

the measurements of the temperature difference between plates ΔT , the convection patterns are investigated using the Digital Particle Image Velocimetry (**DPIV**) technique, [6, 7]. For this purpose, the cavity is illuminated from the side with a vertical laser sheet. The onset of the convective instability is detected by integral measurements of the temperature between plates and local measurements of the velocity field (the amplitude of the convective patterns). The working fluids are seeded with polyamide tracers (20 μm in diameter). The flow patterns are visualised frontally with a digital camera at a 1 fps speed. The spatial structure and temporal dynamics of the flow fields are obtained for Carbopol solutions with different concentrations between 0.05 %wt and 0.11 %wt.

3 Results

The experimental setup and measuring techniques have been first validated by conducting experiments with a Newtonian fluid, pure Glycerin. The Rayleigh number is defined as :

$$Ra = \frac{\alpha \Delta T g d^3}{k \cdot \nu} \quad (1)$$

where α is the coefficient of thermal expansion, g the gravitational constant, k the thermal diffusivity, ν the kinematic viscosity, ΔT the temperature difference measured between the plates and d is the distance between plates. The experimentally measured values of the physical parameters and the Rayleigh number for the Glycerin are : $\alpha = 5 \cdot 10^{-4} \text{K}^{-1}$, $g = 9.8 \text{m}^2/\text{s}$, $k = 1.37 \cdot 10^{-7} \text{m}^2/\text{s}$ and

$\nu = 872 \cdot 10^{-6} m^2 s^{-1}$. Measurements of the temperature difference ΔT between the plates for a Glycerin solution are presented in Fig. 4, for both increasing and decreasing heating powers P . A linear increase of ΔT with P which corresponds to a purely conductive heat transfer regime is observed up to a critical value of the heating power $P_c \approx 16.32 W$. Beyond this onset the dependence becomes sub linear consistently with a mixed conductive-convective heat transfer regime. Based on the material parameters enumerated above, the critical Rayleigh number corresponding to the onset of the thermal convection can be estimated $Ra_c \approx 1774$ which is in a fair agreement with the theoretical value $Ra_c^t = 1708$ given in [8]. The dependence of the reduced temperature difference $\Delta T_r = \frac{\Delta T}{\Delta T_{lin}} - 1$ on to the reduced power $P_r = P/P_c - 1$ is presented on Fig. 5 (a). Here ΔT_{lin} represents the linear temperature difference measured within the conductive regime (see the full line in Fig. 4). In agreement with the theoretical predictions, the reduced temperature difference ΔT_r increases linearly with the heating power. Measurements of the convection amplitude are presented in Fig. 5. Above the onset of convection the velocity amplitude follows a square root dependence on the heat flux $V = A\sqrt{\frac{P}{P_c} - 1}$.

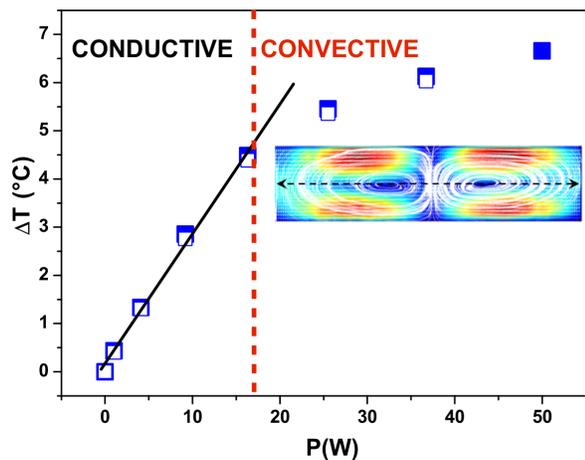


FIGURE 4 – Dependence of the temperature gradient ΔT within Glycerin on the heat flux P . The full/empty symbols refer to increasing/decreasing heat flux. A typical convection pattern is presented in the inset.

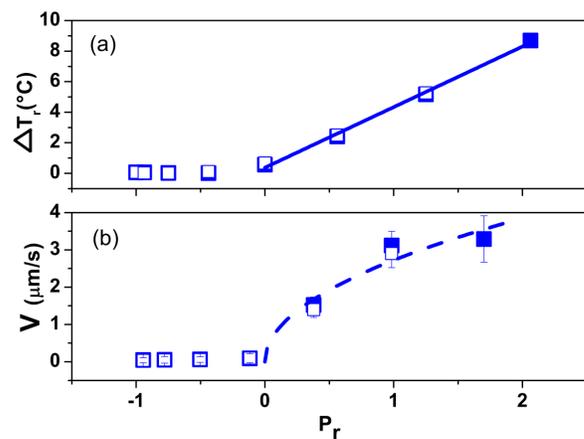


FIGURE 5 – (a) Dependence of the reduced temperature ΔT_r on the reduced power. The line is a linear fit. (b) Dependence of the pattern amplitude V on the reduced power P_r . The line is a Landau fit.

These measurements clearly identify the transition to convective states within a Newtonian fluid as an imperfect bifurcation in agreement with both theoretical predictions and previous experimental results.

After having validated the experimental setup and measuring techniques we repeated the same experiments as glycerin with Carbopol 980. Six concentrations of Carbopol have been exploited from 0.05 %wt to 0.11 %wt. The rheological properties of the Carbopol gels have been investigated using a MAARS rheometer.

The flow curve presented in Fig. 6 reveals three distinct deformation regimes. For low applied stresses, an elastic solid deformation regime is observed (constant deformation rate). For the largest values of the applied stresses a fully yielded (fluid) regime that can be accurately described by the Herschel-Bulkley model is observed. The transition from a solid regime to a fluid regime is not direct but mediated by an intermediate regime where the solid and fluid fingerprints coexist. An important and previously ignored (for a Carbopol gel) feature of the increasing/decreasing stress flow curves is the irreversibility observed within the solid and the intermediate regime. The yield stress of the fluid τ_y is obtained from a Herschel-Bulkley fit of the data acquired within the fully yielded regime. The dependence of the apparent yield stress τ_y is linear with the Carbopol concentration, Fig. 7.

By monitoring the error bars of the yield stress data presented in Fig. 7 one can clearly see that the Carbopol concentrations we have investigated are larger than the overlap concentration and we indeed have a yield stress behaviour.

In Fig. 8 we present the dependence of the temperature difference ΔT on the heating power for six

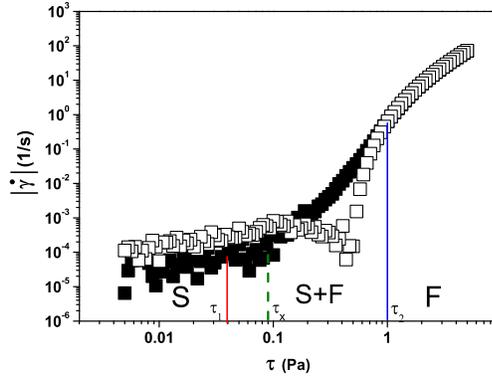


FIGURE 6 – Controlled stress flow curve for a 0.1% wt Carbopol solution. Dependence of the absolute value of the rate of shear on the applied stress τ . The full/empty symbols refer to increasing/decreasing applied stresses.

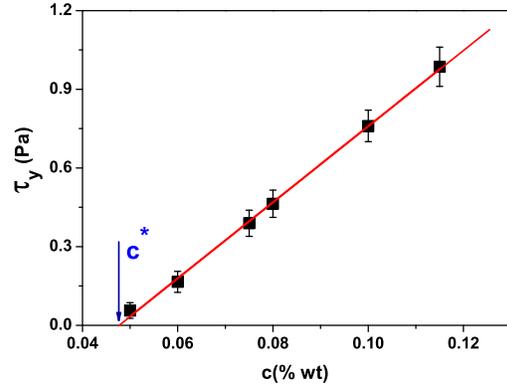


FIGURE 7 – Dependence of the yield stress τ_y on the concentration of Carbopol. The line is a linear fit and c^* is the overlap concentration.

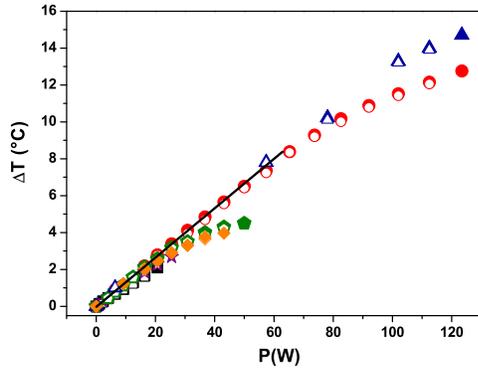


FIGURE 8 – Dependence of temperature gradient on the heat flux for six values of the Carbopol concentration : triangles : $c = 0.11$ %wt, circles : $c = 0.1$ %wt, pentagons : $c = 0.08$ %wt, rhombs : $c = 0.075$ %wt, stars : $c = 0.06$ %wt, squares : $c = 0.05$ %wt. The full/empty symbols refer to increasing/decreasing heat flux.

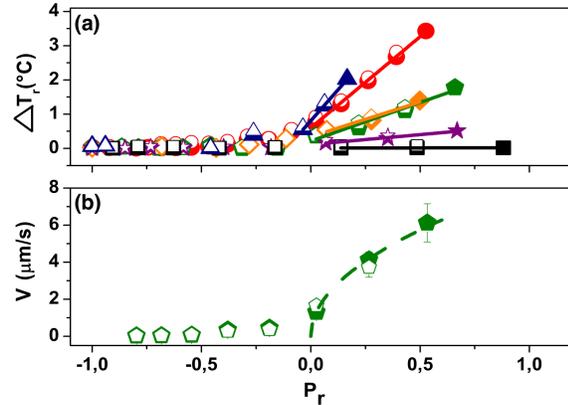


FIGURE 9 – (a) Dependence of the reduced temperature ΔT_r on the reduced power P_r for various Carbopol concentrations, see Fig. 8. The full lines are linear fitting functions. (b) Dependence the convection pattern amplitude V on the reduced power P_r . The dotted line is a square root fit function according to the Landau model.

values of the Carbopol concentration (larger than the overlap concentration c^*) and ranging in between 0.05 %wt and 0.11 %wt. Below a certain onset ΔT_c which increases with the Carbopol concentration, each of these measurements reveal a purely conductive regime characterised by a linear dependence of the temperature difference ΔT on the heating power (heat flux) P . Beyond the onset ΔT_c the convection occurs and the slope of the dependence ΔT vs P decreases. Although the rheological behaviour of the Carbopol 980 gel presents a clear hysteresis (see Fig. 6), the experimental results of Rayleigh-Bénard instability shows a reversible behaviour of temperature gradient on the heating flux. This is in itself an interesting result which awaits a theoretical explanation. We present in Fig. 9(a) the dependence of the reduced temperature difference ΔT_r on the reduced power P_r . Regardless the polymer concentration, a linear correlation is observed beyond the onset of instability $P_r > 0$. As in the Newtonian case, the amplitude of convective pattern V follows a square root scaling with the reduced power in agreement with the Landau theory of continuous (reversible) bifurcations, Fig. 9(b).

To conclude, we have demonstrated experimentally that the Rayleigh-Bénard convection can be triggered with a viscoplastic fluid (Carbopol 980). This result contrasts with the linear stability predictions within the framework of the Bingham model presented in Ref. [4]. This simply indicates that, a theoretical understanding of the Rayleigh-Bénard convection in a yield stress fluid needs to account for a different rheological picture able to properly describe the startup of the fluid deformation within a range of very low rates of deformation (of order of $10^{-3} s^{-1}$ or lower).

Whereas several previous theoretical studies suggest that the yield stress either completely suppresses (see Ref. [4]) or inhibits (see Ref. [5]) the (onset of) convection, our experimental study indicates a possible steady state of convection with a clear apparition of Rayleigh-Bénard patterns can be achieved for several concentrations of Carbopol 980. Quite interestingly, we find that the transition to convective states in a Carbopol gel is a continuous one and it can be accurately described by the Landau theory.

4 Conclusions

An experimental investigation of the Rayleigh-Bénard convection in a yield stress fluid by combined local measurements of the flow fields and integral temperature measurements is presented. First, the experimental system and the measuring techniques are validated by conducting experiments with a Newtonian fluid. The classical continuous imperfect bifurcation to convective states is confirmed (Fig. 2) and the critical Rayleigh number is found in a fair agreement with both theoretical prediction and previous experimental studies. Similar measurements were performed for Carbopol solutions at various concentrations (various yield stresses). For each of these yield stress fluids an imperfect bifurcation to convective states is found. As the rheological behaviour of the solutions is highly nonlinear and irreversible upon increasing/decreasing applied stresses this result is somewhat surprising. In spite of rheological nonlinearity and irreversibility of deformation states illustrated in Fig. 6 the reversibility of the bifurcation towards convective states in a Carbopol gel is preserved. For each of the solutions investigated the dependencies of the local amplitude of the convection (time averaged flow speed) on the heating power P can be described by the Landau model where the temperature difference present a linear behaviour on the heating power.

Future theoretical and numerical developments able to rationalise our experimental findings are needed. What is clear for now, however, is that Rayleigh-Bénard stability problem with elasto-viscoplastic fluids should be tackled via non-linear methods and, perhaps, a more sophisticated rheological picture of the solid-fluid transition in a yield stress fluid should be employed, [3].

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