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# Probabilistic Characterization of Uncertainties and Spatial Variability of Material Properties from NDT measurements

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## Abstract :

*The localization of weak properties or bad behavior of a structure is still a very challenging task that concentrates the improvement of Non Destructive Testing (NDT) tools on more efficiency and higher structural coverage. In case of random loading or material properties, this challenge is arduous because of the limited number of measures and the quasi-infinite potential positions of local failures. The paper shows that the stationary property is useful to find the minimum quantity of NDT measurements and their position for a given quality assessment. It is shown that a two stages procedure allows us (i) to quantify the properties of the ergodic, stationary field (ii) to assess the distribution of the characteristics. The optimization is reached following a criterion based on confidence intervals of the statistics.*

**Key work : Confidence interval, probability of interval, spatial variability, Karhunen-Loève, optimization of inspection**

## 1 Introduction

Industrially developed countries face more and more the maintenance of existing infrastructures. This challenge began in the 80's with nuclear power stations or offshore oil and gas platforms for which safety requirements were very provided very early because of the high level of consequences of failure. Specific methods for Inspection Maintenance and Repair were devoted to each industrial field. However, their generalization to other assets is not immediate because of the required money and human resources. That is why methods based on visual inspection have been developed with a real potential [3] and that new methods and techniques of monitoring, within Bridge Management Systems (BMS), are under development [7-8]. Various other assets than bridges assets are now under consideration: sewer networks [2], wharfs. In the probabilistic thinking, this question is addressed under the methods of "random variable updating" that have been widely addressed during the two last decades. Random variable updating is very useful when a prior distribution is known and data coming from inspections or monitored systems are collected for condition assessment and reliability updating [10]. Basically, the so-called RBI generalizes these approaches in the case of non-perfect inspections by linking inspection and decisions [4].

Moreover, establishing a prior distribution for a whole set of material parameters of existing structures is a very difficult task and probabilistic updating must be applied with care. In parallel, it is more and more feasible and inexpensive to collect a huge quantity of data, by an operator or a new generation of remotely operating vehicles and robots devoted to Non Destructive Testing tools (interferometer robot for instance: [5]). On the whole, the stochastic nature of material properties and deterioration processes as well as the factors that reduce the quality of the inspections that can be used to quantify them, trans-form the management of deteriorating systems in a major challenge for owners and operators. In the light of this, we suggest providing a flowchart for optimizing the number of inspections and provide the statistics of material characteristics on real structures with spatial dependency starting from five main assumptions about the inspection and the random field modeling one-dimensional.

The goal of the paper is to present and solve the optimization problem of the total number of measurements after the two stages. The paper first presents in the second part usual representation of spatial variability in civil engineering through stochastic field modeling and focuses on stationary stochastic fields: the Karhunen-Loève decomposition is reminded for modeling this spatial variability. Then the third part presents the two-

stage inspection strategy in view (i) to assess the correlation function and (ii) to get a set of fairly dependent measurements to compute statistics of the marginal distribution. A criterion of quality of the modeling after inspection is suggested. Original parameters are highlighted in view to represent the role of the distance between inspections. The paper ends in the fourth part with an application to an academic study case in view to illustrate the potential of the methodology and the suggested parameters.

## 2 Spatial random field modeling

### 2.1 Usual approaches for spatial field modeling

The stochastic field could take several forms more or less complicated. The most simple when the degradation can be considered as homogeneous is the stationary stochastic field that can be used, for instance, to model chloride distribution, concrete properties [6], or soil properties. In some cases, when a stochastic process is influenced by several phenomena that vary with time (sea wave for instance) or with space (concreting of a structure in several steps with heterogeneity of these steps), it can be modeled as piecewise stationary. This more sophisticated stochastic field can represent, for example, the variability of concreting materials by pieces or the corrosion of structures located in contiguous environments with different characteristics [1]. In this case, a piecewise stationary stochastic field could be used to have a good representation of the loss of thickness.

### 2.2 Main assumptions for the stochastic modeling

Starting from the previous sections and to limit the study, we consider five main assumptions about the inspection and the random field modeling: (i) the stochastic field is statically homogeneous, and only few information on the marginal distribution is known: the type of the unique probability density distribution. In the following applications we consider a Gaussian field; (ii) a huge number (more than 100) of measurements can be performed; (iii) second order stationary stochastic field can piecewise or totally describe the spatial fields; (iv) inspections are regularly spaced for simplicity of the campaign planning; and (v) inspections are considered as perfect.

Given a probability space  $(\Omega, F, P)$ , a stochastic field or process with state space  $Z$  is a collection of  $Z$ -valued random variables indexed respectively by a set  $s$  “space” or  $t$  “time”. We consider here homogeneous fields only. This means that the marginal distribution of  $Z(x_j, \theta)$  does not depend on the location. A stochastic field is second order stationary if it follows three main properties: (i) expectation  $E[Z(x, \theta)]$  does not depend on the location  $x$  –i.e.,  $E[Z(x, \theta)] = E[Z]$ ; (ii) variance  $V[Z(x, \theta)]$  does not depend on the location  $x$  –i.e.,  $V[Z(x, \theta)] = V[Z]$ ; and (iii) spatial covariance  $COV[Z(x, \theta), Z(x', \theta)]$  depends only on the distance  $(x - x')$ . Thus, the second order stationary is a property restricted to the two first probabilistic moments. It can be shown that geometries of welds for ships or the spatial distribution of chloride concentration in reinforced concrete structures can be represented by stationary stochastic fields. For instance, FIG.1 presents the correlation function of the chloride diffusion coefficient as function of the distance between two points (duratiNet project: <http://www.duratinet.org>, following [6]). In this paper, we select a Karhunen-Loève expansion to represent the stochastic field of resistance of a structure  $Z(x, \theta)$ . This expansion represents a random field as a combination of orthogonal functions on a bounded interval  $[-a, a]$ :

$$Z(x, \theta) = \mu_Z + \sigma_Z \cdot \sum_{i=1}^n \sqrt{\lambda_i} \cdot \xi_i(\theta) \cdot f_i(x) \quad (1)$$

where,  $\mu_Z$  is the mean of the field  $Z$ ,  $\sigma_Z$  is the standard deviation of the statistically homogeneous field  $Z$ ,  $n$  is number of terms in the expansion,  $\xi_i$  is a set of centered Gaussian random variables and  $\lambda_i$  and  $f_i$  are, respectively, the eigenvalues and eigen functions of the correlation function  $\rho(\Delta x)$ . It is possible to analytically determine the eigenvalues  $\lambda_i$  and eigen functions  $f_i$  for some correlation functions. For example, it can be determined if we assume that the field is second order stationary and we use an exponential correlation function:

$$\rho(\Delta x = x_1 - x_2) = \exp\left(-\frac{|\Delta x|}{b}\right); 0 < b \quad (2)$$

where  $b$  is the correlation length and  $\Delta x \in [-a, a]$

:

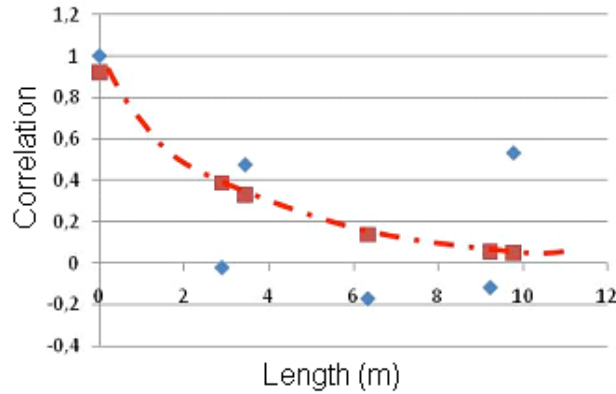


FIG.1-Spatial correlation of the chloride diffusion coefficient in concrete – data Trinity College of Dublin.

### 3 Inspection strategy and goals

#### 3.1 Concept of Inspection Distance Threshold

The two stages inspection we suggest should provide both the parameters of the spatial correlation function and independent events that characterize the marginal distribution of  $Z$ . We consider in this paper a one-dimensional spatial field and we will apply the methodology on a set of trajectories: these trajectories could be a set of 1D components (beams). In view to limit the inspection costs we first inspect a trajectory with a “sufficiently low distance  $L_b$ ” to assess the shape of the correlation function (2): for instance the shape parameter  $b$ . In view to sample independent events for  $Z$ , we need to inspect the trajectories with a “sufficiently high distance  $L_c$ ” to get fairly correlated events. These “sufficiently low and high distances” can be gathered under the generic term *IDT* for Inspection Distance Threshold. Thus  $L_b$  and  $L_c$  should satisfy:  $L_b \in ]0, IDT[$ ;  $L_c \in ]IDT, L[$  where  $L$  is the length of the trajectory. For illustrating the methodology, we consider in the following a set of 1D-components (beams) with infinite length  $L \sim \infty$ : we don’t discuss the case of components with a limited size, especially those where  $L < L_b$  for which it is quite impossible to characterize the spatial variability. The *IDT* is defined by assuming that after a given distance, the events measured from an inspection can be assumed as statistically independent. A Spatial Correlation Threshold *SCT* of the spatial correlation gives this weak correlation. For instance in FIG.1 (red line) for  $SCT=0.4$ ,  $IDT=3$  meters. It is linked with *IDT* by the relationship:

$$IDT = -b \ln(SCT) \quad (3)$$

#### 3.2 Assessment of the correlation function from discrete inspections

We assume that the stationary stochastic field can be characterized by an autocorrelation function (ACF) considered for spatial variability of structures with their parameter, called length of correlation  $b$ . A complete overview of the auto-correlation functions and their application is available in [5]. Let us focus on the assessment of this function from experimental data (sensors or NDT tests). In the first procedure, reported by Li[7], the Maximum Likelihood Estimate method (MLE) is used in which different values for the model parameter of the proposed ACF model is assumed and the value that maximizes the corresponding MLE is taken as the model parameter. In the second procedure, proposed by Vanmarcke [11]. In this paper, we select an exponential ACF (see (2)) and we use the MLE for the estimation of  $b$  (4).

$$L_h = \prod_{i=1}^k \left( \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v_i^2}{2}\right) \right) = \left( \frac{1}{\sqrt{2\pi}} \right)^k \exp\left(-\frac{\sum_{i=1}^k v_i^2}{2}\right) \quad (4)$$

where  $v_i$  is the  $i^{th}$  component of the vector of independent standard values.

#### 3.3 Number of inspections to optimize in the two stages procedure

When focusing on a practical application, we will have to optimize the total number of discrete inspections  $N=Np*Ns*Nt$  where  $Np$  is the number of repetitive tests for reducing the error of inspection,  $Ns$  is the number of inspected sections and  $Nt$  the number of trajectories. Here the error of inspection is neglected and we inspect separately and firstly  $Ns_1$  sections on the first trajectory to assess the spatial correlation function,

thus  $N = N_s + N_s*(N_t - 1)$  (see FIG.2).

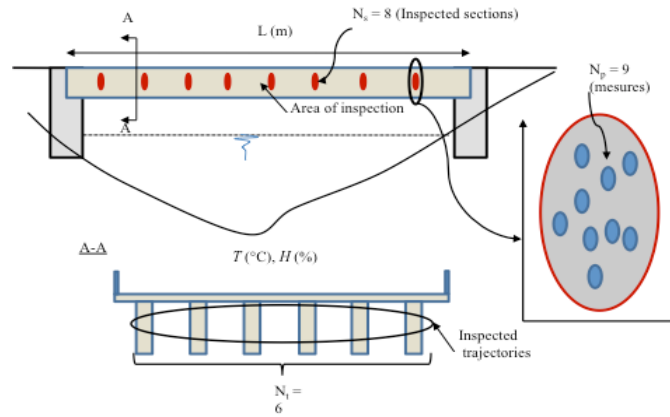


FIG.2-Definition of the number of inspections.

Our objective is to optimize the number of inspections in view to reach a given quality of the result.

### 3.4 Definition of a reliability oriented measure of quality of inspection

The quality of a result of inspection can be expressed by several concepts. If we consider the error of measurement risk oriented measures have been developed: PoD, PFA, PoI, PGA and PWA. We select in this paper a confidence interval of both the mean  $\mu$  and the standard deviation  $\sigma$  expressed as a percentage  $\varepsilon$  of  $\mu$  and  $\sigma$  respectively. We carry out Monte-Carlo simulations to estimate the bounds of the confidence interval for a target probability for the inspection respectively  $P_{ii,\mu}$  and  $P_{ii,\sigma}$ . In a reliability study,  $P_{ii}$  will be provided by the requirements on the accuracy of the probability of failure assessment. Thus we focus on the two estimates:

$$P_z^\mu = P(\sigma_z \in [(1 - \varepsilon_\mu)\sigma_z; (1 + \varepsilon_\mu)\sigma_z]) \quad \text{and} \quad P_z^\sigma = P(\sigma_z \in [(1 - \varepsilon_\sigma)\sigma_z; (1 + \varepsilon_\sigma)\sigma_z]) \quad (5)$$

Where if only a statistical error occurs (no effect of weak dependency between measurements), theory of confidence intervals gives:

$$\varepsilon_\mu = u_{1-\alpha/2} \frac{\sigma_z}{\mu_z \sqrt{N}} \quad \text{and} \quad \varepsilon_\sigma = u_{1-\alpha/2} \frac{1}{\sqrt{2N}} \quad (6)$$

With  $u_{1-\alpha/2} = 1.96$  for  $P_{ii} = 0.95$  ( $\alpha = 0.05$ ).

In the case of measurements on trajectories with a weak dependency between data, these values will be affected.

## 4 Application to a study case

### 4.1 Structure characteristics and inspection goals

We consider a structure for which  $L \gg b$  that allows to assess  $b$  from theoretical point of view. The Gaussian stationary stochastic field is characterized by:  $b = 1$ ,  $\mu_z = 100$  and  $\sigma_z = 20$  and the length of the structure is  $L = 200$ . We aim to provide an inspection protocol that ensures: respectively  $P_{ii,\mu} = P_{ii,\sigma} = 0.95$ .

### 4.2 Effect of SCT on confidence interval

We first analyze the effect of a weak correlation between measurements due to the spatial field on the quality of inspection according to (5). For  $N = 100$ , theoretical values deduced from (6) are:  $\varepsilon_\mu = 3.9\%$  and  $\varepsilon_\sigma = 13.85\%$ . FIG. 3 plots the effect of *SCT* (from 0.1 to 0.9) on  $\varepsilon_\mu$  and  $\varepsilon_\sigma$  respectively. The width of the confidence interval increases obviously with *SCT* and this effect is more pronounced for *SCT* higher than 0.4 for both  $\mu$  and  $\sigma$ . In the following we fix *SCT* = 0.4 and *IDT* is computer from (3). We can also express the result in another way. By fixing the confidence bounds -for instance  $\varepsilon_\mu = 5\%$  and  $\varepsilon_\sigma = 20\%$  - and  $L_c$  ( $L_c \in ]IDT, L[, L_c = b \cdot \ln(0.3) \approx 1.2 \text{ m}$ ), FIG. 4 presents the couple of minimum required values for  $N_s$  and  $N_t$  to ensure  $P_{ii,\mu} = P_{ii,\sigma} = 0.95$ .

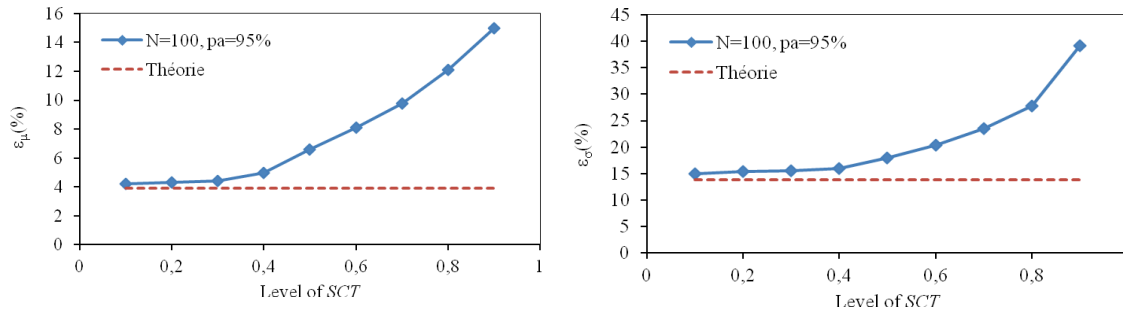


FIG.3 - Effect of SCT on  $\epsilon_\mu$  and  $\epsilon_\sigma$

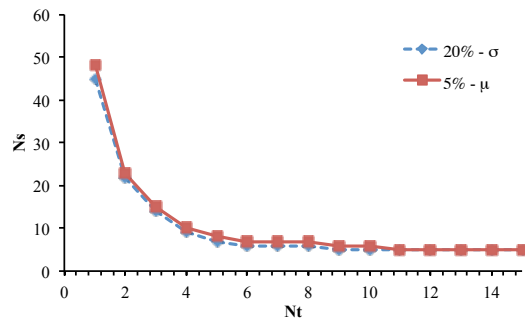


FIG.4 - Number of required  $N_s$  and  $N_t$  to ensure  $P_{ii,\mu} = P_{ii,\sigma} = 0.95$  ( $L_c = 1.2$  m).

The optimal number  $N_{opt}$  is solution of equation (7). Here  $N_{opt} = 40$  if 4 trajectories are available and  $N_s=10$ .

$$N_{opt} = \underset{N}{\operatorname{argmin}} \left( \max(N|\epsilon_\mu; N|\epsilon_\sigma) \right) \tag{7}$$

### 4.3 Inspection methodology: first stage

According to section 3, the first stage of the inspection methodology is devoted to the assessment of  $b$  on a single trajectory according to 3.2. We propose  $L_b = -b \ln(0.5) \approx 0.7$  m. Practically  $b$  can be deduced from expert judgment. Here it is fixed at its theoretical value:  $b=1$ . FIG 5 plots the effects of  $N_{s1}$  on the mean value  $\mu_B$  and the standard deviation  $\sigma_B$  of  $B$ . The convergence of  $\mu_B$  is quick and the error is less than 0.1 for  $N_{s1}=38$  but the convergence of  $\sigma_B$  is very slow and the error remains high (more than 0.3) even for  $N_{s1}=100$ . That underlines the role of the choice of  $L_b$ .

### 4.4 Inspection methodology: second stage and optimization

The uncertainty on the assessment of  $b$  will cause an uncertainty on  $L_c$  and will propagate this uncertainty when assessing statistics of  $\hat{Z}$ . Let us fix for instance  $N_s = 23$  and  $N_t = 2$  to reach  $\epsilon_\mu = 5\%$  and  $\epsilon_\sigma = 20\%$ . FIG.5 gives the effect of the choice of  $N_{s1}$  on  $\epsilon_\mu$  and  $\epsilon_\sigma$  respectively, with  $L_c = 1.2$  m (see section 4.3). That allows us to illustrate that exceeding 70 measurements  $N_{s1}$  for  $\epsilon_\mu$  and 50 for  $\epsilon_\sigma$  doesn't decrease the confidence interval. Note that the values expected for confidence intervals that confirms the good selection of  $L_b$ .

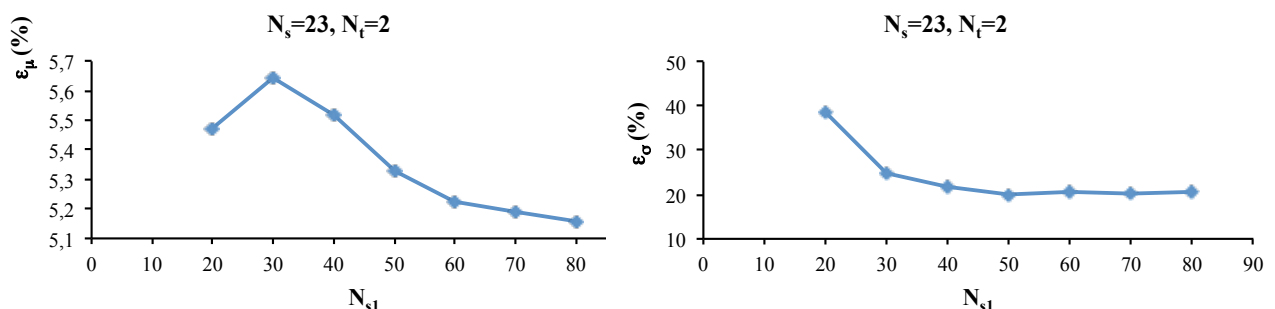


FIG.5- Effect of  $N_{s1}$  on  $\epsilon_\mu$  and  $\epsilon_\sigma$

## 5 Conclusion

We propose in this paper an original two stage method for the optimization of the number of inspection of a Gaussian stationary stochastic field in view to (i) assess the shape of the correlation function (ii) deduce the statistics of the marginal distribution. The role of the distance between measurement for assessment of (i) and (ii) is highlighted and a probabilistic-oriented measure of quality is suggested in terms of confidence intervals of these quantities. That underlines the key role of a Spatial Correlation Threshold. The methodology relies on (1) the measurement of the correlation function on a first trajectory with a given number of inspections and (2) the assessment of statistics of the marginal distribution on a given set of trajectories, knowing (1). The paper ends with a practical applications and suggestions for the spacing between inspection in view to reach (1) and (2).

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