



HAL
open science

Massive multi-missions statistical study and analytical modeling of the Earth magnetopause: 3- An asymmetric non indented magnetopause analytical model

Gautier Nguyen, Nicolas Aunai, Bayane Michotte de Welle, Alexis Jeandet,
Benoit Lavraud, Dominique Fontaine

► To cite this version:

Gautier Nguyen, Nicolas Aunai, Bayane Michotte de Welle, Alexis Jeandet, Benoit Lavraud, et al.. Massive multi-missions statistical study and analytical modeling of the Earth magnetopause: 3- An asymmetric non indented magnetopause analytical model. 2021. hal-03440292

HAL Id: hal-03440292

<https://hal.science/hal-03440292>

Preprint submitted on 22 Nov 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Massive multi-missions statistical study and analytical**
 2 **modeling of the Earth magnetopause: 3- An asymmetric**
 3 **non indented magnetopause analytical model**

4 **G. Nguyen**¹, **N.Aunai**¹, **B.Michotte de Welle**¹, **A.Jeandet**¹, **B.Lavraud**^{2,3} and
 5 **D.Fontaine**¹

6 ¹CNRS, Ecole polytechnique, Sorbonne Université, Univ Paris Sud, Observatoire de Paris, Institut
 7 Polytechnique de Paris, Université Paris-Saclay, PSL Research University, Laboratoire de Physique des
 8 Plasmas, Palaiseau, France

9 ²Laboratoire d'astrophysique de Bordeaux, Univ. Bordeaux, CNRS, B18N, alle Geoffroy Saint-Hilaire,
 10 33615 Pessac, France

11 ³Institut de Recherche en Astrophysique et Plantologie, Universit de Toulouse, CNRS, CNES, Toulouse,
 12 France

13 **Key Points:**

- 14 • We use a multi-mission crossings catalog to develop a new asymmetric, non-indented
 15 magnetopause surface model.
- 16 • The model is parametrized by the upstream solar wind dynamic and magnetic pres-
 17 sures, by the IMF clock angle and by the Earth dipole tilt angle.
- 18 • The model provides a more accurate prediction of the magnetopause location than
 19 current Magnetopause surface models, especially on the night side of the magneto-
 20 sphere.

Corresponding author: Gautier Nguyen, gautier-mahe.nguyen@intra.edef.gouv.fr

21 **Abstract**

22 In a companion statistical study, we showed that the expression of the magnetopause
 23 surface as a power law of an elliptic function of the zenith angle θ holds at lunar distances,
 24 that the flaring of the magnetopause surface is influenced by the Interplanetary Magnetic
 25 Field (IMF) B_y component and that the IMF B_x component had no influence on the stand-
 26 off distance.

27 As a follow-up to these statistical results, this paper presents a new empirical analytical
 28 asymmetric and non-indented model of the magnetopause location and shape. This model
 29 is obtained from fitting of 15 349 magnetopause crossings using 17 different spacecraft and
 30 is parametrized by the upstream solar wind dynamic and magnetic pressures, the IMF clock
 31 angle and the Earth dipole tilt angle.

32 The constructed model provides a more accurate prediction of the magnetopause sur-
 33 face location than current Magnetopause surface models, especially on the night side of the
 34 magnetosphere.

35 **1 Introduction**

36 The empirical modeling of the Earth’s magnetopause surface as a function of solar
 37 wind plasma and magnetic field parameters has been an important topic from the very
 38 first discovery of this boundary (Spreiter & Briggs, 1962) until very recently (Nmeek et al.
 39 (2020); Hasegawa (2012) and references therein).

40 Assuming a quadric shape of this boundary, Fairfield (1971) and Formisano (1979)
 41 developed the very first magnetopause surface model fitted with in-situ IMP observations.

42 These early observations also showed that reconnection eroded the magnetosphere in
 43 a location that depends on the orientation of the Interplanetary Magnetic Field (IMF) re-
 44 sulting in an earthward motion and in the decrease of the level of flaring for a southward
 45 orientation. From then on, numerous analytical empirical models based on a quadric sur-
 46 face, which coefficients depend on both the solar wind dynamic pressure and the IMF B_z
 47 component, were fitted using the observations of a single mission at a time (Sibeck et al.,
 48 1991; Petrinec et al., 1991; Petrinec & Russell, 1993; Roelof & Sibeck, 1993; Petrinec &
 49 Russell, 1996).

50 Using the observations of several missions simultaneously (IMP and ISEE), Shue et
 51 al. (1997) fitted the magnetopause radial distance as the power law of an elliptic function of
 52 the zenith angle θ and developed one of the most popular existing model for its simplicity
 53 to use and its accuracy.

54 Despite of this popularity, this model assumes axisymmetry around the Sun-Earth axis,
 55 an assumption later questioned by the evidence of seasonal variations of the magnetopause
 56 shape by the Hawkeye observations of Boardsen et al. (2000) and Eastman et al. (2000) and
 57 by the Interball observations of Šafránková et al. (2002).

58 This finding, along with the suggestion of a dawn-dusk asymmetry by Kuznetsov
 59 and Suvorova (1998), led Lin et al. (2010) to fit their magnetopause surface model using
 60 crossings observed from the data of 10 different missions. In addition to considering these
 61 two asymmetries, their model also comes with an analytical description of the near-cusp
 62 magnetopause in the form of an inward indentation, parametrized by the IMF B_z and the
 63 dipole tilt angle. Other parameters such as the full clock angle or the cone angle were not
 64 considered yet their effect on the near-cusp magnetopause is unknown. Their expression of
 65 the near-cusp indentation furthermore has the drawback of being non-negligible far from
 66 the cusp and modifies the interpretation of the terms otherwise controlling the stand-off
 67 distance of the level of flaring.

68 Some years later, Wang et al. (2013) developed a non-analytical model resulting from
 69 a support vector regression applied to the combined crossings of 23 different spacecraft. Just
 70 assuming the dawn dusk and the seasonal symmetries of the magnetopause, they recovered
 71 the dependencies on the dynamic pressure and the IMF B_z . However, neither their data
 72 nor their model is shared, the study is consequently hardly reproducible for comparison
 73 purposes.

74 In parallel to these observational studies, Liu et al. (2015) used the magnetopause
 75 detected in MagnetoHydroDynamics (MHD) simulations to adapt the model of Shue et al.
 76 (1997) to the North-South asymmetry of the magnetopause. Compared to Lin et al. (2010),
 77 this model has the advantage of taking into account the influence of the three components
 78 of the IMF while predicting the magnetopause with a slightly increased accuracy (Liu et al.,
 79 2015). Nevertheless, their model indicates that an increasing IMF B_x induces a North-South
 80 asymmetry, which is in opposition with the observational findings of Dušík et al. (2010)
 81 or Grygorov et al. (2017) who rather suggested a sunward motion of the magnetopause.
 82 Furthermore, the crossings used for the observational comparison with Lin et al. (2010)
 83 were by far mostly located in the dayside northern hemisphere region which thus gives poor
 84 evidence on how the model performs in the southern hemisphere or at lunar distances.

85 Using automatically detected events along with observations manually selected by
 86 various experts, Nguyen et al. (2020b) performed a statistical analysis of the position of 15
 87 349 magnetopause crossings that came from 10 different missions. This study confirmed
 88 well-known properties of the magnetopause such as the influence of the solar wind dynamic
 89 pressure, the seasonal variation or the influence of the IMF B_z component. They also showed
 90 that the expression of the magnetopause surface as a power law of an elliptic function of the
 91 zenith angle θ was still holding at lunar distances and evidenced the influence of the IMF
 92 GSM clock angle on the flaring level. Their investigations also suggested the absence of a
 93 sunward motion of the magnetopause with an increasing IMF B_x component.

94 In this third paper of a series of papers dedicated to the magnetopause, we develop a
 95 new asymmetric, non-indented analytical magnetopause model that takes into consideration
 96 these latest findings.

97 The outline of this paper is as follows: Section 2 presents the magnetopause crossings
 98 dataset we use and the associated upstream solar wind and IMF conditions, Section 3 focuses
 99 on the development of the magnetopause model, Section 4 compares it with other existing
 100 magnetopause models and Section 5 presents its different characteristics.

101 2 Dataset

102 We use the same magnetopause crossings dataset as that presented in Nguyen et
 103 al. (2020b). This dataset consists in the combination of 13 181 magnetopause crossings
 104 detected in the data of the most recent near-Earth missions (THEMIS, Cluster, Double
 105 Star, ARTEMIS and MMS) with a gradient boosting classifier (see Nguyen et al. (2020a)
 106 for more details)¹ together with the 2168 events manually identified by various experts in
 107 the data of older missions (OGO, Geotail, Hawkeye, AMPTE and IMP).²

108 The solar wind upstream conditions associated to each crossings were obtained from
 109 OMNI data by applying the two-step propagation algorithm exposed in Šafránková et al.
 110 (2002).

¹ Such crossings can be found online at : <https://github.com/gautiernguyen/in-situ.Events.lists>

² Such crossings can be found online at : <ftp://nssdcftp.gsfc.nasa.gov/spacecraftdata/magnetopausecrossings>)

111 In order to remove the aberration caused by the Earth’s revolution, we correct the GSM
 112 position of each crossing by using a similar approach than Lin et al. (2010) and Boardsen
 113 et al. (2000), assuming a revolution velocity of 30 km/s.

114 In addition to the non-aberrated GSM (X, Y, Z) cartesian coordinates, we use the
 115 spherical counterpart (R, θ, ϕ) as in Nguyen et al. (2020b):

$$\begin{cases} X = R \cos(\theta) \\ Y = R \sin(\theta) \sin(\phi) \\ Z = R \sin(\theta) \cos(\phi) \end{cases} \quad (1)$$

116 Similarly to Wang et al. (2013) and Nguyen et al. (2020b), we balance the hemispheric
 117 distribution of the dataset by assuming a symmetry between the summer northern hemi-
 118 sphere and the winter southern hemisphere and thus by reverting the GSM Z coordinate
 119 and the Earth dipole tilt angle γ : $r(X, Y, Z, \gamma) = r(X, Y, -Z, -\gamma)$.

120 Following the suggestions of Nguyen et al. (2020b) we also assume a dawn-dusk sym-
 121 metry of the magnetopause and thus revert the Y component of each event.

122 Different observations of the polar cusps crossings led to different conclusions regarding
 123 the shape of the boundary in the near-cusp region. Boardsen et al. (2000); Šafránková et
 124 al. (2002); afrnkov et al. (2005) suggested the magnetopause is indented while Zhou and
 125 Russell (1997); Lavraud et al. (2004) suggested a non-indented boundary. This question is
 126 investigated in the fourth paper of our study (Nguyen et al., 2020c). For now, we restrict
 127 the dataset to the 58154 so-called ”out of cusp” events. Those events are defined as those
 128 falling outside of the cusp indentation as defined by Lin et al. (2010), for which θ satisfies:

$$\begin{cases} (\theta - \theta_n)^2 + \phi^2 \geq \left(-\frac{1}{d_n}\right)^{\frac{2}{a_{21}}} & \text{if } Z \geq 0 \\ (\theta - \theta_s)^2 + \phi^2 \geq \left(-\frac{1}{d_s}\right)^{\frac{2}{a_{21}}} & \text{if } Z \leq 0 \end{cases} \quad (2)$$

129 Where $d_{n,s} = a_{16} \pm a_{17}\gamma + a_{18}\gamma^2$, $\theta_{n,s} = a_{19} \pm a_{20}\gamma$, and a_{21} represent the scope,
 130 the zenithal position and the shape of the polar cusps, γ is the Earth dipole tilt angle and
 131 $a_{16}, a_{17}, a_{18}, a_{19}, a_{20}$ and a_{21} are the corresponding coefficients fitted by Lin et al. (2010).
 132 The crossings that are found inside of this so-defined cusp-indentation will constitute the
 133 core of the dataset used in Nguyen et al. (2020c).

134 The obtained symmetrized dataset is randomly split into a training set 43 664 events
 135 that will serve for the fit performed in section 4 and into a test set of 14 490 events used to
 136 evaluate the accuracy of the resulting model in section 5.

137 The distribution of the cartesian position and the associated solar wind physical pa-
 138 rameters of the events that constitute the different sets is shown in Figure 1.

139 In each panel, we notice similar distributions of both the train and the test sets. This
 140 indicates that there is no particular bias between the two sets.

141 3 Construction of the magnetopause model

142 Previous statistical studies of the magnetopause location and shape (Shue et al., 1997;
 143 Liu et al., 2015; Nguyen et al., 2020b) confirmed the relevance of the analytical definition of
 144 the magnetopause surface as the power law of an elliptic function. Thus, we keep this basis
 145 for the construction of our model:

$$r = r_0 \left(\frac{2}{1 + \cos(\theta)} \right)^\alpha \quad (3)$$

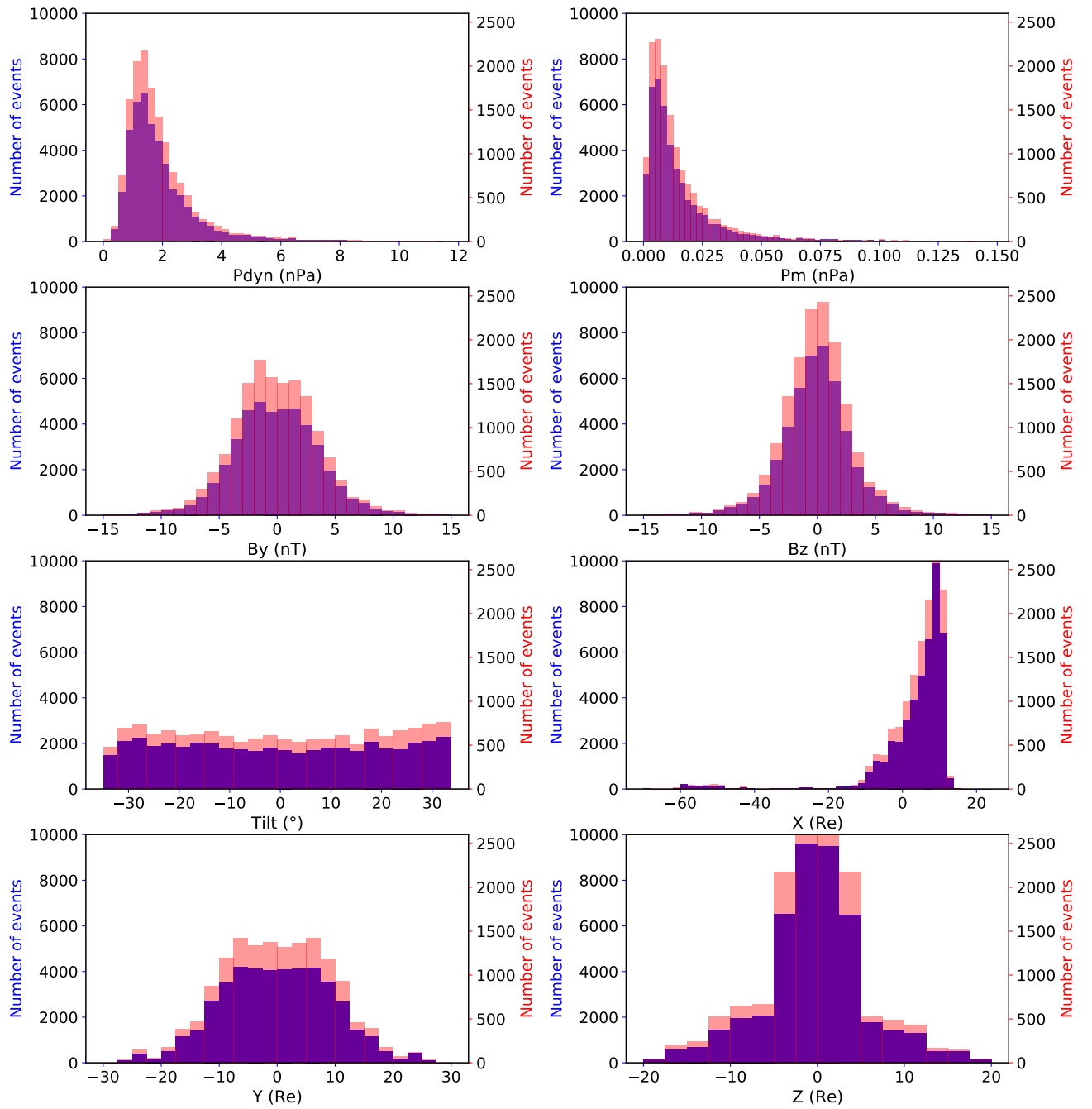


Figure 1. Histogram of the solar wind parameters and the cartesian position of the 43 664 events of the train set (blue) and of the 14 490 events of the test set (blue red): the dynamic pressure P_{dyn} (top left), the magnetic pressure P_m (top, right), the IMF B_y and B_z components (second row, left and right), the Earth dipole tilt angle γ (third row, left) and the GSM cartesian position, X (third row, right), Y and Z (last row, left and right).

146 Where r_0 describes the position of the magnetopause nose and α controls the level of
 147 tail flaring.

148 Following the results of Nguyen et al. (2020b), we define r_0 as:

$$r_0 = a_0(P_{dyn} + P_m)^{a_1}(1 + a_2 \tanh(a_3 B_z) + a_4) \quad (4)$$

149 In the case of an axisymmetric magnetopause, α is independant from the zenith and
 150 the azimuth angles, θ and ϕ . The assumption, espacially made by Shue et al. (1997), is
 151 however not true as the most recent statistical studies (Nguyen et al. (2020b) and references
 152 therein) evidenced an azimuthal and a north-south asymmetry of the magnetopause induced
 153 by the IMF B_y and B_z components and by the Earth dipole tilt angle, respectively.

154 Consequently, we define the flaring level as:

$$\begin{cases} \alpha = \alpha_0 + \alpha_1 \cos(\phi) + \alpha_2 \cos(\phi)^2 + \alpha_3 \sin(\phi)^2 \\ \alpha_0 = a_5 \\ \alpha_1 = a_6 \gamma \\ \alpha_2 = a_7 \cos(\Omega) \\ \alpha_3 = a_8 \cos(\Omega) \end{cases} \quad (5)$$

155 Where α_0 is the average level of flaring expected in the case of an axisymmetric mag-
 156 netopause. α_1 describes the north-south asymmetry induced by seasonal variations through
 157 the variation of the dipole tilt angle γ . α_2 (resp. α_3) describes the variations of the mag-
 158 netopause in the $(X - Z)$ (resp. $(X - Y)$) plane induced by the variations of the IMF clock
 159 angle Ω .

160 The values of the 9 a_i coefficients are initially set with the fitting values found in
 161 Nguyen et al. (2020b). a_0 , a_1 , a_2 , a_3 and a_4 are predetermined by fitting (4) to the 275
 162 events for which $\theta < 7.750^\circ$ and $Z > 0$. The initial values of a_5 and a_7 are determined by
 163 fitting 3, 4, 5, to the 5170 out of cusps crossings for which $|Z| < 1 Re$ and assuming all of
 164 these events belong to the $(X - Y)$ plane. The initial values of a_5 , a_6 and a_8 are determined
 165 by fitting 3, 4, 5 to the 2154 out of cusps crossings for which $|Y| < 2 Re$ assuming all of
 166 these events belong to the $(X - Z)$ plane. The initial fitting values are shown in Table 1.

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
10.75	-0.161	0.050	0.35	1.60	0.55 , 0.51	0.026	0.015	-0.050

Table 1. Initial values of the coefficients of the equations 3 to 5 obtained from the initial fits around the subsolar point of a_0 , a_1 , a_2 , a_3 and a_4 , in the $(X - Y)$ plane for a_5 (*first value*) and a_7 and in the $(X - Z)$ plane for a_5 (*second value*), a_6 and a_8

167 The final values of the 9 a_i coefficients are then obtained by applying the Levenberg-
 168 Marquardt fitting method (Newville et al., 2014) on the 43 664 events of the training set
 169 and are presented in the Table 2.

170 This fitting phase then results in an analytical empirical model of the non-indented
 171 magnetopause shape and location that depends on the solar wind total pressure $P_{dyn} + P_m$,
 172 the IMF B_z and clock angle Ω and the dipole tilt angle γ . A numerical implementation of
 173 this model can be found at: https://github.com/gautiernguyen/magnetopause_models
 174 and will be the one used in the following paragraphs.

a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
10.73	-0.150	0.0208	0.38	2.09	0.55	0.088	0.015	-0.087

Table 2. Final values of the coefficients a_i of the equations 3 to 5 obtained after a total fit on the training set.

	Our model	Liu et al. (2015)	Lin et al. (2010)	Shue et al. (1997)
$X < -30$ (361)	6.06 ± 0.32	15.01 ± 0.53	14.06 ± 0.34	9.19 ± 0.34
$X > -30$ and $X < 0$ (2525)	1.67 ± 0.033	2.39 ± 0.040	2.48 ± 0.047	2.26 ± 0.034
$X > 0$ and $ Z > 7.5$ (1188)	1.84 ± 0.042	1.78 ± 0.042	1.72 ± 0.044	2.24 ± 0.043
$X > 0$ and $ Z < 7.5$ and $ Y > 7.5$ (3992)	1.02 ± 0.016	0.99 ± 0.017	1.22 ± 0.016	1.19 ± 0.016
$X > 0$ and $ Z < 7.5$ and $ Y > 7.5$ (6424)	0.93 ± 0.011	0.86 ± 0.010	0.96 ± 0.010	1.00 ± 0.012
All regions (14490)	1.53 ± 0.013	2.73 ± 0.021	2.65 ± 0.023	2.06 ± 0.015

Table 3. RMSE of the different models in different region for the 14490 crossings of the test set, the uncertainty represents the standard error of mean of the error of each model. The number between brackets in the first column indicate the number of events per region.

4 Comparison with other models

175

176 We evaluate the accuracy of the fitted model by computing its Root Mean Square
 177 Error (RMSE) on the 14490 events of the test set and compare it to the RMSE of the
 178 models of Shue et al. (1997), Lin et al. (2010) and Liu et al. (2015) computed on the same
 179 test set. To keep a consistent comparison, we removed the indentation part of the models
 180 of Lin et al. (2010) and Liu et al. (2015) during the computation of the RMSE. The score
 181 we obtain for the different models for different spatial regions are shown in the Table 3 and
 182 visually represented in the Figure 2.

183 The obtained RMSE in the low-latitude, dayside, subsolar and flank regions, is almost
 184 similar for the four models, this indicates that our model provides an accurate description
 185 of the magnetopause shape and location in those regions and this is not surprising given the
 186 proximity of the expression and coefficients of the stand-off distance of the 4 models. At
 187 high latitudes, we notice a more important error for the model of Shue et al. (1997) that
 188 is not surprising as no high latitude data was considered during the development of this
 189 model and this is particularly reflected by the similarity we find between our RMSE and
 190 the RMSE of Lin et al. (2010) and Liu et al. (2015).

191 Looking at the "close" nightside region, we notice this time a reduced error of our
 192 model in comparison to the three others, than can be explained by our consideration of the
 193 IMF clock angle in the expression of the flaring coefficient α rather than the lone B_z .

194 Finally, the previous models were established without consideration of magnetopause
 195 crossings further than -30 Re, especially the one detected by ARTEMIS and it is thus not
 196 surprising to notice a lower RMSE for our model in the "far" nightside. Naturally, the error
 197 here is possibly substantially higher than in any other region we considered. This could be
 198 explained by the flapping of the magnetotail in the far nightside that could result in a much
 199 more variable boundary (Sergeev et al., 1998). As the position of the magnetopause does
 200 not rely on the solar wind physical parameters alone but also on the magnetotail dynamics,
 201 this necessarily constitute a non negligible source of errors.

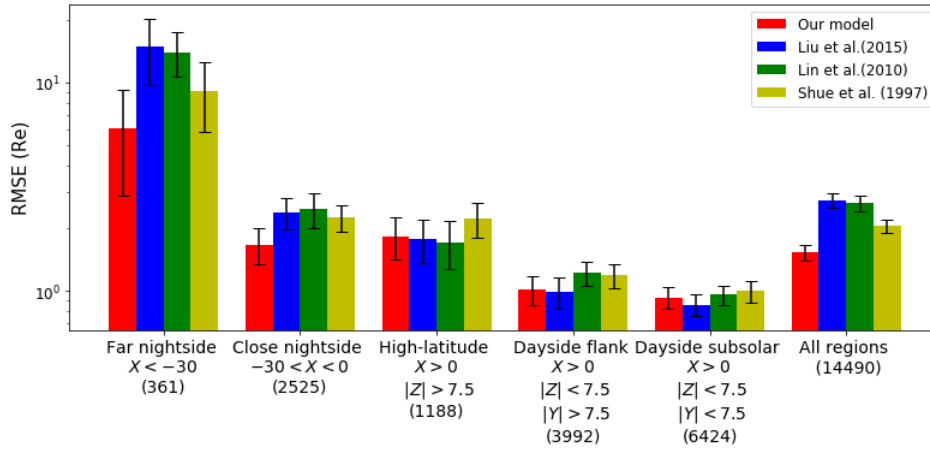


Figure 2. Visual representation of the RMSE of the different models exposed in the Table 3. The error bar represent 10 times the Standard Error of the Mean of the error made by the models on the test set.

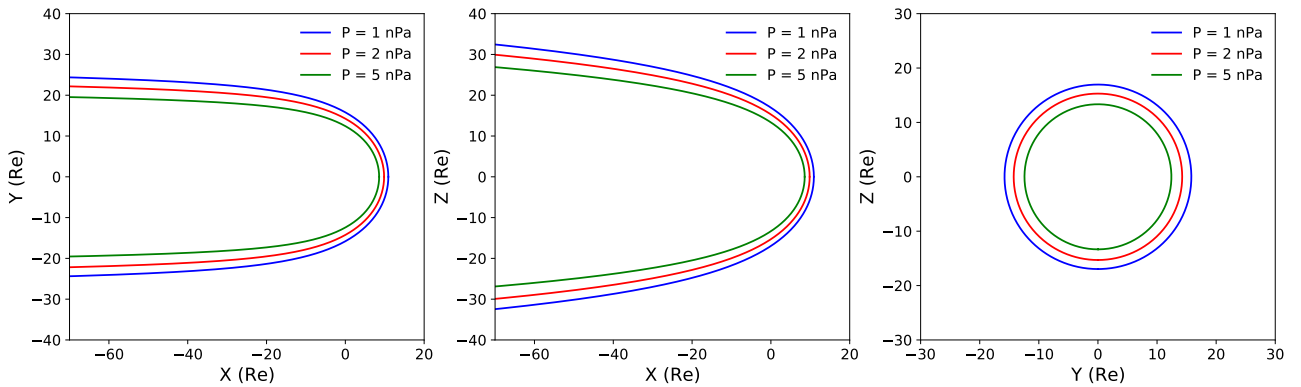


Figure 3. Projection in the $(X - Y)$ (left), $(X - Z)$ (middle), and $(Y - Z)$ (right) planes of our model for varying total pressure $P = P_{dyn} + P_m$. The IMF is purely southward and $B_z = -2$ nT.

202 Combining the regions altogether (last group of bars of Figure 2), we obtain a global
 203 RMSE that is lower for our model in comparison to the others and thus ensures the reliability
 204 of our model and its legitimacy to be exploited in further magnetopause studies.

205 5 Characteristics of the model

206 We show the influence of the total pressure, $P = P_{dyn} + P_m$ on our magnetopause
 207 model with the three panels of Figure 3. Following what we evidenced in Nguyen et al.
 208 (2020b), the total pressure pushes the magnetopause earthward along the X axis without
 209 influencing the flaring. This behavior confirms the findings of the previous models that
 210 showed very little pressure dependency of the flaring. Additionally, we find a power law
 211 index a_1 equal to -0.15 that is very close to the theoretical $-1/6$ for a dipole in vacuum
 212 and in the same orders of magnitude than the values found by Shue et al. (1997), Lin et al.
 213 (2010) and Liu et al. (2015).

214 The three panels of Figure 4 that represent the influence of the IMF clock angle.
 215 Following the suggestions of Nguyen et al. (2020b), the model results in an elliptic magne-
 216 topause YZ cross section on the Z axis for negative B_z . This direction change is consistent

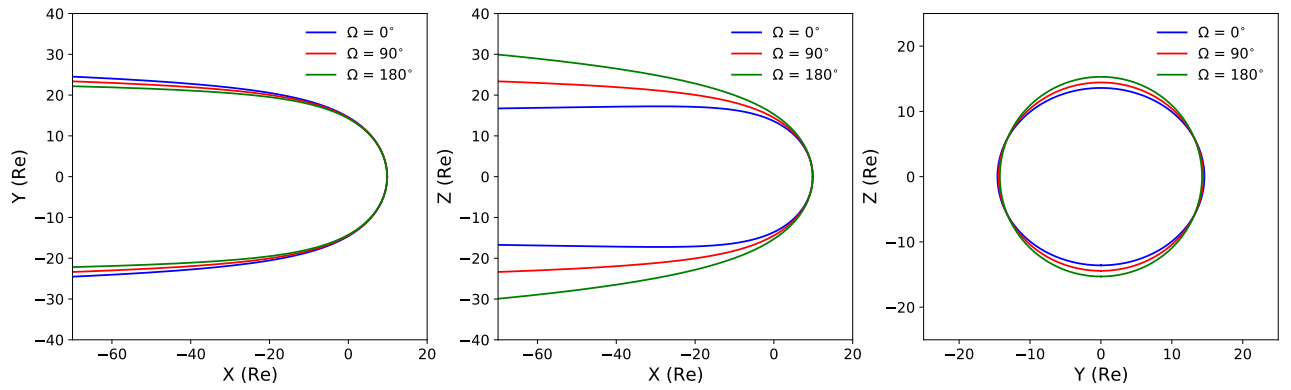


Figure 4. Projection in the $(X - Y)$ (*left*), $(X - Z)$ (*middle*), and $(Y - Z)$ (*right*) planes of our model for varying clock angle Ω . The total pressure is equal to 2 nPa and $|B| = 2nT$.

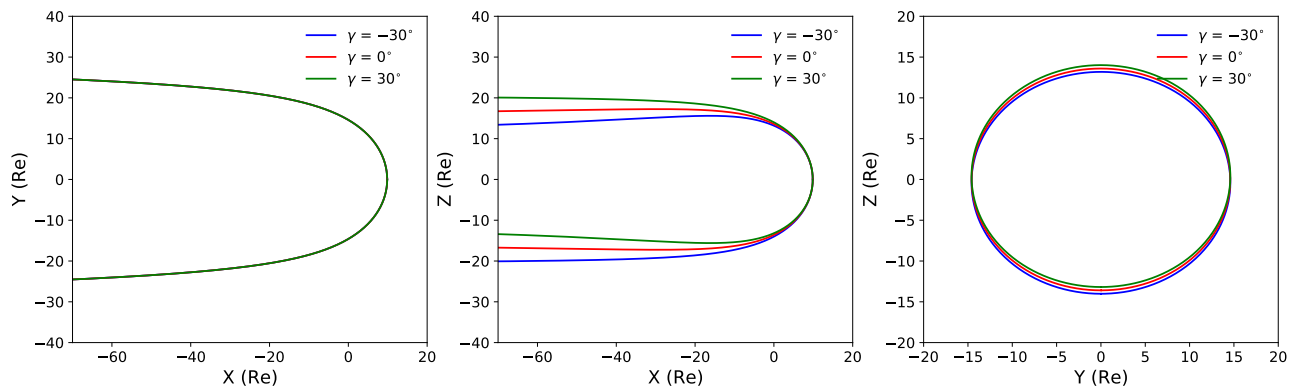


Figure 5. Projection in the $(X - Y)$ (*left*), $(X - Z)$ (*middle*), and $(Y - Z)$ (*right*) planes of our model for varying dipole tilt angle γ . The total pressure is equal to 2 nPa, the IMF is purely northward and $B_z = 2nT$.

217 with the one obtained by (Wang et al., 2013) and could be explained by the tail accumula-
 218 tion of newly reconnected field line that appears in the case of a southward IMF, resulting
 219 in an enhanced level of flaring.

220 We show the influence of the dipole tilt angle γ with the three panels of Figure 5 that
 221 clearly indicates a magnetopause that rotates around the Y axis with a rotating dipole tilt
 222 angle resulting in a north hemisphere summer (respectively south hemisphere winter) shift
 223 of the magnetopause cross section.

224 Finally, computing the magnetopause location at all angles for the same solar wind
 225 conditions results in a misleading picture. The different existing magnetopause surface mod-
 226 els return a static representation of the magnetopause for a permanent upstream solar wind
 227 regime. Actually, the magnetopause has a dynamic motion that follows the variations of the
 228 upstream solar wind. For this reason, it would be interesting to adapt the existing static
 229 model into a dynamic expression of the magnetopause surface able to give an estimation of
 230 the magnetopause shape and location at any time. A simple description of such adaptation
 231 to our model that only consider different upstream solar wind conditions for the different
 232 points of the surface can especially be found in the appendix A.

233 6 Conclusion

234 In this paper, we exploit the statistical findings of Nguyen et al. (2020b), a companion
 235 study, to develop a new asymmetric non indented magnetopause surface model. Just like
 236 the past existing studies (Shue et al., 1997; Lin et al., 2010; Liu et al., 2015), our model
 237 is expressed as the power law of an elliptic function of the GSM zenith angle θ and is here
 238 parameterized by the upstream solar wind dynamic and magnetic pressure, by the IMF
 239 clock angle and by the Earth dipole tilt angle.

240 Comparing our model with the models of Shue et al. (1997), Lin et al. (2010) and
 241 Liu et al. (2015), we found the 4 models to predict the magnetopause location with similar
 242 accuracy on the dayside equatorial part of the magnetopause and the error made by our
 243 model is similar to the one made by Lin et al. (2010) and Liu et al. (2015) on the high-
 244 latitude dayside part of the magnetopause. On the nightside, the consideration of crossings
 245 farther downtail than -30 Re resulted in a reduced error in comparison to the other existing
 246 models. Nevertheless, the lack of data in this region combined to the much more variable
 247 nature of the magnetopause at such distances (Sergeev et al., 1998) indicate further studies
 248 are needed in this specific region of the near-Earth environment.

249 Assuming a dawn-dusk symmetry of the magnetopause, our model predicts a magne-
 250 topause squeezed or stretched in the Y or Z direction, respectively, when the IMF turns
 251 from a northward to a southward orientation. This finding gives clues on the influence of the
 252 Y component of the IMF on the magnetopause and is in agreement with Liu et al. (2015).
 253 However, for statistical reasons, we limited our study to the $(X - Y)$ and the $(X - Z)$
 254 and symmetrized our dataset. As a consequence, neither the statistical investigations we
 255 performed in Nguyen et al. (2020b) nor the model we developed in this paper considered a
 256 continuous twist of the magnetopause with the IMF clock angle as suggested by Liu et al.
 257 (2015) and Lavraud and Borovsky (2008). Future investigations, using more data, should
 258 address this delicate point.

259 Finally, all of the crossings we used in this paper were located outside of the near-
 260 cusp regions and there is thus no clue on how well our non-indented model performs and
 261 the extent to which it has to be adapted in order to provide a precise description of the
 262 magnetopause in this specific region of the near-Earth environment. This issue is addressed
 263 in Nguyen et al. (2020c), the last companion paper of our study.

264 Appendix A From a static to a dynamic model

265 All of the existing analytical magnetopause surface model provide a static view of the
 266 magnetopause for a given upstream solar wind regime, which is far from being the actual
 267 ground truth. This shows the interest we have in adapting the existing static models into
 268 their dynamic counterpart.

269 To do so, we adapt the two-step propagation algorithm of Šafránková et al. (2002) in
 270 order to estimate the temporal shift to OMNI data needed for each zenith angle θ along the
 271 GSM X axis :

- 272 1. At a given time t and a given zenith angle θ , we estimate a first position of the
 273 magnetopause by computing our model for the averaged solar wind conditions in the
 274 interval between t and $t - 30$ min. We chose 30 minutes as this is the typical shifting
 275 time we obtain for $X \sim -70$ Re.
- 276 2. This first position serves to estimate a first value of the X coordinate of the magne-
 277 topause at this value of θ .
- 278 3. We apply the two step propagation algorithm to estimate a first shifting time from
 279 this first X position and compute the associated radial position of the magnetopause.

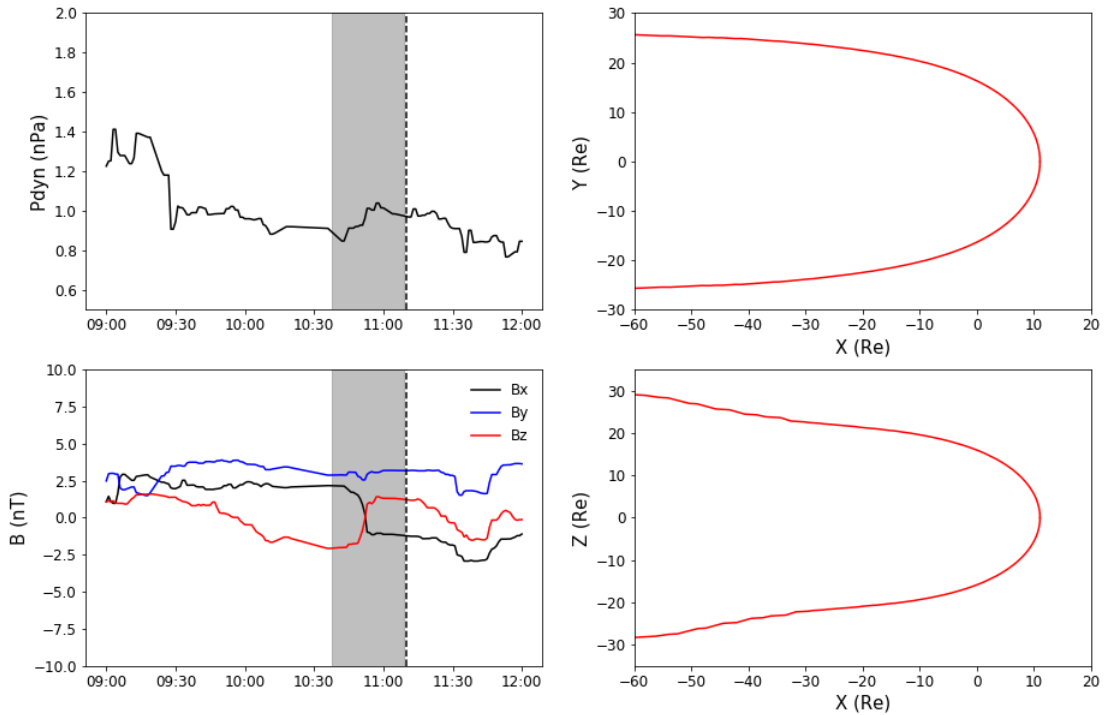


Figure A1. OMNI solar wind dynamic pressure (*top*) and magnetic field (*bottom*) measurement (*left column*) on the 3rd of March 2011 and projection of our dynamical magnetopause model (*right column*) in the ($X - Y$) (*top*) and in the ($X - Z$) (*bottom*) planes at the time corresponding to the black dashed line on the left column. The grey interval in the left panels represent the data interval propagated throughout the whole magnetopause.

- 280 4. We use this second radial position to re-estimate the X coordinate of the magne-
 281 topause at this value of θ .
 282 5. We apply the two-step propagation algorithm a second time to determine the final
 283 shifting time that will be used for this value θ and we compute the associated final
 284 radial position of the magnetopause.

285 At a given time t , we apply this process for every zenithal position θ and end up
 286 with a dynamical magnetopause model similar to the one shown in the two right panels of
 287 Figure A1 where we represented the projection of the magnetopause in the ($X - Y$) and in
 288 the ($X - Z$) planes at the time indicated by the black dashed line on the two left panels.
 289 Naturally, the shifting time increases with the zenith angle and the left boundary of the grey
 290 interval represented in the two left panels then corresponds to the magnetopause computed
 291 at the left border of the two right panels. The obtained magnetopause then considers an IMF
 292 shift from a negative to a positive B_z and the propagation of this transition is reproduced
 293 through the propagation of the erosion we notice in the ($X - Z$) plane.

294 Adapting the two step propagation algorithm, we then elaborated a process useful for
 295 providing a dynamical view of the magnetopause at any time. Additionally, this process
 296 is independent from the used static magnetopause model and thus easily adaptable to the
 297 models of Shue et al. (1997), Lin et al. (2010) and Liu et al. (2015).

298 The numerical implementation of this models can be found at: https://github.com/gautiernguyen/magnetopause_models
 299

300 **Acknowledgments**

301 Enter acknowledgments, including your data availability statement, here.

302 **References**

- 303 Boardsen, S. A., Eastman, T. E., Sotirelis, T., & Green, J. L. (2000). An empirical
 304 model of the high-latitude magnetopause. *Journal of Geophysical Research: Space*
 305 *Physics*, *105*(A10), 23193-23219. Retrieved from [https://agupubs.onlinelibrary](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/1998JA000143)
 306 [.wiley.com/doi/abs/10.1029/1998JA000143](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/1998JA000143) doi: 10.1029/1998JA000143
- 307 Dušík, Š., Granko, G., Šafránková, J., Němeček, Z., & Jelínek, K. (2010, October). IMF cone
 308 angle control of the magnetopause location: Statistical study. *Geophysical Research*
 309 *Letters*, *37*(19), L19103. doi: 10.1029/2010GL044965
- 310 Eastman, T. E., Sotirelis, T., & Green, J. L. (2000). An Empirical Model of the High-
 311 latitude Magnetopause. *Journal of Geophysical Research*, *105*(1998). doi: doi:10.1029/
 312 1998JA000143
- 313 Fairfield, D. H. (1971, Jan). Average and unusual locations of the Earth's magne-
 314 topause and bow shock. *Journal of Geophysical Research*, *76*(28), 6700. doi:
 315 10.1029/JA076i028p06700
- 316 Formisano, V. (1979, Dec). The three-dimensional shape of the bow shock. *Planetary and*
 317 *Space Science*, *2C*, 681-692. doi: 10.1007/BF02558125
- 318 Grygorov, K., afrnkov, J., Nmeek, Z., Pi, G., Pech, L., & Urb, J. (2017). Shape of the equa-
 319 torial magnetopause affected by the radial interplanetary magnetic field. *Planetary*
 320 *and Space Science*, *148*, 28 - 34. Retrieved from [http://www.sciencedirect.com/](http://www.sciencedirect.com/science/article/pii/S0032063317302131)
 321 [science/article/pii/S0032063317302131](http://www.sciencedirect.com/science/article/pii/S0032063317302131) doi: [https://doi.org/10.1016/j.pss.2017](https://doi.org/10.1016/j.pss.2017.09.011)
 322 [.09.011](https://doi.org/10.1016/j.pss.2017.09.011)
- 323 Hasegawa, H. (2012, August). Structure and Dynamics of the Magnetopause and Its Bound-
 324 ary Layers. *Monographs on Environment, Earth and Planets*, *1*(2), 71-119. doi:
 325 10.5047/meep.2012.00102.0071
- 326 Kuznetsov, S. N., & Suvorova, A. V. (1998, January). Solar wind magnetic field and pressure
 327 during magnetopause crossings at geosynchronous orbit. *Advances in Space Research*,
 328 *22*(1), 63-66. doi: 10.1016/S0273-1177(97)01101-0
- 329 Lavraud, B., & Borovsky, J. E. (2008). Altered solar wind-magnetosphere interaction at
 330 low mach numbers: Coronal mass ejections. *Journal of Geophysical Research: Space*
 331 *Physics*, *113*(A9). Retrieved from [https://agupubs.onlinelibrary.wiley.com/](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2008JA013192)
 332 [doi/abs/10.1029/2008JA013192](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2008JA013192) doi: <https://doi.org/10.1029/2008JA013192>
- 333 Lavraud, B., Fedorov, A., Budnik, E., Grigoriev, A., Cargill, P., Dunlop, M., ... Balogh,
 334 A. (2004, August). Cluster survey of the high-altitude cusp properties: a three-year
 335 statistical study. *Annales Geophysicae*, *22*(8), 3009-3019. doi: 10.5194/angeo-22-3009
 336 -2004
- 337 Lin, R. L., Zhang, X. X., Liu, S. Q., Wang, Y. L., & Gong, J. C. (2010, Apr). A three-
 338 dimensional asymmetric magnetopause model. *Journal of Geophysical Research (Space*
 339 *Physics)*, *115*(A4), A04207. doi: 10.1029/2009JA014235
- 340 Liu, Z., Lu, J. Y., Wang, C., Kabin, K., Zhao, J. S., Wang, M., ... Zhao, M. X. (2015).
 341 Journal of Geophysical Research : Space Physics A three-dimensional high Mach
 342 number asymmetric magnetopause model from global MHD simulation. *Journal of*
 343 *Geophysical Research*, 5645–5666. doi: 10.1002/2014JA020961. Received
- 344 Newville, M., Stensitzki, T., Allen, D. B., & Ingargiola, A. (2014, September). *LMFIT: Non-*
 345 *Linear Least-Square Minimization and Curve-Fitting for Python*. Zenodo. Retrieved
 346 from <https://doi.org/10.5281/zenodo.11813> doi: 10.5281/zenodo.11813
- 347 Nguyen, G., Aunai, N., Michotte de Welle, B., Jeandet, A., Lavraud, B., & Fontaine, D.
 348 (2020a). *Massive multi-missions statistical study and analytical modeling of the Earth*
 349 *magnetopause: 1 - A gradient boosting based automatic detection of near-Earth regions*.
 350 (Submitted)
- 351 Nguyen, G., Aunai, N., Michotte de Welle, B., Jeandet, A., Lavraud, B., & Fontaine, D.
 352 (2020b). *Massive multi-missions statistical study and analytical modeling of the Earth*

- 353 *magnetopause: 2 - Shape and location.* (Submitted)
- 354 Nguyen, G., Aunai, N., Michotte de Welle, B., Jeandet, A., Lavraud, B., & Fontaine, D.
355 (2020c). *Massive multi-missions statistical study and analytical modeling of the Earth*
356 *magnetopause: 4- On the near-cusp magnetopause indentation.* (Submitted)
- 357 Nmeek, Z., afrnkov, J., & imnek, J. (2020). An examination of the magnetopause po-
358 sition and shape based upon new observations. In *Dayside magnetosphere interac-*
359 *tions* (p. 135-151). American Geophysical Union (AGU). Retrieved from [https://](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/9781119509592.ch8)
360 agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/9781119509592.ch8 doi:
361 10.1002/9781119509592.ch8
- 362 Petrinec, S. M., & Russell, C. T. (1993, December). An empirical model of the size and
363 shape of the near-Earth magnetotail. *Geophysical Research Letters*, *20*(23), 2695-2698.
364 doi: 10.1029/93GL02847
- 365 Petrinec, S. M., & Russell, C. T. (1996). Near-earth magnetotail shape and size as deter-
366 mined from the magnetopause flaring angle. *Journal of Geophysical Research: Space*
367 *Physics*, *101*(A1), 137-152. Retrieved from [https://agupubs.onlinelibrary.wiley](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/95JA02834)
368 [.com/doi/abs/10.1029/95JA02834](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/95JA02834) doi: 10.1029/95JA02834
- 369 Petrinec, S. P., Song, P., & Russell, C. T. (1991, May). Solar cycle variations in the size
370 and shape of the magnetopause. *Journal of Geophysical Research*, *96*(A5), 7893-7896.
371 doi: 10.1029/90JA02566
- 372 Roelof, E. C., & Sibeck, D. G. (1993, December). Magnetopause shape as a bivariate
373 function of interplanetary magnetic field B_z and solar wind dynamic pressure. *Journal*
374 *of Geophysical Research*, *98*(A12), 21421-21450. doi: 10.1029/93JA02362
- 375 Sergeev, V., Angelopoulos, V., Carlson, C., & Sutcliffe, P. (1998). Current sheet measure-
376 ments within a flapping plasma sheet. *Journal of Geophysical Research: Space Physics*,
377 *103*(A5), 9177-9187. Retrieved from [https://agupubs.onlinelibrary.wiley.com/](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/97JA02093)
378 [doi/abs/10.1029/97JA02093](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/97JA02093) doi: 10.1029/97JA02093
- 379 Shue, J. H., Chao, J. K., Fu, H. C., Russell, C. T., Song, P., Khurana, K. K., & Singer,
380 H. J. (1997, May). A new functional form to study the solar wind control of the
381 magnetopause size and shape. *Journal of Geophysical Research*, *102*(A5), 9497-9512.
382 doi: 10.1029/97JA00196
- 383 Sibeck, D. G., Lopez, R. E., & Roelof, E. C. (1991, Apr). Solar wind control of the
384 magnetopause shape, location, and motion. *Journal of Geophysical Research*, *96*(A4),
385 5489-5495. doi: 10.1029/90JA02464
- 386 Spreiter, J. R., & Briggs, B. R. (1962). Theoretical determination of the form of
387 the boundary of the solar corpuscular stream produced by interaction with the
388 magnetic dipole field of the earth. *Journal of Geophysical Research (1896-1977)*,
389 *67*(1), 37-51. Retrieved from [https://agupubs.onlinelibrary.wiley.com/doi/](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JZ067i001p00037)
390 [abs/10.1029/JZ067i001p00037](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JZ067i001p00037) doi: 10.1029/JZ067i001p00037
- 391 Šafránková, J., Němeček, Z., Dušík, v., Přeč, L., Sibeck, D. G., & Borodkova, N. N. (2002).
392 The magnetopause shape and location: a comparison of the interball and geotail
393 observations with models. *Annales Geophysicae*, *20*(3), 301-309. Retrieved from
394 <https://www.ann-geophys.net/20/301/2002/> doi: 10.5194/angeo-20-301-2002
- 395 Wang, Y., Sibeck, D. G., Merka, J., Boardsen, S. A., Karimabadi, H., Sipes, T. B., ... Lin,
396 R. (2013, May). A new three-dimensional magnetopause model with a support vector
397 regression machine and a large database of multiple spacecraft observations. *Journal*
398 *of Geophysical Research (Space Physics)*, *118*, 2173-2184. doi: 10.1002/jgra.50226
- 399 Zhou, X.-W., & Russell, C. T. (1997). The location of the high-latitude polar cusp and
400 the shape of the surrounding magnetopause. *Journal of Geophysical Research: Space*
401 *Physics*, *102*(A1), 105-110. Retrieved from [https://agupubs.onlinelibrary.wiley](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/96JA02702)
402 [.com/doi/abs/10.1029/96JA02702](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/96JA02702) doi: 10.1029/96JA02702
- 403 afrnkov, J., Dusk, ., & Nmeek, Z. (2005). The shape and location of the high-latitude
404 magnetopause. *Advances in Space Research*, *36*(10), 1934 - 1939. Retrieved from
405 <http://www.sciencedirect.com/science/article/pii/S0273117705004795> (So-
406 lar Wind-Magnetosphere-Ionosphere Dynamics and Radiation Models) doi: [https://](https://doi.org/10.1016/j.asr.2004.05.009)
407 doi.org/10.1016/j.asr.2004.05.009