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## Reply to “Comment on ‘Absence of a Dissipative Quantum Phase Transition in Josephson Junctions’”

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In their Comment [P. Hakonen and E. B. Sonin, preceding comment, Phys. Rev. X **11**, 018001 (2021)], Hakonen and Sonin (HS) object to our conclusion on the absence of a dissipation-induced superconducting-to-insulating quantum phase transition (DQPT) in resistively shunted Josephson junctions (RSJJs) originally predicted by Schmid and Bulgadaev (SB). Their objections are based on the account they make of a theory explaining the DQPT in terms of Bloch bands which was developed in the 1980s and early 1990s. In this Reply, we point to several issues in the Comment which undermine the objections HS formulate against our work.

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### I. WHAT ARE QUANTUM PHASE TRANSITIONS, AND HOW TO PROPERLY ESTABLISH THEIR OCCURRENCE?

A quantum phase transition is, by definition, a change in nature of the system’s ground state occurring at a given value of a system parameter (in the case we consider, the strength of the dissipation supposedly drives the system from a superconducting to an insulating ground state at a given value). Although probing directly the true ground state is impossible experimentally, since it would require operating at zero temperature, a well-established generic theory of quantum phase transitions (Ref. [28] in our article [1]) shows that the existence of such a transition implies that finite-temperature equilibrium properties of the system follow power laws with predictable universal exponents characteristic of universality classes to which the system belongs (e.g., its dimensionality). Such universal power laws are generally regarded as the observable hallmarks of a phase transition [2].

Previous experiments on the DQPT in JJs cited in the Comment [3] did not investigate these hallmarks. Instead, they claim to check the prediction using measurements whose links with the general theory of quantum phase transition are not established [4], and these measurements

are furthermore affected by technical limitations, as explained in Appendix A of the article.

In contrast, in our work, we follow the prescriptions of the general theory, measuring the linear response of JJs supposed to be well in the insulating phase. Unlike what is expected for an insulating ground state, we observe this linear response does not vanish following a power law of temperature; on the contrary, it saturates at low temperature while displaying a flux modulation characteristic of a superconducting system. We therefore conclude, from these experimental observations alone, that the predicted insulating phase does not exist, thereby contradicting the theory defended by HS.

We stress that this conclusion does not need any theoretical input regarding the RSJJ. Thus, any conceptual or technical error we could have made in our subsequent theoretical analysis of the RSJJ would not at all affect this conclusion.

As already discussed in the metadiscussion section of the article, the only way to challenge our conclusion would be to provide an argued explanation why the expected power law is not observed in the range we explore and to predict in which range it would have been observable. However, the Comment [5] fails to do so, as discussed below.

### II. CRITICISM TO HS’S THEORETICAL DESCRIPTION OF THE SB PREDICTION

(valid independently of our work)

In the Comment [5], the core of the argumentation is based on the dissipationless Hamiltonian Eq. (1) [6], although, by construction, this Hamiltonian obviously

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cannot be used to predict the dissipative phase transition. In contrast, the common ground is (as we do in our article) to start from the Caldeira and Leggett (CL) Hamiltonian which correctly implements dissipation. It is well known, as also pointed out in our article, that, within the CL Hamiltonian, the system is invariant with respect to any translation of the quasicharge, hence yielding a perfectly flat ground Bloch band  $E_0(\tilde{Q})$ . Consequently, the “insulating state” invoked by HS, corresponding to the quasicharge at a local minimum of this band, simply does not exist for a shunted junction and is, thus, irrelevant for explaining a possible phase transition in RSJJs.

Moreover, even in the unshunted case, HS’s “effective Hamiltonian” Eq. (3) describes only a semiclassical theory which intrinsically misses relevant quantum effects. Specifically, HS’s Eq. (3) is not a proper Hamiltonian, because  $\varphi$  and  $\tilde{Q}$  are not conjugate variables. In other words, although Hamiltonian Eq. (3) may naively seem a dual form of Hamiltonian Eq. (1) in which  $E_0(\tilde{Q})$  would play the role of  $-E_J \cos \varphi$ , this dual picture is broken, because Eq. (3) notably lacks a “kinetic-energy”-like term (i.e., a concave  $\varphi$  term) providing a finite mass to the  $\tilde{Q}$  “particle,” for yielding the same zero-point phase fluctuations as Eq. (1) which it is supposed to replace. As a consequence of this broken duality, the semiclassical equations of motion Eq. (4) and the linear response Eq. (5) derived from Eq. (3) fail to capture crucial quantum

parts of the system’s dynamics (as is entirely expected in a semiclassical approach). Starting from a proper dual Hamiltonian, the kinetic-energy-like term missing in Eq. (3) would indeed yield an extra term in the equations of motion Eq. (4) which would be nothing else than the inductive response of the junction. Thus, the JJ inductive response (and the associated supercurrent) is unwittingly disregarded in the semiclassical presentation of the DQPT prediction made by HS [7], and it may explain why they believe it is incorrect that an inductive response of the JJ appears in our analysis.

Finally, HS write that the finite-voltage zero-dc-current solution of their Eq. (2) “describes the insulating state” of the unshunted JJ. Actually, the state they refer to corresponds to the well-known ac Josephson effect [see Fig. 1(a)], in which the JJ carries a steady-state sinusoidal supercurrent  $I_0 \sin(2eV_{dc}t/\hbar)$  (with a zero time average, i.e., zero dc current). This oscillatory supercurrent is due to Cooper pairs shuttling through the junction [8], a situation which obviously does not fit the common sense definition an *insulating state* [9]; i.e., charge carriers cannot go through. That standard definition of *insulating* actually forbids resistive or superconducting charge flow and, particularly, implies the actual dc current is always zero. However, the logical contrary does not hold: A zero dc current does not imply that the system is actually *insulating*. Hence, HS’s usage of the terminology “insulating

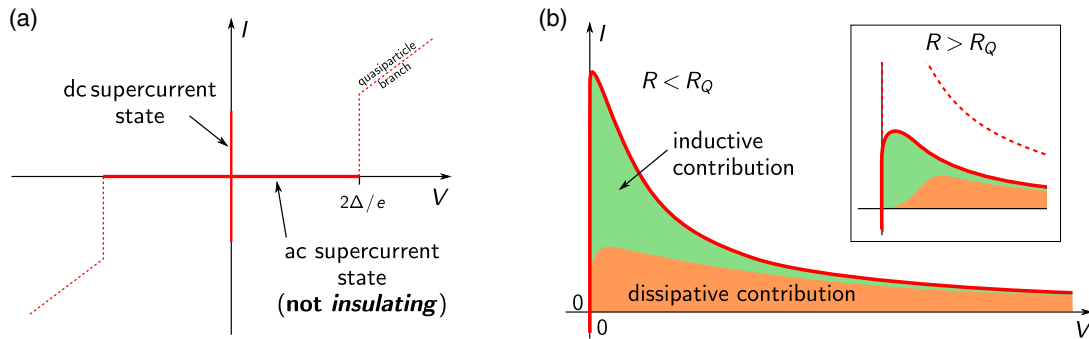


FIG. 1. (a) Predicted  $I - V$  characteristics of an unshunted Josephson junction. In that simple system, evaluating the dc mobility  $\mu = \lim_{V \rightarrow 0^+} (I/V) = 0$  erroneously suggests the junction is insulating, because it completely misses the ac Josephson supercurrent and the dc supercurrent branch, the most remarkable features of that system. (b) Sketch of the top right quadrant of the  $I - V$  characteristic of a RSJJ, close to the origin (for  $R < R_Q$ , see Ref. [47] in our article and, for  $R > R_Q$ , Fig. S15 in Supplemental Material in Ref. [29] of our article). In contrast to the dissipationless unshunted case (a), one can now have a finite dc current at finite voltage, because the resistor can absorb the corresponding power. The total dc current has a contribution from the time evolution of the supercurrent not averaging to zero in the steady state, leading to a ground state inductive contribution  $\propto E_J \langle \cos \varphi \rangle$  at zero bias (green area) and a contribution arising from the dissipative Kubo term proportional to  $E_J^2$  (orange area). Most theory papers on the SB transition evaluate the dc mobility which captures the orange contribution only, overlooking [very much as discussed in (a)] the junction’s dynamical inductive (superconducting) response which dominates transport properties at equilibrium and close to it. Such dc mobility calculations indeed predict a transition, with a low-voltage suppression of the orange contribution at  $R > R_Q$ , as shown in the inset [10]. However, the green contribution has no dissipative transition, because the equilibrium inductive response  $\propto E_J \langle \cos \varphi \rangle$  remains finite for all values of the resistance (although it is reduced at large  $R$ ), so that the RSJJ is always superconducting. Equilibrium transport properties are accurately characterized by dc mobility calculations only in open normal-state systems where one is certain to never have any supercurrent.

state” for the unshunted voltage-biased JJ is arguably incorrect in the standard meaning, and, since no suitable alternate unconventional definition is given, it is a source of inconsistencies and confusion. For instance, HS’s uncommon meaning of “insulating” leads them to consider that an unshunted voltage-biased JJ is simultaneously superconducting and “insulating,” at which point the mere concept they want to defend (that of a superconducting-to-insulating transition) no longer makes sense. Similarly, HS argue that the observation of a superconducting modulation in our junction is compatible with our junction being “insulating,” which makes them object to our conclusions that the junction is not *insulating*. In our article, there is no such confusion or ambiguity: The meanings of *insulating* and superconducting strictly adhere to the common understanding and are, thus, mutually exclusive, as most people expect. Consequently, in our work, a superconducting-to-insulating transition is a squarely defined concept leaving no place for subjective interpretations or byzantine discussions.

### III. DISMISSING HS’S CRITICISM TO OUR WORK

In Appendixes F and G of the article, we explain the predicted phase transition results from a charge mobility calculation that overlooks the inductive response of the JJ tied to its superconducting behavior and that such a mobility argument cannot properly characterize the equilibrium transport properties (see Fig. 1) unless one establishes by other means that the JJ inductive response  $\propto E_J \langle \cos \varphi \rangle$  vanishes (in which case the junction indeed becomes insulating). In their Comment [5], when they explain the SB prediction, HS indeed confirm they overlook the superconducting behavior of the junction (e.g., the junction is already assumed insulating when stating that charge relaxation in the circuit is governed by the  $RC$  timescale) and the electrical equations of motion and linear response they derive also ignore the JJ inductive response, as explained above. They attempt to justify not considering an inductive response in their analysis by suggesting it is rather a spurious term in our analysis due to using an ansatz introduced by one of us in Ref. [23] of our article and/or due to our compact phase approach overlooking phase slips. However, these tentative assertions are both incorrect.

First, it is actually well known that the JJ’s inductive behavior directly emerges from the Josephson Hamiltonian already at the classical level [11] without using any ansatz. Furthermore, we stress that, unlike HS write, the content of the commented article is self-contained and independent of Ref. [23] in our article; in particular, in the present work, the JJ’s environment impedance is just the Ohmic impedance of the plain CL model [12].

Second, HS believe that, since a compact phase description lacks “a procedure to keep track of the rotation angle multiples of  $2\pi$ ,” it cannot capture phase slip physics and is,

thus, inherently flawed. Let us first remark that, if it were true, it is very unlikely that such a fundamental issue would not have been raised in the literature that previously investigated the compact vs extended phase debate (see Ref. [33] in our article; recent works also consider the compact phase is unproblematic [13,14]). In any case, we now show that a compact phase description consistently handles all of the phase slip physics. In the RSJJ system, Kirchhoff’s loop law relates the junction phase  $\varphi$ , the flux across the resistor  $\Phi_{\text{Res}}$  (the time integral of the voltage), and the magnetic flux  $\Phi_{\text{loop}}$  in the loop between the junction and the resistor through  $\Phi_{\text{loop}} = (\hbar/2e)\varphi - \Phi_{\text{Res}}$ , where  $\varphi$ ,  $\Phi_{\text{loop}}$ , and  $\Phi_R$  are all operators. *Phase slip events* are easiest to apprehend when  $\varphi$  is well localized in an extended phase description: In the phase representation, the diagonal of the reduced density matrix  $\text{diag}\rho(\varphi)$  forms a narrow phase “packet” going from one well of the  $\cos\varphi$  potential to the next (for instance, because of a thermal fluctuation in the resistor). During such an event,  $\varphi$ ,  $\Phi_{\text{Res}}$ , and  $\Phi_{\text{loop}}$  evolve jointly, according to current conservation and the constitutive relations of the various elements (Ohm’s law, capacitor’s law, and Josephson’s relation) [15]. In this extended phase description, the index of the well where the phase sits (often called the winding number) is well defined and changes by  $\pm 1$  at each phase slip; to HS, it seems an important parameter that would “keep track” of the state of the RSJJ, but such is not the case, actually. Indeed, the state of the junction depends only on  $\cos\varphi$  and is thus clearly independent of the winding number. As well, the state of the resistor depends only on the instantaneous voltage (the time derivative of  $\Phi_{\text{Res}}$ ) and is, thus, insensitive to the instantaneous or past values of  $\varphi$  or the winding number (the CL Hamiltonian is translationally invariant in phase, too). Finally, in this system where no equilibrium permanent current can exist,  $\Phi_{\text{loop}}$  would always relax to zero (or a fixed, externally imposed value), and it thus cannot keep track of the winding number either. Hence, the winding number is irrelevant in the dynamics of phase slips in the RSJJ, already for an extended phase. We now turn to the compact phase case. As we define them, compact phase states have a  $2\pi$ -periodic  $\text{diag}\rho(\varphi)$ ; i.e., the phase is in all wells of the extended description at the same time. In this description, there is indeed no winding number, but lacking an irrelevant quantity cannot be the flaw HS claim it is. When the periodic  $\text{diag}\rho(\varphi)$  vanishes between each well, a phase slip event such as described above still occurs all the same. Let us now consider the case of high-impedance circuits where the junction’s periodic  $\text{diag}\rho(\varphi)$  nowhere vanishes (i.e., the phase is largely delocalized). Although isolated phase slip events such as above can no longer be well defined in that case, “tunnelings between minima of the phase-dependent potential” do occur in both directions simultaneously, and they are taken into account properly in  $\text{diag}\rho(\varphi)$  itself. Hence, HS’s statement that the junction

retaining a finite inductive behavior in our approach would be due to ignoring phase slip physics appears unfounded.

Although HS believe that our analysis does not respect the charge-phase duality principle, we explain in the article how we exploit precisely that duality to arrive at our Fig. 4, and the symmetry of that figure actually illustrates charge-phase duality. In our depiction of this duality, all states are superconducting, but this omnipresence of superconductivity does not violate anything. That HS expect the dual of a superconducting state to be an “insulating state” probably results from the confusion about the real nature of their “insulating state” and the broken duality picture arising from their “effective Hamiltonian” Eq. (3) which has spurious insulating states and overlooks the JJ superconducting behavior, as explained above.

We mention in the introduction of the article that, in the  $R \rightarrow \infty$  limit, the RSJJ model gives the Cooper pair box (CPB) type of qubits, which poses a continuity problem to the DQPT prediction: On the one side, there are many evidences of an excellent agreement between CPB experiments and theoretical predictions regarding *unshunted* junctions (all predicting superconducting junctions with  $E_J \langle \cos \varphi \rangle > 0$  in the ground state) and even direct experimental evidences of supercurrent flow (Ref. [20] in our article). On the other side, the theory of the DQPT leaves no room for the junction to be superconducting in the  $R \rightarrow \infty$  limit, in order to agree with the above evidences. Yet, HS claim that the dissipationless limit of the CPB is a purely academic result unattainable in practice, implying that in actual CPB experiments the DQPT should always apply and yield insulating junctions, against all the evidences mentioned above which show the opposite. Their reinterpretation of the SB phase diagram (their Fig. 1) does not help resolving the contradiction either: In their diagram, the observable phase boundary intercepts the left axis at  $E_J/E_C \simeq 7$ , implying that only CPBs above that threshold should be observed superconducting and the other ones insulating, but no such change of behavior is observed in CPB-type qubit experiments. Hence, HS neither acknowledge nor address the unresolvable discrepancy between experiments and any version of the theory they want to defend. In contrast, our work proposes a simple solution that resolves all past theoretical inconsistencies and that is compatible with all simple theoretical limits and all experimental observations so far.

Just before their conclusion, HS claim the quantum critical regime (where the universal power laws are observable) of the alleged insulating state would occur at extremely low energies and is, hence, altogether out of the accessible range for our experiment. This claim is based on the same reasoning HS use to plot the apparent phase boundary in their Fig. 1, with equations evaluated for the parameters of our sample 2, at maximum  $E_J$  (where  $E_J \simeq 2.4E_C$  in the convention they use). These parameters are situated in HS’s “apparent superconducting phase,” where,

indeed, our observations are seemingly compatible with their analysis. However, their evaluation assumes  $E_J \gg E_C$  [Eq. (4.18) of Ref. [14] in the Comment]; it does not apply for our sample 1, nor for sample 2 when  $E_J$  is reduced by the flux in the SQUID. Hence, HS’s evaluation by no means proves the alleged quantum critical regime is everywhere out of reach of our experiment.

#### IV. CONCLUSIONS

In summary, in this Reply, we show that

- (i) Our conclusion on the nonexistence of a DQPT in RSJJ is drawn from bare experimental facts in application of a generic theory of quantum phase transitions, needing no model for the RSJJ itself. Hence, even if our theoretical analysis of the RSJJ would be flawed in any way, our experimental results would still contradict the theory defended by HS.
- (ii) We point to issues in HS’s theoretical description which easily explain differences with our analysis and notably why the junction inductive response was overlooked in the old theory HS defend.
- (iii) A semantic issue regarding the meaning of “insulating” in HS’s Comment [5] (also present in part of the corresponding literature) creates confusion and inconsistencies which explain some of their objections to our work.
- (iv) The objections HS raise against our use of a compact phase, the inductive response of the junction, or the duality principle are unfounded.
- (v) The Comment [5] does not even acknowledge the major contradiction raised by the theory HS defend in the  $R \rightarrow \infty$  limit and which motivated our work.
- (vi) The numerical evaluation HS perform is not valid for all the values of the sample parameters we probe; one cannot conclude from this evaluation that the quantum critical regime would be always out of reach of our experiment.

We believe this Reply clarifies in which respect the understanding of the SB prediction in Josephson junctions could be unsatisfactory prior to our work, beyond what is already in our article. We maintain all the conclusions presented in the article and stress that our analysis of the RSJJ is presently the only one compatible with our experimental results and all simple limits.

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Science and Technology (IQST). C. A. acknowledges funding from the European Research Council under the European Union’s Horizon 2020 program (ERC Grant Agreement No. 639039).

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- [2] See Refs. [38, 65–69, 71, 72] in our article but also, for systems belonging to other universality classes, Z. Iftikhar, A. Anthore, A. K. Mitchell, F. D. Parmentier, U. Gennser, A. Ouerghi, A. Cavanna, C. Mora, P. Simon, and F. Pierre, *Tunable Quantum Criticality and Super-Ballistic Transport in a “Charge” Kondo Circuit*, *Science* **360**, 1315 (2018); Z. Iftikhar, S. Jezouin, A. Anthore, U. Gennser, F. D. Parmentier, A. Cavanna, and F. Pierre, *Two-Channel Kondo Effect and Renormalization Flow with Macroscopic Quantum Charge States* *Nature (London)* **526**, 233 (2015); A. J. Keller, L. Peeters, C. P. Moca, I. Weymann, D. Mahalu, V. Umansky, G. Zaránd, and D. Goldhaber-Gordon, *Universal Fermi Liquid Crossover and Quantum Criticality in a Mesoscopic System*, *Nature (London)* **526**, 237 (2015); P. Maher, C. R. Dean, A. F. Young, T. Taniguchi, K. Watanabe, K. L. Shepard, J. Hone, and P. Kim, *Evidence for a Spin Phase Transition at charge Neutrality in Bilayer Graphene*, *Nat. Phys.* **9**, 154 (2013); H. T. Mebrahtu, I. V. Borzenets, D. E. Liu, H. Zheng, Y. V. Bomze, A. I. Smirnov, H. U. Baranger, and G. Finkelstein, *Quantum Phase Transition in a Resonant Level Coupled to Interacting Leads*, *Nature (London)* **488**, 61 (2012).
- [3] Note that Ref. [10] in the Comment is irrelevant for discussing the SB prediction: It is about a nondissipative circuit consisting of a JJ shunted by a large superconducting inductance (also known as a superinductance).
- [4] In particular, HS rely on out-of-equilibrium measurements in Refs. [7] and [5] of the Comment, and they claim that the DPQT “can be detected only in nonlinear response.” However, they do not explain why quantum phase transitions in JJs would not follow general principles.
- [5] P. Hakonen and E. B. Sonin, preceding comment, *Phys. Rev. X* **11**, 018001 (2021).
- [6] All equation numbers refer to the Comment [5], unless specified otherwise.
- [7] Since HS’s Eqs. (4) and (5) in the Comment disregard the junction’s superconducting response, they describe the same situation we consider in our article Appendix G. 1, capturing only the orange contribution in Fig. 1(b).
- [8] One can evidence this ac Cooper pair flow by adding an ac voltage drive at the frequency  $2eV_{dc}/h$ . Under these conditions, the nonlinearity of the first Josephson relation partly rectifies the ac current to yield a nonzero dc current (the so-called Shapiro steps of the ac Josephson effect), clearly showing the junction is not insulating.
- [9] In the remainder of this Reply, *insulating* written in italic refers to the standard meaning of the term (charge carriers do not go through), while “insulating” between quotes refers to HS’s improper description of the finite voltage ac Josephson supercurrent state.
- [10] See Appendix G. 1 of the article for a basic calculation. Note, however, that for a shunt resistance smaller than  $R_Q$  such as depicted in the main panel in Fig. 1(b), more elaborate calculations are required for not having a divergence as  $V \rightarrow 0$  (see Ref. [9] in our article). Note also that such a predicted transition does not take into account the backaction of the junction on the resistor, as mentioned in Appendix G. 2.
- [11] A. Barone and G. Paternò, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
- [12] In Ref. [23] in our article, the backaction of the junction on the environment is treated in a self-consistent linear mean-field approach. In the present article, the backaction manifests through the entanglement of the JJ with the low-frequency bath oscillators in the density matrix of the full system, which is, however, not worked out and left implicit.
- [13] D. T. Le, J. H. Cole, and T. M. Stace, *Building a Bigger Hilbert Space for Superconducting Devices, One Bloch State at a Time*, *Phys. Rev. Research* **2**, 013245 (2020).
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- [15] For the evolution of  $\Phi_{loop}$  to be fully specified, one needs to introduce a loop inductance. In the Caldeira-Leggett model, this inductance is not taken into account (it is considered negligible).