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Warping effect in a multifiber beam model for nonlinear analysis of reinforced concrete structures under torsion

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Abstract :

A procedure is developed to account for torsional warping in the computation of fiber deformations in multifiber beam element models. The theoretical and numerical formulations to compute the cross section warping are presented and validated against axial displacements in a 3D finite element model of a column. The global nonlinear response of a column under torsion is then computed using a multifiber beam model. Differences in global and local behavior between both models with and without warping show the importance of warping effect both in linear and nonlinear behavior.

Keywords : torsion ; warping ; damage

1 Introduction

Non linear behavior of reinforced concrete structures can be accurately computed using continuum based 2D or 3D finite elements. However, this method is computationally expensive. Modeling concrete structures with 1D beam elements on the other hand is not accurate enough to account for damage variation in the transverse directions, which highly impacts the global structural behavior. As a trade-off, multifiber beam elements have been developed. They consist in computing displacements and forces at the beam element nodes, while generalized kinematics is computed at the cross-section level. Thus, based on the cross-section kinematics, stresses and strains are computed at the fiber level, allowing various constitutive laws and material states throughout the section ([4]).

Multifiber beam element have proved their accuracy for slender frame structures subject to bending and/or axial loading, when Euler-Bernoulli beam theory can be reasonably applied. Although Timoshenko's beam elements have been introduced to take shear deformation energy into account (see for example [1]), the method still fails to capture the behavior of shear dominated members. Timoshenko's assumption of a rigid body motion for the cross sections does not longer lead to accurate deformations in the cross section. When the structural member is subject to torsion, Mazars et al. ([3]) show that ignoring warping of a plain concrete square cross section leads to a wrong torsional stiffness and damage profile. To address this problem, Mazars et al. ([3]) take warping into account in the kinematics of the cross section by adding a warping function to Timoshenko's kinematics assumptions for torsion. The warping function is obtained by initially solving a Laplace equation on the homogeneous cross section. In [3], warping profile is kept constant throughout the computation of beam behavior.

In this paper, a method to account for updated torsional warping of a composite cross section is investigated, with the aim of analyzing the nonlinear behavior of a reinforced concrete structural member under torsion using a multifiber beam approach. The theoretical formulation for the warping function is developed. Numerical examples validate the linear elastic kinematics of the cross section. The computation of a multifiber concrete column subject to torsion highlights the importance of warping in the global behavior, both in the linear and non linear ranges.

2 Theoretical background

A straight prismatic column of length L, made of M homogeneous materials is considered here. The column is fixed at the bottom and twisted at the top. Twist angle is denoted by θ. Following Saint-Venant torsion theory, axial displacement u is governed by a warping function $\varphi(y,z)$. Kinematics in the column is given by equation 1, assuming small displacements. The origin of coordinates in the cross section is the shear center.

$$\begin{cases} u(x,y,z) = \frac{\theta}{L}\,\varphi(y,z) \\ v(x,y,z) = -\frac{\theta}{L}\,x\,z \\ w(x,y,z) = \frac{\theta}{L}\,x\,y \end{cases} \tag{1}$$

Knowledge of φ is required to account for warping when computing the deformations in the multifiber beam cross section. Governing equation for φ is presented for an homogeneous material first, following Saint Venant theory. Conditions at material interfaces and boundary conditions are then expressed, in order to build the finite element formulation to solve for the warping function, in the case of a composite cross section.

2.1 Warping function governing equation for an homogeneous material

Following on from kinematics given by equation 1, the linearized strain tensor reads:

$$\boldsymbol{\epsilon} = \frac{1}{2}\frac{\theta}{L}\begin{bmatrix} 0 & \frac{\partial\varphi}{\partial y} - z & \frac{\partial\varphi}{\partial z} + y \\ \frac{\partial\varphi}{\partial y} - z & 0 & 0 \\ \frac{\partial\varphi}{\partial z} + y & 0 & 0 \end{bmatrix} \tag{2}$$

Assuming that the material constitutive behavior follows Hooke's law, Cauchy's stress tensor follows on from strain as:

$$\boldsymbol{\sigma} = G\frac{\theta}{L}\begin{bmatrix} 0 & \frac{\partial\varphi}{\partial y} - z & \frac{\partial\varphi}{\partial z} + y \\ \frac{\partial\varphi}{\partial y} - z & 0 & 0 \\ \frac{\partial\varphi}{\partial z} + y & 0 & 0 \end{bmatrix} \tag{3}$$

Assuming that there is no body force, static equilibrium in the column reads:

$$\mathbf{div}\,(\boldsymbol{\sigma}) = \frac{\partial}{\partial y}\left(G\left(\frac{\partial\varphi}{\partial y} - z\right)\right) + \frac{\partial}{\partial z}\left(G\left(\frac{\partial\varphi}{\partial z} + y\right)\right) = 0 \tag{4}$$

which leads to, in the case of an homogeneous material:

$$\Delta\,(\varphi) = 0 \tag{5}$$

Equation 5 governs the warping function profile in each homogeneous part of the cross section. To formulate the warping function governing equations in the case of a composite cross section, the material interface conditions and section boundary conditions need to be defined for φ.

2.2 Interface conditions

The displacement field is assumed to be continuous in the cross-section. Following on from equation 1, the warping function φ is continuous over the interface between materials m and l, Γ^{lm}. This condition is expressed by equation 6. The stress field is also assumed to be continuous over the interface in the direction of normal $\hat{\mathbf{n}}$, which is expressed by equation 7.

$$\varphi^l = \varphi^m \quad \text{on the interface } \Gamma^{lm} \tag{6}$$

$$\boldsymbol{\sigma}^l \hat{\mathbf{n}}^l = \boldsymbol{\sigma}^m \hat{\mathbf{n}}^l \quad \text{on the interface } \Gamma^{lm} \tag{7}$$

Introducing the expression for $\boldsymbol{\sigma}$ given by equation 3 into equation 7 and taking continuity of φ into account leads to the interface condition on the warping function gradient (equation 8).

$$G^l \left(\frac{\partial \varphi}{\partial y} - z \right) \hat{n}_y^l + G^l \left(\frac{\partial \varphi}{\partial z} + y \right) \hat{n}_z^l = G^m \left(\frac{\partial \varphi}{\partial y} - z \right) \hat{n}_y^l + G^m \left(\frac{\partial \varphi}{\partial z} + y \right) \hat{n}_z^l \tag{8}$$

2.3 Boundary conditions

The lateral surface of the column is assumed to be free of load. This is expressed as a boundary condition for the cross section by equation 9.

$$G \left(\tfrac{\partial \varphi}{\partial y} - z \right) \hat{n}_y + G \left(\tfrac{\partial \varphi}{\partial z} + y \right) \hat{n}_z = 0 \quad \text{on the free boundary } \Gamma \tag{9}$$

2.4 Weak formulation

Starting from the strong form in each material m of the section given by equation 5, the weak formulation for the warping function problem goes as follows:

$$\int_{\mathcal{S}^m} \left(\frac{\partial^2 \varphi^m}{\partial y^2} + \frac{\partial^2 \varphi^m}{\partial z^2} \right) w \, d\mathcal{S} = 0 \tag{10}$$

where w denotes the test function and $\mathcal{S}^m$ denotes the homogeneous part of the section containing material m.

Integrating equation 10 by part and multiplying by the shear modulus of the material G^m leads to:

$$\int_{\mathcal{S}^m} G^m \left(\frac{\partial \varphi^m}{\partial y} \frac{\partial w}{\partial y} + \frac{\partial \varphi^m}{\partial z} \frac{\partial w}{\partial z} \right) d\mathcal{S} = \int_{\Gamma^m} G^m \left(\frac{\partial \varphi^m}{\partial y} \hat{n}_y^m + \frac{\partial \varphi^m}{\partial z} \hat{n}_z^m \right) dl \tag{11}$$

By summing over all the section materials and applying the interface conditions (equation 8), we get:

$$\sum_{m=1}^{M} \int_{\mathcal{S}^m} G^m \left(\frac{\partial \varphi^m}{\partial y} \frac{\partial w}{\partial y} + \frac{\partial \varphi^m}{\partial z} \frac{\partial w}{\partial z} \right) d\mathcal{S} = \sum_{m=1}^{M} \int_{\Gamma^m} G^m \left(z \, \hat{n}_y^m - y \, \hat{n}_z^m \right) dl \tag{12}$$

The finite element method is used to solve for φ, based on equation 12. Each element is assigned a shear modulus G corresponding to its material properties. This enables the warping function to be computed for a cross section with various material properties or to be updated during multifiber calculation if some of the material properties change due to concrete damage for example. Warping profiles obtained by this formulation are shown in section 3.

3 Validation of the kinematics at the cross section level

The 2D warping problem is solved on a $0.3\,\mathrm{m} \times 0.3\,\mathrm{m}$ square section by the method developed in part 2. About 1800 three-node linear triangles are used for the spatial discretization. Figure 1 compares the resulting elastic warping profile with the actual displacement field in the middle cross section of a 3D finite element model of a column under torsion. Two cases are investigated: a column with homogeneous concrete cross section ($G = 12.8\,\mathrm{MPa}$) and a reinforced concrete column ($G = 12.8\,\mathrm{MPa}$ for concrete and $G = 83.3\,\mathrm{MPa}$ for steel).

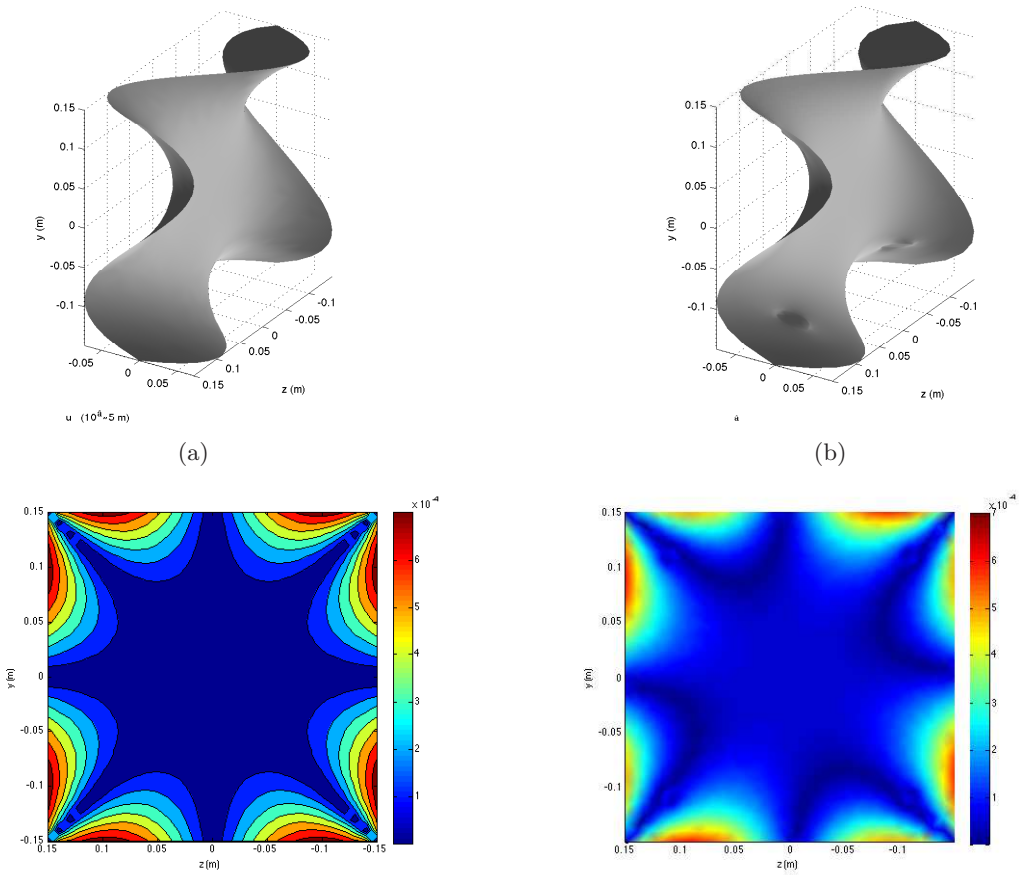


Figure 1: Elastic warping analysis of the cross section: (a) Warping profile of an homogeneous cross section; (b) Warping profile of a reinforced concrete cross section; (c) Error contours map with respect to the 3D analysis for the homogeneous cross section case; (d) Error contours map with respect to the 3D analysis for the reinforced concrete cross section case.

As a result of the comparisons, error contours maps are drawn on figures 1(c) and 1(d). The error measure is chosen as the normalized difference between the axial displacement field resulting from the 2D warping problem analysis and the axial displacement fields coming from the 3D computation. Error contours 1(c) and 1(d) show a good matching between the two solutions.

4 Application to nonlinear analysis of a concrete column

Nonlinear behavior of a plain concrete column under torsion is modeled using a multifiber approach to analyze the influence of warping kinematics on the torsional responses and damage profiles. A Timoshenko beam element is used, with or without adding the contribution of warping to the computation

of deformations in the cross section. Material constitutive behavior is governed by the MU model, 3D concrete damage model developed in [2].

When warping is ignored, the cross section kinematics is kept as Timoshenko's even in the nonlinear range of the behavior. When warping is considered, the solution procedure for φ is kept linear elastic as described in part 2, but the warping function profile is updated at each time step, through the use of a damaged shear modulus in each element of the cross section. The shear modulus is updated according to the material properties from the previous converged time step, enabling the use of an explicit scheme.

Figure 2 shows the torsional response of a twisted column of length $L = 10\,\mathrm{m}$ with a square cross section of dimensions $0.3\,\mathrm{m} \times 0.3\,\mathrm{m}$. The cases with and without warping are considered. The initial stiffness is lowered by about 20% due to the cross section warping. The peak torque is also affected, which is explained by the influence of warping on the damage profile for a given torque. Residual torques when damage of the cross section is complete are similar. These results are consistent with [3], which shows that multifiber approach overestimates stiffness and limit torque when compared to experimental behavior of a twisted concrete beam.

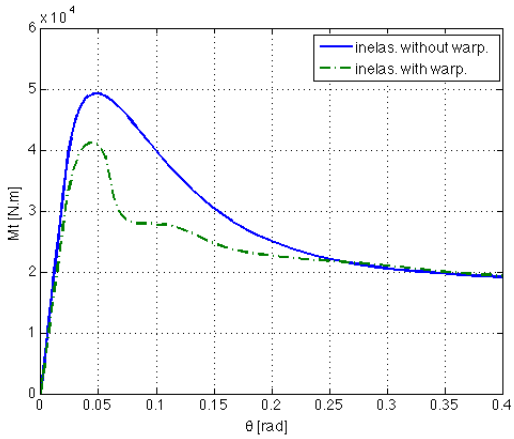
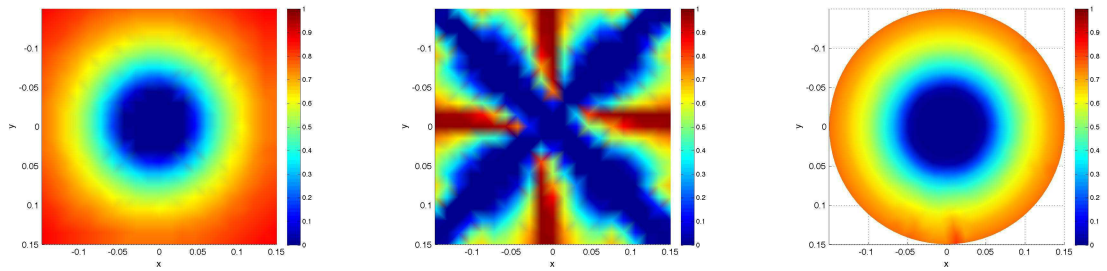


Figure 2: Torque moment versus twist angle curve, for the plain concrete multifiber beam, with and without warping taken into account

Figure 3 shows the damage profiles in the cross section of the concrete column, twisted by an angle $\theta = 0.24\,\mathrm{rad}$. Figure 3(a) shows damage profile in the square cross section, when the model does not take warping into account. As a reference, figure 3(c) shows damage values in a circular cross section. Although warping is activated in the model, the cross section remains plane, which is consistent with its axisymmetric shape. Damage contours in the square plane cross section of figure 3(a) match the ones in the circular cross section. By taking warping is into account however, figure 3(b) shows that the damage profile is sensitively modified.

5 Conclusion

A formulation to account for torsional warping in the deformations of the cross section of Timoshenko beam elements has been developed. The formulation enables to update the warping profile when material properties are affected by damage. When modeling a concrete column subject to severe torsional loading, taking warping into account leads to a more reasonable local and global behavior compared to a plain Timoshenko multifiber elements model.



(a) (b) (c)

Figure 3: Damage profile in the multifiber beam cross section:
(a) Case of a square cross section (model without warping); (b) Case of a square cross section (model with warping); (c) Case of a circular cross section.

With the method developed in this paper, modeling nonlinear behavior of reinforced concrete structural members under torsion is a straightforward step, subject of current work. Comparison with experimental results is planned, to complete the validation of the proposed approach. Aim of future work is further improvement of the method to handle coupled flexure-torsion loads with warping induced by shear forces. The model will be applied to the analysis of structural response to dynamic loading.

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