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Optimization by kriging of a 2D vertical channel asymmetrically heated

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Résumé :

On étudie un écoulement de convection naturelle d'air dans un canal vertical asymétriquement chauffé à flux constant présentant une ouverture sur sa paroi non-chauffée. L'influence de l'ouverture sur la dynamique et la thermique de l'écoulement est caractérisée. La méthode de krigeage est utilisée afin de construire un méta-modèle permettant de prédire le comportement du canal. Plusieurs configurations optimales du point de vue de la position et de la dimension de l'ouverture sont alors proposées afin de répondre à des objectifs précis (maximiser le transfert de chaleur, le débit du fluide aspiré par le bas). Les résultats sont présentés pour un nombre de Rayleigh de 5.10^5 et comparés avec une configuration sans ouverture.

Abstract :

Natural convection of air-flow in a vertical channel asymmetrically heated at a constant heat flux is studied. An opening is placed on the opposite wall to the heated wall. Its influence on the flow dynamics and heat flux is characterized. Kriging is used to build an estimator of some representative values. We seek the optimum configuration in terms of location and dimension of the opening regarding some objective functions as maximize the heat flux exchange or the mass flow rate entering by the inlet. Results are obtained for a Rayleigh number of 5.10^5 and compared with a configuration without opening.

Key words : optimization, kriging, natural convection, vertical channel

1 Introduction

Vertical channel asymmetrically heated is a frequently encountered configuration in industrial systems in which natural convection occurs. This prototype configuration led to many numerical [1, 2] and experimental [3, 4] studies where the heat transfer and the flow dynamics are described. Previous studies highlight two dynamical regimes for vertical channel asymmetrically heated. At low Rayleigh number (Ra) the fluid is entering entirely by the bottom. At high Rayleigh number a boundary layer regime near the heated wall with a recirculation cell at the top of the channel has been observed. A reference solution is given by [2] for $Ra = 5.10^5$. These works are restricted to channel which are open only at the top and bottom. However some industrial systems have an additional opening in lateral wall and have specific goals called here objective functions. Azevedo and Sparrow [5] had performed a parametric study for a configuration with one opening at the unheated wall and characterized its influence on heat transfer and flow dynamics. In this article we are interesting in an opening at the adiabatic wall of a vertical channel asymmetrically heated for $Ra = 5.10^5$. Its effect on heat transfer and flow dynamics is characterized. Different objective functions are proposed : maximize the mass flow rate entering by the bottom of the channel, the Nusselt Number, the bulk temperature and minimize the mass flow rate entering by the top of the channel. We use kriging, a statistical prediction method developed by [6] and adapted by [7], to build a meta-model of objective function. Kriging estimator is used to seek an optimal configuration for a given objective function in terms of location and dimension of the opening. Results are compared with a configuration without opening and with [5].

2 Kriging method : mathematical model and kriging estimator

Kriging is used in order to build a meta-model of an objective function Y which associates to a design variable $x \in E=R^k$, where k is the number of parameters in the design variable, a scalar value $y \in R$,

$$\begin{aligned} Y : R^k &\rightarrow R \\ x &\mapsto y. \end{aligned} \quad (1)$$

Kriging does not require any knowledge about this function except the m evaluations of $y = \{y_1, \dots, y_m\}$ at the points of the experimental design plan $D = \{x_1, \dots, x_m\}$. Knowing this, the method gives informations on $Y(x_0)$ where $x_0 \notin D$ and $x_0 \in E$. The different data are linked with a modelisation of a gaussian process. The objective function Y is written as

$$Y(x) = \sum_{j=1}^k \beta_j f_j(x) + Z(x), \quad \forall x \in E, \quad (2)$$

where $Z(x)$ is a gaussian process, $f(x) = \{f_1(x), \dots, f_k(x)\} = \{1, \dots, 1\}$ the base functions of the regression polynomial and $\beta = \{\beta_1, \dots, \beta_k\}$ the coefficient values of the regression polynomial. We consider a constant regression polynomial. According to the litterature, this choice has not significant losses in precision. More details about the mathematical background are given in [8, 9]. $Z(x)$ is a gaussian process with zero mean. It is expressed as follows,

$$\begin{cases} \text{Cov}(Z(x); Z(x')) &= \sigma_Z^2 K_\theta(x; x') & \text{covariance function,} \\ E(Z(x)) &= 0 & \text{mean.} \end{cases} \quad (3)$$

The space correlation function (SCF), denoted by K_θ is symmetric, strictly positive, and defines such as $K_\theta(x; x) = 1, \forall x \in E$. The variance of gaussian process $Z(x)$ is noted σ_Z . We consider the following space correlation function,

$$K_\theta(x; x') = \prod_{d=1}^k \exp(-\theta_d |x_d - x'_d|^2). \quad (4)$$

The values of the kriging parameters θ and σ_Z are unknow. These ones are estimated using the maximum likelihood estimation (MLE) [10]. According to the litterature the unknown parameters are expressed as

$$\begin{cases} \sigma_Z^2 &= \frac{1}{k} (y - F\beta)^t R^{-1} (y - F\beta), \\ \psi &= \arg \min \left[k * \log(\sigma_Z^2(\psi)) + \log[\det(R(\psi))] \right], \\ \psi &= \{\theta_1, \dots, \theta_k\}, \text{ parameter vector of SCF,} \end{cases} \quad (5)$$

with $\beta = (F^t R^{-1} F)^{-1} F^t R^{-1} y$, $R_{ij} = K_\theta(x_i; x_j)$, and $F = [f(x_1) \dots f(x_m)]^t$. Estimation of the kriging parameters is realized by minimization of Equation (5). We use a pattern search method [11].

We note \hat{y} the kriging estimator of Y . It is a linear estimator without bias which minimizes the mean quadratic error,

$$\begin{cases} \forall x_i \in D, & \hat{y}(x_i) = Y(x_i), \\ & \hat{y} \in \arg \min(EQM(\hat{y})), \\ EQM(\hat{y}) &= E[(\hat{y}(x) - Y(x))^2], \\ \hat{y} &= c^t * y. \end{cases} \quad (6)$$

Kriging estimator \hat{y} can then be written as

$$\hat{y}(x) = f(x)^t * \beta + r(x)^t * \gamma, \quad \forall x \in E, \quad (7)$$

where $\gamma = R^{-1}(y - F\beta)$, and $r(x) = [K_\theta(x; x_1) \dots K_\theta(x; x_m)]^t$.

We define the optimal value x_{opt} as $Y(x_{opt}) = \min(Y(x)), \forall x \in E$. If we search a minimum of the objective function Y , x_{opt} is obtained by the minimization of the kriging estimator \hat{y} using the pattern search method. When we seek a maximum of Y we work with the opposite objective function $Y' = -Y$.

3 Optimization of a vertical channel asymmetrically heated

Physical problem We consider a two-dimensional vertical channel filled with air, heated at a constant heat flux q'' to half of one of walls, other walls being adiabatic. The channel is open at the top and bottom ends on two semi-infinite environment at constant pressure p_0 and temperature T_0 . This case was proposed on experimental study [4] and adopted as numerical test case [1, 2]. Here we investigate the influence of one opening locating on the adiabatic wall. The event is opened on a semi-infinite environment at constant pressure p_0 and temperature T_0 . We note d the size of the opening and e the vertical distance between the inlet of the channel and the opening. We consider an aspect ratio of the heated part $A = \frac{H}{2L} = 5$, the Rayleigh number $Ra = \frac{g\beta q'' L^4}{\alpha\nu\lambda} = 5.10^5$ and the Prandtl number $Pr = 0.71$, as benchmarks [1, 2] do. At this Rayleigh number, the configuration without opening presents a boundary layer regime at the heated wall with a recirculation cell near the opposite wall [2].

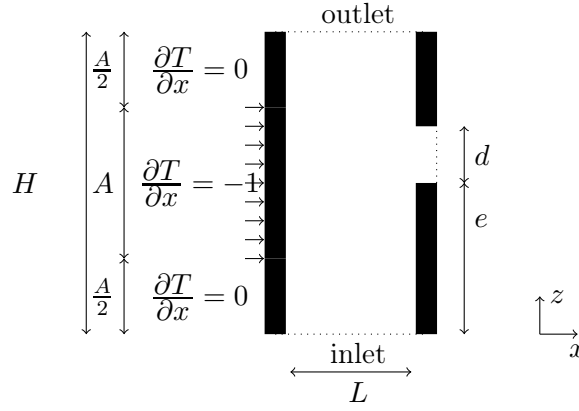


Figure 1: Configuration of the vertical channel asymmetrically heated

Boundaries conditions on the inlet, the outlet and the opening depend on the direction of the fluid,

$$\begin{cases} \text{if the fluid enters } \vec{V} \cdot \vec{n} < 0 & T = T_0, \quad P = -\frac{(\vec{V} \cdot \vec{n})^2}{2} + p_0, \quad \vec{V} \cdot \vec{t} = 0, \quad \frac{\partial \vec{V}}{\partial \vec{n}} = 0, \\ \text{if the fluid exits } \vec{V} \cdot \vec{n} > 0 & \frac{\partial T}{\partial n} = 0, \quad P = p_0, \quad \vec{V} \cdot \vec{t} = 0, \quad \frac{\partial \vec{V}}{\partial \vec{n}} = 0. \end{cases} \quad (8)$$

Objective function Four objective functions are defined in Table 1. Value of objective function is obtained by solving the unsteady Boussinesq equations. These equations are made dimensionless using L as unit length, the reference velocity is defined as $\frac{\alpha}{L} Ra^{\frac{1}{2}}$. They are discretized with a second order in time using an implicit discretization of linear terms and an explicit discretization of nonlinear terms. A second order backward Euler scheme is used for time integration. The problem is discretized spatially using a centered second-order volume finite scheme on a 64×256 staggered grid. The resolution of the energy equation is decoupled from the velocity-pressure problem and requires solving a Helmholtz problem. The coupling velocity-pressure is treated with a prediction-projection method.

Y_1	Y_2	Y_3	Y_4
maximize Nusselt number	maximize mass flow rate entering by the inlet	minimize mass flow rate entering by the outlet	maximize bulk temperature at the outlet
$\int_{A/2}^{3A/2} \frac{1}{T(0, z)} dz$	$\int_0^1 w(x, 0) dx$	$\int_{x_0}^1 w(x, H) dx$	$\frac{\int_0^1 w(x, H) T(x, H) dx}{\int_0^1 w(x, 0) dx}$

Table 1: Definition of objective functions

Design variable Design variable is composed of two parameters : the size and the location of opening. Table 2 shows the lower and upper boundaries of these parameters.

	lower boundary	upper boundary
d	$L/4 = 0,25$	$H - 5L/4 = 6,75$
e	$5L/16 = 0,25$	$5L/4 = 1$

Table 2: Boundaries of the design variable parameters

4 Numerical methodology

The numerical methodology is based on two codes : a volume finite code to solve Boussinesq equations and a code for kriging based on the Fortran Kriging (ForK) Library developped by [12]. The different steps of the numerical methodology are described in Table 3. The experimental design plan is based on 30 evaluations of Y . Use fewer points in design plan induces a loss of precision for the kriging estimator \hat{y} and a significant additionnal cost in computation time is generated by more points although the precision will be improved.

Step n°	Description	Method used
1	compute the m value of Y	volume finite code
2	estimate kriging parameters θ and σ_z	MLE and pattern search
3	build kriging estimator \hat{y}	—
4	find the minimum value of \hat{y}	pattern search
5	compute the optimal solution	volume finite code

Table 3: Steps of the numerical methodology

5 Results and discussions

Results are steady state solutions and they are obtained for $Ra_L = 5.10^5$, $H = 8$, $L = 0.8$, $A = 5$. For all configurations we observe a boundary layer regime near the heated wall with a recirculation cell near the opposite wall.

Objective function	d	e	Nu	G_{in}	G_{out}	T_{bulk}
Y_1	1.27	1	6.86	16.5	14.8	0.24
Y_2	6.75	0.25	6.64	66.7	11.9	$5.93 \cdot 10^{-2}$
Y_3	0.97	1	6.85	16.7	14.3	0.25
Y_4	6.05	1	6.55	52.4	4.2	$7.63 \cdot 10^{-2}$
without openings	—	—	6.68	72.9	14.7	$6.82 \cdot 10^{-2}$

Table 4: Comparison of optimal solutions

Table 4 shows the optimal solutions for the four objective functions. Figure 2 gives a mapping representation of the kriging estimation \hat{y} of the mass flow rate entering by the inlet of the channel. Results highlight that a configuration with an opening in the lower part of the channel tends to decrease the mass flow rate entering by the inlet compared to a configuration without opening. It is even more evident that the opening is larger. This is confirmed by observations made by [5] and explained by

the fact that it is easier to draw fluid through an opening than by the inlet since the frictional forces are reduced. In Figure 2 we identify two optimal solutions: a local optimum when the opening is located in the lowest part of the channel with the smallest dimension and a global optimum when the opening is in the top of the channel. This second optimum corresponds to the one observed by [5]. We show that mass flow rate entering by the inlet is greatly dependent of the opening.

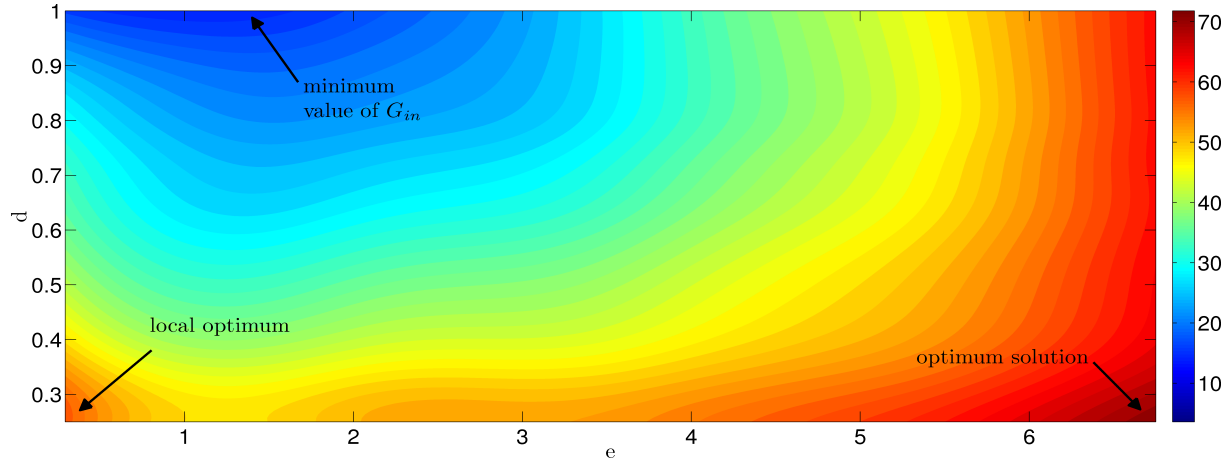


Figure 2: Estimation of the mass flow rate G_{in} entering by the inlet with the kriging estimator

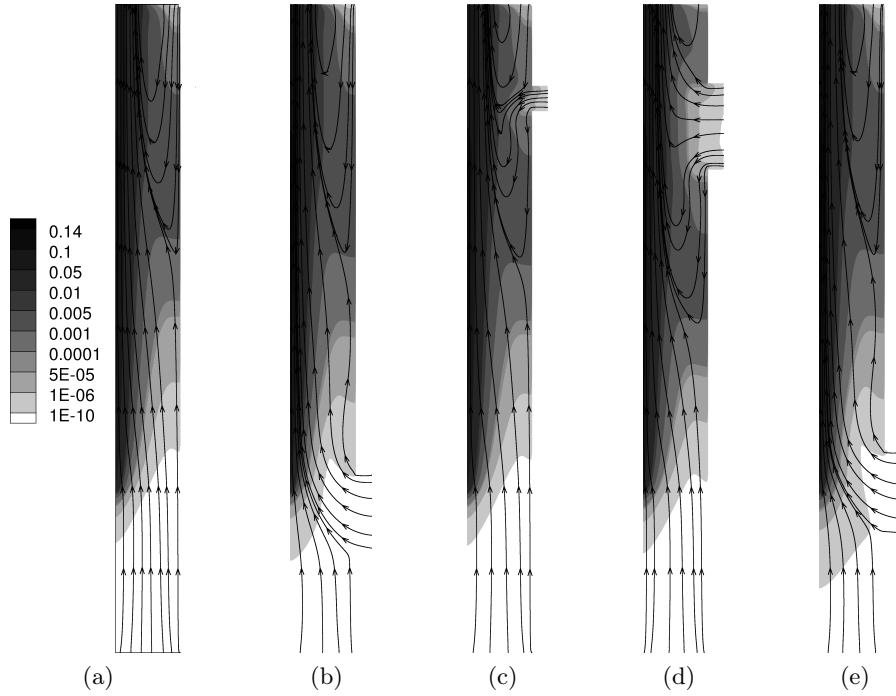


Figure 3: Streamlines and temperature for different configurations : without openings (a), Y_1 (b), Y_2 (c), Y_3 (d) and for Y_4 (e)

The fluid aspiration through the outlet is reduced when the opening is located in the upper part of the channel. In such configuration the fluid drawn through the opening supplies cold fluid to the recirculation cell. V-shaped recirculation observed by [5] is shown in Figure 3c. The width and depth of the recirculation cell is almost unchanged when fluid enters by an opening in the lower part of the channel. The total mass flow rate drawn through the channel remains almost constant with size and location of the opening. The field temperature near the heated wall is hardly influenced by opening. The Nusselt number varies only by 5% with the position and size of the opening. The insensitivity of the average Nusselt number to both the location and the dimension of the opening is confirmed by

[5]. It is important to notice that for all configurations the flow through the opening comes from the environment to the channel and never the opposite way.

6 Conclusions

We investigate the influence of an opening in the unheated wall of a vertical channel asymmetrically heated at constant flux and for a Rayleigh number of $5 \cdot 10^5$. The kriging method is used to build an estimator of some objective functions in order to find optimal configurations in terms of location and size of the opening. Different optimal configuration are presented. We always observe a boundary layer regime near the heated wall with a recirculation cell near the adiabatic wall. Mass flow rate entering by the inlet of the channel is sensitive to size and location of the opening and always less important than in a configuration without opening. The heat transfer and flow dynamics near the heated wall are not influenced by an opening in the unheated wall.

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