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# Reduced Order Modeling of flow fields in cylinder arrangements in the context of fluid structure interaction

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## Résumé :

*Une méthode de réduction de modèles dédiée à des problèmes d'interaction fluide/structure en dynamique en grands déplacements est proposée. La performance de l'approche est évaluée sur des configurations élémentaires mettant en jeu des vibrations de cylindres en réseaux soumis à des écoulements de fluides externes monophasiques. La construction d'un modèle réduit paramétrique est présentée dans la première partie et une étude de sensibilité de la stabilité de ces systèmes dynamiques est menée. L'approche de réduction de modèle est déclinée dans le cadre dynamique au moyen d'une formulation fluide/solide compatible avec la présence d'une interface solide mobile. Une étude paramétrique est ensuite proposée à partir d'un modèle réduit unique de référence; une amélioration possible prévue dans des travaux ultérieurs consistera en une interpolation de modèles réduits pour différentes valeurs d'un paramètre. Le choix de l'interpolation est crucial lorsque l'on remarque que l'ensemble des bases réduites a une structure de variété différentielle et non pas d'un espace vectoriel.*

## Abstract :

*A reduced order modeling method devoted to fluid structure interaction problems is proposed in statics and dynamics, for large magnitude motion. The performance is evaluated through model problems involving vibrations of cylinders under cross flows of external single-phase fluid. A parametric reduced model is also built and applied to the sensitivity analysis of a cylinder arrangement stability to fluid parameters. The reduced order modeling approach is formulated in a dynamic context through a fluid/solid formulation compatible with moving solid interfaces. The parametric model is built by using a single reference reduced basis, but it would be possible to use an interpolation of reduced basis corresponding to several values of the considered parameter. The interpolation choice is a challenge since we remark that the reduced order basis make up a differential manifold and not a vector space.*

**Keywords :** Fluid-Structure Interaction, Reduced-Order Models, Tube Arrays

## 1 Introduction

In this article the application of a Reduced-Order Method to large structure displacements due to the presence of a flowing fluid is investigated. The proposed model, called Multiphase-POD, is based on a classical POD-Galerkin method including adaptation to fluid-structure interaction. It consists in building a global velocity field for both fluid and solid domains, in order to tackle the problem of moving solid interface. A low-order dynamical system is obtained by a Galerkin projection of the global velocity field equations onto a POD basis. This technique is known to give satisfying results for small structure displacements [9]. Its ability to reproduce large displacements is highlighted in the present study. The approximation of flow fields and structure displacements in a cylinder arrangements is investigated. Moreover, as sensitivity studies are an integral part of instable mechanism numerical

studies, their contribution to the total cost has to be taken into account. That is why the last section is devoted to a method enabling to perform parametric analyses.

## 2 Reduced order model formulation

One considers a POD-Galerkin method [4] and its adaptation to fluid-solid interaction through a Multiphase-fluid-solid-POD formulation [6]. Complete calculations are performed with a pseudo-eulerian approach involving a moving grid method for modeling moving interfaces [8]. Thus, in dynamic case, in the presence of flow-induced vibrations, a classic POD-Galerkin method cannot be used because of the presence of a moving interface. POD modes are only spatial and consequently, they do not contain any dynamic information although snapshots have been taken for several positions of mesh nodes. To get round this problem, a method is proposed [6] to address fluid-structure interaction problem with an adaptation of the POD-Galerkin technique. It is called Multiphase-POD method, where a non-moving mesh is involved.

Considering a global domain containing the fluid and the solid, the latter is seen as another fluid with its specific characteristics, particularly the non-deformability. Thus, the whole domain is considered as a zone occupied by a two-phase flow. A unique and fixed mesh is created on this global domain and then, data obtained from a fluid moving-mesh complete calculation are interpolated on this fixed mesh, so that a classic POD basis can be constructed. The description of the Multiphase-POD method is the following : let us consider a global domain  $\Omega$  containing the fluid domain  $\Omega_f(t)$  and the solid domain  $\Omega_s(t)$  at each time step  $t$ , where the solid domain is considered as a particular fluid with its own physical characteristics (density, viscosity). We have  $\Omega = \Omega_f(t) \cup \Omega_s(t) \cup \Gamma_i(t)$ , where  $\Gamma_i(t)$  is the interface between fluid and solid domains. A global velocity field  $u \in H(\Omega)$  (with  $H$  a Hilbert space) is considered :

$$u(x, t) = u_f(x, t)\chi_{\Omega_f}(x, t) + u_s(x, t)\chi_{\Omega_s}(x, t) \quad (1)$$

where  $\chi_{\Omega_f}$  and  $\chi_{\Omega_s}$  are respectively characteristics functions defining if the considered point position is in the fluid or in the solid domain :

$$\chi_{\Omega_s}(x, t) = \begin{cases} 1 & \text{if } x \in \overline{\Omega_s} \\ 0 & \text{if } x \notin \overline{\Omega_s} \end{cases} \quad (2)$$

and  $\chi_{\Omega_f}(x, t) = 1 - \chi_{\Omega_s}(x, t)$ . Taking into account this notations, a global weak form of Navier-Stokes equations on  $\Omega$  is made possible to formulate :

$$\begin{cases} \int_{\Omega} \rho \frac{\partial u(x, t)}{\partial t} u^* dx + \int_{\Omega} (u \cdot \nabla) u \cdot u^* dx = \int_{\Omega} (\nabla \cdot \sigma) u^* dx \\ \int_{\Omega} \nabla \cdot u q dx = 0 \end{cases} \quad (3)$$

where  $u^*$  is a test-function defined as  $u^* \in H(\Omega)$  with the non-deformable solid constraint condition :

$$\tau(u^*) = 0 \text{ in } \Omega_s(t) \quad (4)$$

and  $q \in L_0^2(\Omega)$ . Each variable (density and viscosity) and each field is decomposed following the example of equation (1). Let us define both components of the constraints tensor  $\sigma$  :

$$\sigma_{f,ij}(x, t) = -p\delta_i^j + 2\mu_f\tau_{ij}(u_f) \quad (5)$$

where  $\delta_i^j$  is the Kroenecker symbol and  $\tau_{ij}$  is the deformation velocity tensor. The definition of the structural component  $\sigma_s(x, t)$  allows taking into account that the solid has its specific viscosity and the non-deformable structural condition. For the viscosity, a penalization term is used : in order to specify that the domain  $\Omega_s(t)$  is solid, the viscosity is artificially increased. To insure the non-deformable condition, a Lagrange multiplier  $\Lambda$  is added. Thus, the structural component of the stress tensor is :

$$\sigma_{s,ij}(x, t) = -p\delta_i^j + \Lambda + 2\mu_s\tau_{ij}(u_s) \quad (6)$$

Developing the global weak form with these definitions and making the Proper Orthogonal Decomposition on the global velocity flow field leads to the construction of a dynamical system for the whole domain  $\Omega$  which is fixed all over the time interval of interest. Taking into account the space-time decomposition of the global velocity field as :

$$u(x, t) = \sum_{i=1}^N a_n(t) \Phi_n(x) \quad (7)$$

where  $\Phi_n$ ,  $n = 1, \dots, N$  are elements of the POD basis and  $a_n(t)$ ,  $n = 1, \dots, N$  are time coefficients, the final dynamical system is the following :

$$\begin{cases} \sum_{i=1}^N \frac{da_i}{dt} A_{in} = \sum_{i=1}^N \sum_{j=1}^N B_{ijn} a_i a_j + \sum_{i=1}^N C_{in} a_i + E_n \\ D(u) = 0 \text{ on } \Omega_s(t) & \text{(non deformability)} \\ \frac{\partial \chi_{\Omega_s}}{\partial t} + u \cdot \nabla \chi_{\Omega_s} = 0 & \text{(characteristic function transfer)} \end{cases} \quad (8)$$

for each  $n = 1, \dots, N$  where  $N$  is the number of modes in the POD basis. Coefficients  $A_{in}$ ,  $B_{ijn}$ ,  $C_{in}$ ,  $E_n$  are not detailed here, but we precise that the coefficient  $E_n$  contains a boundary pressure term which is treated thanks to a penalization technique [1]. The interface pressure term disappears since the test functions are the POD modes, and this is precisely the interest of the Multiphase-POD technique. A very important point to notice is that they are not all exclusively spatial coefficients, because some of them contain the physical characteristics  $\rho(x, t)$  and  $\mu(x, t)$ . Thus, they have to be re-calculated at each time step : the time calculation is increased in comparison with a classic POD model without moving structure. But this computational effort is still less than what is required for a complete calculation. Another approach consists in making the proper orthogonal decomposition of the characteristic function  $\chi_{\Omega_f}(x, t)$  also, which allows avoiding the time dependence of all coefficients of the dynamical system. For more precisions, see [7]. Structural properties (damping, stiffness) are contained into the volume force. Practical implementation of the Multiphase-POD technique is described below.

1. Perform a complete fluid calculation of the fluid-structure interaction problem during a time interval  $[0, T]$  using a moving mesh pseudo-eulerian formulation.
2. Extract enough snapshots from this complete calculation.
3. Create a unique Cartesian fixed mesh containing both fluid and solid domains.
4. Interpolate each extracted snapshot onto the fixed reference mesh : new fixed snapshots are created.
5. Apply the classic POD approach for the new snapshots constructed on the reference mesh.
6. Construct the dynamical system following (8) and resolve it with a classic method (Runge-Kutta for example).

### 3 Stability analysis of dynamical system

One considers a configuration involving a square cylinder arrangement subjected to an external single-phase fluid flow characterized by a low Reynolds number  $\mathcal{R}_e = \frac{\rho U_p D}{\mu}$ . The gap fluid velocity  $U_p$  takes into account the confinement and is defined as :  $U_p = U_\infty \frac{P}{P-D}$  where  $U_\infty$  is the equivalent mean flow velocity in an infinite domain and  $P$  the pitch ratio.

Only the central cylinder is moving in the cross direction ( $y$  - direction). A 2D domain including only one cylinder and its neighbours is considered. Thus, the domain is not representing a whole tube bundle but a confined case. Complete calculations are performed with *Code\_Saturne* [2] which has been used in various studies involving FSI in tube bundle systems [5, 8]. Geometrical and physical values are gathered in table 1. Two cases are considered : a case denoted "stable" with limited and

Geometrical values		
Cylinder diameter	$D$	$1.10^{-2} m$
Pitch ratio (center-to-center distance)	$P$	$1,44 D$
Physical values		
Fluid		
Dynamical viscosity	$\mu$	$1.10^{-3} kg.m^{-1}.s^{-1}$
Density	$\rho$	$1.10^3 kg.m^{-3}$
Solid (moving cylinder)		
Mass	$m$	$6,13.10^{-3} kg$
Stiffness	$k$	$1,512 kg.s^{-1}$
Damping in air	$\zeta$	$0,0437 \%$
Frequency in air	$f$	$2,5 Hz$

TABLE 1 – Characteristic values of the problem

Case	Inlet velocity	$\mathcal{R}_e$
Stable	$0.015 m.s^{-1}$	491
Unstable	$0.03 m.s^{-1}$	982

TABLE 2 – Definition of the flow-induced vibrations studied cases

decreasing vibrations of the central cylinder and a case where instability is clear, called “unstable”, where the amplitude of vibrations is increasing. For these different cases, we choose low Reynolds numbers since the computation is bidimensional. Here the goal is not to work on a case validated by comparison with experiments or with specific models, but to observe the ability of the Multiphase-POD method to reproduce several configurations with their own properties (amplitude, frequency, etc.). Table 2 gathers the chosen Reynolds numbers corresponding to the incoming flow, and associated inlet velocities.

To study these cases, we modify the inlet flow velocity in order to obtain various Reynolds numbers. The mesh contains 633,000 cells. The goal here is to observe the ability of the Multiphase-POD method to fulfill two functions : reproduce complete calculation features (flow and structure velocity, structure displacement) and be predictive in the case of a parameter modification. That is why the 2D configuration seems to be a good compromise, giving low cost computations and memory storage to construct the correlation matrices. The application to a more realistic case (3D computation, high Reynolds numbers) represents an advanced step in this work. Here we work on simple cases in order to observe the Multiphase-POD method behaviour. Results of complete calculations required to collect snapshots and obtained for the two chosen cases are displayed. Fig. 1 shows snapshots of the global velocity field for each case.

The Multiphase-POD method is applied to the stable and unstable cases and one observes the reconstruction of the gravity center displacement of the cylinder by this method over one displacement period. Complete calculations are performed with the CFD code *Code\_Saturne* [2] and data at the interface are interpolated to the cylinder gravity center. No stabilization technique has been applied to dynamical systems here, in order to respect the numerical behaviour of these systems : for example, a stabilization could false the conclusion of a stable case reconstruction. However, it is very interesting to use stabilization techniques in order to offset the destabilization induced by the POD basis truncature (very useful and simple to use stabilization methods such as Cazemier-Rempfer type are presented in [3]). The results are presented with snapshots extracted over one period of the displacement. In the stable case, the reduced-order model is constructed with the following characteristics : 148 snapshots

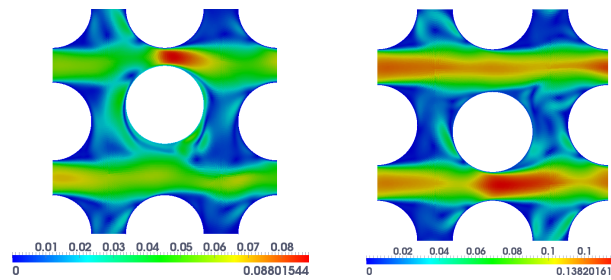


FIGURE 1 – Snapshots of the global velocity flow field for the two cases : left stable interaction, right unstable interaction

are extracted from the complete fluid calculation, 4 POD modes are constituting the POD basis. The fixed reference mesh contains  $200 \times 200$  points. If one observes the behaviour of the dynamical system over a longer time (several cycles), we can see that, for the stable case, the Multiphase-POD method is predictive since snapshots have been captured only over the first period (see Fig. 2 left). This predictivity is very interesting : engineers ensure that they can perform long computations with this POD basis (here we had to perform the complete computation on all the long time interval just to check the system behaviour). On the contrary, the unstable case is less predictive since only the two first cycles are correctly reconstructed by the dynamical system (snapshots are also gathered over the first cycle), see Fig. 2 right).

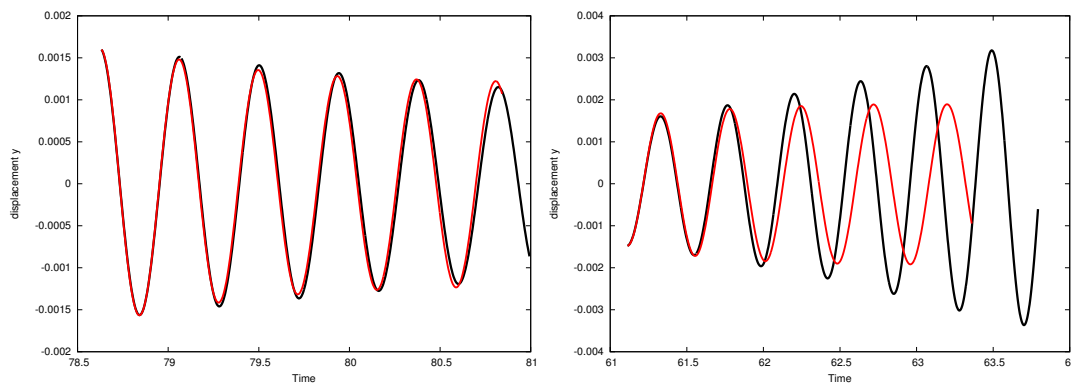


FIGURE 2 – Gravity center displacement reconstruction of the cylinder for the stable case over several cycles. — complete calculation ; — multiphase-pod reconstruction (4 modes left, 7 modes right)

Now, for the willingness of studying instability behaviours with parametric studies, a question arises : is the Multiphase-POD basis used for the stable case able to reproduce an unstable behaviour ? This question is the basis of an on-going work presented in the next section.

## 4 Parametric analysis using ROM

If we consider the Reynolds number as a changing parameter, we want to know if it is possible to cover an instability mechanism with few POD computations. A first step consists in taking the Multiphase-POD basis associated with the stable case and to project the discrete Navier-Stokes equations associated with the unstable Reynolds number onto this basis, and observing if the dynamical system thus obtained can correctly reproduce the fields (velocity and displacement). Fig. 3 shows the reconstruction of the unstable cylinder displacement with 9 POD modes from the stable basis, and even if one observes a delay between both complete and Multiphase-POD computations, the amplitude increase is reproduced. The above presented test is a first step in a work-in-progress : the stable POD basis does not contain a rich enough amount of information, thus features of the fluid flow could

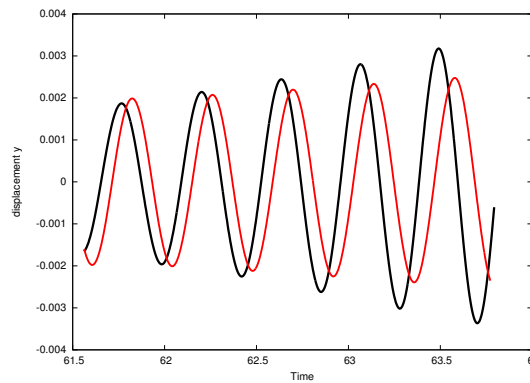


FIGURE 3 – Gravity center displacement reconstruction of the cylinder over several cycles, stable basis and unstable case (parametric study). — complete calculation ; — multiphase-pod reconstruction (9 modes)

be not captured with this method. The next step will consist in making a collection of various POD basis (each is associated with a value of the considered parameter) and constructing an interpolation between them in order to obtain a sustainable Multiphase-POD basis which could cover a range of values of the studied parameter. The challenge is to find the interpolation which is the best in terms of efficiency (gather a lot of information by keeping short computations).

## Conclusion

In this paper, the Multiphase-POD method is presented and applied to the case of a confined cylinder in tube array under cross-flow. The method was already shown to be efficient in the case of small and large displacements of a structure under flow solicitations and here, we show its efficiency in the case of a stable then unstable phenomenon. The application of parametric studies with the help of POD and Multiphase-POD consists in a work-in-progress : the main interest of reduced-order models consists in their ability to reconstruct various solutions of a system where one or several parameters have been changed. Indeed, the reconstruction of a solution for which we already have the complete calculation is not sufficient. Our goal consists in the application of these methods to the case of a fluid-elastic instability, in order to capture the critical velocity.

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