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Spacetime curvature, a potential result of the quantum mass/energy Information Entropy

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Abstract

his work proposes a possible theory and a proof on how the spacetime curvature is created by any object (mass/energy) and suggests a mathematical model that may conciliate quantum mechanics and general relativity. As each elementary particle is a fluctuation in its particular field in Quantum Field Theory and follows quantum mechanics properties, the wave function of each elementary particle of an object is deformed at each spacetime point in the presence of another massive object (set of particles). This comes from the fact that the wave functions of the particles of the two massive objects in their respective quantum fields may be aggregated and hence modifies in time and space the wave function values. This could subsequently change the location of the object less massive as the *probabilities* of the presence of the elementary particles are getting higher next to the highly massive objects: *the less massive objects are correlated to the highly massive one*. In this study we suppose that between two nearest spatial points N_x and N_2 there is an infinity of quantum-mechanically possible Time paths in quantum vacuum as assumed in path integrals theory. Using a modified Information Entropy of Shannon with complex probabilities on the Time paths arriving to N_2, we prove that this entropy is equivalent to the Hilbert-Einstein Lagrangian density which leads to the Einstein general relativity equation by minimizing this entropy or in other words by applying the principle of least actionhis work proposes a possible theory and a proof on how the spacetime curvature is created by any object (mass/energy) and suggests a mathematical model that may conciliate quantum mechanics and general relativity. As each elementary particle is a fluctuation in its particular field in Quantum Field Theory and follows quantum mechanics properties, the wave function of each elementary particle of an object is deformed at each spacetime point in the presence of another massive object (set of particles). This comes from the fact that the wave functions of the particles of the two massive objects in their respective quantum fields may be aggregated and hence modifies in time and space the wave function values. This could subsequently change the location of the object less massive as the *probabilities* of the presence of the elementary particles are getting higher next to the highly massive objects: *the less massive objects are correlated to the highly massive one*. In this study we suppose that between two nearest spatial points N_x and N_2 there is an infinity of quantum-mechanically possible Time paths in quantum vacuum as assumed in path integrals theory. Using a modified Information Entropy of Shannon with complex probabilities on the Time paths arriving to N_2, we prove that this entropy is equivalent to the Hilbert-Einstein Lagrangian density which leads to the Einstein general relativity equation by minimizing this entropy or in other words by applying the principle of least actionT

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I. INTRODUCTION

In the Universe, an Information or what we will call in all this paper the signal-Information can be an elementary particle, a set of elementary particles (atoms, molecules...) or anything that has the property of wave-particle. If the information *still exist* at $t+1$, i.e it can be detected at time $t+1$, travels **inevitably** in time from t to $t+1$ and may move also in space if a *correlated external additional information* exists: interactions, forces...or any other type of *correlation*. As in Quantum Field Theory (QFT)([1],[2]), we assume that all the other elementary particles are an excitation of their respective field, they all have the same mass and they are the same fluctuation in their field.

When the electron is at its wave property i.e. it can be in a superposition of locations, it cannot has the repulsive force to itself in other positions otherwise there will be no creation of the electron when detected. We suppose then that there is the summation/interference of wave functions in their fields. Two electrons when in their wave property that cannot be differentiated, their mutual wave functions hence may be added and hence may interfer. But as soon as they lose their wave property to become particles they repel each other thanks to the exchanged photons. This interfering or wave summation effect between same elementary particles is what

we will demonstrate to be the Gravity, and the entropy or the deformation of the information brought by all the wave functions at a spacetime location in the respective field is the well known spacetime curvature.

In this paper we will take the fact that the elementary particles in their respective fields have wave functions which can interfere one another as long as they are in their wave property. Moreover, as in QFT we consider that the vacuum ([3],[4]) is not truly empty but contains virtual particles and antiparticles that are able to *exist spatially* thanks to Heisenberg energy-time uncertainty principle $\Delta E \Delta t \geq \hbar$ and the equivalence energy-mass equation $E = mc^2$ for a short time Δt depending on the *mass* of the particle: particles and their respective antiparticles are constantly popping into existence and popping back out in a quite short time. We suppose that these virtual particles always exist in other dimensions, called Time dimensions i.e. the quantum vacuum dimensions, different from the three spatial dimensions (**3D**). Virtual particles would be the *excitatory mediators* and the virtual antiparticles the *inhibitory*. If one virtual particle and its one respective antiparticle exist in the same spacetime location they cancel each other directly. We assume that the virtual particles, as quantum particle in space dimensions, have also the wave properties.

We suppose now that there is a real particle present in a spacetime *around* the position (N_x, t), and at the same time and space location, in a *quite short time*, virtual particles and antiparticles in the same field may appear and could be similar to this real particle. As all the particles including the virtual ones have the wave properties, a *part of* a virtual antiparticle, thanks to the superposition property, may cancel the same proportion of the real particle. Hence we suppose in this paper at any spatial point there is a kind of aggregation of the real particles' and virtual particles'/antiparticles' wave functions. This aggregation of all these *signal-Information*, and the deformation of the amplitude sum are what we will study here and we will demonstrate that this deformation is the spacetime curvature.

II. Mathematical model for the information dissimination in the Universe

Model description: at each spatial point N_2 we aggregate around its virtual surface $\partial\Omega$ during Δt (respective to the studied field) all the signal-Information or in other words all the wave functions from the neighboring existed virtual particles and virtual particle/antiparticles. Δt and $\partial\Omega$ will be defined thereafter.

A. Information Reactions system model in Spacetime dimensions in Universe

As in quantum mechanics, we will suppose that between two nearest (in term of Planck length) spatial points N_x and N_2 there is an infinity of quantum-mechanically possible Time dimensions paths created by virtual particles, antiparticles and real particles. Note that one virtual/real particle can create an infinity of Time paths thanks to the superposition property of the quantum particles. Each from these paths has a particular Time action noted S_r that arrives with incidence r to N_2 from a particular point N^r (avirual image of the neighboring node N_x) by assuming that N_2 is bounded by a virtual surface $\partial\Omega$ which forms a border with the all N_x. Two different N^r can be virtual images of one N_x. Each Time path can be either an excitatory path if there is a real particle at N_x or if the acted virtual particle is a potential real particle (by energy-time incertitude principle) or an inhibitory path if the acted virtual particle is a potential real antiparticle Fig.1.

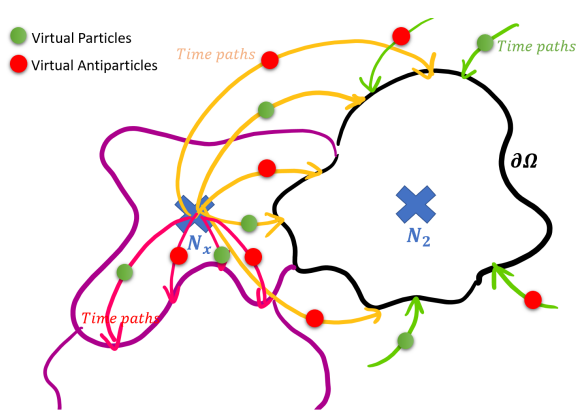

Fig. 1. Time dimensions paths created by virtual particles and antiparticles

All the signals coming from all the nearest spatial points N_x, if a Signal-Information SI (any energy-matter object) exists at this location, to N_2 are remodulated by each Time path and reach the point N_2 where they are assumed to be aggregated in a particular time duration noted Δt and on a virtual surface edge $\partial\Omega$ that limits the point N_2. This is similar on what Dirac and Feynman have done in the paths integral (see for example [5]). If two signals far in time arrives in a same point they could not be aggregated, so we add an additivity

time tolerance of signals Δt and it depends on the field type. In a particular field if during its respective time Δt (or less) at N_2, the energy aggregated is the energy of the particle of this field hence a real particle is *re*-created or created thanks to these two formulas: $\Delta E \Delta t \geq \hbar$ and $E = mc^2$.

We recall that between any N_x and N_2 there are no other points in the 3 dimensions of space and therefore this border $\partial\Omega$ is like a virtual border which limits N_2 to all the closest points N_x (Fig.1). The initial SI coming from a particular position N_1 to a particular nearest point N_2 is not only deformed due to different action of the paths (excitatory and inhibitory) but also due to the SI coming from the other neighboring nodes N_x to N_2 arriving during the time tolerance laps Δt of the signals aggregation. Δt could be transformed to a shorter duration $\delta t'$ if extra energy arrives. This deformation is what we will see is the gravity effect.

Each 3 dimensions spatial point N_2 in a particular field (QFT) is *excited* by the signal transmission aggregated coming from the neighborhood N_x and also by the quantum vacuum and its virtual particles/antiparticles, and if a particular threshold of energy is exceeded during Δt or less, a new Signal-Information deformed or not is created. This threshold is the parameter m in QFT that encodes a lower energy threshold (see for example the Klein-Gordon Lagrangian in [6]) from which a point in that field is excited. m could be equivalent to the time duration Δt in the case where there is no other mass/energy around. Therefore one can assume that the signal brought by the Time paths is a signal of the quantity of mass/energy per unit surface of $\partial\Omega$ and per time of Δt.

The SI amplitude coming from N^r, a particular N_x with incidence r to the border $\partial\Omega$ of N_2 at time t, denoted by $\psi(N^r, t)$, is the mass/energy wave function of the signal that existed in the node N_x at time t with incidence r that may be complex, as signals can interfer with one another thanks to time and space summation (signals far apart in time or/and space are less aggregated than the ones closer), remodulated in Time paths. Hence the main difference between $\psi(N^r, t)$ and $\psi(N_x, t)$ is that $\psi(N^r, t)$ takes into account also the action of the virtual particles and antiparticles. Real particles are assumed to be transformed to virtual ones between N_x and N_2.

This function would have the same definition as the wave function in quantum physics if we compute it in a 3D locations N_x at time t. Recall that N^r are not real spatial 3D points but the virtual images projection of the real 3D point N_x on the virtual surface $\partial\Omega$. In this paper, we suppose the square module of $\psi(N^r, t)$ **is equal to the density of energy/mass per surface $\partial\Omega$ and per time Δt** of the signal-Information coming with incidence r to N_2. If there is no particular real signal-Information (no energy or matter) existing at the upstreaming spatial point N_x i.e. $|\psi(N_x, t)| = 0$, the amplitude would be $|\psi(N^r, t)| = 1_{Time\ Dimensions}$ coming from the virtual particles/antiparticles in the quantum vacuum thanks to the time-energy principle incertitude that ensures quantum particles creation during less than Δt. $1_{Time\ Dimensions}$ is a wave function where $1^2_{Time\ Dimensions}$ is the density function in time and space (explained later) of virtual particles-antiparticles couple. Otherwise, if an object exists in N_x at time t hence $|\psi(N^r, t)| = 1_{Time\ Dimensions} + |\psi(N_x, t)|$, where $|\psi(N_x, t)|$ defines here the square root of the density of energy/mass per surface and time coming from N_x at time t which will be equal by value to the wave function amplitude in quantum mechanics theory.

We introduce now the *normalized* action induced by each Time-path, either there is a signal-Information SI or not in the upstream node N_x at time t, which will be defined by the integral in time of three Lagrangians: the Kinetic Lagrangian in Time dimensions created by the virtual particles and the real existed particles and may be equivalent to the well known mc^2, the Matter Lagrangian created by the usptream SI in the upstream point N_x at time t and the Inhibitory/Excitatory Lagrangian of the path induced by the type of virtual particles or antiparticles. Hence we define three normalized actions:

The normalized Time Kinetic Action:

$$\frac{S_r}{\hbar} = \frac{1}{\hbar}\int_t \frac{1}{2}\rho(\frac{dq_r}{du})^2 du. \tag{1}$$

where ρ represents the potential *mass* of the virtual particle and previous real particles, i.e. the potential forthcoming real particle/antiparticle, or the equivalent energy. $\frac{dq}{du}$ is the potential speed of that quantum of the signal-Information in Time dimensions. One can note that $\frac{dq}{du}$ may be equals to c the speed of light, however if we take the threshold time Δt equal for any field, $\frac{dq}{du}$ would be different. In this paper we fix Δt for all the quantum fields as the maximum one of all over type of fields in the vacuum, and let $\frac{dq}{du}$ depends on the field. The photon would have hence the maximum $\frac{dq}{du}$ wich is equal to c and would not experience the chosen maximal time Δt...our time.

$\frac{1}{\hbar}$ acts as the normalization of the energy density in time where if Lagrangian $L = \frac{1}{2}\rho(\frac{dq_r}{du})^2$ is equal to one energy unit L_1 for one time unit t_1, hence:

$$\int_0^{t_1} \frac{L_1}{\hbar} du = 1 \tag{2}$$

where $\frac{L_1}{\hbar}$ would be equivalent to the density to find a quantum of energy at u in probability theory, with $\hbar$ the maximal quantity of energy that can hold a temporal distance unit, the Time distance between two spatial 3D points N_x and N_2. With this definition, one can note that $\frac{S}{\hbar}$ is the mean of $\frac{L}{L_1}$ which is the number of energy units which is in the Physics laws, is equivalent to the frequency.

The normalized Matter Action:

This action is the phase $Ł_{m_r}$ of the initial signal coming from the point N_x at time t with incidence r. This Lagrangian represents the *density of* (explained later) the Lagrangian density of matter defined in the standard model which takes into account propagations and interactions and etc. (see for example [7]). $Ł_{m_r}$ is the density of the potential real information transferred in time to N_2 i.e. A virtual particle in Time dimensions does not have any effect on the real particles existed in space dimensions only if it exists at space dimensions. $Ł_{m_r}$ role is to keep the initial information taken from N_x at time t.

Between two nearest nodes N_x and N_2 all the signals $\psi(N^r, t)$ from N_x with incidence r propagate in all the possibles Time paths, and all the signals are summed in time and space at N_2. These additive action paths will be called Time excitatory paths and this is a result of the virtual particles.

So when aggregating all the SI, the inhibitory Time paths (created by the virtual antiparticles) reduces the total aggregated wave functions in N_2 and the excitatory Time paths enhance this aggregated wave function.

The normalized Inhibitory/Excitatory Action:

It is the action of a particular path in addition that describes *Time dimensions action* or what we could call the quantum vacuum action. If there is no matter/energy present in the space dimensions, the virtual particles present in the quantum vacuum defined in Quantum Field Theory (QFT) ([3],[2],[4]), present in pairs of virtual particles and antiparticles, may give the Inhibitory/Excitatory action property to a time path between the two nearest spatial points. These virtual particles give the definition of the Time dimensions or the time paths. Virtual particles and antiparticles are respectively the excitatory and inhibitory (E/I) time particles of the paths that will bring the E/I propriety of each path. In this paper we suppose the fact that Time paths may have excitatory or inhibitory effect i.e. $\psi(N^r, t)$ can be transformed to $-\psi(N^r, t)$: it can cancel the same type signal, and this effect is due to the similar virtual antiparticles.

If the path is excitatory, we add nothing, this action is equal to 0 i.e. everything is going as we know and we do not add anything more, but if the Time path is inhibitory the E/I action would be equal to $-\pi$ which ensures the transformation of $\psi(N^r, t)$ to $-\psi(N^r, t)$. Hence this action is defined by:

$$S_{I/E,r} = -\pi\delta_{inh,r,t}, \tag{3}$$

where $\delta_{inh,r,t} = 0$ if the action is excitatory and $\delta_{inh,r,t} = 1$ if it is inhibitory.

However in the presence of matter and/or energy, the Time path created thanks to this object is an excitatory Time path with incidence r and $\delta_{inh,r,t} = 0$.

To explain the model, we give the analogy of the brain on how a signal-information propagates between two nearest neurons. Between two neurons there are multiple paths with at each path there is a synapse with neurotransmitters: if the neurotransmitters in particular path (defined by one synapse) are inhibitory then the signal inhibites the signal aggregation at the arrival neuron soma, otherwise it is additive [8]. The virtual particles in the brain are these neurotransmitters, the time paths are the dendrites and the spatial 3D points are the axonal cones of the neurone where a potential signal may be recreated by the aggregation of all the signals coming from the dendrites if a particular threshold is exceeded.

The total normalized action of a path with incidence r is therefore as follows:

$$A_r = \frac{1}{\hbar}\int_t \frac{1}{2}\rho(\frac{dq_r}{du})^2 du + Ł_{m_r} - \pi\delta_{inh,r,t} \tag{4}$$

One can note here that the $S_{I/E,r} = -\pi\delta_{inh,r,t}$ if $\delta = 1$ ensures the sign modification of the signal as $\psi = |\psi|e^{-i\pi} = -|\psi|$

The signal-Information propagating with amplitude $\psi(N_x, t)$ in a particular Time path with incidence r therefore undergoes a total time phase shift (Fig.2) and the resulted Information signal defined by $\psi(N^r, t)$ is:

$$\psi(N^r, t) = (1_{r,Time\ Dimensions} + |\psi(N_x, t)|) \times e^{i(\frac{S_r}{\hbar} - \delta_{inh,r,t}\pi + Ł_{m_r})} \tag{5}$$

where we recall that $1_{Time\ Dimensions}$ represents the normalized square root of the density of mass/energy per virtual surface and per time of incoming virtual particles or symetrically antiparticles in N_2 from incidence r.

B. Mathematical Model for Information propagation in spacetime dimensions in Universe

At the arrival to the surface border $\partial\Omega$ of the node or the spatial point N_2, all the signals-information coming from all the virtual neighboring points N^r are summed according to their spatial and temporal proximity. This is called the *spatial-temporal summation* where the signals not far apart temporally and spatially are summed. This property can be written as a spatial and temporal sum on the signals arriving at each incident point r of the surface $\partial\Omega$ which defines the N_2 3 dimensions spatial point Fig.2. The temporal summation is on a time duration Δt, the time duration that a particular possible signal-Information may be created i.e. if two signals are coming far apart with time lag higher than Δt, then the signals cannot be aggregated. The wave amplitude function of a signal-Information $\psi(N_2, t_f)$ in location N_2 at time $t_f = t + \Delta t$ is therefore:

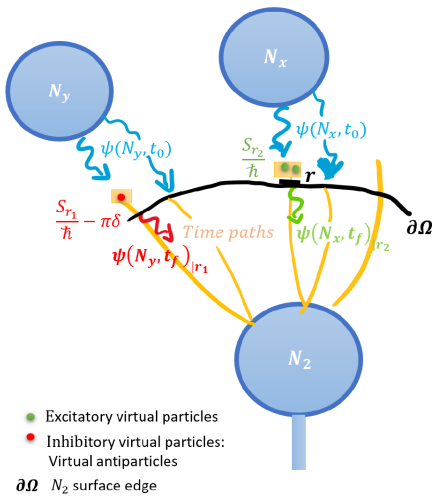

Fig. 2. Signal-Information propagation with Time path action

$$\psi(N_2, t_f) = \int_{\partial\Omega} \int_{\Delta t} |\psi(N^r, t)| e^{i\frac{S_r}{\hbar} - \delta_{inh,r,t}\pi + Ł_{m_r}} D[q_r(t)]\ dtdr \tag{6}$$

where S_r the Time kinetic action of a path arriving to the border point N_2 with incidence r. $D[q_r(t)]dtdr$ is the measure of the path q_r arriving with incidence r in the N_2 point. One may suppose that it is a constant if all the paths are equivalent. $|\psi(N^r, t)|$ is the initial wave function module coming from the virtual point N^r respectively to the path q_r since time t where we recall that $|\psi(N^r, t)| = 1_T + |\psi(N_x, t)|$ with $1_T = 1_{r,Time\ Dimensions}$ in the Time dimensions, and $|\psi(N_x, t)|$ the module of the SI existing in 3D space point N_x at time t.

$|\psi(N^r, t)|^2$ describes the density of virtual information quantity (mass or energy of virtual or real particles) per virtual surface and per time coming to N_2.

Note that if there is no signal-Information coming from any N_x, we have $|\psi(N^r, t)| = 1_T$ as $Ł_{m_r} = 0$, and if we see just the signal coming from only one incidence point r (the path can be either excitatory or inhibitory, and we assume that here it is excitatory $\delta_{inh,r,t}\pi = 0$), and supposing that in this case $D[q_r(t)] = 1$ as there is only one excitatory path (and one inhibitory path), we have that the wave amplitude in N_2 at time t_f is:

$$\psi(N_2, t_f)_{r,exc} = \int_{\Delta t} 1_T e^{i\frac{\iota_K . U_t}{\hbar}}\ dt \tag{7}$$

where $\iota_K . U_t = \int_t \frac{1}{2}\rho(\frac{dq_r}{du})^2 du$ the scalar product of Time kinetic Lagrangian of virtual particles and time unity U_t. We can notice that the formula (7) is a kind of a Fourier Transform of $H(\frac{t}{\Delta t})$ (the Top-hat function) in the excitatory Time path with incidence r if 1_T is assumed to be constant. Therefore we have $\psi(N_2, t_f)_{r,exc} \equiv 2\Delta t \times sinc(\Delta t \frac{\iota_K}{\hbar})$ ([10]). This shows how a virtual particle with frequency $\frac{\iota_K}{\hbar}$ may potentially appear in spacetime location at (N_2, t_f) in during Δt and similarly for the virtual antiparticles. Moreover 1_T is not just equal to 1 but it's a kind of a square root of a mass/energy density. By taking the square of 1_T as the density of mass/energy per time and surface with incidence r of virtual particles we set here $1_T^2 = \frac{1}{\Delta t \times 4\pi} \times \frac{1}{2}$ if it is uniform (demonstrated thereafter). The factor $\frac{1}{2}$ comes from that the chance to have an excitatory path is equivalent to have an inhibitory one in the case of vacuum. Note that if we want to compute the mean value of $\delta_{inh,r,t}$ it will be equivalent to $\frac{1}{\Delta t \times 4\pi} \times \frac{1}{2}$ which is the density of inhibitory Time paths with incidence r. As $\int sinc(X)^2 dX = \pi$ and by setting the variable change $X = \Delta t \times \frac{\iota_K}{\hbar}$, we have:

$$\int |\psi(N_2, t_f)_{r,exc}|^2 \equiv \frac{1}{\Delta t \times 4\pi} \times \frac{1}{2} \int (2\Delta t \times sinc(\Delta t \frac{\iota_K}{\hbar}))^2 d\frac{\iota_K}{\hbar} = \frac{1}{\pi} \times \pi \times \frac{1}{2} = \frac{1}{2} \tag{8}$$

which verifies the normalized choice of the 1_T^2 expression if we also add $\frac{1}{2}$ of the virtual antiparticles part.

As soon as we sum over all the incidence virtual points r in the virtual surface $\partial\Omega$ the signals ψ cancel one another as there are also the virtual antiparticles that are added if they are of the same proportion as the virtual particles. Hence in case when there is no energy/matter in any location the total $\psi(N_2, t_f)$ over Δt time and virtual surface is equal to 0 and hence the probability to have present virtual particles in N_2 at t_f

is $|\psi(N_2, t_f)|^2 = 0$ wich will not cause the problem of normalization for the probability densities $|\psi|^2$ in presence of matter/energy in spacetime dimensions.

While observing closely the equation (6), one can note that $\psi(N_2, t_f)$ is a particular kind of conditional expected value of a random variable $D[q_R(T)]$ knowing that the trajectory with incidence R is either excitatory or inhibitory (i.e. knowing the value of $\delta_{inh,r,t}$), time T between 0 and Δt, and a kind of density $|\psi(N^r, t)|e^{i\frac{S_r}{\hbar} - \delta_{inh,r,t}\pi + Ł_{m_r}}$. R and T are the random variables defined as follows:

- R represents the random variable of the signal existence or the Time path existence coming from the virtual point N^R including the virtual signals from the virtual particles and antiparticles,
- T is the random variable of the summation possibility of signals coming from the neighboring points i.e it represents the time summation capacity or the interference capacity in time.

Hence we can see mathematically $\psi(N_2, t_f)$ as:

$$\psi(N_2, t_f) = \mathbb{E}[D(q_R(T))|\delta_{inh,R,T}] \tag{9}$$

Summary and deduction:

To analyze what is happening in the spacetime location N_2, t_f we aggregated all what is happening in the neighboring spacetime locations N_x, t during Δt unit of time including the virtual particles/antiparticles pairs created by the quantum vacuum. If we sum over this Δt, one can note that the effect of virtual particles/antiparticles is null. However if we see it in time unit and in unit virtual surface edge of N_2, the effect changes: the virtual antiparticles created may create the inhibition effect as it can delete the real existing particles in N_2 for a quite short moment. But thanks to the presence at the same time of the virtual particles this effect is stopped. The real existing particles in N_2 stays in its initial equilibrium state in the absence of other energy/mass around.

Link with Gravity Effect:

*Now we assume that there are two objects **A** and **B** in different space locations respectively in N_x and N_y at the same time t where **B** is much heavier/more energetic than **A**. We want to analyze what happens to the **A** object. We suppose that between **A** and **B** there is an N_2 space point location, the nearest point to the N_x. We study the $\psi(N_2, t_f)$ for the object **A**: at each N_2 we sum all the incident $\psi(N^r, t)$ during Δt. The only difference here is that N^r incidence points at N_2 not only transfers the signal-Information of the **A** object present in N_x and all the information of virtual particles/antiparticles but also the **B** information that has travelled point by point since the first time of the **B** object existence. The signal-Information coming from N_y is heavier in N_2 than in N_x (recall that $\psi(N^r, t)$ is the mass/energy density by Time path) as N_2 is closer to N_y than N_x. If **A** is at an optimal particular location N_x in time t where the total energy/mass $|\psi(N_2, t_f)|$ exceeds the threshold m in the particular field before ending the duration Δt thanks to the **B** object influence, hence **A** moves to N_2 at time t_f. The duration threshold Δt is therefore modified to a shorter threshold duration $\Delta t'$ in the presence of the heavy object **B**. If we considered that Δt is the unit of our time and is modified by the heavy object **B** to a shorter $\Delta t'$ then there will be in one hour more units of time $\Delta t'$ ...time is slower to pass one hour in this case then if it was the threshold duration Δt. This is **the gravity effect**: time is slower and objects are deformed in space and time dimensions next to an intense signal-Information field.*

Now that an Information may be reconstructed thanks to temporal and spatial summation of signal-Information coming from different Time paths from any spacetime locations, we want to know if this Information has been distorted due to different Time paths between two spatial nodes N_x and N_2 and also due to other time paths coming from other signal-Information from other spatial neighboring nodes N_x. In the Information theory, we may compute the deformation rate of random variables thanks to the Entropy of Shannon. As we deal here with a particular random variable $D(q_R(T))$ with complex density, one can deduce an entropy of the signal-Information present in spacetime dimension.

In the next section, we will show that a particular entropy of the signal-Information brought by Time paths between N_1 (and all the neighboring points N_x) and N_2 define the scalar curvature term $\mathcal{R}$ [11] in Hilbert-Einstein action [12], [13] and the cosmological constant.

III. Information Entropy: The Universe Spacetime curvature

In this section we want to measure the average level of disorder of the signal-Information *received virtually* by the point N_2 in other words the entropy of $D(q_R(T))$. We introduce here an entropy similar to Shannon entropy in information theory [14]. We recall that Shannon entropy measures the information quantity sent by a information source. The more different information the source emits, the greater the entropy (or uncertainty

about what the source emits). *Note that Δt is not really in time dimension but a quantity of unit of time, and this is what we call time in our Universe.*

In quantum mechanics, we work with wave functions, a particular probability functions. Any complex wave function $\psi(r,t)$ can be rewritten as following $\psi(r,t) = R(r,t)e^{i\theta(r,t)}$ with the probability density $p(r,t) = R^2(r,t)$ and its phase $\theta(r,t)$. Recall that $\psi(N^r,t)$ in our study is the mass/energy wave function of the signal-Information coming to N_2 at time t from the particular Time path with incidence r, and r is a point on the virtual surface border $\partial\Omega$ of N_2. Nalewajski in [15] gives a definition of the complex entropy by first defining the *overall entropy*:

$$s(\psi) = \int -p(r,t)[ln(p(r,t)) + 2i\theta(r,t)]dtdr \equiv s(p) + i\ s(\theta), \tag{10}$$

where the integral is overall the surface $\partial\Omega$ of the N_2 virtual border and over a time duration Δt. Here we note $p(r,t)$=$p(N^r,t)$.

The real part $p(r,t)ln(p(r,t))$ does not take into account the phase of the Time-paths which is very important to compute the entropy in our study. However the imaginary part $-p(r,t)\theta(r,t)$ (without the 2) does. One can note that the imaginary part is the part represents the link between the past t and the future $t+\Delta t$. The real part represents the thermodynamic entropy in space at time between t and $t+\Delta t$. In this section we will study the imaginary part of the entropy.

In the paper [16] the entropy considered is equivalent to the actions, and in our study we will choose also the part that takes into account the Time path actions with the probabilities: the Imaginary part of the entropy (10) without the 2 factor.

Hence we introduce a new type of entropy, a *virtual entropy*, on the phases of signal-Information coming into the point N_2 from each virtual Time path with extremety r at time t. The spacetime entropy of $D(q_R(T))$ at time t_f in location N_2 is therefore:

$$\S[D(q_R(T))] = \mathcal{L}_m\sqrt{|g|} + \int_{\partial\Omega}\int_{\Delta t} -p(r,t)\times\gamma(r,t)dtdr \tag{11}$$

$p(r,t)$ is the mass/energy (we will choose mass thereafter) density of the incoming signal amplitude per surface and time at N_2 with incidence r. For the Universe system, in the mass field $p(r,t)$ represents the incoming mass density Information for the signal-Information per time and surface. $p(N^r,t)$ or $p(r,t)$ encompass the probability information of the existence of the initial incoming Information from N_x with incidence r and also the probability information of the existence of Time path with incidence r. $\gamma(r,t)$ is the total Time phase shift at time t of the path incoming from r without the Matter Lagrangian part $Ł_m$:

$$\gamma(r,t) = \frac{S}{\hbar} - \delta_{inh,r,t}\pi \tag{12}$$

S, or noted before S_r, is the time kinetic action on the signal-Information unit in a particular Time path with incidence r that is defined the formula (1).

Here $\mathcal{L}_m\sqrt{|g|} \sim \int_{\partial\Omega}\int_{\Delta t} -p(r,t)Ł_{m_r}dtdr$ where $Ł_{m_r}$ is the density that represents the Lagrangian density $\mathcal{L}_m$, the Standard Model Lagrangian density ([7]), of the initial signal-Information coming from the point N_x at time t with incidence r and defined in $\psi(N_x,t) = R(r,t)_x e^{iŁ_{m_r}}$ and from the potential virtual particles/antiparticles. $\mathcal{L}_m$ would be a kind of an expected value of the $Ł_{m_r}$ density. $\sqrt{|g|}$ represents the spacetime volume unit at the studied location which will be explained later, but one can say that this comes from the effect of the mass incoming density p.

As $p(r,t)$ represents the incoming mass/energy density per virtual surface and time, to normalize $p(r,t)$, we multiply and divide by $\frac{M_p}{S_p t_p}$, the maximum mass that we could have in one space point N_2. In fact the maximum of mass quantity that a time path of time Planck t_p can carry is the mass of Planck M_p, and S_p is the surface of Planck. We consider thereafter $\S[D(q_R(T))]$ without the Lagrangian density of matter:

$$\S[D(q_R(T))] = \frac{M_p}{S_p t_p}\int_{\partial\Omega}\int_{\Delta t} -P(r,t)\times\gamma(r,t)dtdr \tag{13}$$

where $P(r,t) = \frac{p(r,t)}{\frac{M_p}{S_p t_p}}$.

In the virtual surface border $\partial\Omega$ of N_2 there is a temporal summation of all the action potentials arrived during Δt, a duration which after the *space location* N_2 may *re*-create a new signal-Information unit. Recall that this duration is the maximal time *tolerance* for all the fields in the vacuum that any information unit (the respective particle in its field) may be recreated in any spatial location. Recall that a particle can be recreated in its field at N_2 with a less duration than Δt noted $\Delta t'$ if there is another correlated Information around in space coming to N_2 during this time laps Δt.

We rewrite the entropy without the Lagrangian of the matter:

$$\S[D(q_R(T))] = \frac{M_p}{S_p t_p}\int_{\partial\Omega}\int_{\Delta t} -P(r,t)\times[\frac{1}{\hbar}\int^t \frac{1}{2}\rho(\frac{dq_r}{du})^2 du\ -\ \delta_{inh,r,t}\pi]dtdr.$$

By normalizing the mass ρ by Planck mass M_p (an elementary virtual particle/antiparticle is assumed to not have a mass higher than Planck mass), thus:

$$\S[D(q_R(T))] = \frac{M_p}{S_p t_p}\int_{\partial\Omega}\int_{\Delta t} -P(r,t)\times[\frac{M_p}{\hbar}\int^t \frac{1}{2}\frac{\rho}{M_p}(\frac{dq_r}{du})^2 du \ - \ \delta_{inh,r,t}\pi]dtdr$$

By setting $\frac{M_p}{\hbar}$ outside the double integral, we have:

$$\S[D(q_R(T))] = \frac{M_p^2}{S_p t_p \hbar}\int_{\partial\Omega}\int_{\Delta t} -P(r,t)\times[\int^t \frac{1}{2}\frac{\rho}{M_p}(\frac{dq_r}{du})^2 du \ - \ \frac{\hbar}{M_p}\delta_{inh,r,t}\pi]dtdr \tag{14}$$

We note that $-P(r,t)[\int_t \frac{1}{2}\frac{\rho}{M_p}(\frac{dq_r}{du})^2 du - \delta_{inh,r,t}\pi]$ may represent the density (in space and time) of the variation in time of spatio-temporal square distance that a signal-Information travels from two nearest points N_x and N_2 i.e. the variation of square distance that an Information arrives with incidence r in the virtual border $\partial\Omega$ of N_2 at time t_f Fig.3.

We note the variation in time of square distance of the particular excitatory Time path with incidence r i.e. without the inhibition part $\frac{\partial g_{x_r,t}}{\partial t}$:

$$\frac{\partial g_{x_r,t}}{\partial t} = -P(r,t)\int^t \frac{\rho}{M_p}(\frac{dq_r}{du})^2 du, \tag{15}$$

and the variation in time of square distance of the inhibitory Time path effect with incidence r:

$$\frac{\partial \overline{g}_{x_r,t}}{\partial t} = P(r,t)\frac{\hbar}{M_p}\delta_{inh,r,t}\pi. \tag{16}$$

The distance between two points N_x and N_2 is infinite if no Information can propagate between them i.e. if $P(r,t) = 0$ for all Time incidence r at N_2. Which is impossible with our definition of P if the virtual particles may exist in spacetime dimensions thanks to the energy-time principle incertitude. Note that $\frac{\partial g_{x_r,t}}{\partial t}$ is surely to be negative (as $\delta_{inh,r,t}\pi = 0$) and as the virtual mass ρ of a virtual particle is as positive as $(\frac{dq_r}{du})^2$. This means that the distance between the two points N_x and N_2 is decreasing between times if $P(r,t)$ is stricly positive.

However if the mediators are inhibitory then $-\delta_{inh,r,t}\pi = -\pi$ and hence the square distance is getting higher as the variation in time of the square distance $\overline{g}_{x_r,t}$: $\frac{\partial \overline{g}_{x_r,t}}{\partial t} = P(r,t)\frac{\hbar}{M_p}\pi$ is always positive. This leads to a kind of a virtual spatio-temporal expansion between the two nearest points N_x and N_2: this may be the result of the spacetime expansion and this what we will also prove later.

We rewrite the mass entropy or the Information entropy:

$$\S[D(q_R(T))] = \frac{M_p^2}{S_p t_p \hbar}[\frac{1}{2}\int_{\partial\Omega}\int_{\Delta t}\frac{\partial g_{x_r,t}}{\partial t}\ dtdr + \frac{\hbar}{M_p}\int_{\partial\Omega}\int_{\Delta t} P(r,t)\delta_{inh,r,t}\pi\ dtdr] \tag{17}$$

We consider now the following vectors (see Fig.3):

$$\begin{aligned} -\overrightarrow{n}dS \equiv & \quad dr,\ the\ surface\ unit \\ -\overrightarrow{n} = & \quad incoming\ normal\ vector\ to\ dS \\ \overrightarrow{F} = & \quad \overrightarrow{grad}\frac{\partial g_{x_r,t}}{\partial t} \\ \overrightarrow{\Pi} = & \quad \overrightarrow{grad}(\frac{\hbar}{M_p}P(r,t)\delta_{inh,r,t}\pi) = \overrightarrow{grad}\frac{\partial \overline{g}_{x_r,t}}{\partial t} \end{aligned} \tag{18}$$

$\overrightarrow{F}$ is the resulted vector that shrinks distances of a particular path between N_x and N_2 spatial points. $\overrightarrow{\Pi}$ is the vector that increases this distance. One can note that to N_2 the distance is identical to any direct neighboring neuron N_x as the distance is only the sum of all these vectors that represent the incoming Time-paths to N_2 Fig.3. The distance between two any linked neighboring points N_x and N_2 depends on the Time-paths and on the initial incoming signal-Information. Space and Time are hence relative on what is going on the Time dimensions and Time paths (quantum vacuum) between the nearest two spatial points and on the incoming/present signal-Information in space: the matter and/or the energy.

The importance of the sign $-$ next to π in the inhibitory part choice is materialized here: in the wave function ψ it has no impact but in signal-Information flow at point N_2 it precises that the Information is removed from N_2 which leads to a probable lengthening of the distance between two neighboring points for the studied signal-Information.

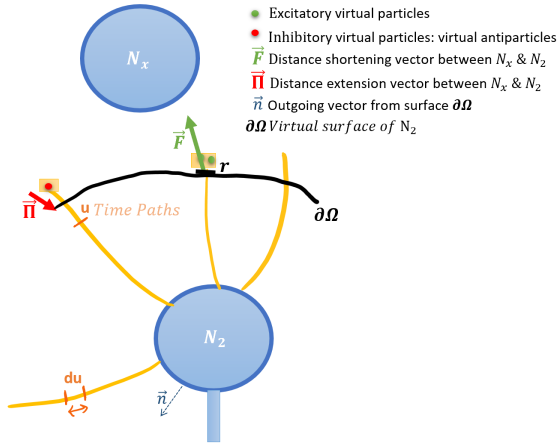

Fig. 3. Creation of the distance between two linked points N_x and N_2

The Mass-Information entropy thanks to Gradient theorem can be rewritten as follows:

$$\S[D(q_R(T))] = \frac{M_p^2}{S_p t_p \hbar}[\frac{1}{2}\int_{\partial\Omega}\int_{\Delta t}\int^t -\overrightarrow{F}.\overrightarrow{n}\ dudtdS - \frac{\hbar\pi}{M_p}\int_{\partial\Omega}\int_{\Delta t} P(r,t)\delta_{inh,r,t}\ dtdS] \tag{19}$$

Thanks to Green-Ostragradski theorem in the first integral, by interverting integrals and by using Gradient theorem while replacing in the second integral du by $\frac{1}{c}\times cdu$ and hence $dV = cdu \times dS$, we have:

$$\S[D(q_R(T))] = \frac{M_p^2}{S_p t_p \hbar}[\frac{1}{2}\int_{\Omega}\int_{\Delta t}\int^t -div(\overrightarrow{F})\ dudtdV - \frac{\hbar\pi}{M_p c}\int_{\Delta t}\int_{\Omega}\overrightarrow{grad}(P(r,t)\delta_{inh,r,t})).\overrightarrow{n}\ dVdt]$$

Using that $\overrightarrow{F} = \overrightarrow{grad}\frac{\partial g_{x_r,t}}{\partial t}$, by interverting div and $\frac{\partial}{\partial t}$ and integrating in time in the first integral, we have:

$$\S[D(q_R(T))] = \frac{M_p^2}{S_p t_p \hbar}[\frac{1}{2}\int_{\Omega}\int_{\Delta t} -div(\overrightarrow{grad}(g_{x_r,t}))\ dtdV - \frac{\hbar\pi}{M_p c}\int_{\Delta t}\int_{\Omega}\overrightarrow{grad}(\delta_{inh,r,t}P(r,t)).\overrightarrow{n}\ dVdt] \tag{20}$$

Using also the Green-Ostragradski theorem for the second integral and knowing that $div(\overrightarrow{grad}(N) = \Delta N$ the Laplacian operator of N, we have:

$$\S[D(q_R(T))] = \frac{M_p^2}{S_p t_p \hbar}[\frac{1}{2}\int_{\Omega}\int_{\Delta t} -(\Delta g_{x_r,t})\ dtdV - \int_{\Omega}\int_{\Delta t}(\Delta\overline{g})\ dtdV] \tag{21}$$

The first integral of the formula (21) **study**:

By considering the approximation $Ric_{x_r,t} \simeq -\frac{1}{2}\Delta g_{x_r,t}$ in the first integral that we will note A of (21), where $Ric_{x_r,t}$ is a kind of the Ricci curvature defined in Riemannian geometry at the virtual location x_r, t. Therefore we have:

$$A \simeq \frac{M_p^2}{S_p t_p \hbar}\int_{\Omega}\int_{\Delta t} Ric_{x_r,t}\ dtdV$$

By normalyzing dV with ${l_p}^3$ the Planck volume we have:

$$A \simeq \frac{M_p^2 {l_p}^3}{S_p t_p \hbar}\int_{\Omega}\int_{\Delta t} Ric_{x_r,t}\ dtdv \tag{22}$$

where $dv = \frac{dV}{{l_p}^3}$ is the normalized volume unit. Using another time Ostrogradski theorem in the integral we have:

$$A \simeq \frac{M_p^2 {l_p}^3}{S_p t_p \hbar}\int_{\nu}\int_{\Delta t} div(Ric_{x_r,t})dtd\nu \tag{23}$$

where $d\nu = dv \times dt$ a four dimension volume. And from the contracted Bianchi identities where $R_{x_r,t}$ is similar to the scalar curvature, we have $div(Ric_{x_r,t}) = \frac{1}{2} dR_{x_r,t}$ hence:

$$\int_{\nu}\int_{\Delta t} div(Ric_{x_r,t})\ dtd\nu = \frac{1}{2}\int_{\nu}\int_{\Delta t} dR_{x_r,t}\ dtd\nu \sim \frac{\mathcal{R}}{2}\sqrt{|g|} \tag{24}$$

where $\mathcal{R}$ is the defined scalar curvature of Einstein equation in general relativity, and $\sqrt{|g|}$ the four dimension volume unit with the effect of P (explained in the second integral study). $\mathcal{R}$ is a kind of the mean value of $dR_{x_r,t}$ over the spacetime volume ν during a threshold Δt. Finally we deduce that:

$$A \sim \frac{M_p^2 {l_p}^3}{S_p t_p \hbar}\frac{\mathcal{R}}{2}\sqrt{|g|} \tag{25}$$

From Planck units where G is the gravitational constant, we have:

$$\begin{aligned} M_p &= \sqrt{\frac{c\hbar}{8\pi G}} \\ S_p &= l_p^2 \\ c &= \frac{l_p}{t_p} \end{aligned} \tag{26}$$

we can deduce that:

$$\frac{M_p^2 {l_p}^3}{S_p t_p \hbar} = \frac{c^2}{8\pi G} = \frac{1}{\kappa} \tag{27}$$

where κ the Einstein gravitational constant [17], as In Einstein's original publication, the choice is $\kappa = \frac{8\pi G}{c^2}$, in which case the stress–energy tensor components have units of mass density what we indeed chose for the initial $p(r,t)$ definition: p was the mass Information density. Therefore adding the matter effect, the entropy with the curvature effect and the matter effect is hence:

$$A_m \sim \frac{1}{\kappa}\frac{\mathcal{R}}{2}\sqrt{|g|} + \mathcal{L}_m\sqrt{|g|} \tag{28}$$

Which is the Lagrangian density of the Einstein-Hilbert action plus the Langrangian matter. To find how an object or a signal-Information moves over spacetime we minimize the entropy A_m ([16]) or in other words we apply the action principle by minimizing the action A_m ([18]). This calculus gives the Einstein equation of the general relativity with T the energy-matter tensor:

$$Ric - \frac{1}{2}\mathcal{R} = \frac{1}{\kappa}T \tag{29}$$

What about the second integral of the formula (21) **? :**

Now we observe the second integral of the formula (19), which is the same second integral of (21), and its meaning. We note this integral B:

$$B = -\frac{\hbar\pi}{M_p}\int_{\partial\Omega}\int_{\Delta t} P(r,t)\delta_{inh,r,t}\ dtdS \tag{30}$$

We suppose that we have $\delta_{inh,r,t}$ that is equals to its mean that we have defined in the previous section i.e. $\delta_{inh,r,t} = \frac{1}{4\pi\Delta t} \times \frac{1}{2}$ that represents the mean density of the incoming virtual antiparticles at N_2 in the case where there is no signal-Information i.e. there is no matter nor energy. Therefore by interverting the integrals and using the Gradient theorem:

$$B \equiv -\frac{\hbar\pi}{M_p}\frac{1}{4\pi\Delta t}\frac{1}{2}\int_{\partial\Omega}\int_{\Delta t} P(r,t)\ dtdS = -\frac{\hbar}{8M_p\Delta t}\int_{\Delta t}\int_{\partial\Omega}\int^{t}\overrightarrow{grad}(P).\overrightarrow{n}du\ dSdt. \tag{31}$$

By replacing du by $\frac{1}{c} \times cdu$ (c is the maximal speed -the speed of light) and hence $dV = cdu \times dS$, we have:

$$B \equiv -\frac{\hbar}{8cM_p\Delta t}\int_{\Delta t}\int_{\Omega}\overrightarrow{grad}(P).\overrightarrow{n}dVdt. \tag{32}$$

We normalize dV with $dv = \frac{dV}{l_p^3}$, dt with $d\tau = \frac{dt}{\Delta t}$ and using the Ostrogradski theorem in four dimensions:

$$B \equiv -\frac{\hbar l_p^3\Delta t}{8cM_p\Delta t}\int_{\Delta t}\int_{\nu} div(\overrightarrow{grad}(P))d\nu d\tau \tag{33}$$

Note that $d\tau$ is not the unity of the time but the quantity of unit of times. Hence:

$$\frac{B}{l_p^3} \sim -\frac{\hbar}{8cM_p}\sqrt{|g|} \simeq -1.0128 \times 10^{-35}\sqrt{|g|} \sim -\Lambda\sqrt{|g|} \tag{34}$$

Which is quite representative of the cosmological constant value Λ in time metric, and which if we divide by c^2 we have the cosmological constant with the space metric: $\Lambda = 1.1269 \times 10^{-52}$ which is quite representative of the cosmological constant value. $\sqrt{|g|}$ would be equivalent to $\int_{\Delta t}\int_{\nu} div(\overrightarrow{grad}(P))d\nu d\tau$.

Finally we will have the total entropy taking into account the second integral which is the virtual antiparticles effect, and factoring by l_p^3, we have:

$$\S[D(q_R(T))] \sim \frac{1}{\kappa}[\frac{\mathcal{R}}{2}\sqrt{|g|} - \Lambda\sqrt{|g|}] + \mathcal{L}_m\sqrt{|g|} \tag{35}$$

By minimazing this entropy or the Lagrangian density of the Einstein-Hilbert action (see for example [19]) the calculus gives the Einstein equation of the general relativity with T the energy-matter tensor:

$$Ric - \frac{1}{2}\mathcal{R} + \Lambda g = \frac{1}{\kappa}T \tag{36}$$

We finally can conclude that the Time phase entropy of the signal-Information at any point represents the spacetime curvature modelization added to the cosmological constant a result of the virtual antiparticles. The gravity could be assimilated to the degree of correlation between massive objects and not a kind of interaction. A highly massive object **B** or signal-Information impacts highly the Time field of any other closed correlated signal-Information **C** (less massive) as Time paths coming from particles of **B** are added to the paths of the studied object **C** Fig.4. Hence **C** object existence is highly to be next to the object **B** at time $t+1$ than its past location at time t: this is the gravity effect.

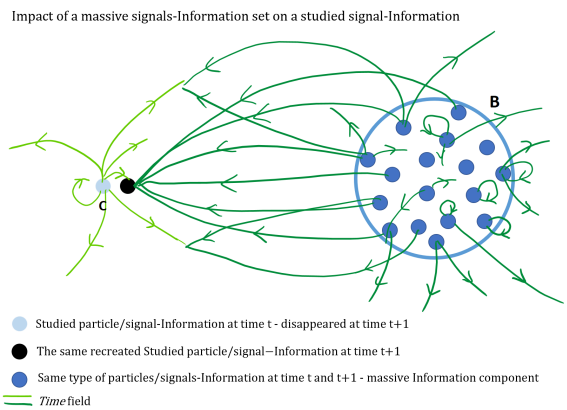

Fig. 4. Impact of a massive set of signals-Information on another studied Information

IV. Implications on Universe laws

- **Gravity:** In universe, Gravity is the result of massive matter correlation, and the spacetime curvature is the massive Time phase entropy created by the neighboring massive objects.
- **Black Holes:** There is no more correlation between objects, even the information with itself. In fact all any possible Time path is absorbed by the massive point inside the black hole. No such quantum fields could exist nor virtual particles: the information does not exist without the vacuum, as no Time path could exist in black holes. There has been a tearing of quantum fields as any probable created field is absorbed by the massive point.
- **Universe expansion:** The cosmological constant as demonstrated in the article may be a result of the virtual antiparticles when they appear in spacetime dimension thanks to energy-time principle incertitude.
- **The Heisenberg incertitude principle $\Delta x \Delta p \geq \frac{\hbar}{2}$:** This formula may be explained as following: if we try to locate a particle in a particular position then we give a high intensity of Information concentrated into this location which may enhance the density of Time paths from this location to this same location. We have inhibited the other paths to other locations and we have enhanced at the same time the energy of the particle localization.

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