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Spacetime curvature, a potential result of the quantum mass/energy Information Entropy

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Abstract

his work proposes a possible proof on how the spacetime curvature is created by any object (mass/energy) and suggests a mathematical model that may conciliate quantum mechanics and general relativity. As each elementary particle is a fluctuation in its particular field in Quantum Fiel Theory and follows quantum mechanics properties, the wave function of each elementary particle of an object is deformed at each spacetime point in the presence of another massive object (set of particles). This comes from the fact that the wave functions of the particles of the two massive objects in their respective quantum fields may be aggregated and hence modifies in time and space the wave functions values. This could subsequently change the location of the object less massive as the probabilities of the presence of the elementary particles are getting higher next to the highly massive objects: the less massive objects are correlated to the highly massive one. In this study we suppose that between two nearest spatial points N_x and N_2 there is an infinity of quantum-mechanically possible Time paths in quantum vacuum (or what we will call Time paths) as assumed in path integrals theory. Using a modified Information Entropy of Shannon into the complex probabilities on the Time paths arriving to N_2 from any N_x , we prove in this paper that this entropy on the trajectories is equivalent to the Hilbert-Einstein Lagrangian density which leads to the Einstein general relativity equation by minimizing this entropy or in other words by applying the principle of least action.his work proposes a possible proof on how the spacetime curvature is created by any object (mass/energy) and suggests a mathematical model that may conciliate quantum mechanics and general relativity. As each elementary particle is a fluctuation in its particular field in Quantum Fiel Theory and follows quantum mechanics properties, the wave function of each elementary particle of an object is deformed at each spacetime point in the presence of another massive object (set of particles). This comes from the fact that the wave functions of the particles of the two massive objects in their respective quantum fields may be aggregated and hence modifies in time and space the wave functions values. This could subsequently change the location of the object less massive as the probabilities of the presence of the elementary particles are getting higher next to the highly massive objects: the less massive objects are correlated to the highly massive one. In this study we suppose that between two nearest spatial points N_x and N_2 there is an infinity of quantum-mechanically possible Time paths in quantum vacuum (or what we will call Time paths) as assumed in path integrals theory. Using a modified Information Entropy of Shannon into the complex probabilities on the Time paths arriving to N_2 from any N_x , we prove in this paper that this entropy on the trajectories is equivalent to the Hilbert-Einstein Lagrangian density which leads to the Einstein general relativity equation by minimizing this entropy or in other words by applying the principle of least action.T

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I. INTRODUCTION

In the Universe, an Information or what we will call in all this paper the signal-Information can be an elementary particle, a set of elementary particles (atoms, molecules...) or anything that has the property of wave-particle. If the information *still exist* at t+1, i.e it can be detected at time t+1, travels **inevitably** in time from t to t+1 and may move also in space if a *correlated external additional information* exists: interactions, forces...or any other type of *correlation*. In this paper we call a correlated object A to an object B any object B that may lead a change in structure or motion of the object A and reciprocally. For example two electrons are highly correlated as they have repulsive effect to each other. But at the same time electrons are identical, they all have the same mass and they are the same fluctuation in their field in Quantum Field Theory (QFT)([1],[2]). When the electron is at its wave property i.e. it can be in a superposition of locations at the same time, it cannot has the repulsive force to itself in other positions otherwise there will be no interference/asummation of wave function of the electron. We cannot have at the same time the photon exchange for the repulsive effect and the spatial superposition. For the example of the two electrons in their wave property that we cannot

differentiate, their mutual wave functions hence may be added and hence may interfer. But as soon as they lose their wave property to become particles they repel each other thanks to the exchanged photons. This interfering or wave summation effect between same elementary particles is what we will demonstrate to be the Gravity or the matter correlation, and the entropy or the deformation of the information brought by all the wave functions at a spacetime location in the respective field is the well know spacetime curvature.

In this paper we will take the fact that the elementary particles in their respective fields have wave functions which can interfere one another as long as they are in their wave property. Moreover, as in Quantum Field Theory we consider that the vacuum ([3],[4]) is not truly empty but contains virtual particles and antiparticles that are able to *exist spatially* thanks to Heisenberg energy-time uncertainty principle $\Delta E \Delta t \geq \hbar$ and the equivalence energy-mass equation $E = mc^2$: particles and their respective antiparticles are constantly popping into existence and popping back out in a quite short time. We suppose that these virtual particles always exist in other dimensions, called Time dimensions i.e. the quantum vacuum dimensions, different from the three spatial dimensions (**3D**). Virtual particles would be the excitatory mediators and the virtual antiparticles the inhibitory. If one virtual particle and its one respective antiparticle exist in the same spacetime location they cancel each other directly. We will call this property the spacetime summation of virtual particles.

We introduce now a theory of what may happen between one particular spatial point N_2 and its nearest neighboring points N_x of any particular field in QFT and hence a theory on what causes the spacetime curvature. Between the two nearest points, i.e. a particular object cannot be seen travel as in its shape between these two points, virtual particles/antiparticles exist in the Time dimensions (quantum vacuum) and thanks to energy-time uncertainty principle they can exist in any location of spacetime dimensions and can act as excitatory mediators and antiparticles as inhibitory mediators. The uncertainty principle describes how the known *time* could *move inevitably*.

We suppose now that there is a real particle present in a spacetime *around* (incertitude principle) the position (N_x, t) , and at the same time and space location, in a quite short time, virtual particles and antiparticles may appear and could be similar to this real particle. What prevents the real particle from being annihilated by the antiparticle created by the vacuum, and that the former virtual particle takes place of the real particle ? We assume hence that during their appearance in the spatial dimensions, virtual particles/antiparticles also may have the wave-particle properties and can act on the real particles everywhere in spacetime.

II. MATHEMATICAL MODEL FOR THE INFORMATION DISSIMINATION IN THE UNIVERSE

A. Information Reactions system model in Spacetime dimensions in Universe

As in quantum mechanics, we will suppose that between two nearest spatial points N_x and N_2 there is an infinity of quantum-mechanically possible Time dimensions trajectories created by virtual particles and antiparticles. Note that one virtual particle can create an infinity of Time paths thanks to the superposition property of the quantum particles. Each from these paths has a particular action noted S_r that arrives with incidence r to N_2 from a particular point N^r (from the neighboring nodes N_x) by assuming that N_2 is bounded by a virtual surface which forms a border with the all N_x . Two different N^r can be one N_x . Each Time path can be either an excitatory path if the acted virtual particle is a potential real particle (by energy-time incertitude principle) or an inhibitory path if the acted virtual particle is a potential real antiparticle Fig.1.

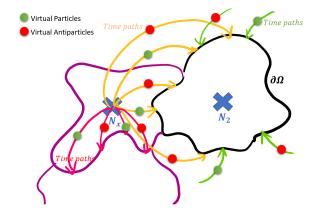


Fig. 1. Time dimensions paths created by virtual particles and antiparticles

All the signals coming from all the nearest spatial points N_x , if a Signal-Information SI (any energymatter object) exists at this location, to N_2 (Note that N_2 is in the set of the neighboring points N_x) are hence remodulated by each Time path and reach the point N_2 where they are assumed to be aggregated in a particular time duration noted Δt and on a virtual surface edge $\partial\Omega$ that limits the point N_2 . This is similar on what Dirac and Feynman have done in the paths integral (see for example [5]). If two signals far in time arrives in a same point they would not be aggregated, so we add an additivity time tolerance of signals Δt that can depend on the field type. It is what we call signals time interference. We recall that between any N_x and N_2 there are no other points in the 3 dimensions of space and therefore this border $\partial\Omega$ is like a virtual border which limits N_2 to all the closest points N_x . The initial **SI** coming from a particular position N_1 to a particular nearest point N_2 is not only deformed due to different action of the paths (excitatory and inhibitory) but also due to the **SI** coming from the other neighboring nodes N_x to N_2 arriving during the time tolerance laps Δt of the signals aggregation. This deformation is what we will see the gravity effect.

Each 3 dimensions spatial position N_2 in a particular field (QFT) is *excited* by the signal transmission aggregated coming from the neighborhood N_x and also by the quantum vacuum (or the Time paths without any real matter or energy), and if a particular threshold is exceeded: a new Signal-Information deformed or not is created. This threshold is the parameter m in QFT that encodes a lower energy threshold (see for example the Klein-Gordon Lagrangian in [6]) from which a point in that field is excited. Hence one can assume that the signal bringing by the Time paths is a signal of the quantity of mass/energy per unit surface from $\partial\Omega$ and per time of Δt .

The signal-Information amplitude coming from N^r , a particular N_x with incidence r to the border $\partial\Omega$ of N_2 at time t denoted by $\psi(N^r, t)$, is the wave function of the signal that existed in the node N_x at time t with incidence r that may be complex as signals can interfer with one another thanks to time and space summation. Note that N_r can come from N_2 itself. This function will have the same definition as the wave function in quantum physics if we compute it in a 3D locations N_x at time t. Note that N^r are not real spatial 3D points but the virtual images projection of real 3D point N_x and hence the surface $\partial\Omega$ is a virtual surface border. There are as much N_r for a particular N_x as much possible Time paths between N_x and N_2 . Here the square module of $\psi(N^r, t)$ is equal to the density of mass/energy per surface and per time of the signal-Information coming with incidence r to N_2 . If there is no particular signal-Information (no energy or matter) existing at the upstreaming spatial point N_x of N^r then $|\psi(N^r, t)| = 1_{Time \ Dimensions}$ which means that in all the cases there is always a Time dimensions signal-Information coming from the virtual particles and antiparticles that may exist in the final point N_2 ...This is the main result of the time-energy principle incertitude. $1_{Time \ Dimensions}$ is a normalized 1 in time and space (explained later). Otherwise, if an object exists in N_x an time t hence $|\psi(N^r, t)| = 1_{Time \ Dimensions} + |\psi(N_x, t)|$, where $|\psi(N_x, t)|$ defines here the square root of the density of energy/mass per surface and time coming from N_x at time t which will be equal by value to the wave function amplitude in quantum mechanics theory.

We introduce now the *normalized* action induced by each Time trajectory, either there is a signal-Information SI (energy/matter or any object) or not in the upstream node N_x at time t, which will be defined by the integral in time of three Lagrangians: the kinetic Lagrangian in Time dimensions created by the virtual particles, the Matter Lagrangian created by the usptream SI in the upstream point N_x at time t and the Inhibitory/Excitatory Lagrangian of the path induced by the type of virtual particles (particles or antiparticles). Hence we define three normalized actions, the kinetic one:

$$\frac{S_r}{\hbar} = \frac{1}{\hbar} \int_t \frac{1}{2} \rho \left(\frac{dq_r}{du}\right)^2 du. \tag{1}$$

where ρ represents the potential *mass* of the virtual particle, i.e. the potential forthcoming real particle/antiparticle, or the equivalent energy (thanks to Einstein formula $E = \rho c^2$). Virtual particle may be renamed quantum signal-Information in Time dimensions. $\frac{dq}{du}$ is the potential speed of that quantum of the signal-Information in Time dimensions. $\frac{1}{h}$ acts as the normalization of the energy density in time where if Lagrangian $L = \frac{1}{2}\rho(\frac{dq_T}{du})^2$ is equal to one energy unit L_1 for one time unit t_1 , hence:

$$\int_0^{t_1} \frac{L_1}{\hbar} du = 1 \tag{2}$$

where $\frac{L_1}{\hbar}$ would be equivalent to the density to find a quantum of energy at u in probability theory, with \hbar the maximal quantity of energy that can hold a temporal distance unit, the Time distance between two spatial 3D points N_x and N_2 . With this definition, one can note that $\frac{S}{\hbar}$ is the mean of $\frac{L}{L_1}$ which is the number of energy units wich is in the Physics laws, is equivalent to the frequency.

The normalized Matter action is the phase L_{m_r} of the initial signal coming from the point N_x at time t with incidence r. It defines the This Lagrangian represents the *density of* (explained later) the Lagrangian density of matter defined in the standard model which takes into account propagations and interactions and etc. (see for example [7]).

Between two nearest nodes N_x and N_2 all the signals $\psi(N^r, t)$ from N_x with incidence r propagate in all the possibles Time paths, and all the signals are summed at N_2 . These additive action paths will be called Time excitatory path and this is a result of the virtual particles. However while existing in space dimensions, a virtual particle is created in pair with its respective antiparticle. Therefore, in this paper we suppose the fact that Time paths may have also time inhibitory effect i.e. $\psi(N^r, t)$ can be transformed to $-\psi(N^r, t)$: it can cancel the same type signal, and this effect is due to the similar virtual antiparticles. If a real particle existes in N_2 at time t and at the same time a virtual particle and antiparticle with the same type as the real one appear, we recall that the virtual antiparticle can annhilate the real particle and the virtual one becomes the real one.

So when adding all the signal, the inhibitory Time paths (created by the virtual antiparticles) reduces the total aggregated wave functions in N_2 and the excitatory Time paths enhance this aggregated wave function.

The normalized Inhibitory/Excitatory action of a path is the action in addition that describes *Time dimensions action* or what we could call the quantum vacuum action. It is excitatory if this action is equal to 0 i.e. everything is going as we know, and it is inhibitory if the action is equal to $-\pi$. Hence this action is defined by:

$$S_{I/E,r} = -\pi \delta_{inh,r,t},\tag{3}$$

where $\delta_{inh,r,t} = 0$ if the action is excitatory and $\delta_{inh,r,t} = 1$ if it is inhibitory. But what may give the excitatory or inhibitory time property of a path? If there is no matter/energy present in the space dimensions, the virtual particles present in the quantum vacuum defined in Quantum Field Theory (QFT) ([3],[2],[4]), present in pairs of virtual particles and antiparticles, may give this property to a time path between the two nearest spatial points. These virtual particles give the definition of the Time dimensions or the time paths. Virtual particles and antiparticles are respectively the excitatory and inhibitory (E/I) time particles of the paths that will bring the E/I propriety of each path. However in the presence of matter and/or energy, the Time path created thanks to this object is an excitatory Time path with incidence r and $\delta_{inh,r,t} = 0$. In the presence of antimatter (anti-energy?), the Time path with incidence r created is inhibitory and $\delta_{inh,r,t} = 1$.

To explain the model, we give the analogy of the brain on how a signal-information propagates between two nearest neurons. Between two neurons there are multiple paths with at each path there is a synapse with neurotransmitters: if the neurotransmitters in particular path (synapse) are inhibitory then the signal inhibites the signal aggregation at the arrival neuron soma, otherwise it is additive [8]. The virtual particles in the brain are these neurotransmitters, the time paths are the dendrites and the spatial 3D points are the axonal cones of the neurone where a potential signal may be recreated by the aggregation of all the signals coming from the dendrites if a particular threshold is exceeded.

The total normalized action of a path with incidence r is therefore as follows:

$$A_r = \frac{1}{\hbar} \int_t \frac{1}{2} \rho \left(\frac{dq_r}{du}\right)^2 du + \mathcal{L}_{m_r} - \pi \delta_{inh,r,t} \tag{4}$$

One can note that the $S_{I/E,r} = -\pi \delta_{inh,r,t}$ if $\delta = 1$ ensures the sign modification of the signal: $\psi = Ae^{-i\pi} = -A$

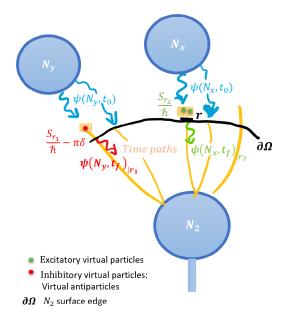


Fig. 2. Signal-Information propagation with Time path action

The signal-Information propagating with amplitude $\psi(N_x, t)$ in a particular Time path with incidence r therefore undergoes a total phase shift (Fig.2) and a module shift (explained in the next subsection) and the resulted Information signal defined by $\psi(N^r, t)$ is:

$$\psi(N^{r},t) = (1_{r,Time\ Dimensions} + |\psi(N_{x},t)|)e^{iA_{r}} = \psi(N_{1},t_{0})e^{i(\frac{D_{r}}{\hbar} - \delta_{inh,r,t}\pi + \mathbf{L}_{m_{r}})}$$
(5)

where $1_{Time \ Dimensions}$ represents the normalized square root of the density of mass/energy per virtual surface and per time of incoming virtual particles or symetrically antiparticles in N_2 from incidence r. Note that $1_{r,Time \ Dimensions} = 0$ if $|\psi(N_x, t)| \neq 0$ (a real object exists in N_x at time t) where N_x the real image of N^r .

Note that:

 \hbar can be defined as the maximal quantity of energy that can hold a unity of temporal distance t_p , the smaller time that two different events can be seen in space 3D which is the Planck time t_p [9]. Moreover, this Planck time is equivalent to a Planck distance l_p that can contain a maximum mass equivalent to Planck mass M_p . The maximal energy, thanks to mass-energy equivalence, is therefore equals to M_pc^2 in the shortest temporal distance t_p , therefore:

$$\hbar = M_p c^2 t_p = 1.0564 \times 10^{-34} kg \ m^2 \ s^{-1} \tag{6}$$

which is equal to the value of the reduced Planck constant.

B. Mathematical Model for Information propagation in spacetime dimensions in Universe

At the arrival to the surface border $\partial\Omega$ of the node or spatial point N_2 , all the signals-information coming from all the virtual neighboring points N^r are summed according to their spatial and temporal proximity. This is called the *spatial-temporal summation* where the signals not far apart temporally and spatially are summed. This property can be written as a spatial and temporal sum on the signals arriving at each incident point r of the surface $\partial\Omega$ which defines the N_2 3 dimensions spatial point Fig.2. The temporal summation is on a time duration Δt , the time duration that a particular possible signal-Information may be created i.e. if two signals are coming far apart with time lag higher than Δt , then the signals cannot be aggregated. The wave amplitude function of a signal-Information $\psi(N_2, t_f)$ in location N_2 at time t_f is therefore:

$$\psi(N_2, t_f) = \int_{\partial\Omega} \int_{\Delta t} |\psi(N^r, t)| e^{i\frac{S_r}{\hbar} - \delta_{inh, r, t}\pi + \iota_{mr}} D[q_r(t)] dt dr$$
(7)

where S_r the virtual kinetic action of a path arriving to the border point N_2 with incidence r. $D[q_r(t)]dtdr$ is the measure of the path q arriving with incidence r is the N_2 point. One may suppose that it is a constant. $|\psi(N^r, t)|$ is the initial wave function module coming from the virtual point N^r respectively to the path q_r since time t where we recall that $|\psi(N^r, t)| = 1_T + |\psi(N_x, t)|$ with $1_T = 1_{r,Time \ Dimensions}$ in the Time dimensions, and $|\psi(N_x, t)|$ the module of the SI existing in 3D space point N_x at time t.

 $|\psi(N^r, t)|^2$ describes the density of virtual information quantity (mass or energy) per virtual surface and per time.

Note that if there is no signal-Information coming from any N_x , we have $|\psi(N^r, t)| = 1_T$, $\iota_{m_r} = 0$ and if we see just the signal coming from only on incidence point r (the path can be either excitatory or inhibitory, and we assume that here it is excitatory $\delta_{inh,r,t}\pi = 0$), and supposing that in this case $D[q_r(t)] = 1$ as there is only one excitatory path (and one inhibitory path), we have that the wave amplitude in N_2 at time t_f with restrecting oneself to the Time path with incidence r and knowing that the property of this path is excitatory, is therefore:

$$\psi(N_2, t_f)_{r, exc} = \int_{\Delta t} 1_T e^{i\frac{\iota_K \cdot U_t}{\hbar}} dt$$
(8)

where $\iota_K.U_t = \int_t \frac{1}{2}\rho(\frac{d_a}{d_u})^2 du$ the scalar product of kinetic Lagrangian of virtual particles and time unity U_t . We can notice that the formula (8) is a kind of a Fourier Transform of $H(\frac{t}{\Delta t})$ (the Tophat function) in the excitatory Time path with incidence r. Therefore we have $\psi(N_2, t_f)_{r,exc} \equiv 2\Delta t \times sinc(\Delta t \frac{\iota_K}{\hbar})$ ([10]). This shows how a virtual particle with frequency $\frac{\iota_K}{\hbar}$ may potentially appear in spacetime location at (N_2, t_f) in a quite short time during Δt and similarly for the virtual antiparticles. Moreover 1_T is not just equal to 1 but it's a kind of a square root of a probability density. By taking the square of 1_T as the density of mass/energy per time and surface with incidence r of virtual particles hence we have $1_T^2 = \frac{1}{\Delta t \times 4\pi} \times \frac{1}{2}$ if it is uniform. The factor $\frac{1}{2}$ comes from that the chance to have an excitatory or an inhibitory Time path is supposed equal and because here we took just the case of the excitatory Time path with incidence r. Note that if we want to compute the mean value of $\delta_{inh,r,t}$ it will be equivalent to $\frac{1}{\Delta t \times 4\pi} \times \frac{1}{2}$ which is the density of inhibitory Time path with incidence r. As $\int sinc(X)^2 dX = \pi$ and by setting the variable change $X = \Delta t \times \frac{\iota_K}{\hbar}$, we have:

$$\int |\psi(N_2, t_f)_{r, exc}|^2 \equiv \frac{1}{\Delta t \times 4\pi} \times \frac{1}{2} \int (2\Delta t \times sinc(\Delta t \frac{\iota_K}{\hbar}))^2 d\frac{\iota_K}{\hbar} = \frac{1}{\pi} \times \pi \times \frac{1}{2} = \frac{1}{2}$$
(9)

which verifies the normalized choice of 1_T if we also add the virtual antiparticles part.

Note that Δt is not the time that we know but the quantity of unit of time hence $\Delta t \times u$ is the *time* that we know. Moreover, as soon as we sum over all the incidence virtual points r in the virtual surface $\partial \Omega$ the signals ψ cancel one another as there are also the virtual antiparticles that are added if they are of the same proportion as the virtual particles. Hence in case when there is no energy/matter in any location the total

 $\psi(N_2, t_f)$ over Δt time and virtual surface is equal to 0 and hence the probability to have present virtual particles in N_2 at t_f is $|\psi(N_2, t_f)|^2 = 0$ wich will not cause the problem of normalization for the probability densities $|\psi|^2$ in presence of matter/energy in spacetime dimensions.

While observing closely the equation (7), one can note that $\psi(N_2, t_f)$ is a conditional expected value of the function $D[q_R(T)]$ knowing that the trajectory with incidence R is either excitatory or inhibitory (knowing the value of $\delta_{inh,r,t}$), with complex density where R and T are the random variables defined as follows:

- R represents the random variable of the signal existence coming from the virtual point N^R including the virtual signals of the virtual paticles and antiparticles,
- T is the random variable of the summation possibility of signals coming from the neighboring points i.e it represents the time summation capacity: interference in time.

Hence we can see mathematically $\psi(N_2, t_f)$:

$$\psi(N_2, t_f) = \mathbb{E}[D(q_R(T))|\delta_{inh, R, T}]$$
(10)

Summary and deduction:

To analyze what is happening in the spacetime location N_2, t_f we aggregated all what is happening in the neighboring spacetime locations N_x, t during Δt unit of time including the the virtual particles/antiparticles pairs created by the quantum vacuum. If we sum over this Δt , one can note that the effect of virtual particles/antiparticles is null. However if if we see it in time unit and in unit virtual surface edge of N_2 , the effect changes: the virtual antiparticles created may create the antigravity effect as it can delete the real existing particles in N_2 for a quite short moment. But thanks to the presence at the same time of the virtual particles this effect is stopped. The real existing particles in N_2 stays in its initial equilibrium state.

Now we assume that there are two objects \mathbf{A} and \mathbf{B} in different space locations N_x and N_y at the same time t where \mathbf{B} is much heavier/more energetic than \mathbf{A} . We want to analyze what happens to the \mathbf{A} object. We suppose that between \mathbf{A} and \mathbf{B} there is an N_2 space location, the nearest point to the N_x . We study the $\psi(N_2, t_f)$ for the object \mathbf{A} : at each N_2 we sum all the incident $\psi(N^r, t)$ during Δt . The only difference here is that N^r incidence points at N_2 may come from also N_y (that had travelled in the past spatial point by point) plus the N_x ones. The signal-Information coming from N_y is *heavier* (recall that $\psi(N^r, t)$ is the mass/energy density by Time path) in N_2 than in N_x as N_2 is closer to N_y . If \mathbf{A} is at particular location N_x in time t where $|\psi(N_2, t_f)|$ can be higher than $|\psi(N_x, t_f)|$ thanks to the \mathbf{B} object, than \mathbf{A} moves to N_2 at time t_f . This is not symetric to \mathbf{B} as the $\psi(N_2, t_f)$ for the \mathbf{B} is much higher next to the N_y and hence \mathbf{B} stays at N_y .

Now that an Information may be reconstructed thanks to temporal and spatial summation of signal-Information coming from different Time paths form any spacetime locations, we want to know if this Information has been distorted due to different Time paths between two spatial nodes N_x and N_2 and also due to other time paths coming to N_2 from other spatial nodes N_x . In the Information theory, we may compute the deformation rate of random variables thanks to the Entropy of Shannon. As we deal here with a particular random variable $D(q_R(T))$, one can deduce an entropy of the signal-Information present in spacetime dimension.

In the next section, we will show that a particular entropy of the signal-Information brought by Time paths between N_1 (and all the neighboring points N_x) and N_2 define the scalar curvature term \mathcal{R} [11] in Hilbert-Einstein action [12], [13].

III. INFORMATION ENTROPY: THE UNIVERSE SPACETIME CURVATURE

In this section we want to measure the average level of disorder of the signal-Information *received virtually* by the point N_2 . We introduce here an entropy similar to Shannon entropy in information theory [14]. We recall that Shannon entropy measures the information quantity sent by a information source. The more different information the source emits, the greater the entropy (or uncertainty about what the source emits). We note that Δt is not really in time dimension but a quantity of unit of time, and this is what we call time in our Universe.

In quantum mechanics, we work with wave functions, a particular probability functions. Any complex wave function $\psi(r, t)$ can be rewritten as following $\psi(r, t) = R(r, t)e^{i\theta(r,t)}$ with the probability density $p(r,t) = R^2(r,t)$ and its phase $\theta(r,t)$. Recall that $\psi(N^r,t)$ in our study is the wave function of the signal-Information coming to N_2 at time t from the particular Time path with incidence r, and r is a point on the virtual surface border $\partial\Omega$ of N_2 . Nalewajski in [15] gives a definition of the complex entropy by first defining the *overall entropy*:

$$s(\psi) = \int -p(r,t)[ln(p(r,t)) + 2i\theta(r,t)]dtdr \equiv s(p) + i \ s(\theta), \tag{11}$$

where the integral is overall the surface $\partial \Omega$ of the N_2 virtual border and over a time duration Δt . Here we note $p(r,t)=p(N^r,t)$.

The real part p(r,t)ln(p(r,t)) does not take into account the phase which is very important to compute the entropy in our study. However the imaginary part $-p(r,t)\theta(r,t)$ (without the 2) does. In the paper [16] the entropy considered is equivalent to the actions, and in our study we will choose also the part that takes into account the Time path actions with the probabilities: the Imaginary part of the entropy (11) without the 2.

Hence we introduce a new type of entropy, a virtual entropy, on the phases of signal-Information coming into the point N_2 from each virtual Time path with extremety r at time t. The spacetime entropy of a signal ψ at time t_f in location N_2 is therefore:

$$\S(\psi(N_2, t_f)) = \mathcal{L}_m \sqrt{|g|} + \int_{\partial\Omega} \int_{\Delta t} -p(r, t) \times (\gamma(r, t)) dt dr$$
(12)

p(r, t) is the mass/energy (we will choose mass in the following) density of the incoming signal amplitude per surface and time at N_2 with incidence r. For the Universe system, in the mass field p(r,t) represents the incoming mass density Information for the signal-Information per time and surface. We do not mean that the mass of an Information is travelling but the signal-Information that cares the mass/energy/interaction information of an object that travels in Time dimensions, namely the carrier could be virtual particle/antiparticle or any virtual gauge boson. $p(N^r, t)$ or p(r, t) encompass the probability of the existence of the initial incoming Information from N_x with incidence r and also the probability of existence of Time path with incidence r. $\gamma(r,t)$ is the Time phase shift at time t of the path incoming from r without the matter lagrangian part:

$$\gamma(r,t) = \frac{S}{\hbar} - \delta_{inh,r,t}\pi$$
(13)

S, or noted before S_r , is the action on the signal-Information unit in a particular Time path with incidence r

that is defined the formula (1).

Here $\mathcal{L}_m \sqrt{|g|} \sim \int_{\partial\Omega} \int_{\Delta t} -p(r,t) \iota_{m_r} dt dr$ where ι_{m_r} is the density of the Lagrangian density \mathcal{L}_m , the Standard model Lagrangian density ([7]), of the initial signal-Information coming from the point N_x at time t with incidence r and defined in $\psi(N_x, t) = R(r, t)_x e^{i\iota_{m_r}}$. $\sqrt{|g|}$ represents the volume unit at the studied location which will be explained later, but one can say that this comes from the effect of the mass incoming density p.

In the Universe, If the signal-Information is a real particle, its mediators would be virtual particles and antiparticles. Particles and antiparticles of the same type have same mass and hence same mass Information. The difference between the inhibitory and the excitatory mediators Lagrangian is the $-\pi$ factor added in the path action explained previously.

As p(r,t) represents the incoming mass/amplitude density per virtual surface and time, to normalize p(r,t), we multiply and divide by $\frac{M_p}{S_p t_p}$, the maximum mass that we could have in one space point N_2 . In fact the maximum of mass quantity that a time path of time Planck t_p can carry is the mass of Planck M_p , and S_p is the surface of Planck. We consider now $\S(\psi(N_2, t_f))$ without the Lagrangian density of matter:

$$\S(\psi(N_2, t_f)) = \frac{M_p}{S_p t_p} \int_{\partial\Omega} \int_{\Delta t} -P(r, t) \times \gamma(r, t) dt dr$$
(14)

where $P(r,t) = \frac{p(r,t)}{\frac{M_p}{S_p t_p}}$. In the virtual surface border $\partial \Omega$ of N_2 there is a temporal summation of all the action potentials arrived during Δt , a duration which after the space location N_2 may create a new signal-Information unit that will be sent after to Time paths linked to the neighboring spatial points N_2 with the condition that the sum of signal amplitude exceeds a certain threshold which depends on the studied field. Recall that this duration is the tolerance time that any information unit (the respective particle in its field) may be recreated in any spatial location. Note that a particle can be recreated in its field at N_2 with a less duration than Δt if there is another correlated Information coming to N_2 during this time laps Δt .

More explicitly, the entropy, without the Lagrangian of the matter, can be rewritten as follows:

$$\S(\psi(N_2, t_f)) = \frac{M_p}{S_p t_p} \int_{\partial\Omega} \int_{\Delta t} -P(r, t) \times \left[\frac{1}{\hbar} \int^t \frac{1}{2} \rho(\frac{dq_r}{du})^2 du - \delta_{inh, r, t} \pi\right] dt dr$$

By normalizing the mass ρ by Planck mass M_p (an elementary virtual particle/antiparticle cannot have a mass higher than Planck mass), thus:

$$\S(\psi(N_2, t_f)) = \frac{M_p}{S_p t_p} \int_{\partial\Omega} \int_{\Delta t} -P(r, t) \times \left[\frac{M_p}{\hbar} \int^t \frac{1}{2} \frac{\rho}{M_p} (\frac{dq_r}{du})^2 du - \delta_{inh, r, t} \pi \right] dt dr$$

By setting $\frac{M_p}{\hbar}$ outside the double integral, we have:

$$\S(\psi(N_2, t_f)) = \frac{M_p^2}{S_p t_p \hbar} \int_{\partial\Omega} \int_{\Delta t} -P(r, t) \times \left[\int^t \frac{1}{2} \frac{\rho}{M_p} (\frac{dq_r}{du})^2 du - \frac{\hbar}{M_p} \delta_{inh, r, t} \pi\right] dt dr \tag{15}$$

We note that $-P(r,t)\left[\int_t \frac{1}{2} \frac{\rho}{M_p} \left(\frac{dq_r}{du}\right)^2 du - \delta_{inh,r,t}\pi\right]$ may represent the density of the variation in time of spatio-temporal square distance that a signal-Information travels from two nearest points N_x and N_2 i.e. the variation of square distance that an Information arrives with incidence r in the virtual border $\partial\Omega$ of N_2 at time t_f Fig.3.

We note the variation in time of square distance of the particular excitatory Time path with incidence r i.e. without the inhibition part $\frac{\partial g_{x_r,t}}{\partial t}$:

$$\frac{\partial g_{x_r,t}}{\partial t} = -P(r,t) \left[\int_{t_1}^t \frac{\rho}{M_p} (\frac{dq_r}{du})^2 du \right],\tag{16}$$

and the variation in time of square distance of the inhibitory Time path effect with incidence r:

$$\frac{\partial \overline{g}_{x_r,t}}{\partial t} = P(r,t) \frac{\hbar}{M_p} \delta_{inh,r,t} \pi.$$
(17)

The distance between two points N_1 and N_2 is infinite if no Information can propagate between them i.e. if P(r,t) = 0 for all Time incidence r at N_2 . Which is impossible with our definition of P if the virtual particles may exist in spacetime dimensions thanks to the energy-time principle incertitude. Note that $\frac{\partial g_{x,r,t}}{\partial t}$ is surely to be negative (as $\delta_{inh,r,t}\pi = 0$) and as the virtual mass ρ of a virtual particle is positive as $(\frac{dq_r}{du})^2$. This means that the distance between the two points N_x and N_2 is decreasing between times if P(r,t) is stricly positive.

However if the mediators are inhibitory then $-\delta_{inh,r,t}\pi = -\pi$ and this distance is getting greater than the previous one as $\frac{\partial \overline{g}_{x_r,t}}{\partial t}$ would be always positive. This leads to a kind of a virtual spatio-temporal expansion between the two nearest points N_x and N_2 : this may be the result of the spacetime expansion.

We rewrite the mass entropy or the Information entropy:

$$\S(\psi(N_2, t_f))) = \frac{M_p^2}{S_p t_p \hbar} \left[\frac{1}{2} \int_{\partial\Omega} \int_{\Delta t} \frac{\partial g_{x_r, t}}{\partial t} dt dr + \frac{\hbar}{M_p} \int_{\partial\Omega} \int_{\Delta t} P(r, t) \delta_{inh, r, t} \pi dt dr\right]$$
(18)

We consider now (see Fig.3):

$$\begin{array}{ll}
-\overrightarrow{n} dS \equiv & dr, \ the \ surface \ unit \\
-\overrightarrow{n} = & incoming \ normal \ vector \ to \ dS \\
\overrightarrow{F} = & \overrightarrow{grad} \frac{\partial g_{x_r,t}}{\partial t} \\
\overrightarrow{\Pi} = & \overrightarrow{grad} (\frac{\hbar}{M_p} P(r,t) \delta_{inh,r,t} \pi) = \overrightarrow{grad} \frac{\partial \overline{g}_{x_r,t}}{\partial t}
\end{array}$$
(19)

where $\overrightarrow{\Pi}$ can be rewritten as follows:

$$\overrightarrow{\Pi} = \frac{\hbar}{M_p} \pi \overrightarrow{grad}(P(r,t)\delta_{inh,r,t})$$
(20)

and the scalar product of $\overrightarrow{n} dS$ and $\overrightarrow{\Pi}$:

$$-\overrightarrow{n}dS.\int^{t}\overrightarrow{\Pi}du = -\frac{\hbar}{M_{p}}\pi P(r,t)\delta_{inh,r,t}dS$$
(21)

 \vec{F} is the resulted vector of a particular path which narrows the distance between N_x and N_2 spatial points and $\vec{\Pi}$ is the vector that increases this distance. One can note that to N_2 the distance is identical to any direct neighboring neuron N_x as the distance is only the sum of all these vectors from any incoming path to N_2 Fig.3. The distance between two any linked neighboring points N_x and N_2 depends on the Time paths and on the initial incoming signal-Information. Space and Time are hence relative on what is going on the Time dimensions and Time paths (quantum vacuum) between the nearest two spatial points and on the incoming/present signal-Information. The importance of the sign – next to π in the inhibitory part choice is materialized here: in the wave function ψ it has no impact but in signal-Information flow at point N_2 it precises that the Information is removed from N_2 which leads to a probable lengthening of the distance between two neighboring points for the studied signal-Information.

The Mass-Information entropy thanks to Gradient theorem can be rewritten as follows:

$$\S(\psi(N_2, t_f)) = \frac{M_p^2}{S_p t_p \hbar} \left[\frac{1}{2} \int_{\partial \Omega} \int_{\Delta t} \int^t -\overrightarrow{F} \cdot \overrightarrow{n} \, du dt dS - \frac{\hbar \pi}{M_p} \int_{\partial \Omega} \int_{\Delta t} P(r, t) \delta_{inh, r, t} \, dt dS \right]$$
(22)

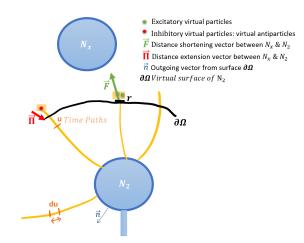


Fig. 3. Creation of the distance between two linked points N_x and N_2

Thanks to Green-Ostragradski theorem in the first integral, by interverting integrals and by using Gradient theorem while replacing in the second integral du by $\frac{1}{c} \times cdu$ and hence $dV = cdu \times dS$, we have:

$$\begin{split} \S(\psi(N_2, t_f)) &= \frac{M_p^2}{S_p t_p \hbar} \Big[\frac{1}{2} \int_{\Omega} \int_{\Delta t} \int^t -div(\overrightarrow{F}) \ dudt dV \\ &- \frac{\hbar \pi}{M_p c} \int_{\Delta t} \int_{\Omega} \overrightarrow{grad}(P(r, t) \delta_{inh, r, t})). \overrightarrow{n} \ dV dt \Big] \end{split}$$

By interverting div and $\frac{\partial}{\partial t}$ and integrating in time in the first integral, we have:

$$\begin{split} \S(\psi(N_2, t_f)) &= \frac{M_p^2}{S_p t_p \hbar} \Big[\frac{1}{2} \int_{\Omega} \int_{\Delta t} -div(\overrightarrow{grad}(g)) \ dudV \\ &- \frac{\hbar \pi}{M_p c} \int_{\Delta t} \int_{\Omega} \overrightarrow{grad}(\delta_{inh,r,t} P(r,t)) \cdot \overrightarrow{n} \ dV dt \Big] \end{split}$$
(23)

Using also the Green-Ostragradski theorem for the second integral and knowing that $div(\overrightarrow{grad}(N) = \Delta N)$ the Laplacian operator of N, we have:

$$\S(\psi(N_2, t_f)) = \frac{M_p^2}{S_p t_p \hbar} \Big[\frac{1}{2} \int_{\Omega} \int_{\Delta t} -(\Delta g) \ du dV - \int_{\Omega} \int_{\Delta t} (\Delta \overline{g}) \ du dV \Big]$$
(24)

The first integral of the formula (24) study:

By considering the approximation $Ric \simeq -\frac{1}{2}\Delta g$ in the first integral noted A of (23), where Ric is the Ricci curvature, given in local coordinates by $R_{ij} = (Ric)_{ij}$ so that Ric is a trace of the Riemann curvature, hence:

$$A \simeq \frac{M_p^2}{S_p t_p \hbar} \int_{\Omega} \int_{\Delta t} Ric \ dudV \tag{25}$$

By normalyzing dV with $l_p{}^3$ the Planck volume we have:

$$A \simeq \frac{M_p^2 l_p^{-3}}{S_p t_p \hbar} \int_{\Omega} \int_{\Delta t} Ric \ du dv \tag{26}$$

where $dv = \frac{dV}{l_p^3}$ is the normalized volume unit. Using another time Ostrogradski theorem in the integral we have:

$$A \simeq \frac{M_p^2 l_p^{-3}}{S_p t_p \hbar} \int_{\nu} \int_{\Delta t} div(Ric) du d\nu$$
⁽²⁷⁾

where $d\nu = dv \times dt$ a four dimension volume. And from the contracted Bianchi identities where R is the scalar curvature, we have $div(Ric) = \frac{1}{2}d_uR$ hence:

$$\int_{\nu} \int_{\Delta t} div(Ric) \ dud\nu = \frac{1}{2} \int_{\nu} \int_{\Delta t} d_u R \ dud\nu \sim \frac{\mathcal{R}}{2} \sqrt{|g|}$$
(28)

where \mathcal{R} is the scalar curvature of Einstein equation in general relativity, and $\sqrt{|g|}$ the four dimension volume unit with the effect of P. Finally we deduce that:

$$A \simeq \frac{M_p^2 l_p^{-3}}{S_p t_p \hbar} \frac{\mathcal{R}}{2} \sqrt{|g|} \tag{29}$$

From Planck units where G is the gravitational constant, we have:

$$M_{p} = \sqrt{\frac{c\hbar}{8\pi G}}$$

$$S_{p} = l_{p}^{2}$$

$$c = \frac{l_{p}}{t_{p}}$$
(30)

we can deduce that:

$$\frac{M_p^2 l_p^{\ 3}}{S_p t_p \hbar} = \frac{c^2}{8\pi G} = \frac{1}{\kappa}$$
(31)

where κ the Einstein gravitational constant [17], as In Einstein's original publication, the choice is $\kappa = \frac{8\pi G}{c^2}$, in which case the stress–energy tensor components have units of mass density what we have indeed choose for the initial p(r,t) definition: p was the mass Information density. Therefore adding the matter effect, the entropy with the curvature effect and the matter effect is hence:

$$A_m \sim \frac{1}{\kappa} \frac{\mathcal{R}}{2} \sqrt{|g|} + \mathcal{L}_m \sqrt{|g|} \tag{32}$$

Which is the Lagrangian density of the Einstein-Hilbert action. To find the how an object or a signal-Information moves over spacetime we minimize the entropy A_m ([16]) or in other words we apply the action principle by minimizing the action A_m ([18]). This calculus gives the Einstein equation of the general relativity with T the energy-matter tensor:

$$Ric - \frac{1}{2}\mathcal{R} = \frac{1}{\kappa}T\tag{33}$$

What about the second integral of the formula (24) ? :

Now we observe the second integral of the formula (22) and its meaning before all the previous computation. We note this integral B

$$B = -\frac{\hbar\pi}{M_p} \int_{\partial\Omega} \int_{\Delta t} P(r,t) \delta_{inh,r,t} \, dt dS \tag{34}$$

We suppose that we have $\delta_{inh,r,t}$ that is equals to its mean that we have defined in the previous section i.e. $\delta_{inh,r,t} = \frac{1}{4\pi\Delta t} \times \frac{1}{2}$ that represents the mean density of the incoming virtual antiparticles at N_2 in the case where there is no signal-Information i.e. there is no matter nor energy. Therefore by interverting the integrals and using the Gradient theorem:

$$B \equiv -\frac{\hbar\pi}{M_p} \frac{1}{4\pi\Delta t} \frac{1}{2} \int_{\partial\Omega} \int_{\Delta t} P(r,t) \, dt dS = -\frac{\hbar}{8M_p\Delta t} \int_{\Delta t} \int_{\partial\Omega} \int^t \overrightarrow{grad}(P) \cdot \overrightarrow{n} \, du \, dS dt. \tag{35}$$

By replacing du by $\frac{1}{c} \times cdu$ (c is the maximal speed -the speed of light) and hence $dV = cdu \times dS$, we have:

$$B \equiv -\frac{\hbar}{8cM_p\Delta t} \int_{\Delta t} \int_{\Omega} \overrightarrow{grad}(P) \cdot \overrightarrow{n} \, dV dt.$$
(36)

We normalize dV with $dv = \frac{dV}{l_p^3}$, dt with $d\tau = \frac{dt}{\Delta t}$ and using the Ostrogradski theorem in four dimensions:

$$B \equiv -\frac{\hbar l_p^3 \Delta t}{8cM_p \Delta t} \int_{\Delta t} \int_{\nu} div(\overrightarrow{grad}(P)) d\nu d\tau$$
(37)

Note that $d\tau$ is not the unity of the time but the quantity of unit of times. Hence:

$$\frac{B}{l_p^3} \sim -\frac{\hbar}{8cM_p}\sqrt{|g|} \simeq -1.0128 \times 10^{-35}\sqrt{|g|} \sim -\Lambda\sqrt{|g|}$$
(38)

Which is quite representative of the cosmological constant value Λ in time metric, and which if we divide by c^2 we have the cosmological constant with the space metric: $\Lambda = 1.1269 \times 10^{-52}$

Finally we will have the total entropy taking into account the second integral which is the virtual antiparticles effect, and factoring by l_p^3 , we have:

$$\S(N_2, t) \sim \frac{1}{\kappa} \left[\frac{\mathcal{R}}{2} \sqrt{|g|} - \Lambda \sqrt{|g|} \right] + \mathcal{L}_m \sqrt{|g|}$$
(39)

And using the action principle or the entropy minimizing we have the total Einstein equation of general relativity:

$$Ric - \frac{1}{2}\mathcal{R} + \Lambda g = \frac{1}{\kappa}T \tag{40}$$

We finally can conclude that the entropy of the signal-Information at any point represents the scalar curvature and an approximation of the cosmological constant. Hence the cosmological constant could define the potential virtual antiparticles. The gravity could be assimilated to the degree of correlation between massive objects and not a kind of interaction. A highly massive object **B** or signal-Information impacts highly the Time field of any other closed correlated signal-Information **C** (less massive) as Time paths coming from particles of **B** are added to the paths of the studied object **C** Fig.4. Hence the probability of **C** object existence is much higher next to the object **B** at time t + 1 than its past location at time t: this is the gravity effect. Finally

Impact of a massive signals-Information set on a studied signal-Information

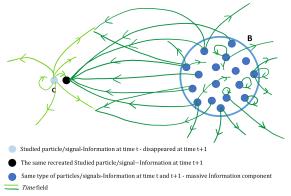


Fig. 4. Impact of a massive set of signals-Information on another studied Information

we conclude that the entropy due to the term probability P, which encompass the existence of the signal-Information (massive object in gravity) and the possible existence of Time paths between any spacetime points, is the gravity impact of the massive objects.

IV. IMPLICATIONS ON UNIVERSE LAWS

Gravity: In universe, Gravity is the result of massive matter correlation, and the spacetime curvature is the massive Information entropy created by the neighboring massive objects.

Black Holes: There is no more correlation between objects, even the information with itself. In fact all any possible Time path is absorbed by the massive point inside the black hole. No such quantum fields could exist: the information is detructed as no Time path could exist in black holes. There has been a tearing of quantum fields as any probable created field is absorbed by the massive point.

Universe expansion: The cosmological constant as demonstrated in the article can be a result of the virtual antiparticles when they appear in spacetime dimension thanks to energy-time principle incertitude. Moreover it could be also a result of the disappeared antimatter since the Big Bang that could be present between two set of matters and their fields.

The Heisenberg incertitude principle $\Delta x \Delta p \ge \frac{\hbar}{2}$: This formula may be explained as following: if we try to locate a particle in a particular position then we give a high intensity of Information concentrated into this location which may enhance the density of Time paths from this location to this same location. We have inhibited the other paths to other locations and we have enhanced at the same time the energy of the particle localization.

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