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## ► To cite this version:

Kheira Camellia Nehar, Mohamed Badaoui, Brahim Elkhail Hachi, Mohamed Guesmi, Abdelkader Benmessaoud. Modal spectral analysis of planar cracked structures subjected to seismic excitations by the eXtended Finite Element Method. CFM 2013 - 21ème Congrès Français de Mécanique, Aug 2013, Bordeaux, France. <hal-03439052>

**HAL Id: hal-03439052**

**<https://hal.science/hal-03439052v1>**

Submitted on 22 Nov 2021

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# Modal spectral analysis of planar cracked structures subjected to seismic excitations by the eXtended Finite Element Method

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## Abstract:

*The aim of this paper is to model planar structures containing stationary cracks and submitted to dynamic loads of seismic nature. This modeling aims to evaluate the dynamic stress intensity factor (DSIF), characterizing the resistance to brutal fracture of cracked structures, using two analysis strategies; the first is a modal (costs less and as precise as a method of direct resolution) and the other one is modal spectral (much less costly but not as much precise as the latter).*

*The evaluation of the dynamic modal stress intensity factor DMSIF is performed using the extended finite element method (X-FEM) coupled with the modal and spectral modal analysis. The main advantage of the latter is its ability to model cracks independently of the mesh at a reduced computational time compared to the conventional finite element method. The proposed procedure is applied to a reference problem (Sailing cracked). Comparisons between DSIF Spectral and maximum of modal DSIF are discussed. In addition, the effects of orientation, length and location of crack on the variation of these DSIF are tested.*

**Key words:** The extended finite element method (X-FEM), Cracked structures, Modal analysis, Spectral modal analysis, Stress intensity factor, Dynamic loads.

## 1 Introduction

The presence of cracks in structures weakens their strength and may cause serious disasters due to their ruin. To avoid them, scientists should be able to predict the behavior of cracked structures. To solve this problem a new method appeared in 1999 called the extended finite element method (X-FEM) developed by Belytschko and Black [1], and Moës et al. [2], which allows to represent the crack independently of the mesh, in the base finite element, by adding enrichment functions, that take account of the discontinuity of the displacement along the crack and the asymptotic form singular stress at the crack tip.

In dynamics, determining the stress intensity factor (SIF) for a cracked structure presents an important step for the fracture prediction. Experimental determination of the dynamic stress intensity factors has been proposed by Bui et al. [3]. Modal approaches have been proposed by Doyle et al. [4] has shown, using finite element method, that the behavior of SIF as a function of frequency is very similar to a modal response.

This method involves a modal analysis of the cracked plate and normalize the DSIF to the static stress intensity factor. A continuation of the latter study is done by Galenne et al.[5]. In this context, the objective is to evaluate the dynamic stress intensity factor by the X-FEM coupled with the modal analysis and the spectral modal analysis. Then, benefiting from all the advantages of X-FEM (particularly the treatment of singularities and discontinuities independently of the mesh) and those of numerical simplicities of modal presentation.

## 2 The X-FEM approach

In order to model the presence of the discontinuity, the finite element approximation is enriched in XFEM by two additional terms [2]:

$$u = \sum_{i \in n} N_i u_i + \sum_{j \in N_j} N_j H a_j + \sum_{p \in N_p} N_p \left( \sum_{l=1}^4 F_l b_p^l \right) \quad (1)$$

Where  $N_i$  is the standard shape function associated with node  $i$  (classical finite element),  $n$ , the set of nodes of the discretized domain,  $N_j$ , the set of nodes whose corresponded elements are cut by crack,  $N_p$ , the set of nodes whose corresponded elements contain the crack tip,  $u_i$ , the displacement at node  $i$ ,  $a_j$ , the additional degrees of freedom which are related to the displacement jump induced by the crack (not crack tips) and  $b_p^l$ , the additional degrees of freedom for modeling crack-tips. The Heaviside enrichment function  $H$  is a discontinuous function which its value is (+1) on one side of the crack and (-1) on the other. The crack tip enrichment functions  $F_l$  are defined as:

$$F_l(r, \theta) = \left[ \sqrt{r} \sin\left(\frac{\theta}{2}\right), \sqrt{r} \cos\left(\frac{\theta}{2}\right), \sqrt{r} \sin\left(\frac{\theta}{2}\right) \sin(\theta), \sqrt{r} \cos\left(\frac{\theta}{2}\right) \cos(\theta) \right] \quad (2)$$

Where  $(r, \theta)$  are the local polar coordinates which their origin is at the crack tip (Figure 1).

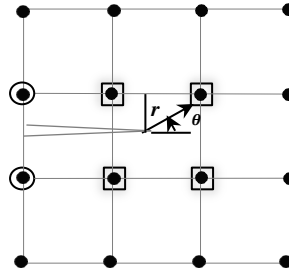


FIG. 1-Variety types of enrichment: ○ Nodes enriched by the Heaviside functions, ◻ nodes enriched by the asymptotic functions.

## 3 Numerical integration

The integration scheme used to evaluate the elementary matrices depends on whether or not the element is enriched and also on the type of the enrichment. The integration of unenriched elements is done the usual way using the Gauss quadrature method. Elements cut by the crack (i.e. those enriched by the Heaviside function) are divided into four triangles and those containing a crack tip (i.e. those enriched by the asymptotic crack tip functions) are divided into six triangles (figure 2). A high order Gauss quadrature formula is then applied in each triangle [5].

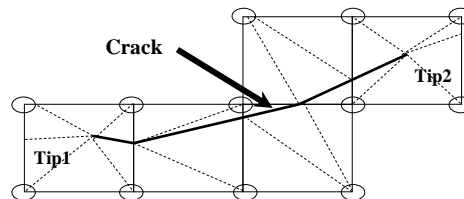


FIG. 2-Sub-triangles used for numerical integration.

## 4 Interaction integral

The stress intensity factors are computed using domain form of the interaction integral [2]. For general mixed-mode problems in linear fracture mechanics, the relationship between the value of the J-integral and the stress intensity factors is:

$$J = \frac{K_I^2}{E^*} + \frac{K_{II}^2}{E^*} \quad (3)$$

Where  $E^*$  is a function of Young's modulus  $E$  and poisson's ratio  $\nu$ , while  $E^* = E$  in plane stress and  $E^* = E/(1 - \nu^2)$  in plane strain.

Two states of a cracked body are considered. The state 1 is the actual state and state 2 is an auxiliary fictive state which satisfies the boundary conditions of the problem. The  $J$  integral of the two superposed states is expressed by:

$$J^{(1+2)} = \int_{\Gamma} \left[ \frac{1}{2} (\sigma_{ij}^{(1)} + \sigma_{ij}^{(2)}) (\varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)}) \delta_{1j} - (\sigma_{ij}^{(1)} + \sigma_{ij}^{(2)}) \frac{\partial (u_i^{(1)} + u_i^{(2)})}{\partial x_1} \right] n_j d\Gamma \quad (4)$$

The interaction integral for states 1 and 2 are defined as:

$$I^{(1,2)} = \int_{\Gamma} \left[ W^{(1,2)} \delta_{1j} - \sigma_{ij}^{(1)} \frac{\partial u_i^{(2)}}{\partial x_1} + \sigma_{ij}^{(2)} \frac{\partial u_i^{(1)}}{\partial x_1} \right] n_j d\Gamma \quad (5)$$

which leads to:

$$I^{(1,2)} = 2 \left( \frac{K_I^{(1)} K_I^{(2)} + K_{II}^{(1)} K_{II}^{(2)}}{E^*} \right) \quad (6)$$

If the auxiliary state is the pure mode  $I$  with  $K_I^{(2)}=1$  and  $K_{II}^{(2)}=0$ , then the SIF in terms of the interaction integral becomes:

$$K_I^{(1)} = \frac{I^{(1, \text{Model})} E^*}{2} \quad (7)$$

## 5 Modal stress intensity factor

The displacement solution  $u(t)$  of a dynamic linear problem may be approximated by its decomposition on a truncated basis of eigenmodes  $\phi^j$  [4,6]. Then, it can be expressed as:

$$u(t) = \sum_{j=1}^N y_j(t) \phi^j \quad (8)$$

Where  $y_j(t)$  are the generalized coordinates which are evaluated by using the Duhamel's integral [6]. The Natural Frequencies  $\omega_j$  and eigenmodes  $\phi^j$  are determined from the following relationship:

$$\det[K - \omega_j^2 M] = 0, \text{ et } [K - \omega_j^2 M] \phi^j = 0 \quad (9)$$

The stiffness and mass matrices  $K$  and  $M$  of structure taking into account the existing of crack are constructed from an assemblage of elementary stiffness and mass matrices  $k_e$  and  $m_e$  respectively, represented as follows [7]:

$$k_e = \begin{bmatrix} k_{uu} & k_{ua} & k_{ub} \\ k_{au} & k_{aa} & k_{ab} \\ k_{bu} & k_{ba} & k_{bb} \end{bmatrix}, m_e = \begin{bmatrix} m_{uu} & m_{ua} & m_{ub} \\ m_{au} & m_{aa} & m_{ab} \\ m_{bu} & m_{ba} & m_{bb} \end{bmatrix} \quad (10)$$

$$\text{Where: } k_{\alpha\beta} = \int_{\Omega} B_{\alpha}^T C B_{\beta} d\Omega, m_{\alpha\beta} = \int_{\Omega} N_{\alpha}^T \rho N_{\beta} d\Omega \quad (11)$$

$B$  is a matrix defined in terms of derivatives of the shape functions,  $C$  is the elastic coefficient matrix, and  $\rho$  is the material density.

A new expression proposed (in 2007) by Galenne et al. [4] for computing the MSIF given for mode  $I$ :

$$K_I(t) = \sum_{j=1}^N y_j(t) K_I^j \quad (12)$$

Where  $K_I^j$  is the modal stress intensity factor of the  $j^{\text{th}}$  eigenmode.

## 6 Spectral stress intensity factor

Similarly for the stress intensity factor modal, we can extend the concept to obtain stress intensity factor spectral modal. The maximum displacement solution  $u_{\max}$  of a dynamic problem can be expressed as:

$$u_{\max} = \sqrt{\sum_{j=1}^N (y_{j\max} \phi^j)^2} \quad (13)$$

Where  $y_{j\max}$  is the generalized coordinates maximum evaluated by the response spectrum [8]. Where the expression of the stress intensity factor for the modal spectral mode  $I$  becomes:

$$K_I = \sqrt{\sum_{j=1}^N (y_{j\max} K_I^j)^2} \quad (14)$$

Where  $K_I^j$  is the modal spectral stress intensity factor of the  $j^{\text{th}}$  eigenmode.

## 7 Numerical example

In this work, a structure "on headed columns" was the subject of a parametric study of the proposed model. It consists of a concrete wall element width  $2w = 2$  m, height  $2h = 8$  m and thickness  $e_p = 0.10$  m, and a square slab of 10 meters by side and 0.10 m of thickness, considered infinitely rigid.

The wind bracing element contains a crack length  $2C$ . This crack is taken at first edge crack and then internal (Figure.4. (a)). The structure is excited by the El Centro earthquake of 1940 and RPA99 v2003 (Algerian Earthquakes Codes 99 version 2003 /Réglement Parasismique Algerienne 99 v 2003) (see Figure.4. (b), (c)).

Material characteristics are: - Modulus of elasticity  $E = 3.10^{10}$  Pa.

-Density of concrete  $\rho = 2400$  kg/m<sup>3</sup>.

-Poisson's ratio  $\nu = 0.18$ .

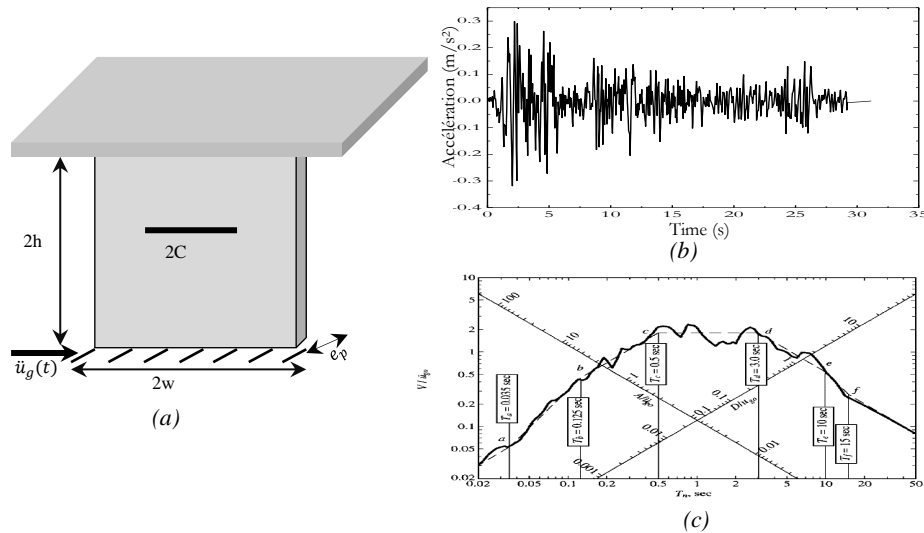


FIG.4-Structure "On headed columns" with crack: (a), (b) Earthquake Elcentro1940, (c) Response spectrum of El Centro.

### 7.1 Effect of the crack length

The figures below represent the values of Dynamic SIF obtained according to the normalized length  $C/w$  for two cases of position of the crack (edge, internal):

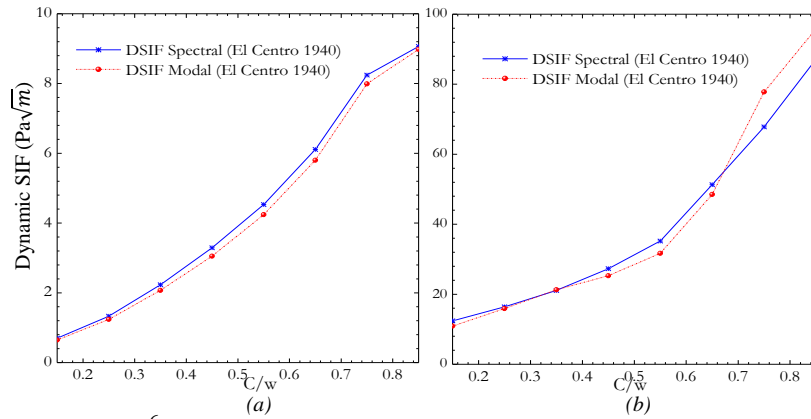


FIG.5: Dynamic SIF ( $\times 10^6$ ) according to the crack length adimensional  $C/w$ , (a) Crack internal, (b) Crack edge.

## 7.2 Effect of the inclination of crack

Calculating the Dynamic SIF modal and spectral varying the angle of inclination of the crack from the horizontal may be presented by the following figures:

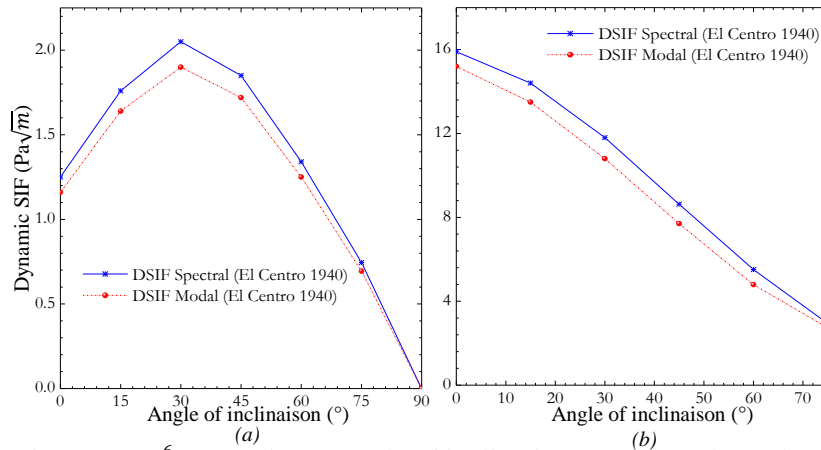


FIG.6-Dynamic SIF ( $\times 10^6$ ) according to angle of inclination: (a) Internal Crack, (b) Edge Crack.

## 7.3 Effect of the intensity of earthquake and site support structure

The response spectrum of RPA99 v2003 is used to calculate DSIF modal spectral (Figures.7 (a) and (b)):

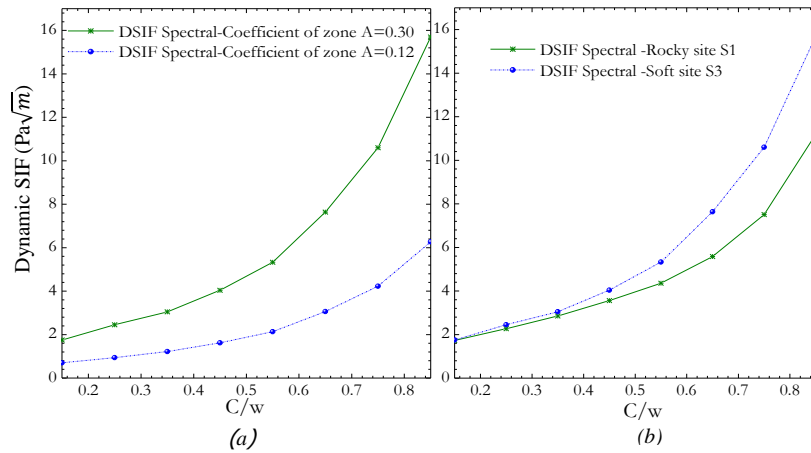


FIG.7: Dynamic Spectral SIF ( $\times 10^7$ ) according to the length adimensional  $C/w$  of crack edge.

## 8 Comments of the obtained results

From the calculation of stress intensity factors dynamic modal DMSIF for different positions, lengths, angles, intensity and site of the earthquake, we found that:

- The DMSIF increases with the crack length (edge and internal).
- The DMSIF decrease with the growth of angles for the edge crack, which leads to the conclusion that the DMSIF is maximum when the crack is horizontal. But in the case of internal crack, the factors increase up to an angle of 30 ° of inclination and then decreases for greater angles.
- The DMSIF's for an edge crack are higher than those for an internal crack in the same length and inclination angle of the crack.
- The DMSIF's calculated by the spectral modal analysis are very similar but they are slightly higher than those determined by modal analysis for the same position, length and angle.
- The spectral modal analysis using the response spectrum of RPA99 v2003 (Algerian Earthquakes Codes 99 version 2003 /Réglement Parasismique Algerienne 99 v 2003) for two cases of acceleration zone 0.12 and 0.30 showed that increasing the intensity of earthquake causes a growth in the DSIF modal spectral.
- As far as the support site of the structure is loose, the DSIF modal spectral increase.

## 9 Conclusion

This work involves the modeling of crack using the extended finite element method (X-FEM) to quantify its effect on the behavior of structure by the stress intensity factor dynamic modal spectral.

In example of a civil engineering structure containing a crack and subjected to a seismic excitation, the calculation of the modal spectral stress intensity factor is performed, using the accelerogram and response spectrum of El Centro 1940 and the response spectrum of RPA99 v2003.

This factor is used to measure the effects of length, position and angle of the crack, and also the effects of the earthquake intensity and the soil supporting the structure. Given the quality and richness of the results, we can conclude that the model presented has given us full satisfaction.

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