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# On the Robustness of the Linear Prediction Method for Blind Channel Identification with Respect to Effective Channel Undermodeling/Overmodeling

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TABLE I  
VALUES OF  $\phi_{\max}$  CORRESPONDING  
DAUBECHIES SCALING FUNCTION THROUGH  $N = 2 \sim 5$

$N$	2	3	4	5
$\phi_{\max}$	1.73205080756590	1.76320523091274	1.09116496568688	1.00048422469579

TABLE II  
COMPARISON OF THE APPROXIMATION ERRORS OF THE ORTHOGONAL  
PROJECTION

$N$	$E_o^{(2)}(f_1)$	$\hat{E}_o^{(2)}(f_1)$	$E_o^{(0)}(f_2)$	$\hat{E}_o^{(0)}(f_2)$
2	0.0303941162039	0.03561386723603	0.02058189833741	0.02791730842698
3	0.0087900005079	0.01468691131114	0.00945146526398	0.01802061589772
4	0.0027705547550	0.00397135019388	0.00516599819353	0.01343595552278
5	0.000911412144300	0.00140859211097	0.00315028315124	0.01112484670610

TABLE III  
COMPARISON OF THE SYSTEMATIC ERRORS

$N$	$E_s^{(2)}(f_1)$	$\hat{E}_s^{(2)}(f_1)$	$E_s^{(0)}(f_2)$	$\hat{E}_s^{(0)}(f_2)$
2	0.06714880126461	0.07963502808127	0.04488877472676	0.06242499939145
3	0.01725001412647	0.02306191961446	0.01780321791304	0.03602837963144
4	0.00317231037715	0.00461325466474	0.00572367587279	0.01560766023259
5	0.000306524353272	0.00145402415052	0.00204837183965	0.01148366205909

TABLE IV  
COMPARISON OF THE APPROXIMATION ERRORS OF THE PREFILTERED  
PROJECTION

$N$	$E_o^{(2)}(f_1)$	$\hat{E}_o^{(2)}(f_1)$	$E_o^{(0)}(f_2)$	$\hat{E}_o^{(0)}(f_2)$
2	0.07370823122739	0.08723580249536	0.04938235145921	0.06838316063790
3	0.01936045186155	0.02578584978889	0.02015650673344	0.04028382722880
4	0.004211831761173	0.00608717840745	0.00771025309781	0.02059427004645
5	0.000961576453500	0.00202443028168	0.00375767363225	0.01598864311071

V. CONCLUSIONS

In this correspondence, we analyze the approximation performance of a special FIR prefilter. The results show that the convergent rate of the prefiltered projection is the same as that of the orthogonal projection. In addition, for bandlimited signals, the quantitative estimates of the upper bounds of the three types of errors are obtained. Particularly for the Daubechies' orthogonal wavelet base, the estimated constant is optimal. Using these results, we can perform an initialization for the DWT.

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REFERENCES

[1] P. Abry and P. Flandrin, "On the initialization of the discrete wavelet transform algorithm," *IEEE Signal Processing Lett.*, vol. 1, pp. 32–34, Feb. 1994.  
 [2] B. Delyon and A. Juditsky, "Estimating wavelet coefficients," in *Recent Advances in Wavelet Analysis*, L. L. Schumaker and G. Webb, Eds. New York: Academic, 1993, pp. 151–168.  
 [3] M. J. Shensa, "The discrete wavelet transform: Wedding the atrous and Mallat algorithm," *IEEE Trans. Signal Processing*, vol. 40, pp. 2464–2482, Oct. 1992.  
 [4] W. Sweldens and R. Piessens, "Quadrature formulae and asymptotic error expansions for wavelet approximations of smooth functions," *SIAM J. Numer. Anal.*, vol. 31, no. 4, pp. 1240–1264, 1994.

[5] X.-G. Xia, C. C. J. Kuo, and Z. Zhang, "Wavelet coefficient computation with optimal prefiltering," *IEEE Trans. Signal Processing*, vol. 42, pp. 2191–2197, Aug. 1994.  
 [6] X.-P. Zhang, L.-S. Tian, and Y.-N. Peng, "From the wavelet series to the discrete wavelet transform-the initialization," *IEEE Trans. Signal Processing*, vol. 44, pp. 129–133, Jan. 1996.  
 [7] M. Unser, "Approximation power of biorthogonal wavelet expansions," *IEEE Trans. Signal Processing*, vol. 44, pp. 519–527, Mar. 1996.

**On the Robustness of the Linear Prediction Method for Blind Channel Identification with Respect to Effective Channel Undermodeling/Overmodeling**

Athanasios P. Liavas, P. A. Regalia, and Jean-Pierre Delmas

*Abstract*—We study the performance of the linear prediction (LP) method for blind channel identification when the true channel is of order  $M$ , whereas the channel model is of order  $m$ , with  $m < M$ . By partitioning the true channel into the  $m$ th-order significant part and the unmodeled tails, we show that the LP method furnishes an approximation to the  $m$ th-order significant part. The closeness depends on the diversity of the  $m$ th-order significant part and the size of the unmodeled tails. Furthermore, we show that two frequently encountered claims concerning the LP method, namely, that a) the method is robust with respect to channel overmodeling and b) the performance of the method depends critically on the size of the first impulse response term, are not correct in realistic scenarios.

*Index Terms*—Communications, multichannel system identification.

I. INTRODUCTION

Many methods can claim exact SIMO channel identification, in the noiseless case, under the so-called length and zero conditions [1]–[4]. However, their behavior may change dramatically under more realistic conditions, including the presence of tails of "small" leading and/or trailing impulse response terms [5]. Robustness issues of blind channel identification methods in such scenarios are very important from a practical point of view [6] but are less well understood. We study the mean asymptotic performance offered by the linear prediction (LP) method when the true channel is of order  $M$ , whereas the channel model is of order  $m$ , with  $m < M$ . We term this case the  $m$ th-order LP method. For ease of presentation, we adopt a single-input/two-output channel setting. Extension to the case of  $p$  output channels with  $p > 2$  is straightforward.

In Section II, we review the LP method for the exact order, exact statistics, noiseless case. In Section III, we bound the distance between the impulse response estimate furnished by the  $m$ th-order LP method and the  $m$ th-order significant part of the true channel, which will be defined in Section III. This distance depends on the diversity of the

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$m$ th-order significant part, as measured by the smallest nonzero singular value of a certain filtering matrix, and the size of the unmodeled tails. In the course of our analysis, we show that two frequent claims concerning the LP method, namely, that a) the method is robust with respect to channel overmodeling and b) the performance of the method depends critically on the size of the first impulse response term, are not correct in realistic scenarios. In Section IV, we present simulations, and some concluding remarks are summarized in Section V.

## II. LP METHOD: EXACT ORDER CASE

In this section and in order to fix notation, we review the basic steps of the LP method for blind channel identification, under the one-input/two-output noiseless channel setting. If the true order of the subchannels is  $M$ , then the output of the  $j$ th subchannel  $x_i^{(j)}$ , for  $j = 1, 2$ , is given by

$$x_i^{(j)} = \sum_{k=0}^M h_k^{(j)} s_{i-k}$$

where  $h_k^{(j)}$  is the impulse response of the  $j$ th subchannel, and  $s_i$  is the input sequence, which is assumed to be zero-mean unit-variance i.i.d. The data vector  $\mathbf{x}_L(i) \triangleq [\mathbf{x}_i^T \cdots \mathbf{x}_{i-L}^T]^T$ , where superscript  $T$  denotes transpose, with  $\mathbf{x}_i \triangleq [x_i^{(1)} \ x_i^{(2)}]^T$ , can be expressed as  $\mathbf{x}_L(i) = \mathcal{T}_L(\mathbf{h}_M) \mathbf{s}_{L+M}(i)$ , where  $\mathbf{h}_M \triangleq [\mathbf{h}_{(0)}^T \cdots \mathbf{h}_{(M)}^T]^T$  denotes the impulse response vector, with  $\mathbf{h}_{(k)} \triangleq [h_k^{(1)} \ h_k^{(2)}]^T$ ,  $\mathcal{T}_L(\mathbf{h}_M)$  is the  $2(L+1) \times (L+M+1)$  generalized Sylvester matrix

$$\mathcal{T}_L(\mathbf{h}_M) \triangleq \begin{bmatrix} \mathbf{h}_{(0)} & \cdots & \mathbf{h}_{(M)} & & \\ & \ddots & & \ddots & \\ & & & & \mathbf{h}_{(0)} & \cdots & \mathbf{h}_{(M)} \end{bmatrix}$$

and  $\mathbf{s}_{L+M}(i) \triangleq [s_i \cdots s_{i-L-M}]^T$ . It is well known that if the subchannels of  $\mathbf{h}_M$  do not share common zeros and  $L \geq M-1$ , then  $\mathcal{T}_L(\mathbf{h}_M)$  has full-column rank.

In the sequel, we review how the  $M$ th-order linear prediction error filter associated with  $\mathbf{x}_i$  can be used for the identification of the  $M$ th-order impulse response  $\mathbf{h}_M$  [3]. At first, we compute the coefficients of the  $2 \times 2M$  minimum mean-square error multichannel linear predictor

$$\underbrace{[\mathbf{A}_1 \cdots \mathbf{A}_M]}_{\mathcal{A}_M} = - \underbrace{[\mathbf{r}_1 \cdots \mathbf{r}_M]}_{\mathcal{R}_M} \mathbf{R}_{M-1}^{-1}$$

where

$$\mathbf{R}_{M-1} \triangleq E\{\mathbf{x}_{M-1}(i)\mathbf{x}_{M-1}^T(i)\} = \mathcal{T}_{M-1}(\mathbf{h}_M)\mathcal{T}_{M-1}^T(\mathbf{h}_M)$$

$$\mathbf{r}_k \triangleq E\{\mathbf{x}_i\mathbf{x}_{i-k}^T\}.$$

If we define  $\mathbf{D} \triangleq \mathbf{r}_0 + \mathcal{A}_M \mathbf{R}_M^{-1}$ , then it can be shown that  $\mathbf{D} = \mathbf{h}_{(0)}\mathbf{h}_{(0)}^T$ . If  $\lambda$  and  $\mathbf{v}$  are, respectively, the nonzero eigenvalue of the rank-one matrix  $\mathbf{D}$ , and its associated unit two-norm eigenvector, i.e.,  $\lambda = \|\mathbf{h}_{(0)}\|_2^2$  and  $\mathbf{v} = \mathbf{h}_{(0)}/\|\mathbf{h}_{(0)}\|_2$ , then an

$M$ th-order zero-forcing zero-delay equalizer is given by the row vector  $\mathbf{g}_M = (1/\sqrt{\lambda})\mathbf{v}^T[\mathbf{I} \ \mathcal{A}_M]$ . The impulse response  $\mathbf{h}_M$  can be identified via

$$\mathbf{h}_M = \mathbf{S}_M \mathbf{g}_M^T, \quad \text{with } \mathbf{S}_M \triangleq \begin{bmatrix} \mathbf{r}_0 & \cdots & \mathbf{r}_M \\ \vdots & \ddots & \\ \mathbf{r}_M & & \end{bmatrix}.$$

In [7, Eq. (3.10)], it is shown that  $\mathbf{S}_M = \mathcal{H}_L(\mathbf{h}_M) \mathcal{T}_M^T(\mathbf{h}_M)$ , with the  $2(L+1) \times (L+M+1)$  matrix  $\mathcal{H}_L(\mathbf{h}_M)$  defined, for  $L \leq M$ , as

$$\mathcal{H}_L(\mathbf{h}_M) \triangleq \begin{bmatrix} \mathbf{h}_{(0)} & \cdots & \mathbf{h}_{(L)} & \cdots & \mathbf{h}_{(M)} \\ \vdots & \ddots & & \ddots & \\ \mathbf{h}_{(L)} & \cdots & \mathbf{h}_{(M)} & & \mathbf{0} \end{bmatrix}.$$

In [3], it was shown that the LP method is able to identify the unknown channel in the overmodeled, with respect to identically zero impulse response terms, exact statistics case. Furthermore, it was claimed that the algorithm is expected to perform well in the estimated statistics, noisy, overmodeled cases, although this was not supported by theoretical results.

In [3] and [4], it was claimed that if the first impulse response term  $\mathbf{h}_{(0)}$  is “small,” then the algorithm is expected to perform poorly. In [6] and [7], it has been argued that microwave radio channel impulse responses usually possess “small” leading and/or trailing terms. This comes from the fact that the impulse response  $\mathbf{h}_M$  models both the shaping filters and the propagation through the channel. Does this mean that the LP method generically performs poorly in realistic cases? More generally, which are the factors that determine the robustness of the LP method in undermodeled/overmodeled cases? These are the questions we address in the sequel.

## III. $m$ TH-ORDER LP METHOD

In order to study the  $m$ th-order LP method, we partition the true channel  $\mathbf{h}_M$  into [5] [8]:

- 1) the  $m$ -th order significant part, which is the  $m$ th-order contiguous part of the true channel that has the largest energy among all its  $m$ th-order contiguous parts; it usually lies near the middle of the impulse response;
- 2) the *unmodeled tails*, which is the complementary part to the  $m$ th-order significant part; the unmodeled tails usually contain “small” leading and/or trailing terms.

Notationally, we express this partitioning for  $0 \leq m_1 < m_2 \triangleq m_1 + m \leq M$ , as

$$\mathbf{h}_M = \mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z$$

where superscript  $z$  denotes “appropriately zero-padded,” and  $\mathbf{h}_{m_1, m_2}^z$  and  $\mathbf{d}_{m_1, m_2}^z$  are defined by the expression at the bottom of the page. With  $\mathbf{h}_{m_1, m_2}$ , we denote the truncated  $m$ th-order significant part  $\mathbf{h}_{m_1, m_2} \triangleq [\mathbf{h}_{(m_1)}^T \cdots \mathbf{h}_{(m_2)}^T]^T$ . Our study proceeds in two steps. We first consider the  $m$ th-order LP method, assuming that the true channel is the  $m$ th-order significant part  $\mathbf{h}_{m_1, m_2}$ , and then, we study the behavior of the  $m$ th-order LP method upon augmenting this significant part with the tails.

$$\mathbf{h}_{m_1, m_2}^z \triangleq \begin{bmatrix} \mathbf{0}^T & \cdots & \mathbf{0}^T & \mathbf{h}_{(m_1)}^T & \cdots & \mathbf{h}_{(m_2)}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\ \underbrace{\hspace{1.5cm}}_{m_1} & & \underbrace{\hspace{1.5cm}}_{m+1} & & \underbrace{\hspace{1.5cm}}_{M-m_2} \end{bmatrix}^T, \quad \mathbf{d}_{m_1, m_2}^z \triangleq \begin{bmatrix} \mathbf{h}_{(0)}^T & \cdots & \mathbf{h}_{(m_1-1)}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T & \mathbf{h}_{(m_2+1)}^T & \cdots & \mathbf{h}_{(M)}^T \\ \underbrace{\hspace{1.5cm}}_{m_1} & & \underbrace{\hspace{1.5cm}}_{m+1} & & \underbrace{\hspace{1.5cm}}_{M-m_2} \end{bmatrix}^T.$$

Thus, let us initially assume that the true impulse response is  $\mathbf{h}_{m_1, m_2}$ . Then, the  $m$ th-order autocorrelation matrix is

$$\mathbf{R}_m = \begin{bmatrix} \mathbf{r}_0 & \mathbf{r}_m \\ \mathbf{r}_m^T & \mathbf{R}_{m-1} \end{bmatrix} = \mathcal{T}_m(\mathbf{h}_{m_1, m_2}) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}). \quad (1)$$

If the subchannels of  $\mathbf{h}_{m_1, m_2}$  do not share common zeros, the  $m$ th-order LP method furnishes

$$\mathcal{A}_m = -\mathbf{r}_m \mathbf{R}_{m-1}^{-1}, \quad \mathbf{D} = \mathbf{r}_0 + \mathcal{A}_m \mathbf{r}_m^T = \mathbf{h}_{(m_1)} \mathbf{h}_{(m_1)}^T \quad (2)$$

$$\lambda = \|\mathbf{h}_{(m_1)}\|_2^2, \quad \mathbf{v} = \frac{\mathbf{h}_{(m_1)}}{\|\mathbf{h}_{(m_1)}\|_2}$$

$$\mathbf{g}_m = \frac{1}{\sqrt{\lambda}} \mathbf{v}^T [\mathbf{I} \mathcal{A}_m] \quad (3)$$

$$\mathbf{h}_{m_1, m_2} = \mathbf{S}_m \mathbf{g}_m^T \quad \text{with } \mathbf{S}_m = \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}). \quad (4)$$

Now, let us assume that the true impulse response is  $\mathbf{h}_{m_1, m_2}^z$ . Since

$$\mathcal{T}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z) = \mathcal{T}_m(\mathbf{h}_{m_1, m_2}) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2})$$

it is simple to show that by following the same sequence of steps, we can “identify” the nonzero part of  $\mathbf{h}_{m_1, m_2}^z$ , namely,  $\mathbf{h}_{m_1, m_2}$ .

Finally, let us consider our augmented problem, in which the true impulse response is  $\mathbf{h}_M$ , which, without loss of generality, is considered to be normalized to unit 2-norm, i.e.,  $\|\mathbf{h}_M\|_2 = 1$ , under the assumption that  $\mathbf{d}_{m_1, m_2}^z$  is small, i.e.,

$$\|\mathbf{d}_{m_1, m_2}^z\|_2 = \epsilon_m \ll 1. \quad (5)$$

In this case, the  $m$ th-order autocorrelation matrix is the perturbed version of  $\mathbf{R}_m$ , as shown in (6) at the bottom of the page. The linear predictor and the associated prediction error power are given by

$$\tilde{\mathcal{A}}_m = -\tilde{\mathbf{r}}_m \tilde{\mathbf{R}}_{m-1}^{-1}, \quad \tilde{\mathbf{D}} = \tilde{\mathbf{r}}_0 + \tilde{\mathcal{A}}_m \tilde{\mathbf{r}}_m^T. \quad (7)$$

Terms  $\tilde{\lambda}$  and  $\tilde{\mathbf{v}}$  are, respectively, the largest eigenvalue of  $\tilde{\mathbf{D}}$  and its associated eigenvector, and

$$\tilde{\mathbf{g}}_m = \frac{1}{\sqrt{\tilde{\lambda}}} \tilde{\mathbf{v}}^T [\mathbf{I} \tilde{\mathcal{A}}_m]. \quad (8)$$

Finally, the  $m$ th-order impulse response estimate is given by

$$\tilde{\mathbf{h}}_{m_1, m_2} = \tilde{\mathbf{S}}_m \tilde{\mathbf{g}}_m^T, \quad \text{with } \tilde{\mathbf{S}}_m = \mathcal{H}_m(\mathbf{h}_M) \mathcal{T}_m^T(\mathbf{h}_M). \quad (9)$$

In the sequel, we provide a first-order upper bound for  $\|\tilde{\mathbf{h}}_{m_1, m_2} - \mathbf{h}_{m_1, m_2}\|_2$ .

#### A. First-Order Analysis

*Result 1:* If  $\mathbf{h}_{m_1, m_2}$  is the truncated  $m$ th-order significant part of the true channel  $\mathbf{h}_M$ , with  $\|\mathbf{h}_M\|_2 = 1$ , and the size of the unmodeled tails is  $\|\mathbf{d}_{m_1, m_2}^z\|_2 = \epsilon_m$ , then the  $m$ th-order LP method furnishes  $\tilde{\mathbf{h}}_{m_1, m_2}$  for which there holds to a first-order, with respect to  $\epsilon_m$

$$\|\tilde{\mathbf{h}}_{m_1, m_2} - \mathbf{h}_{m_1, m_2}\|_2 \leq \frac{(2m+3)\epsilon_m}{\|\mathbf{h}_{(m_1)}\|_2} \sqrt{1 + \frac{1}{\delta_m^2}} + \frac{\sqrt{m+1}\epsilon_m}{\delta_m} \quad (10)$$

with  $\delta_m \triangleq \sigma_{2m}(\mathcal{T}_{m-1}(\mathbf{h}_{m_1, m_2}))$ , where  $\sigma_i(\mathcal{M})$  denotes the  $i$ th singular value of matrix  $\mathcal{M}$ .

The proof is given in the Appendix.

Using arguments similar to those used in [5, Th. 2], it can be shown that

$$\delta_m \leq \min(\|\mathbf{h}_{(m_1)}\|_2, \|\mathbf{h}_{(m_2)}\|_2). \quad (11)$$

From these two bounds, it becomes clear that if  $\mathbf{h}_{(m_1)}$  and  $\mathbf{h}_{(m_2)}$  are “large,” i.e.,  $O(1)$ , then the performance limitations of the  $m$ th-order LP method are dominated by  $\delta_m$ . This can happen when  $m$  is chosen small enough so that the  $m$ th-order significant part of the true channel does not contain any “small” leading and/or trailing terms. If  $\delta_m$  is sufficiently large, with respect to  $\epsilon_m$ , then the  $m$ th-order LP method provides good channel approximations; otherwise, its performance may be poor. Since  $\delta_m$  is the distance, in the matrix 2-norm, of  $\mathcal{T}_{m-1}(\mathbf{h}_{m_1, m_2})$  from the matrices with rank one less than the assumed rank, it may be considered to be a measure of *diversity* of  $\mathbf{h}_{m_1, m_2}$ .

If, on the other hand,  $\mathbf{h}_{(m_1)}$  and/or  $\mathbf{h}_{(m_2)}$  are “small,” i.e.,  $O(\epsilon_m)$ , then  $\delta_m = O(\epsilon_m)$ , that is,  $\delta_m$  becomes of the order of the perturbation, making the approximation problem *ill-conditioned* and leading to potentially poor performance of the  $m$ th-order LP method. This happens when  $m$  is greater than the *effective channel order* [8] and means that we try to model not only the significant part of the true channel but also some “small” leading and/or trailing impulse response terms. This case is termed *effective overmodeling*.

The sensitivity of the LP method, with respect to effective overmodeling, runs counter to recent claims of robustness of the LP method, with respect to overmodeling [3], [4]. The explanation is simple. In these works, overmodeling has been defined with respect to identically zero impulse response terms. As a result, the multichannel linear predictor is a solution to a rank-deficient system of linear equations. In [3], a solution has been computed by using pseudoinversion, whereas, in [4], by using order-recursions. However, a more natural definition of (effective) overmodeling is with respect to “small” impulse response terms [5], [8]. In this case, the linear predictor is the solution of a close to rank-deficient linear system [recall that in this case,  $\delta_m = O(\epsilon_m)$ ]. In order to avoid large enhancement of inaccuracies, which are due to

$$\begin{aligned} \tilde{\mathbf{R}}_m &= \begin{bmatrix} \tilde{\mathbf{r}}_0 & \tilde{\mathbf{r}}_m \\ \tilde{\mathbf{r}}_m^T & \tilde{\mathbf{R}}_{m-1} \end{bmatrix} = \mathcal{T}_m(\mathbf{h}_M) \mathcal{T}_m^T(\mathbf{h}_M) = \mathcal{T}_m(\mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z) \\ &= \mathbf{R}_m + \underbrace{\left\{ \mathcal{T}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) + \mathcal{T}_m(\mathbf{d}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z) + O(\epsilon_m^2) \right\}}_{\Delta \mathbf{R}_m}. \end{aligned} \quad (6)$$

the influence of the tails, both the pseudoinversion and the order-recursions should be implemented with great care. Before the pseudoinversion, we should compute the effective rank of the data covariance matrix, whereas the order recursions should terminate at the right point by using an appropriate stopping criterion. These regularizing actions demand, explicitly or implicitly, the detection of the effective rank of the data covariance matrix, which is synonymous with effective channel order detection [8]. If we can estimate accurately the effective channel order, then we can implement these regularization techniques successfully. However, in this case, overmodeling seems superfluous. Consequently, the LP method, contrary to current beliefs, is *not* inherently robust with respect to effective channel overmodeling.

From Result 1, it becomes clear that if  $m$  is chosen favorably, the model quality depends on the size of the first “significant” term  $\mathbf{h}_{(m_1)}$ . Thus, in our study, term  $\mathbf{h}_{m_1}$  plays the role that the first nonzero term  $\mathbf{h}_{(0)}$  played in previous studies [3], [4], which assumed exact knowledge of the channel length and no tails.

The important problem of effective channel order detection is studied in [8].

One may ask how can we estimate the start and end points of the  $m$ th-order significant part  $m_1$  and  $m_2$ . The answer is that their values are insignificant since during the equalization step, the fact that  $m_1 \geq 0$  changes the solution by adding a delay of  $m_1$  time units [5].

#### IV. SIMULATIONS

In the previous section, we derived bound (10), which provides significant insight into the performance of the LP method in realistic cases. This bound is derived by repeated application of the triangle and submultiplicative inequalities (see the Appendix). Thus, it is, in general, loose. However, it is given by a reasonably simple expression, identifying the cases in which the LP method performs well or may perform poorly. For example, it reveals the instability related to effective overmodeling.

In our simulation, we consider the mean asymptotic performance of the second-order LP method by varying the size of the tails. The significant part of the channel is

$$\mathbf{h}_{2,4} = [-0.6804 \ 0.4281; \ 0.1777 \ -0.2446; \ -0.0902 \ -0.5043].$$

We construct  $\mathbf{h}_{10} = \mathbf{h}_{2,4}^z + \mathbf{d}_{2,4}^z$  by using random “tails”  $\mathbf{d}_{2,4}^z$  with

$$20 \leq 20 \log_{10} (\|\mathbf{h}_{2,4}^z\|_2 / \|\mathbf{d}_{2,4}^z\|_2) \leq 60.$$

We scale  $\mathbf{h}_{10}$  so that  $\|\mathbf{h}_{10}\|_2 = 1$ . Then, we apply the second-order LP method, and we compute  $\hat{\mathbf{h}}_{2,4}$ . In Fig. 1, we plot the first-order bound (10) (thick line) and the actual error. We observe that although the bound is loose, it provides an indication of the quality of the estimator.

#### V. CONCLUSION

We considered the performance of the  $m$ th-order LP method for blind channel identification, when the true channel is of order  $M$ , with  $m < M$ . We showed that the  $m$ th-order LP method furnishes an approximation to the  $m$ th-order significant part of the true channel. The closeness depends on the diversity of the  $m$ th-order significant part and the size of the unmodeled tails. Furthermore, we showed that, contrary to current beliefs, the LP method is not inherently robust with respect to effective channel overmodeling.

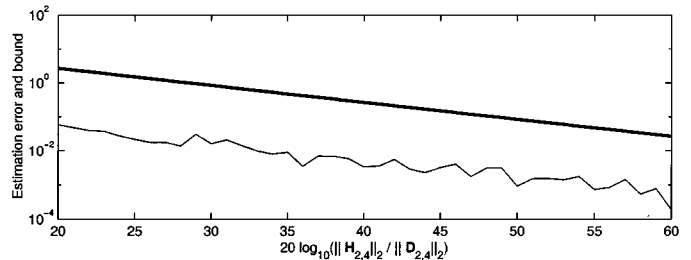


Fig. 1. Estimation error bound (10) (thick line) and measured error.

#### APPENDIX

Before proceeding to the proof of Result 1, we state the following Lemma.

*Lemma:* If  $\mathbf{h}_{m_1, m_2}$  is the truncated  $m$ th-order significant part of the true channel, then the  $m$ th-order LP method furnishes  $\hat{\mathbf{h}}_{m_1, m_2}$ , for which there holds, to a first-order approximation, with respect to the size of the unmodeled tails  $\epsilon_m$

$$\begin{aligned} \hat{\mathbf{h}}_{m_1, m_2} - \mathbf{h}_{m_1, m_2} &= \Delta \mathbf{S}_m \mathbf{g}_m^T - \frac{1}{2} \mathbf{g}_m \Delta \mathbf{R}_m \mathbf{g}_m^T \mathbf{h}_{m_1, m_2} \\ &\quad - \mathbf{S}_m \begin{bmatrix} \mathbf{0} \\ \mathbf{R}_{m-1}^{-1} [\Delta \mathbf{r}_m^T \ \Delta \mathbf{R}_{m-1}] \mathbf{g}_m^T \end{bmatrix}. \end{aligned} \quad (12)$$

The proof can be derived by following steps analogous to those of ([3, App. A]). The error in quantity  $X$  is  $\Delta X \triangleq \hat{X} - X$ , with  $\hat{X}$  defined in (6)–(9) and  $X$  defined in (1)–(4). Furthermore, instead of pseudoinversion, we use inversion.

*Proof of Result 1:* In order to prove Result 1, we will bound the 2-norm of each term of the right-hand side of (12). From (4) and (9), we deduce

$$\begin{aligned} \Delta \mathbf{S}_m &\triangleq \hat{\mathbf{S}}_m - \mathbf{S}_m = \mathcal{H}_m(\mathbf{h}_M) \mathcal{T}_m^T(\mathbf{h}_M) \\ &\quad - \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}) \\ &= \mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z + \mathbf{d}_{m_1, m_2}^z) \\ &\quad - \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}) \\ &= \mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \\ &\quad + \mathcal{H}_m(\mathbf{d}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z) + O(\epsilon_m^2) \end{aligned}$$

and thus

$$\begin{aligned} \Delta \mathbf{S}_m \mathbf{g}_m^T &= \mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \mathbf{g}_m^T \\ &\quad + \mathcal{H}_m(\mathbf{d}_{m_1, m_2}^z) \mathbf{e}_{m_1} + O(\epsilon_m^2) \end{aligned}$$

where  $\mathbf{e}_i$  is the canonical vector with 1 at the  $(i+1)$ st position and zeros elsewhere. It is easy to see that the second term vanishes. Using the matrix 2-norm/F-norm inequality, the structure of  $\mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z)$  and  $\mathcal{T}_m(\mathbf{d}_{m_1, m_2}^z)$ , and (5), we obtain the first-order bound

$$\begin{aligned} \left\| \Delta \mathbf{S}_m \mathbf{g}_m^T \right\|_2 &\leq \left\| \mathcal{H}_m(\mathbf{h}_{m_1, m_2}^z) \right\|_F \left\| \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \right\|_F \|\mathbf{g}_m\|_2 \\ &\leq \epsilon_m (m+1) \|\mathbf{g}_m\|_2. \end{aligned} \quad (13)$$

For the second term, we have, from (6)

$$\begin{aligned} \mathbf{g}_m \Delta \mathbf{R}_m \mathbf{g}_m^T &= \mathbf{g}_m \left\{ \mathcal{T}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \right. \\ &\quad \left. + \mathcal{T}_m(\mathbf{d}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z) + O(\epsilon_m^2) \right\} \mathbf{g}_m^T \\ &= 2 \mathbf{e}_{m_1}^T \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \mathbf{g}_m^T + O(\epsilon_m^2) \\ &= 2 \mathbf{g}_m \begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{m_1-1} \\ \vdots \\ \mathbf{h}_{m_1-m} \end{bmatrix} + O(\epsilon_m^2) \end{aligned}$$

with  $\mathbf{h}_k = \mathbf{0}$ , for  $k < 0$ . Thus, to a first-order with respect to  $\epsilon_m$

$$\left\| \frac{1}{2} \mathbf{g}_m \Delta \mathbf{R}_m \mathbf{g}_m^T \mathbf{h}_{m_1, m_2} \right\|_2 \leq \left\| \frac{1}{2} \mathbf{g}_m \Delta \mathbf{R}_m \mathbf{g}_m^T \right\|_2 \leq \epsilon_m \|\mathbf{g}_m\|_2. \quad (14)$$

The third term of (12), which is denoted by  $T_3$ , can be written as

$$\begin{aligned} T_3 &= \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}) \\ &\times \begin{bmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathcal{T}_{m-1}^{-T}(\mathbf{h}_{m_1, m_2}) \mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1, m_2}) \end{bmatrix} \\ &\times \left\{ \mathcal{T}_m(\mathbf{h}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \right. \\ &\left. + \mathcal{T}_m(\mathbf{d}_{m_1, m_2}^z) \mathcal{T}_m^T(\mathbf{h}_{m_1, m_2}^z) + O(\epsilon_m^2) \right\} \mathbf{g}_m^T \\ &= \mathcal{H}_m(\mathbf{h}_{m_1, m_2}) \left\{ \begin{bmatrix} \mathbf{0}^T & \mathbf{0}^T & \mathbf{0}^T \\ \mathbf{O} & \mathbf{I}_{2m} & \mathbf{O} \end{bmatrix} \mathcal{T}_m^T(\mathbf{d}_{m_1, m_2}^z) \mathbf{g}_m^T \right. \\ &\left. + \begin{bmatrix} \mathbf{0} \\ \mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1, m_2}) \begin{bmatrix} \mathbf{h}_{m_1-1} \\ \vdots \\ \mathbf{h}_{m_1-m} \end{bmatrix} \end{bmatrix} \right\}. \end{aligned}$$

Finally

$$\begin{aligned} \|T_3\|_2 &\leq \|\mathcal{H}_m(\mathbf{h}_{m_1, m_2})\|_F \left\{ \|\mathcal{T}_m(\mathbf{d}_{m_1, m_2}^z)\|_F \|\mathbf{g}_m\|_2 \right. \\ &\left. + \|\mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1, m_2})\|_2 \left\| \begin{bmatrix} \mathbf{h}_{m_1-1} \\ \vdots \\ \mathbf{h}_{m_1-m} \end{bmatrix} \right\|_2 \right\} \\ &\leq (m+1)\epsilon_m \|\mathbf{g}_m\|_2 + \frac{\sqrt{m+1}\epsilon_m}{\delta_m}. \quad (15) \end{aligned}$$

In order to bound the term  $\|\mathbf{g}_m\|_2$  appearing in (13)–(15), we proceed as follows. Since

$$\begin{aligned} \mathcal{A}_m &\left\{ \mathcal{T}_{m-1}(\mathbf{h}_{m_1, m_2}) \mathcal{T}_{m-1}^T(\mathbf{h}_{m_1, m_2}) \right\} \\ &= -[\mathbf{h}_{(m_1+1)} \quad \cdots \quad \mathbf{h}_{(m_2)} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}] \mathcal{T}_{m-1}^T(\mathbf{h}_{m_1, m_2}) \end{aligned}$$

and  $\mathcal{T}_{m-1}(\mathbf{h}_{m_1, m_2})$  is nonsingular, we obtain

$$\begin{aligned} \|\mathcal{A}_m\|_2 &\leq \left\| [\mathbf{h}_{(m_1+1)} \quad \cdots \quad \mathbf{h}_{(m_2)}] \right\|_F \|\mathcal{T}_{m-1}^{-1}(\mathbf{h}_{m_1, m_2})\|_2 \\ &\leq \frac{1}{\delta_m}. \end{aligned}$$

Using the expression for the equalizer  $\mathbf{g}_m = (1/\|\mathbf{h}_{(m_1)}\|_2) \mathbf{v}^T [\mathbf{I} \quad \mathcal{A}_m]$  and the fact that  $\|\mathbf{v}\|_2 = 1$ , we obtain

$$\begin{aligned} \|\mathbf{g}_m\|_2 &\leq \frac{1}{\|\mathbf{h}_{(m_1)}\|_2} \|\mathbf{I} \quad \mathcal{A}_m\|_2 = \frac{1}{\|\mathbf{h}_{(m_1)}\|_2} \sqrt{1 + \|\mathcal{A}_m\|_2^2} \\ &\leq \frac{1}{\|\mathbf{h}_{(m_1)}\|_2} \sqrt{1 + \frac{1}{\delta_m^2}}. \quad (16) \end{aligned}$$

Putting (16) in (13)–(15) and adding the derived bounds, we prove Result 1.

## REFERENCES

[1] G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, pp. 2982–2993, Dec. 1995.

[2] E. Moulines, P. Duhamel, J. F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, pp. 516–525, Feb. 1995.

[3] K. Abed-Meraim, E. Moulines, and P. Loubaton, "Prediction error method for second-order blind identification," *IEEE Trans. Signal Processing*, vol. 45, pp. 694–705, Mar. 1997.

[4] C. B. Papadias and D. T. M. Slock, "Fractionally spaced equalization of linear polyphase channels and related blind techniques based on multichannel linear prediction," *IEEE Trans. Signal Processing*, vol. 47, pp. 641–653, Mar. 1999.

[5] A. P. Liavas, P. A. Regalia, and J.-P. Delmas, "Robustness of least-squares and subspace methods for blind channel identification/equalization with respect to effective channel undermodeling/overmodeling," *IEEE Trans. Signal Processing*, vol. 47, pp. 1636–1645, June 1999.

[6] J. R. Treichler, I. Fijalkow, and C. R. Johnson Jr., "Fractionally spaced equalizers: How long should they really be?," *IEEE Signal Processing Mag.*, pp. 65–81, May 1996.

[7] Z. Ding, "Matrix outer-product decomposition method for blind multiple channel identification," *IEEE Trans. Signal Processing*, vol. 45, pp. 3053–3061, Dec. 1997.

[8] A. P. Liavas, P. A. Regalia, and J.-P. Delmas, "Blind channel approximation: Effective channel order determination," *IEEE Trans. Signal Processing*, vol. 47, pp. 3336–3344, Dec. 1999.

## Interference Blanking Probabilities for SLB in Correlated Gaussian Clutter Plus Noise

Alfonso Farina and Fulvio Gini

**Abstract**—This work presents closed-form expressions of the probability  $P_{FA}$  of false alarm, the probability  $P_{TB}$  of blanking a target received in the main lobe, and the probability  $P_B$  of blanking a coherent repeater interference (CRI) via an SLB device operating in correlated Gaussian clutter with known Doppler spectrum plus white Gaussian thermal noise.

**Index Terms**—Coherent interference rejection, radar clutter, sidelobe blanking.

## I. INTRODUCTION AND WORKING PRINCIPLE

To counter impulsive-type sidelobe interference, a radar receiver usually employs a sidelobe blanking (SLB) system, which blanks the radar receiver output in those range cells where the unwanted signal appears in the radar antenna sidelobes. The purpose is reached via a low gain auxiliary antenna that is located close to the main antenna, which receives the same interfering signals of the main antenna. By comparing the signals captured by the radar and auxiliary antennas, we may ascertain whether the impulsive signal is received through the radar antenna side lobes. In such a situation, the radar signal is blanked in the range cell affected by the interference. The processing scheme connecting the radar and the SLB device is outlined in Fig. 1 [1], [2]. The SLB decides whether or not to blank the main channel

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