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**Book Reviews: "Uncertainty Theory. Second Edition.
By Baoding Liu"**

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► **To cite this version:**

Didier Dubois. Book Reviews: "Uncertainty Theory. Second Edition. By Baoding Liu". SIAM Review, 2009, 51 (2), pp.436-441. hal-03434433

HAL Id: hal-03434433

<https://hal.science/hal-03434433>

Submitted on 19 Nov 2021

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Uncertainty Theory. Second Edition. By Baoding Liu. Springer, Berlin, 2007. \$129.00. x+255 pp., hardcover. ISBN 978-3-540-73164-1.

Situation. Uncertainty is a topic that has received tremendous interest over the last twenty years, due to the questioning of the dominant position of standard probability approaches in fields like decision theory and risk analysis. The advent of computer science has also prompted research in nonprobabilistic uncertainty for knowledge representation and automated reasoning, even if the technology of Bayesian nets is the most popular to date. Various theories have been proposed that account for new facets of uncertainty in cognition, like fuzzy sets, rough sets, the theory of evidence, possibility theory, and imprecise or indeterminate probability theory, among other variants. This book is an addition to this literature, and is a compendium of research results obtained by the author over the last 10 years or so. It is also the second augmented edition of a book whose first edition was published in 2004. In this review, we consider the point of view of knowledge representation and reasoning about uncertainty. Another review from a purely mathematical standpoint by a specialist in nonadditive measures would provide a complementary point of view.

Outline. The contents section of the book is very clear and focuses on mathematical developments of basic notions. It is faithful to the measure-theoretic approach to probability and proposes its counterpart for new set functions. After a short preface placing the inspiration for the book in the works of Kolmogorov and Zadeh [13, 14], the book starts with a technical refresher on measures and integrals. The rest of the book consists of four chapters, each with almost the same structure and the same subsection titles.

These chapters are devoted, respectively, to probability theory and, in the author's words, "credibility theory," "chance theory," and "uncertainty theory." The chapter on probability theory contains 18 subsections and contains a technical textbook-style development in the form of definitions, theorems, and proofs with very little discussion. Standard topics are covered, like random variables, distribution, independence, expected value, variance, moments, entropy, inequalities, convergence, characteristic function, basic limit theorems, Monte Carlo simulation, and conditional probability. It is like a short mathematical vademecum on probability theory in 60 pages.

Chapter 3 basically contains the same subsection titles (but for limit theorems), and here a so-called credibility measure is used instead of a probability measure. A credibility measure is a set function that differs from a probability by the so-called maximality axiom, i.e., the credibility of an arbitrary disjunction is the supremum of the credibilities of the disjuncts *if this supremum is strictly less than 0.5*. In particular it possesses the self-duality property: namely, denoting Cr the credibility measure, $Cr(A^c) = 1 - Cr(A)$, where A^c is the complement of set A . The author claims (and in some sense proves) that a credibility measure is to a fuzzy variable what a probability measure is to a random variable. This chapter has the same style as the previous one: definitions, theorems, proofs, and almost no discussions. A very short motivation is given at the beginning of the chapter: the natural set function induced by fuzzy sets is a possibility function, as proposed by Zadeh [14], but according to the author (p. 81),

it has no self-duality property. However, a self-dual measure is absolutely needed in both theory and practice.

We shall comment on this brief assertion in our later critique.

Chapter 4 considers a “chance theory” that puts together probability measures and credibility measures. The author declares that (p. 157)

chance theory is a hybrid of probability theory and credibility theory.

It is, according to Liu, needed when

fuzziness and randomness simultaneously appear in a system.

So, the author views a chance space as the Cartesian product of a credibility space and a probability space and an event as a relation between both spaces. A chance measure of an event is then computed by combining probability and credibility by means of a max-min combination in the style of fuzzy logic. The author develops counterparts of the 17 sections of the previous chapters in this rather baroque setting.

Finally Chapter 5 is titled “Uncertainty Theory.” It considers so-called uncertain measures, by which the author denotes a generalization of his credibility measures, replacing the maxitivity axiom by a weaker condition satisfied by the latter, namely, the assumption that the uncertain measure of a countable disjunction is not greater than the countable sum of the uncertain measures of the disjuncts—the author calls this “countable subadditivity.” The author defines uncertainty theory as (p. 205)

the branch of mathematics based on normality, monotonicity, self-duality and countable subadditivity.

However, it seems that up to now, this branch of mathematics has been studied only by the author and his group. This chapter again contains material similar to that of the previous chapters, in the same order, this time for uncertain measures. The last chapter contains only a few comments wrapping up the basic notions introduced, along with a figure relating random, fuzzy, hybrid, and uncertain variables to probability, credibility, chance, and uncertain measures, respectively, in the author’s terminology.

Critique. The author has published many papers in fuzzy logic journals, developing the framework of credibility and uncertain measures and their applications (to optimization under uncertainty, essentially), and he thus benefits from some recognition in the areas chartered by some of these journals. This book is thus a welcome detailed exposition of his viewpoint. It also provides the opportunity to look closer at the basic contents of the author’s research, and it is now possible to provide a critique. Indeed, in this reviewer’s opinion, there are many aspects of this book that are questionable.

First, the author chooses to provide a purely technical account of his favorite set functions, allegedly representing non-probabilistic uncertainty. Beyond short introductory paragraphs to chapters (of which the above quotations are excerpts) there is no epistemological analysis of the significance of the proposed tools. The explanations are very brief and several seemingly unsustainable claims are made as to the appropriateness of credibility measures or other set functions for modeling uncertainty. For instance:

- Why is a self-dual measure absolutely needed in both theory and practice (p. 81)? The key point of most important current theories of uncertainty (imprecise probability [12], evidence [10], and possibility theories [4]) is precisely to do away with the self-duality property of probability, so as to open the door to a faithful account of incomplete knowledge. Of course, one might argue that the self-duality property is advocated by a stream of publications starting with Richard Cox’s reasonable expectation [1] and culminating with Edwin Jaynes’s [7] defense of Bayesian probability. Strangely enough, only a paper on statistical mechanics by Jaynes appears in the reference list of the book, seemingly without comment. But the author of *Uncertainty Theory* does not seem to appeal to this tradition at all.
- Why should chance theory be “a hybrid of probability theory and credibility theory”? The word “chance” in the literature (and the history of probability) refers to objective probability, and

is always opposed to an epistemic notion of probability. So the terminology of the author goes against all kinds of tradition in the area. Moreover, the mathematical setting proposed, assuming that objective probability and credibility belong to the same scale (the minimum of a probability degree and a credibility degree is supposed to make sense), is stated out of the blue.

- The term “uncertain measure” appears to be too wide to be restricted to self-dual set functions, as most modern approaches to uncertainty give up the self-duality assumption. In fact, the author essentially cites fuzzy set theory as his main source of inspiration, where a reasoned discussion of the positioning of credibility theory and its extensions with respect to subjective probability, decision theory, and the aforementioned uncertainty theories would be needed. There is no reference to Bruno De Finetti, Leonard Savage, G.L.S. Shackle, Henry Kyburg, Isaac Levi, or Peter Walley, and only Glenn Shafer appears in the reference list. So it is not yet clear what the formal tools presented with a wealth of technical details in the book have to offer to the uncertain reasoner.

It is interesting to take a closer look at credibility measures and figure out where they come from. Indeed, the author does not provide much explanation about his sources of inspiration even when they appear in the reference list. On page 81, the author mentions the step made by Zadeh in 1978 [14] interpreting fuzzy set membership functions in terms of possibility, thus paving the way to possibility theory. It is now well known that possibility theory is closely related to the basic concepts of the Shackle theory of surprise [9], but the author does not refer to Shackle. However, we read on the same page that

[a]lthough possibility measure has been widely used, it has no self-duality property.

So it is clear that self-duality, for a reason never clearly explained, is considered to be of primary importance. Some hints given on

page 206 are not illuminating: a set function taking value 1 for each set (except the empty one, I suppose) is declared pathological. Why is this so? This is precisely a possibility measure.

Even taking self-duality for granted, the axiomatic presentation of credibility measures may baffle the probabilistically sophisticated reader, as it is totally unmotivated. This is the case for the maximality axiom on page 82:

AXIOM 4 (Maximality):
 $Cr(\cup_i A_i) = \sup_i Cr(A_i)$, for any events $\{A_i\}$ with $\sup_i Cr(A_i) < 0.5$.

However, it is clear that what the author calls “credibility measure” is the arithmetic average of a possibility measure $\Pi(A)$ and a necessity measure $N(A)$:

$$Cr(A) = \frac{\Pi(A) + N(A)}{2},$$

where $\Pi(A) = \sup_{x \in A} \pi(x)$, $N(A) = 1 - \Pi(A^c)$, π being a fuzzy set membership function standing for a possibility distribution. This kind of average ensures self-duality. This relation to basic notions in possibility theory is *never* explicitly laid out in the book, even if it becomes clear when checking the mathematical derivations (for instance, eq. 3.27, p. 93). Moreover, this kind of arithmetic average is not new, and was suggested as an aside by Dubois and Prade [4, p. 125] more than 20 years ago; it can be traced back to the certainty factor of the MYCIN expert system [11] described in the early seventies (the certainty factor is a rescaled version of the credibility measure, namely, $2Cr(A) - 1$; see the discussion in the appendix of [5]). As a consequence, all the information contained in a credibility measure is contained in the associated possibility distribution (given as $\pi(x) = \min(1, 2Cr(\{x\}))$), as pointed out in the book).

Once the (expert) reader becomes aware of this link to set functions Π that satisfy $\Pi(A \cup B) = \max(\Pi(A), \Pi(B))$, then the motivation for the strange maximality axiom is very clear, as a mere consequence of the (more natural) maximality axiom of possibility measures. It is also very clear that a credibility measure is completely defined by a possibility distribution (eq. 3.26, p. 93), a

terminology widely used in possibility theory but ignored by the author, who prefers the neutral term “membership function,” despite Zadeh’s terminology in [14].

So, the author’s credibility theory is not that new, but is just another way of presenting numerical possibility theory without clearly acknowledging it.

Moreover, the author adopts, again without clear acknowledgment, the framework of fuzzy variables proposed by Nahmias [8] in 1978 (even if his name appears in the reference list). The idea is to define a possibility space as a triple made of a space, a set of subsets, and a possibility measure, in place of a probability space. A fuzzy variable ξ is then a mapping from a possibility space to the reals. Liu just replaces a possibility measure with its symmetrized credibility measure. However, Nahmias’s framework was never considered to be a serious foundation for possibility theory in the subsequent literature. A probability space is the clear abstraction of a random phenomenon (with a sample space and a random data collection process). However, it is not clear of what a possibility space (hence a credibility space) is the abstraction. The author seems to refer to a “fuzziness” phenomenon, which would parallel random phenomena but differs from them. This reviewer has no idea what a fuzziness phenomenon is within this metaphor. Fuzziness is pervading subjective information, and has little to do with any repetitive phenomenon present in the outside world. Interestingly, subjective probability theory (for instance, see De Finetti [2]) never starts from the definition of probability spaces.

Another step is taken when the author introduces the “cumulative” credibility distribution of a fuzzy variable, of the form $\Phi(x) = Cr(\xi \leq x)$. This is the arithmetic average of upper and lower “possibilistic distribution” functions $\Pi(\xi \leq x)$ and $N(\xi \leq x)$ [4]. Since this credibility distribution has all the properties of a standard distribution (up to discontinuity points), there is an underlying probability measure in many cases, the density of which may exist. The author calls this the “credibility density.” On this basis, he can define standard probabilistic notions, dubbing them credibility-based expectation, variance, moments, in-

equalities, and so on. Indeed, what happens here behind the credibility curtain is that everything is defined from the probability measure, whose cumulative distribution is derived from a credibility measure. So, for instance, the expected value of a fuzzy variable is defined by a Choquet integral (eq. 3.51, p. 108) that can be expressed by means of the credibility distribution. Theorem 3.34 makes it clear that this expected value is nothing more than the standard expected value of the probability measure acting in place of the credibility measure. In the discrete case of a membership function interpreted as generating a discrete consonant Shafer belief function with interval focal sets, the probability distribution underlying a credibility measure is one obtained by equally sharing the mass of each focal set between its two end points. However, the author does not explicitly say that the expected value with respect to a credibility measure is a Choquet integral, even though Choquet is credited with the invention of capacities in the introduction of Chapter 5, along with Sugeno. It seems that, from section 3.4 to the end of the chapter, many of the notions introduced (especially density and moments) are just a rehashing of standard probabilistic notions, couched in the language of credibility measures. It would make sense if the credibility measure could be seen to model a family of probabilities. However, it can be shown that the sets of probabilities dominating a credibility measure and dominated by it are empty. In fact, the probability function underlying the cumulative credibility distribution does belong to the set of probability functions encoded by the possibility measure $\Pi(A) = \min(1, 2Cr(A))$, from which Cr is constructed.

In the chapter on uncertain measures, the same approach is used. First, the countable subadditivity axiom is introduced as being natural (p. 205):

In many cases, the author thinks that self-duality plus countable subadditivity is more essential. . .

But why this is so is not at all explained. In fact, it turns out that it is a property of the credibility measure (see Thm. 3.3, p. 83) that the author seems to find good

enough to pose as a basic condition. This is where a comparison with imprecise probability measures would be of interest. Indeed, given a set of probability measures (representing incomplete information) an upper envelope P^* computed from it satisfies a subadditivity property for a finite union of *disjoint sets*, and is thus at first glance a weaker property than the one proposed by the author. In fact, a subadditive set function g is classically defined by the property $g(A \cup B) \leq g(A) + g(B)$ for $A \cap B = \emptyset$ (see Denneberg [3], another basic reference absent in the book). It is clear that the standard subadditivity notion is equivalent to the finite subadditivity in the sense of B . Liu, since when A and B are not disjoint, $g(A \cup B) \leq g(A) + g(B \setminus A)$ implies $g(A \cup B) \leq g(A) + g(B)$.

However, since the self-duality condition prevents uncertain measures from being interpreted as probability bounds, the author proposes as a general theory of uncertainty an approach that is at odds with imprecise probability theories, without mentioning them or providing a rationale for why self-duality conjoined with subadditivity sounds more reasonable. For instance, a belief function of Shafer [10] is not subadditive, but it makes perfect sense for representing uncertainty. A belief function is standard super-additive, i.e., $g(A \cup B) \geq g(A) + g(B)$ for $A \cap B = \emptyset$, a property that does not extend to nondisjoint sets and generally conflicts with self-duality. In this chapter, the same trick as in Chapter 3 is used for deriving a density from a cumulative uncertain distribution, thus enabling classical notions to be defined for the uncertain measure, substituting it with a standard measure. Again a comparison with standard results in non-additive measure and integrals [3] would be in order, as nonadditive integration theory is based on distribution functions.

Overall, are Liu's credibility and uncertain measures likely to be useful at all? Reading this book does not really help in imagining their applications, given the scarcity of discussions advocating the significance of the approach. Yet, a connection can be made between credibility theory and decision theory, via ideas proposed by Hurwicz criticizing the maximin criterion for

decision under uncertainty. It is clear that, choosing $1/2$ as the coefficient of optimism, the Hurwicz criterion (a weighted average of the maximin and the maximax criteria) coincides with the Liu expected value of the utility function with respect to the credibility measure representing ignorance (allocating a weight $1/2$ to any contingent event). This fact is never mentioned in this book, although it is quite obvious to anybody familiar with decision theory and possibility theory. First, it points out that many properties (other than self-duality) of credibility measures would remain, changing the arithmetic average into the weighted arithmetic average in the definition of credibility. It would provide a more convincing rationale for credibility measures, then seen as a substitute for upper and lower probabilities accounting for the decision-maker attitude in the face of uncertainty. (An interesting axiomatic proposal along this line is provided by Jaffray [6] using belief functions, whereby the mass of each focal element is shared between its best and its worst compatible outcomes.) In fact, the idea of moving away from an interval representation when making decisions is something widely acknowledged in decision theory. One can view Savage theory as a full-fledged justification of the selection of a probability measure when no information is available, so as to get precise estimates of the value of decisions. It accounts for the fact that, even in the face of ignorance, individuals do make choices. Hurwicz's method is another way of acquiring decisiveness in the face of ignorance, as does Liu's credibility measure-based expected value if substituted for upper and lower possibilistic expected values, thus providing a pragmatic argument in favor of the self-duality condition (as one way of escaping the indeterminacy of interval-valued estimates). However, such self-dual functions could more generally be chosen as the arithmetic average of coherent upper and lower probabilities in Walley's sense [12], and beyond self-duality, a weighted average thereof.

Finally, despite its dry presentation, its repetitive style, its blunt claims, and its weak motivation, this book can be thought-provoking, as long as the reader's scientific

background includes the many references on uncertainty that are not discussed by the author.

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