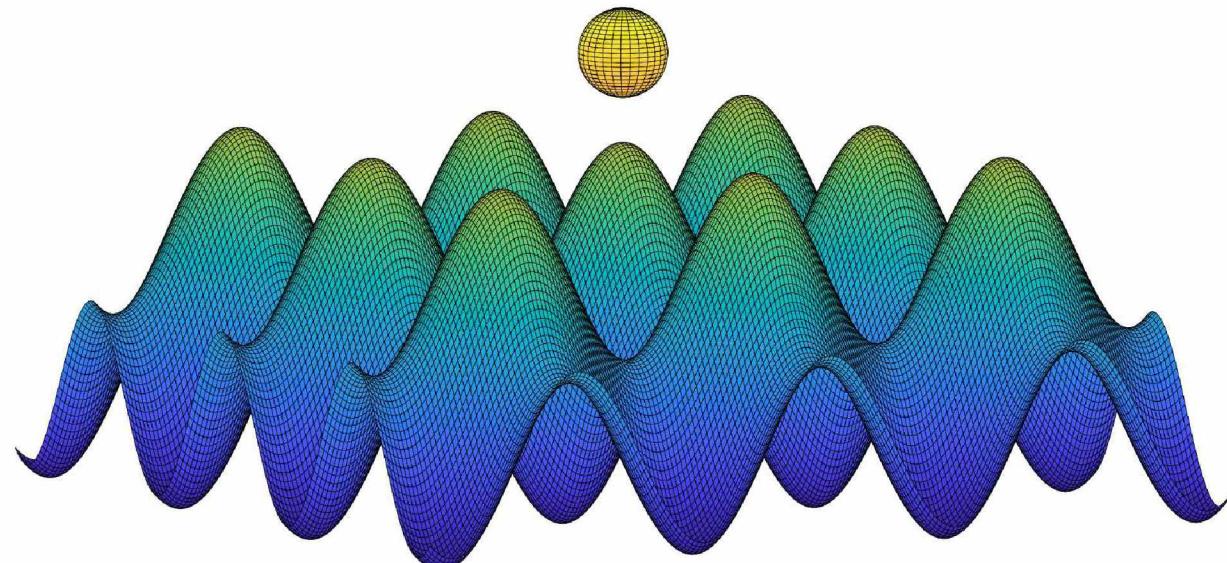


# What force causes particles to stick to surfaces?



Siddharth Rajupet<sup>1</sup>

Adriaan Riet<sup>1</sup>

Qizan Chen<sup>1</sup>

Mamadou Sow<sup>2</sup>

Daniel J. Lacks<sup>1</sup>

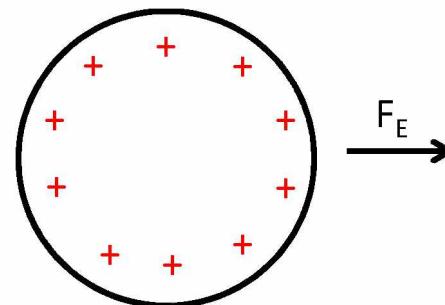
<sup>1</sup>Case Western Reserve University, Cleveland OH, USA

<sup>2</sup>Institut de Radioprotection et de Sécurité Nucléaire,  
Gif-sur-Yvette, France

# What force dominates in particle adhesion?

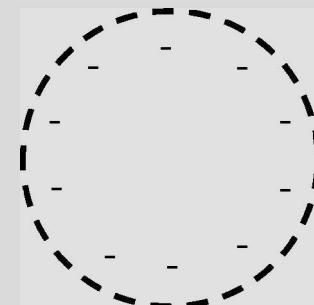
$$\text{van der Waals} \quad \text{Electrostatic}$$
$$F_{\text{adh}} = F_V + F_E$$

Charged particle



$F_E$

Image charge on  
conducting surface



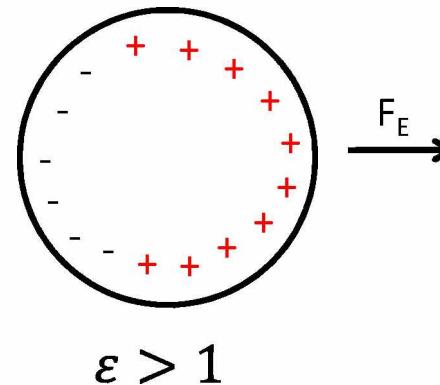
The grounded conductor induces charge to balance charge from the particles. The image charge causes an attractive electrostatic force

# What force dominates in particle adhesion?

van der Waals      Electrostatic

$$F_{\text{adh}} = F_V + F_E$$

Polarizable  
charged particle



Polarized image charge  
on conducting surface

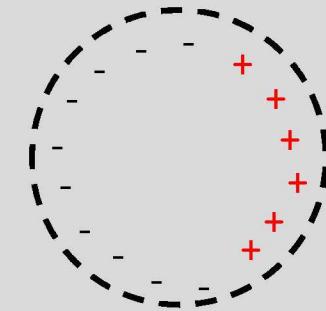


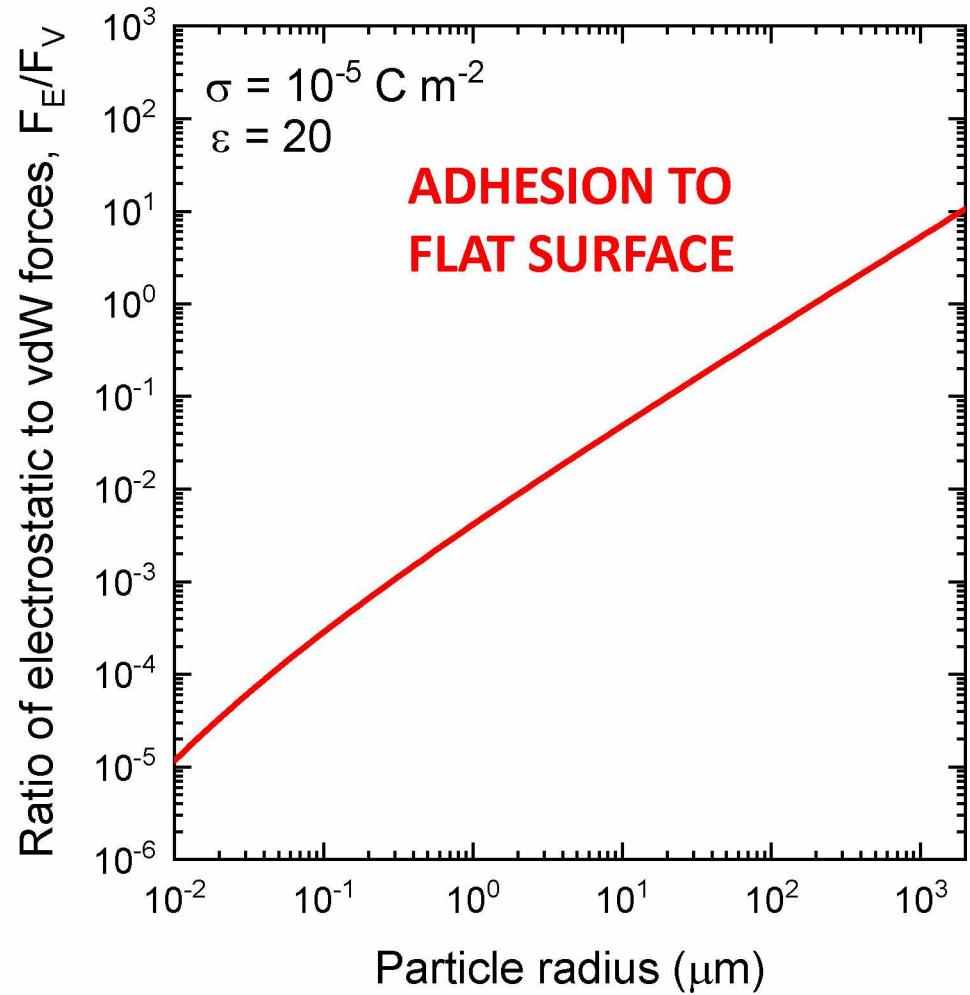
Image charge on conducting surface creates  
higher order polarizations increasing adhesion

# What force dominates in particle adhesion?

van der Waals      Electrostatic

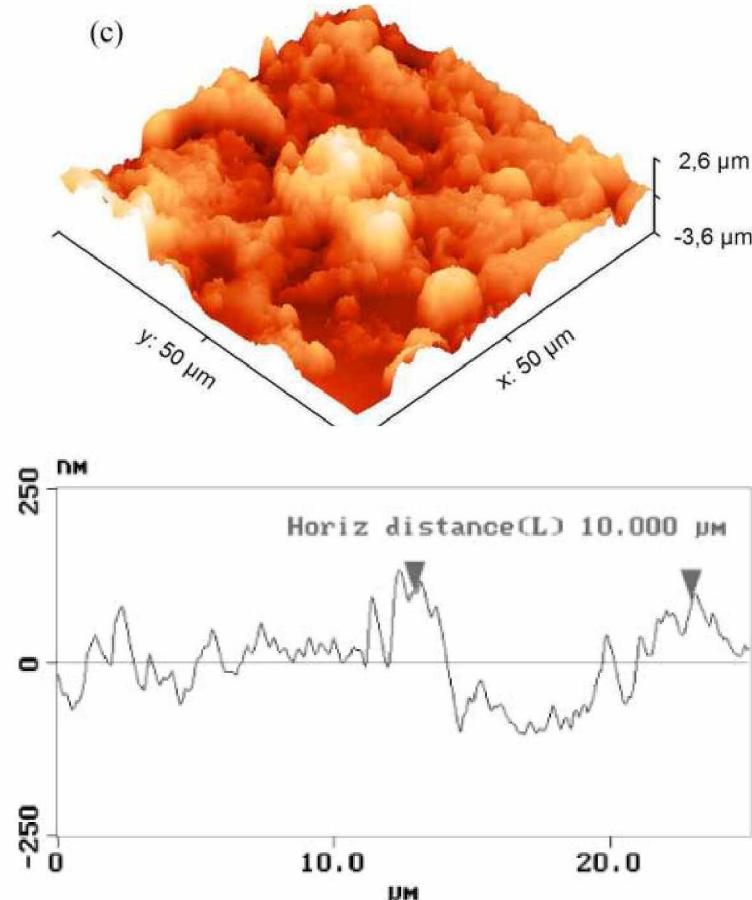
$$F_{\text{adh}} = F_V + F_E$$

Generally assumed van der Waals dominate



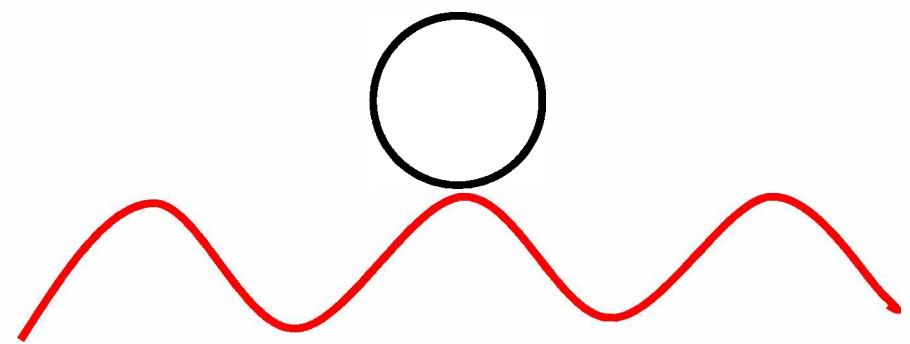
# Simplifying surface roughness

## Complex roughness in real systems



## Model system:

- Spherical particle positioned directly above an asperity of a sinusoidal surface



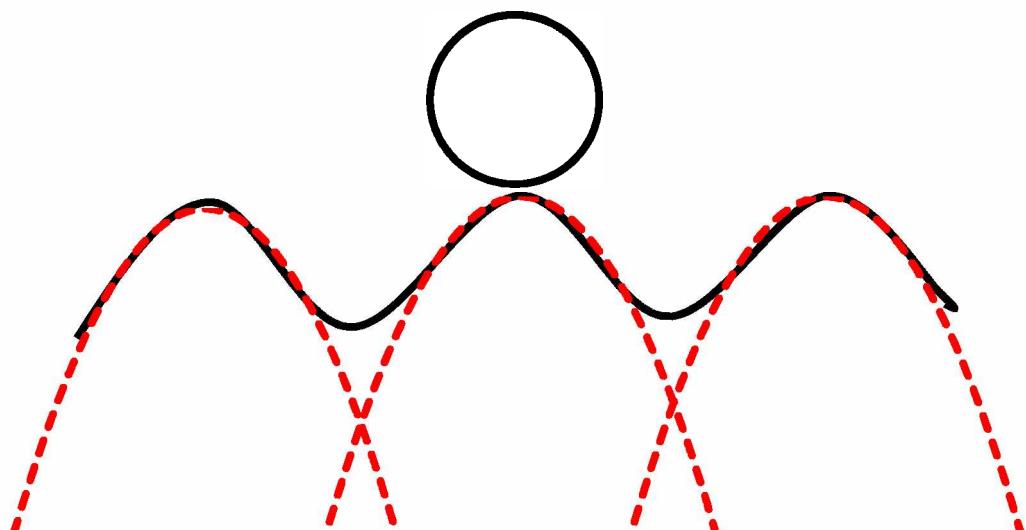
# van der Waals force calculations

Start with fundamental vdW relation:

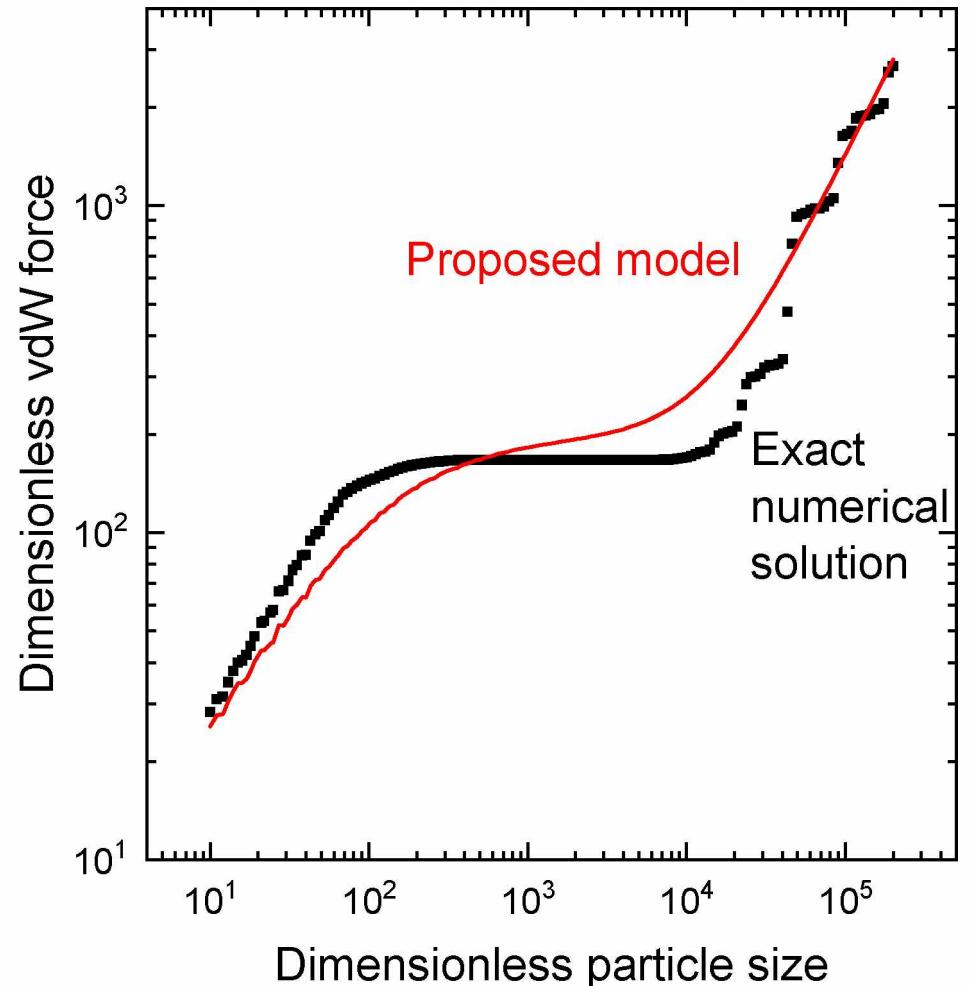
$$F_V = \sum_j \sum_k \frac{6Cz_{jk}}{r_{jk}^8}$$

↓ Use Taylor series to derive analytic expression

$$F_{V,model} = \frac{HR}{6d^2} \left( \frac{1}{1+R/R_a} + \frac{16\pi d_c^2(R+R_a+d)^2}{\lambda^2(1+R/R_a)(\lambda^2+8d_c(R+R_a+d))} \right)$$



Model agrees well with exact solution



Rajupet, Sow, Lacks, *Phys. Rev. E* 2020

# Electrostatic force calculations

## Model system:

- Spherical dielectric particle with uniform  $\sigma$  positioned directly above an asperity of a grounded sinusoidal surface

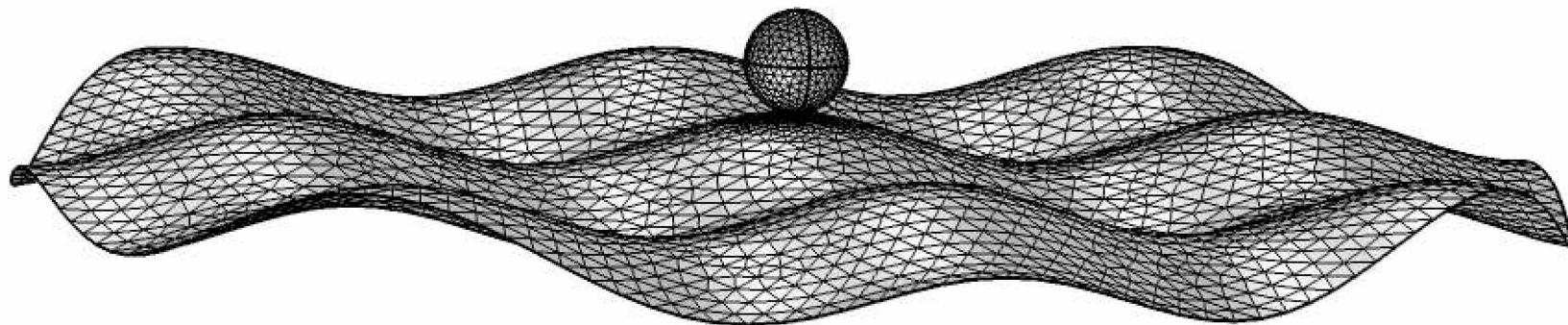
## Calculations:

- Find potential distribution by solving Poisson's equation using FEA:

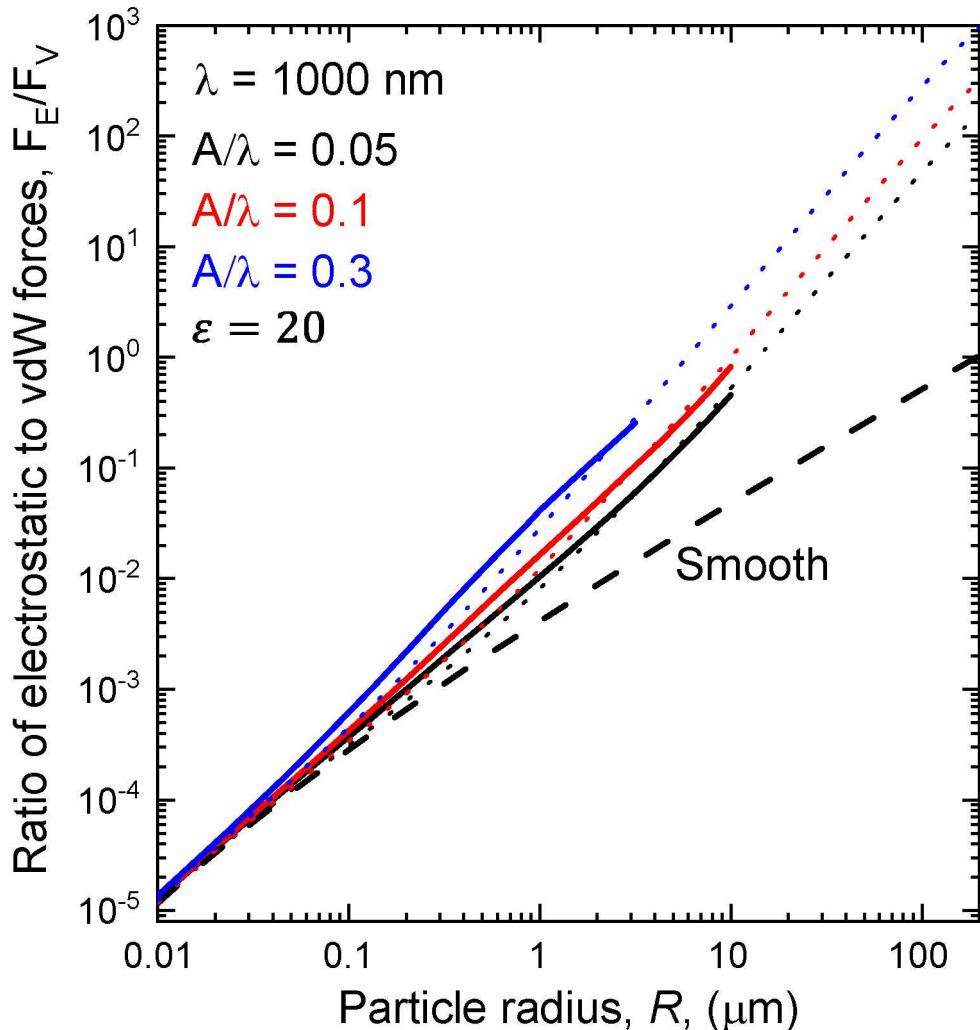
$$\nabla^2 \phi = 0$$

- Integrate Maxwell stress tensor over particle surface for the force:

$$F_E = \iint_S n_z \epsilon_0 (E_z^2 - \frac{1}{2} (E_x^2 + E_y^2 + E_z^2)) dS$$



# Roughness reduces $R$ where electrostatics dominant



Roughness lowers the particle size where electrostatic forces are comparable to vdW forces by up to two orders of magnitude

Electrostatic forces should be considered in micron-sized particle adhesion

# Acknowledgments



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DEPARTMENT OF  
EDUCATION

Supplemental

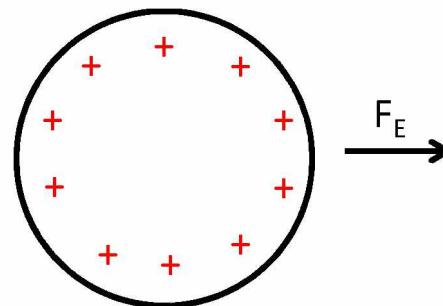
# What causes particle adhesion?

van der Waals

Electrostatic

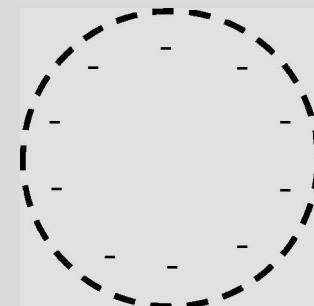
$$F_{\text{adh}} = F_V + F_E$$

Charged particle



$F_E$

Image charge on  
conducting surface



The grounded conductor induces charge to balance charge from the particles. The image charge causes an attractive electrostatic force

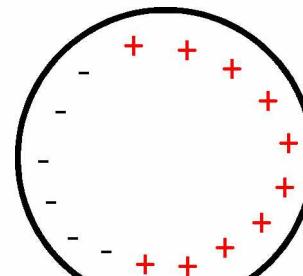
# What causes particle adhesion?

van der Waals

Electrostatic

$$F_{\text{adh}} = F_V + F_E$$

Polarizable  
charged particle



$$\epsilon > 1$$

$$F_E \rightarrow$$

Polarized image charge  
on conducting surface

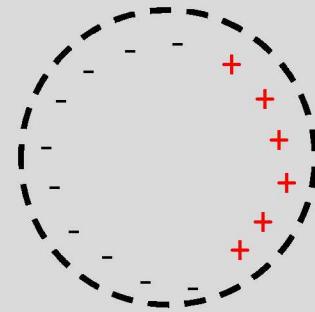


Image charge on conducting surface creates  
higher order polarizations increasing adhesion

# What causes particle adhesion?

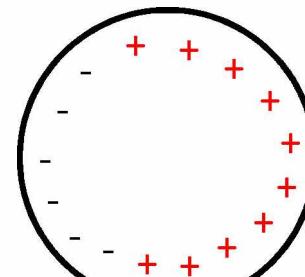
van der Waals

Electrostatic

$$F_{\text{adh}} = F_V + F_E$$

Generally assumed van der Waals dominate

Polarizable  
charged particle



$$\epsilon > 1$$

$$F_E \rightarrow$$

Polarized image charge  
on conducting surface

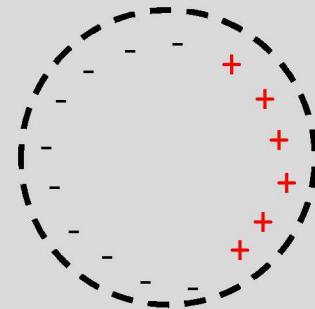


Image charge on conducting surface creates  
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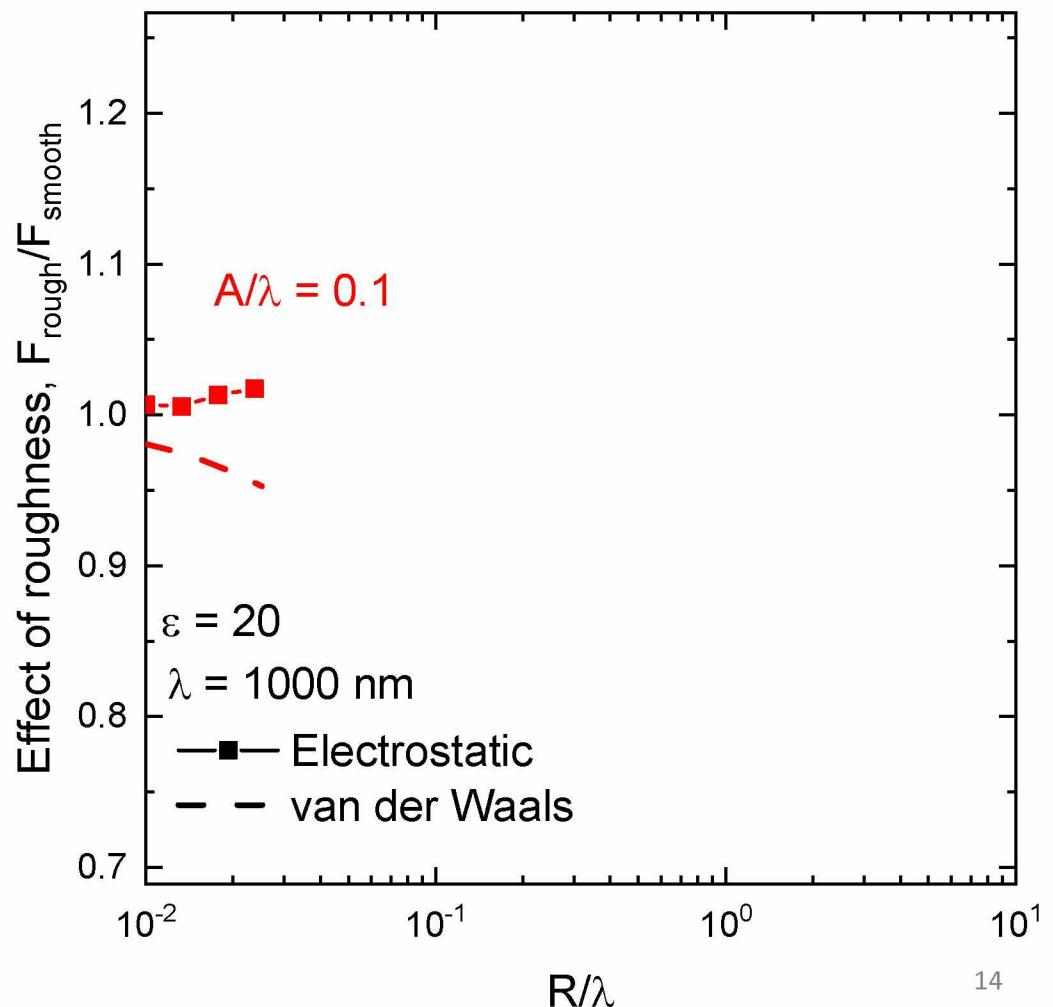
# $F_{rough}/F_{smooth}$ : effect of roughness on adhesion

$F_{rough}$  - Force due to rough surface

$F_{smooth}$  - Force due to smooth surface

If  $F_{rough}/F_{smooth} > 1$  roughness enhances electrostatic adhesion

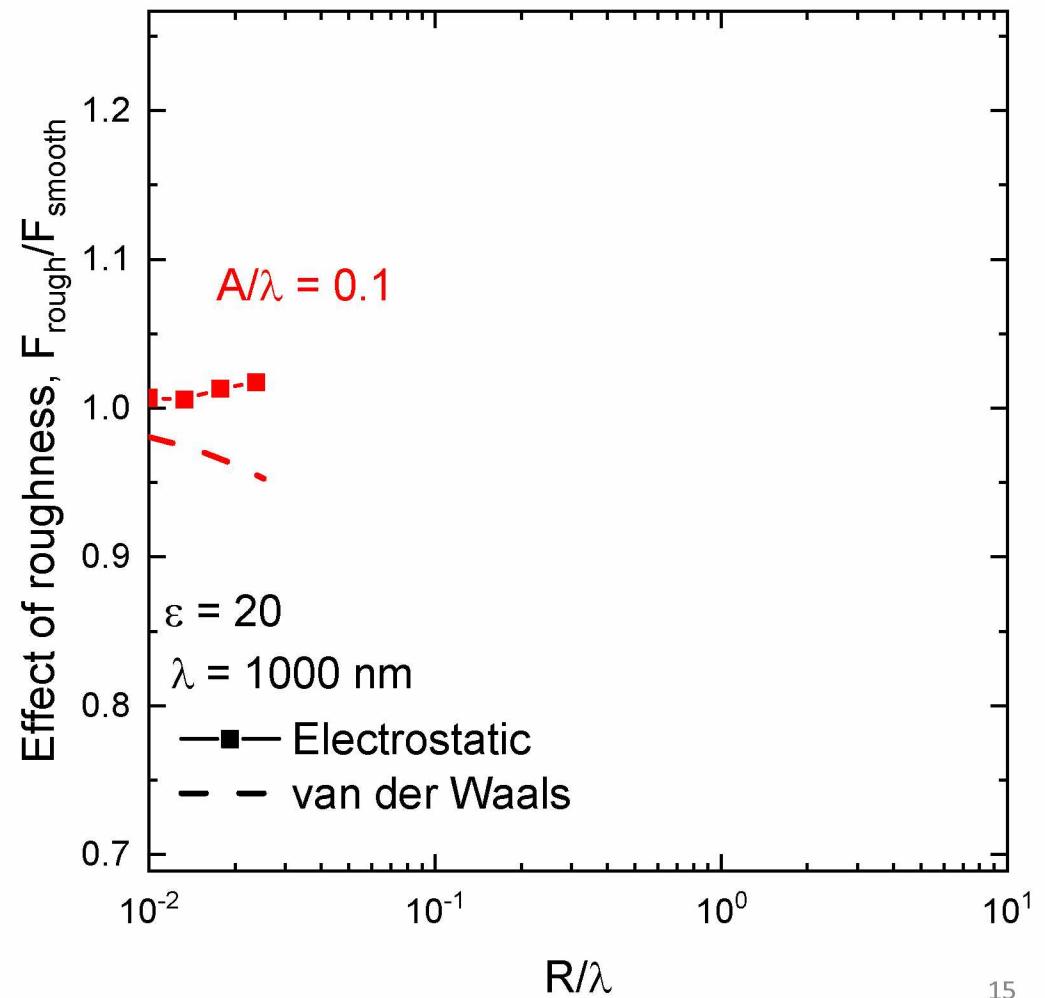
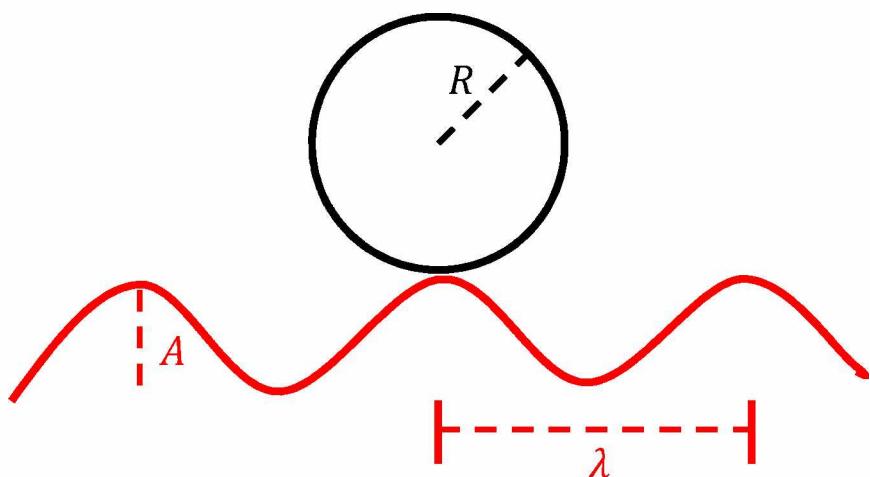
If  $F_{rough}/F_{smooth} < 1$  roughness diminishes electrostatic adhesion



# $F_{rough}/F_{smooth}$ : effect of roughness on adhesion

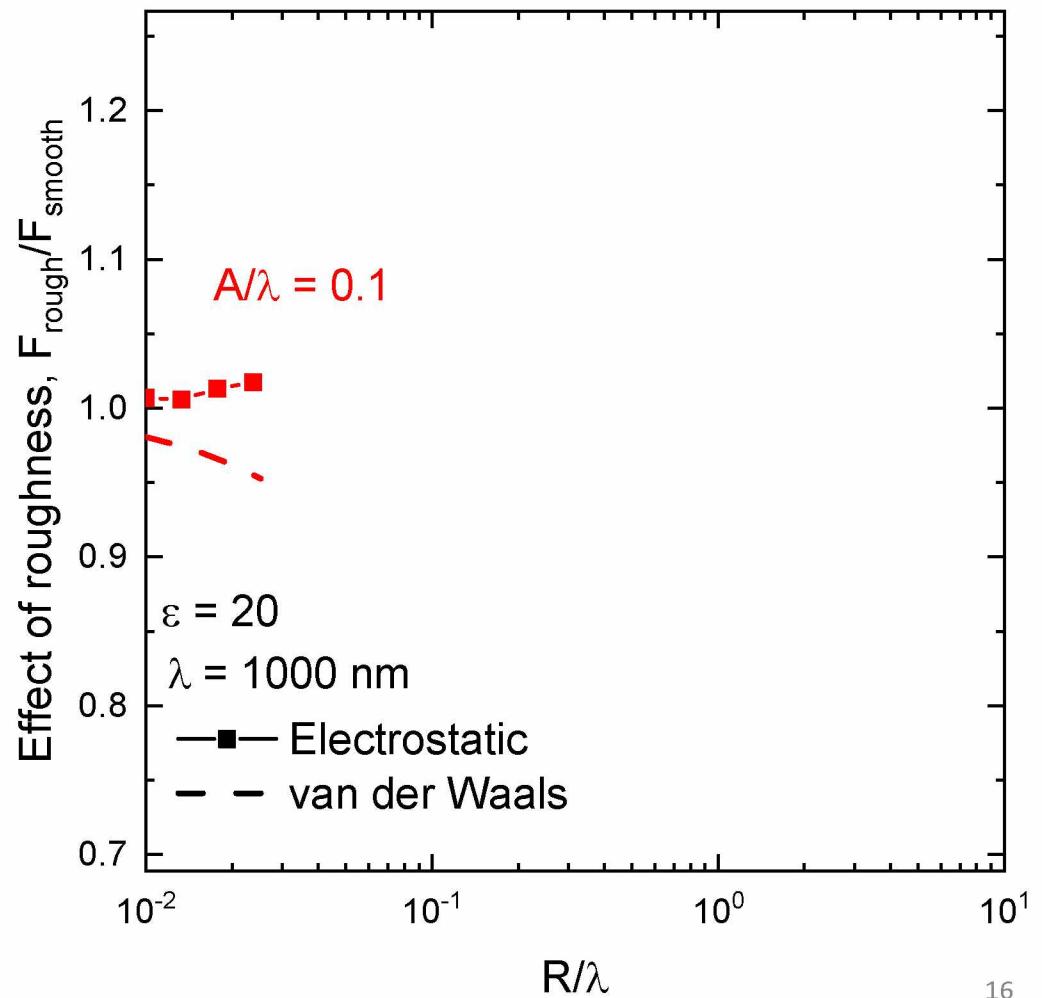
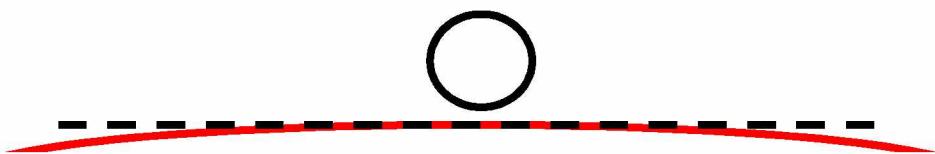
$R/\lambda$  determines size of particle relative to asperities

$A/\lambda$  determines “pointiness” asperities



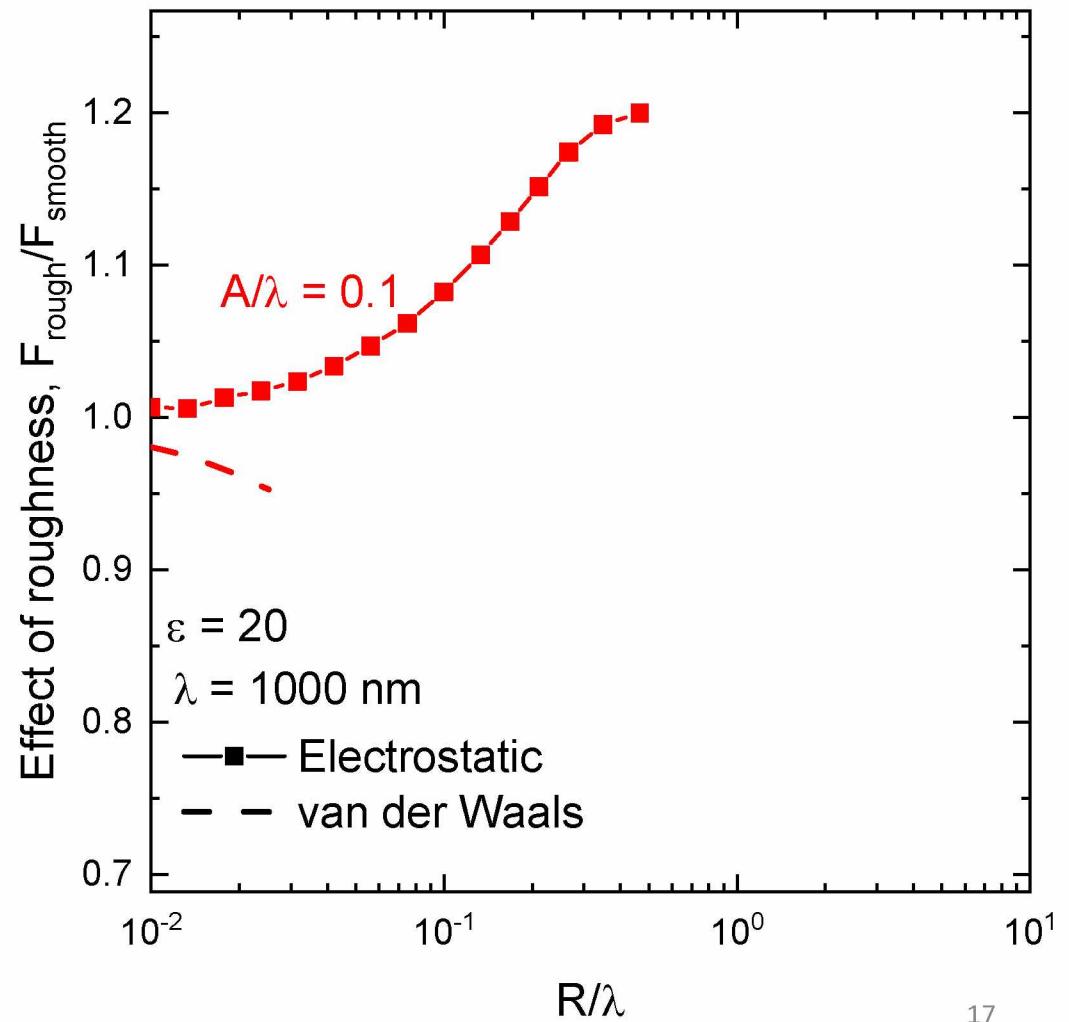
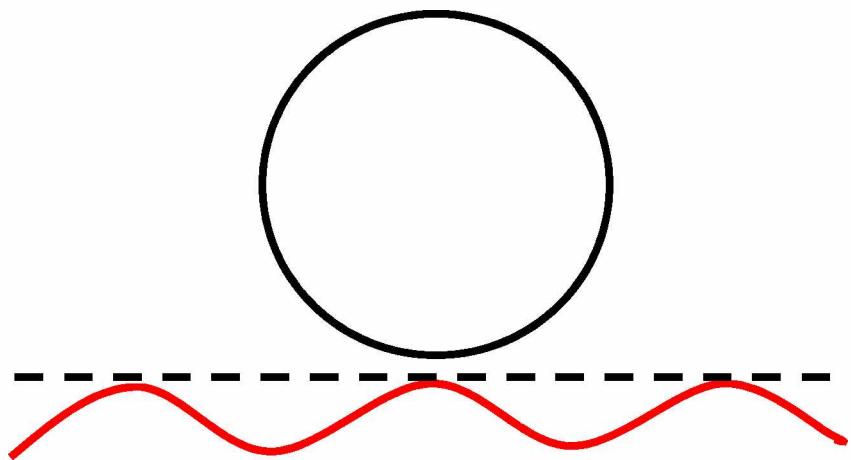
$R \ll \lambda$  : roughness has no effect on  $F_V$  or  $F_E$

When  $R \ll \lambda$ , the asperity appears like a flat surface causing  $F_{\text{rough}} \rightarrow F_{\text{smooth}}$



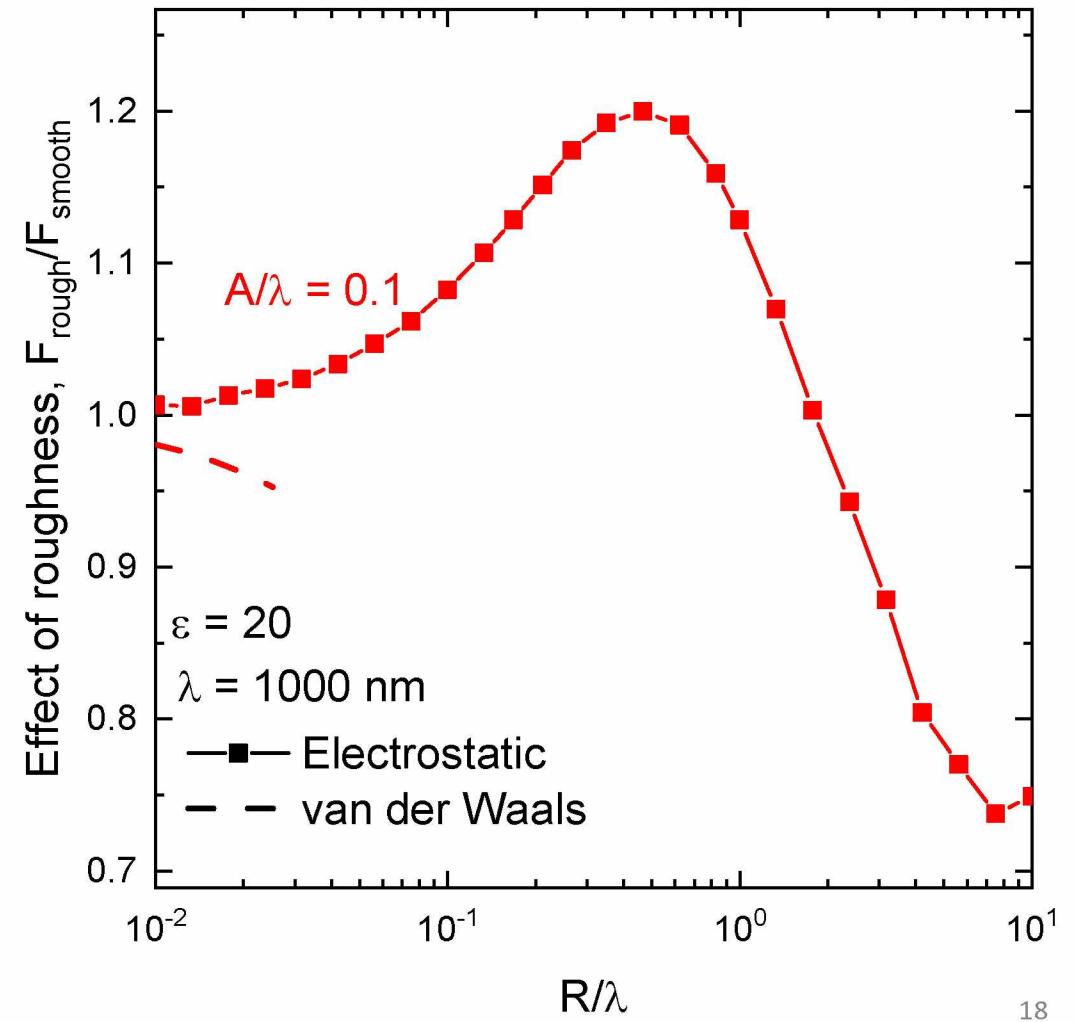
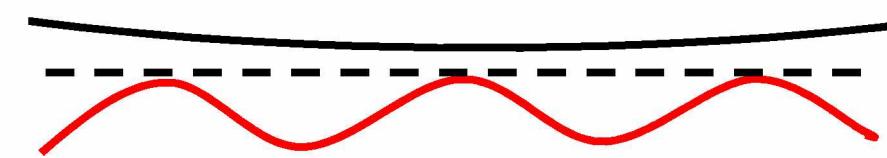
# Roughness enhances $F_E$ when $R < \sim\lambda$

Charge accumulates on peaks of the conducting surface, amplifying polarization and enhancing electrostatic adhesion



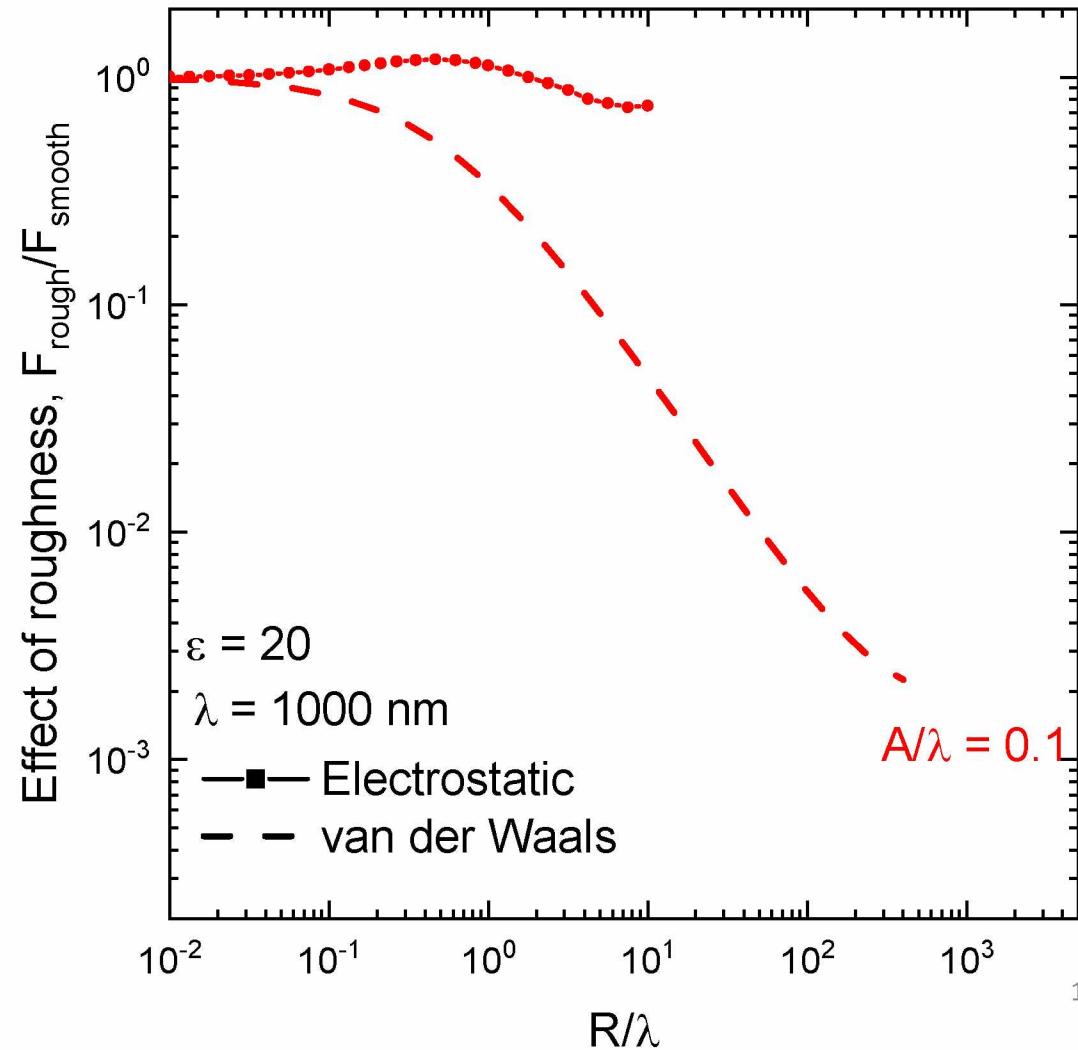
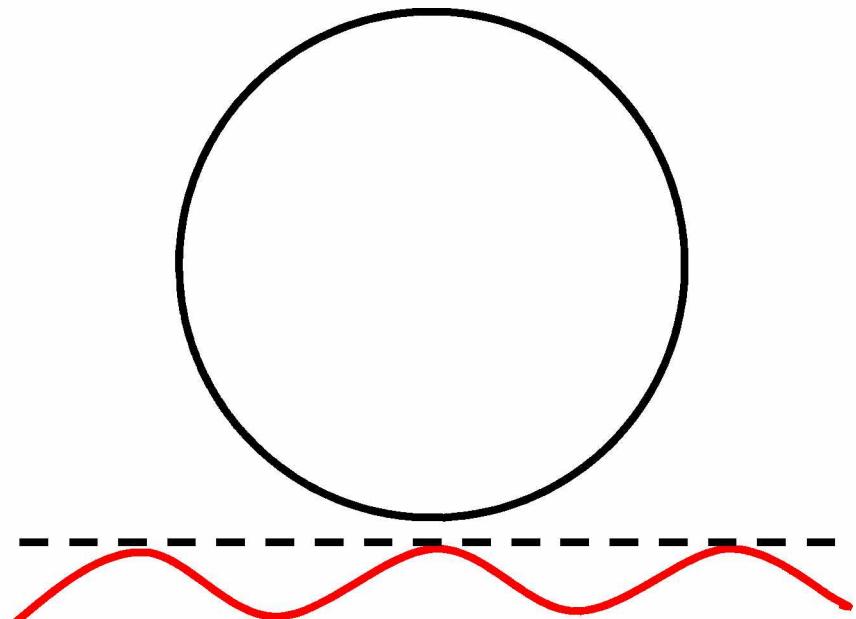
# Roughness diminishes $F_E$ when $R > \sim\lambda$

Increased interaction distance due to roughness causes electrostatic adhesion to be diminished



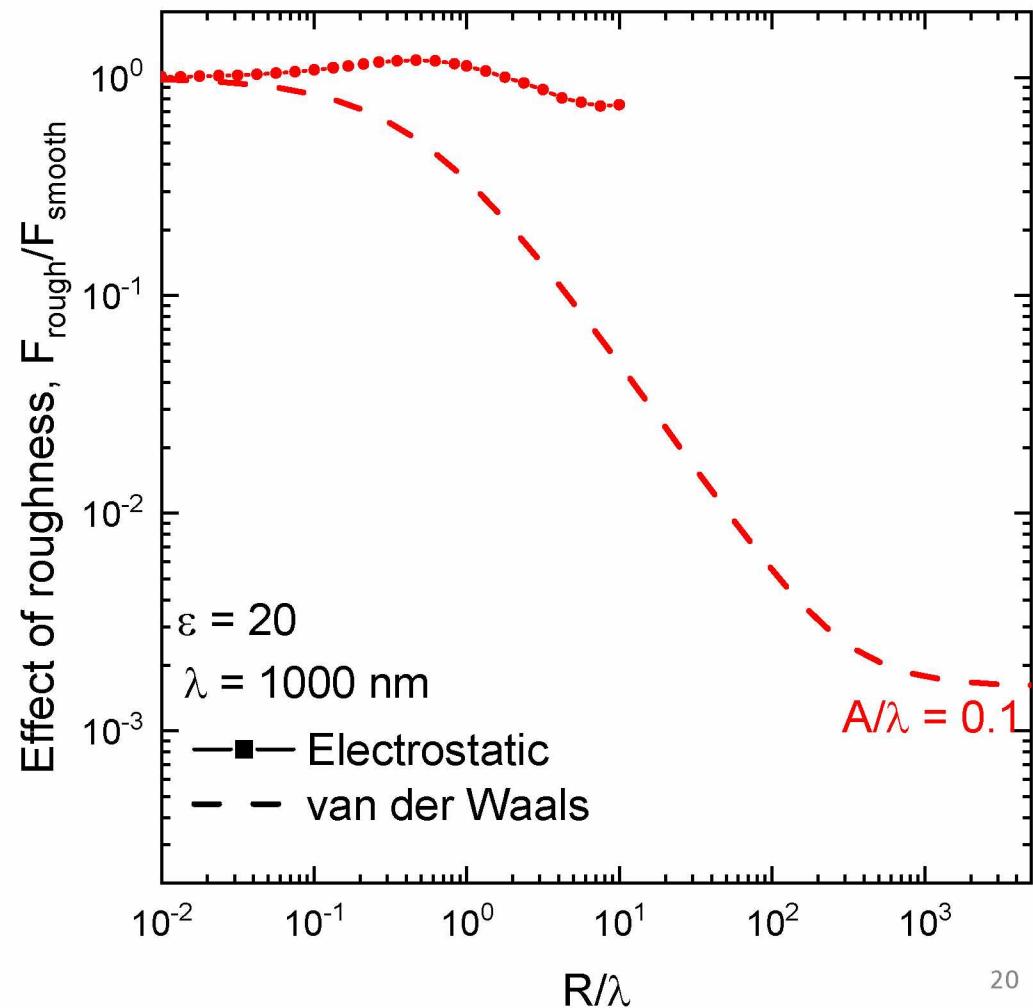
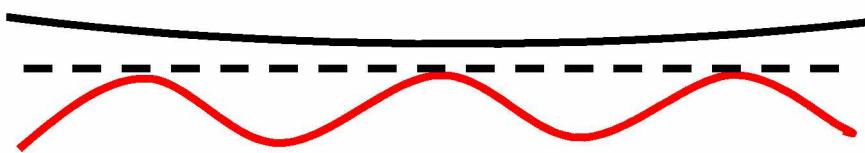
# Roughness diminishes $F_V$ when $R > \sim\lambda$

Increased interaction distance  
causes roughness to diminish  $F_V$

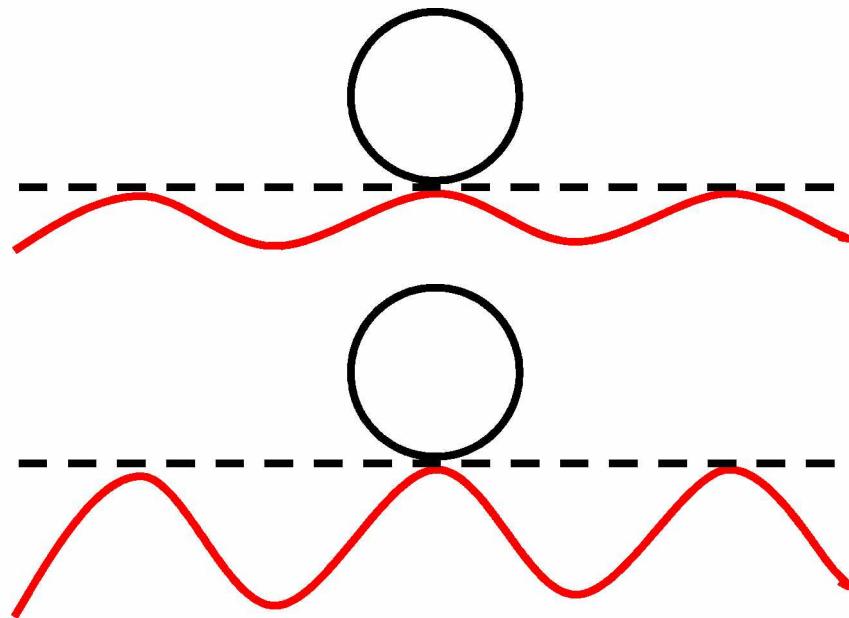


# When $R \gg \lambda$ , effect of roughness on $F_V$ stabilizes

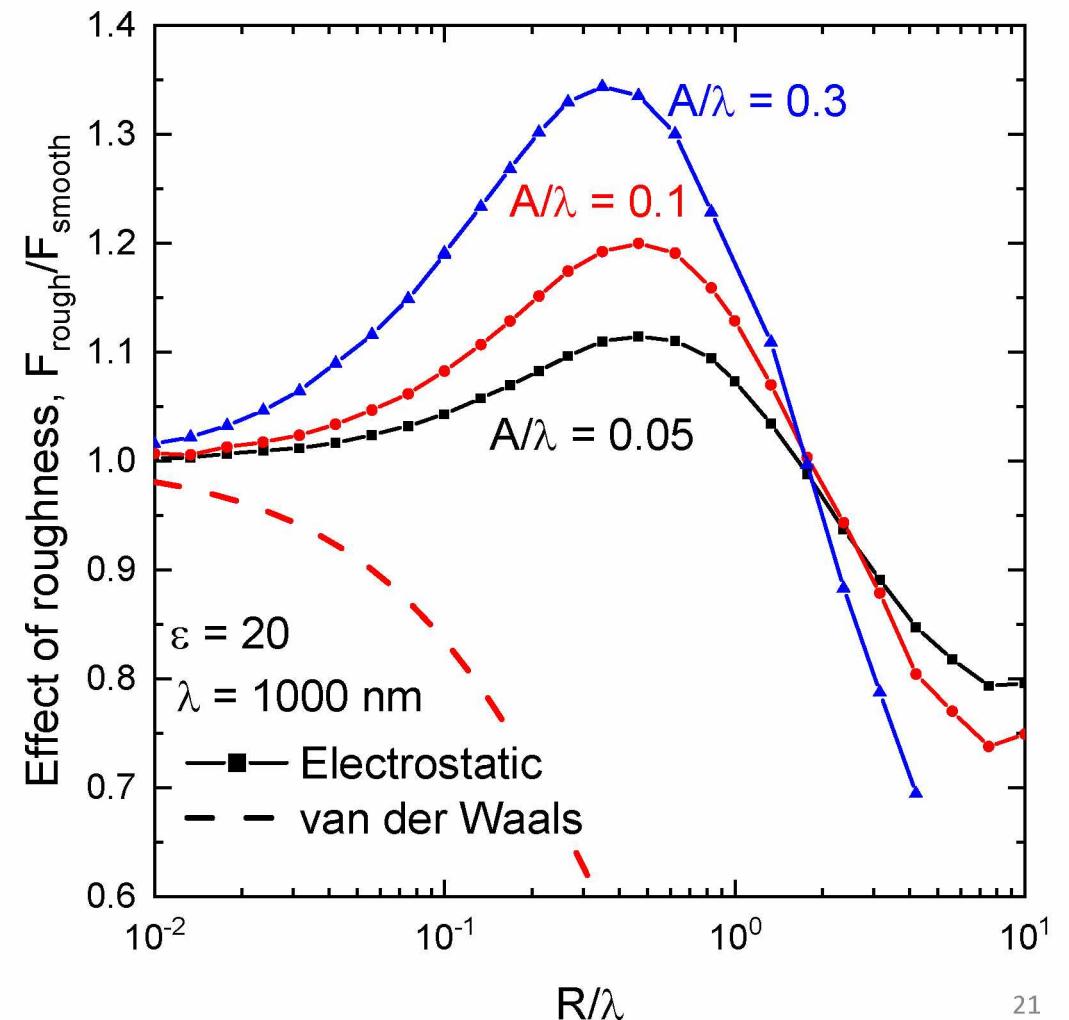
At very large particle sizes, the particle interacts with several asperity and the effect of roughness on the vdW force stabilizes



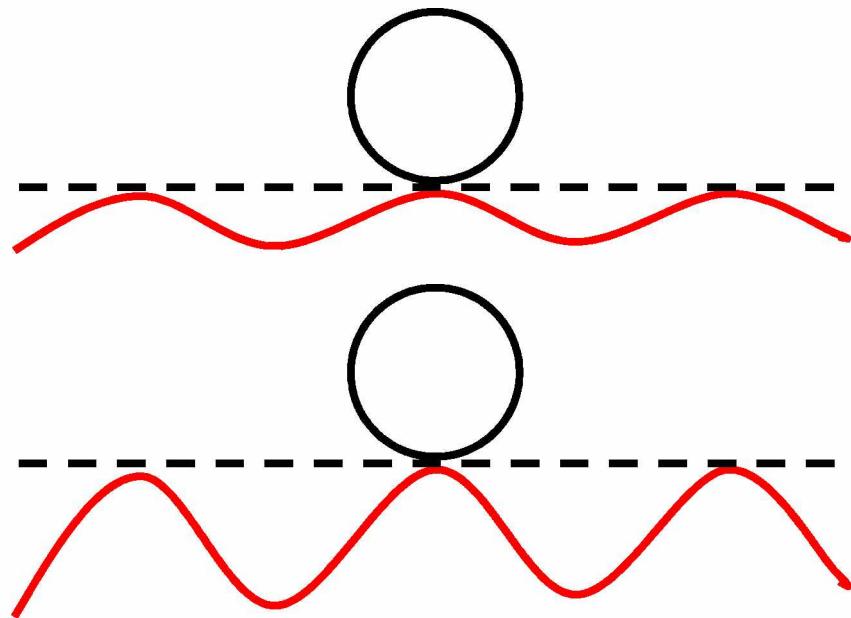
# Increasing amplitude amplifies effect of roughness



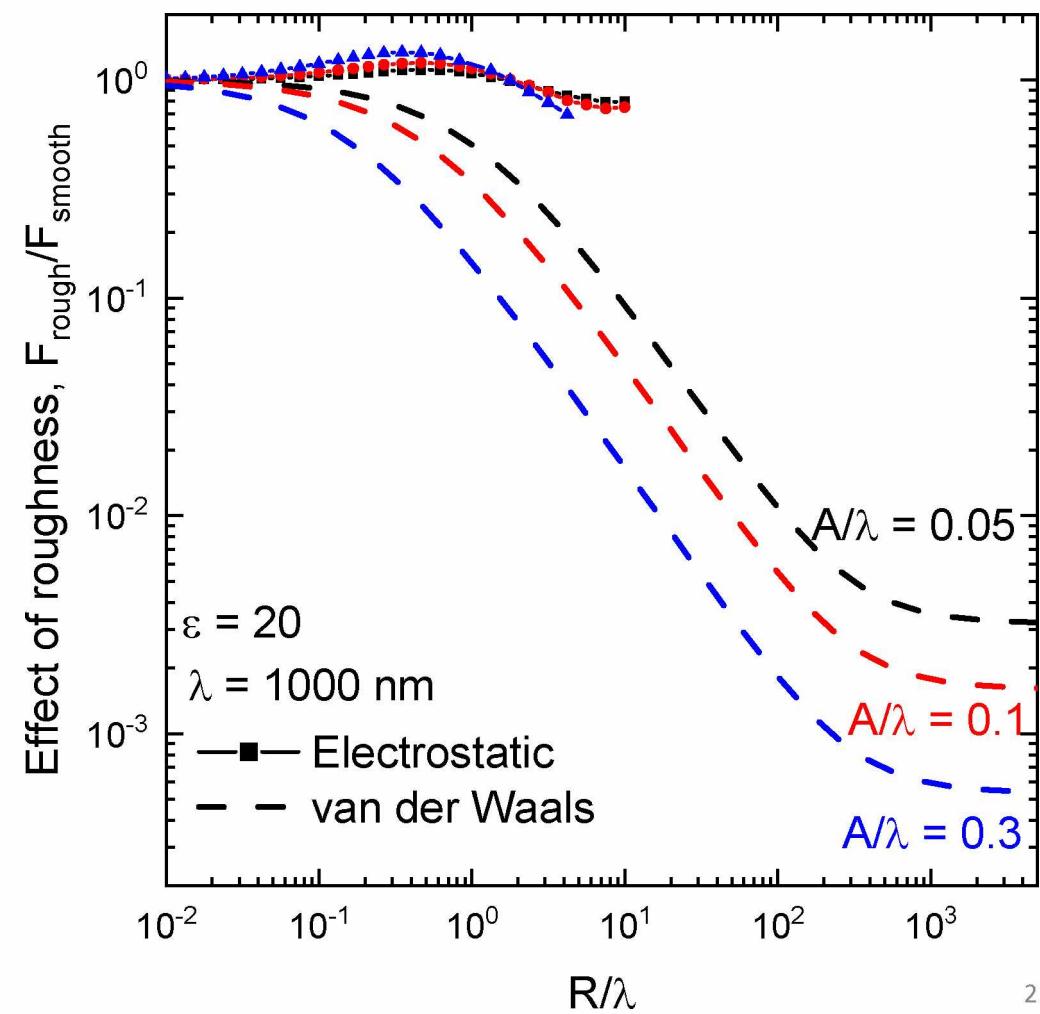
Larger  $A/\lambda$  makes asperities more “pointy”. More charge accumulates at peaks, causing more enhancement of  $F_E$ . Valleys of the surface are further from the particle, causing greater diminishment of  $F_E$ .



# Increasing amplitude amplifies effect of roughness



Larger  $A/\lambda$  diminishes the vdW force more significantly since the interaction distance is increased.



# Roughness diminishes vdW more than electrostatic

Roughness diminishes vdW forces  
by several orders of magnitude  
but scales the electrostatic force  
by less than a factor of 2

