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Associate member of CRLAO (CNRS – UMR 8563), Paris, France

ABSTRACT: This paper proposes a cross-linguistic typology of fractional numbers. A few fractional numbers can be expressed by suppletive (non-systematic) forms, whereas analytical (systematic) linguistic patterns of formation produce what I call "bi-dimensional" numerical forms which refer to both the numerator and the denominator (double argument) or "mono-dimensional" forms which refer to only one of these numbers (single argument). Moreover, a fraction in a partitive expression can be an indivisible semantic unit or may on the contrary have a noun or a classifier inserted between its constituents.

KEY WORDS: Fractions; Numerals; Partitive expressions; Divisible semantic units; Language planning.

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1. DESCRIBING FRACTIONAL NUMBERS IN LANGUAGES

This paper accounts for the information available in secondary literature on the very particular subject of fraction names.

Actually, I tried to wider the scope of languages I would account for. But I faced face the lack of data in the languages of cultures with no strong mathematical traditions, and there is nothing to elaborate on when there is no information. It just shows that some cultures did not develop linguistic means to express arithmetical matters they were not interested in, or that the influence of the people who cared for these issues was not strong enough to pervade the whole society nor the general language which was documented. However, these sociolinguistic aspects are outside the scope of this paper.

Large-scale cross-linguistic surveys were completed regarding the expression of integers, however much less was accomplished on the linguistic expression of fractional numbers. For

instance, Greenberg (1978, 2000) dealt with the expression of integers; he mentioned instances of some fraction names used within the expressions of some integers – e.g. *halv* {1/2}[1] in the construction of some cardinals in Danish – however did not address the general expression of fractions, which was not his subject.

The most inclusive source for the general history of fractions today is still Benoit, Chemla & Ritter's *History of Fractions, Fractions of History*, it focused on the *concept* of fractional numbers in ancient civilizations and only incidentally collected data on their linguistic expression. Moreover, the linguistic information is often irretrievable from the symbolic notations of numbers – we do not know how they were read and we are not even sure they reflected spoken words (Ritter 2001) – making the available written material at times inadequate for a linguistic study. Our modern notation 100 for *one hundred* {1}{100} in English with *one* {1} and *cent* {100} in Modern French with no {1} gives a simple illustration of how the numerical symbols can hide the difference between various linguistic patterns[2]. A more complex example is found with the late Egyptians, who manipulated what we conceptualize as unit-fractions[3] and wrote them in hieratic with a sign which meant *part* placed on top of a sign denoting the denominator, however we are not sure of the morpho-syntactic construction of the *linguistic* expressions corresponding to these notations[4].

I believe the main reason why fractional numbers were a seldom-considered aspect of natural languages is not because we are challenged by the variety of the morpho-syntactic means used to build both the expressions of fractional numbers and the partitive expressions including a fraction. It is more likely because ultimately in this field the specifically linguistic questions seem secondary to the conceptual frameworks. In other words, diachronic changes in

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[1] The following Leipzig Glossing Rules' abbreviations are used throughout this paper: ABL: ablative; ACC: accusative; ACT: active; ADJ: adjective; ART: article; CARD: cardinal; COL: collective; DAT: dative; DECL: declarative; DEF: definite; DET: determiner; DU: dual; F: feminine; FRAC: fractional; FUT: future; GEN: genitive; IND: indicative; M: masculine; MID: middle voice; MW: measure word; NEG: negation; NOM: nominative; OBJ: object; ORD: ordinal; PART: particle; PASS: passive; PERF: perfective; PL: plural; POSS: possessive; PRS: present; PST: past; PTCP: participle; SG: singular; the numbers 1, 2 and 3 followed by a grammatical indication (e.g. OBJ, SG or PL) not restricted to numerals: respectively 1st, 2nd and 3rd person pronoun; an integer *n* followed by a grammatical indication related to numerals (e.g. CARD, FRAC, ORD, etc.): a numeral that expresses the integer *n*.
These standard abbreviations are augmented with: {*n*}: an unanalysable (or unanalysed) form for the integer *n*; {*n*/*d*}: a suppletive form for the fraction *n*/*d*.

[2] Following Huddleston & Pullum (2002), one needs to differentiate *numerals* which denote linguistic expressions (e.g. English '*one hundred*' or French '*cent*') versus *numbers* which are the numerical meaning or content (e.g. the quantity 100). Note that 19th century French had '*un cent*' {1}{100}.

[3] Egyptian calculations with fractions resulted in unit-fractions, while non-unit fractions were expressed as sums of unit-fractions; e.g. 4/7 was conveyed as the sum of 1/2 and 1/14 both written in hieratic (Gardiner 1957: 196). The only exceptions were the use of special signs for the two non-unit fractions 2/3 and 3/4.

[4] Sethe (1916: 84–86) relied on Coptic sources and conjectured a genitive construction starting with '*r*' (*part*) and linked by a preposition to a cardinal which expressed a denominator; e.g. 1/5 would have been expressed as '*part of five*'.

the expression of fractions were driven much more by conceptual developments among restricted groups of technically trained people – and of what (Haugen 1983) calls the *corpus planning*[5] they would initiate – than by any *natural* or *normal* language evolution, be that phonetic, morphosyntactic or lexical. Incidentally, most our evidence, either formal or linguistic, in this domain come from technical texts, e.g. mathematics, architecture, geography, hydraulics. Thus, distinctions between various linguistic formulations of fractions might be understood in terms of logical categories as well as linguistic categories.

This situation might have led to a case of what biologists call convergent evolution, that is the independent evolution of similar features in species of different periods or epochs in time. Similar forms or functions were created that were not present in the last common ancestor of those species. In the field of linguistics, we would talk of similar patterns of expression which are not necessarily the result of a genetic relation nor of a process of borrowing.

Therefore, we suggest a typology which is based on the shape of the expressions and is independent from the morpho-syntactic intricacy of the data. We propose to categorize any expression of a fractional number as either a *suppletive* form (non-systematic) or an *analytic* form (systematic). The set of suppletive forms in a given language maps a finite set of fractions, whereas various analytical forms can define limited analytic sets or even general analytical series. The analytic form can split into *mono-dimensional* (systematic, single argument) and *bi-dimensional* (systematic, double argument). Then we see that bi-dimensional forms can further split up on the basis of its divisibility into constituent parts.

To assess the validity of this typology, I gathered cross-linguistic data which are already available and which can help to refine practical description parameters by contrasting various situations. However, producing a large-scale survey or providing an organized language corpus is not within the scope of this paper.

2. SUPPLETIVE FORMS

Suppletive forms can be used to express a few particular fractions, they are not obviously derived from cardinal forms, at least from a synchronic view-point.

Let us exemplify such suppletive forms for 1/2:

[5] Haugen distinguished four steps: *selection of norm* (which is societal and somehow exterior to the language); *codification of the norm*; *implementation of function* (includes the activities of writers and institutions); and *elaboration of function* (involves the production of a linguistic corpus complying with the norm).

	The fraction 1/2	The cardinal 2
English	*one-half*	*two*
Latin (Maher and Makowski 2001)	*semis*	*duo*
Standard Arabic (Schulz et al. 2000: 111, 214)	*niṣf* [6] (ADJ)	*iṯnāni* (ADJ.M)
Thai (Smyth 2002: 178)	*khrueng*[7]	*song*
Tibetan (Wang Zhijing 1994: 109)	*phye-ka*[8]	*gnyis*
Contemporary Standard Chinese	*bàn* or *yī bàn*[9]	*èr* (cardinal or ordinal) *liǎng* (cardinal only)

Tibetan *phye-ka* and Thai *khrueng* are juxtaposed with a noun or measure word in the same manner as names of integers. Arabic *niṣf* requires nouns to bear the definite article as shown in (1). English *one half* can make use of a preposition in (2) forming a usual partitive expression, however the juxtaposition *one half the population* is possible also, perhaps more colloquial.

(1) *niṣf* *aš-šaʿb*
{1/2}.ADJ ART.DEF-population
'half of the population'

(2) *one half* *of* *the* *population*
{1}{1/2} *of* ART.DEF *population*

Latin (Maher & Makowski 2001) had quite an impressive list of suppletive forms for 1/2, 1/3, 2/3, 1/4, 3/4, 1/6, 5/6, 1/8, 1/12. The names of the multiples of 1/12 up to 11/12, and the names of 1/24, 1/36, 1/72, 1/144 and 1/288 (all fractions of 1/12) are suppletive forms themselves or can be analysed with the fraction name *uncia* {1/12} (the cardinal 12.CARD was *duodecim*). For example, in (3), taken from a passage about arithmetical education in Rome, we can see the suppletive forms *semis* {1/2}, *triens* {1/3} and *uncia* {1/12}, while 5/12 can be construed with 5.CARD and {1/12}:

[6] The transliteration DIN 31635 is used for Arabic.
[7] ISO 11940-2 is used in this paper to transcribe Thai.
[8] The Wylie transliteration is used for Tibetan.
[9] The *Hànyǔ Pīnyīn* transliteration is used for Chinese characters. According to the rules of this transcription, the digit [1] is always Romanized *yī* with a first tone mark regardless of the actual tone in Contemporary Chinese. This tone depends on that of the following syllable and *yī bàn* can actually be pronounced *yí bàn*.

(3) Si de quinc-unce
if of 5.CARD-{1/12}.ABL.SG
'If from 5/12

remo-t-a est uncia,
remove-PTCP.PERF.PASS-F.NOM.SG be.IND.PRS.3SG {1/12}.NOM.SG
is taken 1/12,

quid supera-t? […] Triens. […]
what remain-IND.PRS.3SG […] {1/3}[NOM.SG] […]
what is left?' […] '1/3.' […]

Redi-t uncia,
return-IND.PRS.ACT.3SG {1/12}.NOM.SG
'Now 1/12 comes back,

quid fi-t? Semis.
what be done-IND.PRS.ACT.3SG {1/2}[NOM.SG]
what is obtained? 1/2.'
(Horace [1st c. BCE], *Ars poetica* [*The art of poetry*], verses 327–330)

The two terms *uncia* and *scripulum*, besides their numerical values, were also – or *originally* – used in metrological scales: *uncia*[10] was the suppletive form {1/12}, a unit of area (1/12 of 1 *iugerium*), a monetary unit and a unit of weight (1/12 of 1 *as*); *scripulum* was {1/288} and also a unit of area (1/288 of 1 *iugerium*). However, according to (Maher and Makowski 2001), in example (3) the two words *uncia* and *scripulum* are to be understood as fraction names, not as submultiples within a metrological scale[11].

Contemporary Chinese has a suppletive term *bàn* for 1/2 which cannot be analysed from the names *èr* and *liǎng* of the integer 2. Excavated Chinese texts from the 3rd and 2nd centuries

[10] The Latin word *uncia*, through two different processes of phonological changes, has given the English length unit *inch* (1/12 of a *foot*) and the mass unit *ounce* (it has at times been defined as 1/16 or 1/12 of a *pound*).
[11] Incidentally, another case of *fluidity* between fraction names and submultiples of a metrological scale can be observed in Egyptian (Ritter 2003). This lexical interchangeability can be understood in terms of the conceptual continuum between subunits and fractions, see Ritter (1992, 2001, 2003).

BCE[12] contain suppletive forms for 1/2, 1/3 and 2/3, respectively *bàn* (half), *shǎobàn* (the smaller half) and *tàibàn* (the larger half), both derived from *bàn* {1/2}; they are shown in the examples (4) and (5). The words *shǎobàn* {1/3} and *tàibàn* {2/3} could be replaced by analytical forms constructed with 3.CARD *fēn*[13], however no analytical form for 1/2 was found.

(4) *yī* *bàn* *chéng* *yī,* *bàn* *yě,*
1.CARD {1/2} multiply 1.CARD {1/2} DECL
'1/2 times 1 is 1/2,

chéng *bàn* *sì* *fēn* *yī* *yě.*
multiply {1/2} 4.CARD part 1.CARD DECL
times 1/2 is 1/4.'
(*Suàn shù shū*, strip 3)

(5) *shǎobàn* *chéng* *tàibàn,* *jiǔ* *fēn* *èr* *yě.*
{1/3} multiply {2/3} 9.CARD part 2.CARD DECL
'1/3 times 2/3 is 2/9.'
(*Suàn shù shū*, strip 8)

Later texts, transmitted, and dated to the 11th and 13th centuries, had also *ruòbàn* (the weak half) for 1/4 and *qiángbàn* (the strong half) for 3/4, and a new term *zhōngbàn* (the middle half) for 1/2 had appeared[14]. Most of these terms were lost in Contemporary Chinese: only *bàn* {1/2} and *yī bàn* i.e. 1.CARD {1/2} remain for 1/2, however now the regular analytical form 2.CARD *fēn zhī* 1.CARD is also available. The term *tàibàn* (or *dàbàn*) remains as an approximate number meaning *most*, no longer as an exact number.

[12] These include mathematics texts; at the time of writing two long mathematical texts were available: *Shù* [*Numbers*] dated to the 3rd century BCE and the *Suàn shù shū* [*A Book on the Reckonings with Counting Rods*] dated to the beginning of the 2nd century BCE. There are also texts of law (e.g. the *Èr nián lǜ lìng* [*Statutes and Ordinances of the Second Year*] or the *Qín lǜ shí bā zhǒng* [*Eighteen Statutes of Qin*]. There are also texts concerned with other matters such as medicine. All together contain hundreds of instances of fractions expressed in natural language. One may see Anicotte (2017) for a description of the fraction numbers in Qin-Han excavated texts, and Anicotte (2015 b) about historical changes. As for Chinese integer names, readers may refer to Anicotte (2015 a).

[13] For instance there are three occurrences of 3.CARD *fēn*, on the strips 119, 138 and 139 of the *Suàn shù shū*.

[14] The term *ruòbàn* for 1/4 is found in the first chapter of the *Xiàhóu Yáng suàn jīng* [*Xiahou Yang's Mathematical Manual*] (1084). The terms *qiángbàn* for 3/4 and *zhōngbàn* for 1/2 are found in the *Shù shū jiǔ zhāng* [*Mathematical Treatise in Nine Sections*] (1247) (Libbrecht 1973: 70–71).

3. ANALYTICAL FORMS

Using non-analytical expressions appears viable for a finite list of fractions, but a *generic* linguistic pattern capable of expressing any fractional number would be analytic and account for both the numerator and the denominator as we conceptualize them today. This requirement may suggest that expressions involving two integer names should cover all situations. However, besides the numerical phrases which I call *bi-dimensional* because they do refer to both the denominators and the numerators, we also encounter *mono-dimensional* phrases referring only to their denominators or their numerators.

The bi-dimensional patterns are the most general ones, in the sense that they can potentially express any fraction; or at least any *proper fractions* (smaller than 1, e.g. 1/3). The expression of so-called *improper fractions* (greater than 1, e.g. 4/3) can rely on the same pattern or not, they can also be expressed as *mixed numbers* (the sum of an integer and a proper fraction, e.g. 1+1/3 instead of 4/3).

Benoit et al. (1992) point out that the modern concept of a fraction, that is an object of the form p/q where p and q are integers, is a late-comer to history. Almost all literate ancient societies conceived of fractions in the form now called *unitary*, that is, with numerator 1 – or more precisely, conceptualized without numerator.

3-1 *Mono-dimensional forms*

Some analytical mono-dimensional patterns can express potentially infinite sets of fractions. For example, unit-fractions in Latin, Sanskrit, and in Chinese excavated manuscripts dated to the 3rd and 2nd centuries BCE, are mono-dimensional forms which mention the denominator only, while the numerator 1 is not stated. Note that this description does not intend to reflect the conceptualization at the time, and that the pattern described here was likely not perceived as the *non-statement* of anything.

There are also finite mono-dimensional series which can express a given number of fractions, e.g. the tenths with the phrases *n*.CARD *chéng* (*n* being an integer from 1 to 9) in Chinese which state only the numerator, while the denominator is understood by a linguistic convention applicable to this particular pattern[15].

The unit-fractions (let us say $1/d$ with the numerator 1 and the denominator d) in Chinese texts from the 3rd and 2nd centuries BCE would usually take the form *d*.CARD *fēn* stating the cardinal *d*, and *fēn*, a term whose original meaning is *part* (noun and verb), forming compounds like 4.CARD *fēn* in (6). I call 4.CARD *fēn* a *mono-dimensiona*l phrase because only

[15] See the second entry for *chéng* in the *Xiàndài hànyǔ guīfàn cídiǎn* [Dictionary of Contemporary Chinese], 2010, Beijing: Wàiyǔ jiāoxué yǔ yánjiū chūbǎnshè, p.163.

the numeral 4.CARD is present. The numerator's name 1.CARD was not compulsory and was usually omitted when the fraction name occurred as a factor in a multiplication. However, it was indeed possible to use it, typically when communicating the result of an operation. This is the case in (6) with the *bi-dimensional* phrase 16.CARD *fēn* 1.CARD.

(6) *sì* *fēn* *chéng* *sì* *fēn* *shí liù* *fēn* *yī*
4.CARD part multiply 4.CARD part 16.CARD part 1.CARD
'1/4 times 1/4 are 1/16'
(*Suàn shù shū*, strip 9)

As for Latin, in *De architectura*, Vitruvius used ordinals associated with the word *pars* (part) to express unit-fractions. On one occasion given in (7), this pattern was even used to express 1/8 instead of using the suppletive form *sescuncia*.

(7) *octav-a* pars
8.ORD-NOM.F part[NOM.SG.F]
'1/8'
(Vitruvius [1st c. BCE], *De architectura* [*On architecture*], 1.6.9)

Pliny the Elder made great use of this analytical expression of unit-fractions:

(8) Longissim-a die-i spati-a
longest-NOM.PL day-GEN.SG extent-NOM.PL
The longest extensions of the day are

hor-arum aequinoctial-ium quindecim addi-t-a
hour-GEN.PL equinoctial-GEN.PL 15.CARD add-PTCP.PERF.PASS.NOM.PL
15 equinoctial hours added of

non-a parte un-ius hor-ae
9.ORD-ABL.SG.F part.ABL.S.F 1.CARD-GEN.SG hour-GEN.SG
1/9 of an hour

aut, ut Nigidio plac-u-it, quint-a.
or as Nigidius.ABL.SG please-PERF.3SG.ACT 5.ORD-ABL.SG.F
or 1/5 according to Nigidius.'
(Pliny the Elder [1st c. CE], *Naturalis Historia* [*Natural history*], 6.39)

Note that, in the corpus available online or in printed editions, *quindecim* and *nona* in (8) are actually respectively written XV and IX, but *quinta* is written in full letters, not as V, this is said for the sake of precision but is of no consequence for this paper. The word *pars* would even be dropped when the context was clear enough to indicate that fractions were involved, as it is with *quinta* [*parte*] in (8).

Frontinus used the same pattern, and *pars* would be dropped occasionally, as for example once in (9):

(9) Est autem digit-us ut conveni-t
be.IND.PRS.3SG however digit-NOM.SG.M as suit-IND.PRS.3SG
'However, by convention, the digit is

sexta decima pars ped-is,
16.ORD.NOM.F part[NOM.SG.F] foot-GEN.SG
1/16 of a foot,

uncia duodecim-a.
inch.NOM.F 12.ORD-NOM.F
the inch [is] 1/12 [of a foot].' (Frontinus, *De aquae ductu urbis Romae* [*Water management of the city of Rome*] [cerca 98 CE], 1.24)

Sanskrit[16] names for unit-fractions were also mono-dimensional compounds formed with the nouns *bhāga-* or *aṃśa-*, both meaning *part*, and added to either the cardinal form as in (10) and (11) or the ordinal form[17] of the denominator's name as in (12).

[16] Datta & Singh (1935: 185–186), Filliozat & Mazars (1987), Mazars (1992). The expression of fractions has most likely evolved during the history of Sanskrit and other Indic languages, however available instances are too scarce for us to perceive the changes.
[17] Or rather a form derived from the ordinal according to the grammarian Pāṇini who commented that the term used in fraction names could show phonetic alterations with respect to the original ordinal form (Filliozat & Mazars 1987); Pāṇini is reputed to have been active around the 5th century BCE.

(10) *sapta* *bhāga*
7.CARD part
‘1/7’

(11) *pañcadaśa* *bhāga*
15.CARD part
‘1/15’

(12) *pañcama* *bhāga*
5.ORD part
‘1/5’

Arabic gives a special case of mono-dimensional fraction names which are not the expressions of unit-fractions.

Firstly, to the names of denominators 3 through 10 are specific words built on the basis of a vocalic shift on the cardinal forms (Schulz et al. 2000: 214). For example the word 5.FRAC for 1/5 in (14) is a mono-dimensional form derived from the cardinal 5.CARD given in (13). Then, the name 5.FRAC.DU of 2/5 in (15) is also a mono-dimensional form.

(13) *ḥamas*
5.CARD
‘5’

(14) *ḥumus*
5.FRAC.SG
‘1/5’

(15) *ḥumusain*
5.FRAC.DU
‘2/5’

But the name of 3/5 in (16) is made with the adjectival form of the cardinal for 3 and the plural form of 5.FRAC. That is to say that the fraction name is bi-dimensional.

(16) *ṯālṯat* *ʾaḥmās*
3.ADJ 5.FRAC.PL
‘3/5’

3-2 ***Cross-linguistic variety of bi-dimensional forms***

First of all, we need to be aware that not all bi-dimensional numerical phrases have to do with fractions: there are also those expressing *proportions*, e.g. ‘a 40-60 split’ in English or its Chinese equivalents ‘4 6 *kāi*’ and ‘4 6 *fēn chéng*’. They may imply fractions (40% and 60% for the English phrase, 4/10 and 6/10 for the Chinese one), however the denominators 100 or 10 are not expressed; this is why I think such phrases are not relevant in this study.

Now let us illustrate – in a limited extent – the morpho-syntactic variety of the bi-dimensional phrases for fractional numbers.

Variety can be seen in the way the two numerical items are joined together, that is to say in the morphology specific to the expression of fractions. This can at times reflect patterns for quantification phrases with adjectival or genitive constructions, and this shows with the order in which the numerator and the denominator are enunciated. The denominator may be in second position, this is exemplified in this paper by Arabic, Modern Greek, Hebrew, Latin and

proper fractions in Sanskrit; and it is also the case in English, French, German, Spanish, Swedish, Thai[18], etc. The reverse order, with the denominator put in the first place, is exemplified in this paper by Chinese, Ancient Greek, Japhug, Tibetan and improper fractions in Sanskrit; it is also found in Japanese (Martin 1975: 767) and Korean (Martin 1992: 188).

For instances with the denominator placed in the second position, we had the Arabic example (16) above with an adjectival form of the numerator's name preceding the plural form of the numeral expressing the denominator.

German and Swedish respectively possess the specific suffixes *-tel* and *-del* to produce denominator names[19]; numerator names are put in adjectival position before the denominator's name. Here is an example in Swedish:

(17) två femte-del-ar
2.CARD 5.CARD-*del*-PL
'2/5'

The suffix *-del* is used to form fraction numerals, therefore *femtedelar* in (17) could alternatively be construed as *femtedel-ar* and glossed 5.FRAC-PL.

A generic pattern for proper fractions in Latin can be seen in (18) and (19) cited by (Maher and Makowski 2001) who explain that these are the earliest known written examples of this pattern. It relies on the cardinal form of the numerator followed by the ordinal form of the denominator, which takes a plural mark.

(18) quadrat-us tribus quartis decumis suis
square-NOM.SG 3.CARD.ABL 14.ORD.ABL 3.POSS.ABL.PL
'the square is larger than the round by 3/14 of its own size'

rotund-o maior est
round-ABL.SG larger[NOM.SG] be.IND.PRS.3SG
(*De aquae ductu urbis Romae* [*Water management of the city of Rome*], 1.24)

[18] Smyth (2002: 177–199): The bi-dimensional expressions are "numerator's name + *nai* + denominator's name" built with the preposition *nai* (in, of).
[19] These forms are invariable in German and can take the plural mark *-ar* in Swedish. Attention however to German irregular form *Drittel* (3.FRAC) and variant *Siebtel* (7.FRAC) for *Siebentel* (7.CARD-*tel*).

(19) rotund-us tribus undecumis suis
round-NOM.SG 3.CARD.ABL 11.ORD.ABL 3.POSS.ABL.PL
'the round is smaller than the square by 3/11 of its own size'

quadrat-o minor est
square-ABL.SG smaller[NOM.SG] be.IND.PRS.3SG
(*De aquae ductu urbis Romae* [*Water management of the city of Rome*], 1.24)

The construction of fraction names in Modern Hebrew takes its roots in Mishnaic Hebrew (1st to 4th centuries CE). The fraction name for 2/5 in (20) can be construed as a noun phrase with the name of the numerator and the name of the denominator, in this order. They are linked using the so-called *construct* pattern (based on the genitive form of the first constituent) and is to be understood as the genitival *two of five*.

(20) *štei* *ḫamišiot*
2.CARD.F.GEN 5.CARD.F.PL
'2/5

The Modern Greek fraction name for 2/5 in (21) can be interpreted as a quantification phrase, with the denominator's name playing the role of the quantified item and bearing a plural marker.

(21) *dyo* *pempt-a*
2.CARD 5.CARD-NOM.PL
'2/5'

Sanskrit names for the proper fractions 3/8 and 2/7 are shown in (22) and (23), and they put the numerator in first position:

(22) *tri* *aṣṭama*
3.CARD 8.ORD
'3/8'
(*Śulbasūtras*[20] [Datta & Singh 1935: 186])

[20] The *Śulbasūtras* is a collection of transmitted texts generally dated to the period between the 9th and 2nd century BCE; dates are tentative because the transmission of the corpus was only oral for centuries.

(23) *dvi* *saptama*
2.CARD 7.ORD
‘2/7’
(*Śulbasūtras* [Datta & Singh 1935: 186])

However Sanskrit fractions are not limited to proper fractions (smaller than 1). They can also be improper fractions (greater than 1) like 5/4 in (24), showing the name for 5/4 with the numerator put in final position, formed with the suffix *-ka* added to the cardinal 5, and meaning a collection of 5 items.

(24) *catur*[21] *pañca-ka*
{1/4} 5.CARD-COL
‘5/4’ (literally: ‘a fivesome of fourths’ according to [Filliozat & Mazars 1987])
(*Bakhśalī manuscript*[22])

Let us observe other examples of expressions with the denominator in first position starting with Ancient Greek[23] in (25) which shows a noun phrase expressing 2/5. It relied on the juxtaposition of the genitive form 5.CARD.GEN of the denominator’s name followed by the noun *moira* (part), itself quantified by the numerator’s name 2.CARD.

(25) *Lakedaimon-iōv* […] *Peloponnes-ou*
Lakedaimon-ADJ.GEN.PL […] Peloponnes-GEN.SG
The *Lacedaemons* […] of the Peloponnese
‘The Lacedaimons […] occupy two fifths of the Peloponnese’

t-ōn	*pente*	*t-as*	*dyo*	*moir-as*
ART.DEF-GEN.PL	5.CARD	ART.DEF-ACC.F.PL	2.CARD	part-ACC.F.PL
of the five		the two parts		

[21] The final *r* in *catur* (a word for 1/4) could alternatively be written as its allophone *ḥ* forming the word *catuḥpañcka*.
[22] The *Bakhśalī manuscript* is a mathematical text on pieces of birch bark which was excavated in 1881, it was arguably written around the 7th century C.E. in a late or hybrid variety of Sanskrit (Hayashi 1995).
[23] Waanders (1992) implies that very few instances of bi-dimensional expressions for fractions are known in Ancient Greek.

nem-ontai
occupy-PRS.IND.MID.3PL
occupy
(Thucydides [5th c. BCE], *The History of the Peloponnesian War*, 1-1-10)

The Tibetan bi-dimensional pattern for fractions is explained by Goldstein et al. (1991: 200) and Wang Zhijing (1994: 108–109). The syllable *cha*[24] is placed after the denominator's name, and the compound "denominator's name-*cha*" is inserted before the numerator's name as shown in (26). This respects the order "*noun* + *numeral*" of the language's quantification phrases if we construe the compound suffixed with *-cha* as a nominal form and the numerator's name as the quantifier:

(26) *lgna* *cha* *gnys*
5.card *cha* 2.CARD
'2/5'
(Goldstein et al. 1991: 200)

Japhug (Jacques 2008) is a Tibeto-Burman language of the rGyalrong branch (not a Tibetic language) spoken in Sichuan Province, China. The Japhug bi-dimensional expression of 1/3 in (27) relies on the words tɯ-tɯcɯr (part) and ŋgɯ (interior):

(27) χsɯ tɯcɯr ɣɯ ɯ ŋgɯ tɯ tɯcɯr
3.CARD part GEN 3SG.POSS interior 1.CARD part
'1/3' (expressed as '*one part within three parts*')
(Jacques [personal communication by e-mail, March 2014])

3-3 *Changes in the bi-dimensional forms in Chinese*

The Chinese language has records on almost three thousand years. Egyptian, Akkadian and other Mesopotamian languages had comparable historical depth, but often wrote numbers with symbolic notations, whereas earlier Chinese sources did not rely on symbolic notations and wrote the words which expressed numbers, so that the linguistic information is readily available.

[24] In Tibetan, *fraction* is said *cha-grangs* i.e. 'number of parts' from *cha* 'part' and *grangs* 'number'. The Tibetan-Chinese dictionary *Zàng Hàn dà cí diǎn* (Beijing: Mínzú chūbǎnshè [The Ethnic Publishing House], 1993, vol.1: 772) states that *cha* is a noun meaning *part* and a measure word meaning *pair*. It is used in the expression of fractions and also appears as a suffix in some nouns formed from other nouns.

Nevertheless, we do not see a clear path of diachronic evolution due to syntactic changes or language planning. Often, we observe the synchronic coexistence of various patterns of expression at a given time in non-technical texts, and even in formal mathematical ones.

In excavated texts dated to the 3rd and 2nd centuries BCE, the bi-dimensional pattern stated the denominator and the numerator in this order; see examples (28)–(33):

(28) *jiǔ* *fēn* *èr*
9.CARD part 2.CARD
'2/9'
(*Suàn shù shū*, strip 8)

(29) *qī* *fēn* *zhū* *liù*
7.CARD part *zhū* 6.CARD
'6/7 *zhū*' (*zhū* is a unit of weight)
(*Suàn shù shū*, strip 28)

When a measure word was involved, it was put directly after *fēn* as above in (29) and below in (31) and (33). The genitive morpheme *zhī* could optionally be inserted in the bi-dimensional construction, as can be seen in (30), (31) and (33):

(30) *gè* *shòu* *sān shí* *fēn* *zhī* *èr shí sān*
each get 30.CARD part GEN 23.CARD
'each gets 23/30' (about the sharing of a sum of money)
(*Suàn shù shū*, strip 26)

(31) *jīn* *qī* *fēn* *zhū* *zhī* *sān*
gold 7.CARD part *zhū* GEN 3.CARD
'3/7 *zhū* of gold' (*zhū* is a unit of weigh)
(*Suàn shù shū*, strip 30)

No improper fraction would be mentioned, however there were mixed numbers (sums of an integer and a proper fraction); as in (32), the sum of 12 and 11/72 with no measure word expressed by juxtaposition, and in (33) with a measure word. In other cases, there can be a conjunction *yòu* between the integer and the proper fraction.

(32) *shí èr* *qī shí èr* *fēn* *shí yī*
12.CARD 72.CARD part 11.CARD
'12 11/72'
(*Suàn shù shū*, strip 36)

(33) *zòng* *yī* *bù* *liù* *fēn* *bù* *Zhī* *yī*
length 1.CARD *bù* 6.CARD part *bù* GEN 1.CARD
'a length of 1 *bù* 1/6 *bù*' (*bù* is a unit of length)
(*Suàn shù shū*, strip 121)

Some five hundred years later, in the version of the *Jiǔ zhāng suàn shù* [*Nine Chapters on the Mathematical Art*] compiled by Liu Hui in the 3rd century CE[25], the item *zhī* had become systematic even though no syntactic change had made it necessary. This form was also used by the 13th century mathematician Qin Jiushao in his *Shù shū jiǔ zhāng* [*Mathematical Treatise in Nine Sections*][26] (1247) and it was transmitted to Korea and Japan[27].

However, when no measure word was involved, the pattern "denominator's name + *zhī* + numerator's name" (without *fēn*) was also found during the Qin-Han period (however not in mathematical texts), and until the 19th century (in both non-mathematical and mathematical texts[28]). The existence of a variety of different forms at a given time means that the attempts to standardize the pattern of expression remained in the realm of literati who devoted their work to mathematics, while the general corpuses would exhibit free formation.

In today's Chinese, the phrases "denominator's name + *fēn zhī* + numerator's name" contain the sequence *fēn zhī* and are indivisible: measure words are placed after these fraction names and no longer between their constituents "denominator's name + *fēn*" and "*zhī* + numerator's name". It is hard to assess the detailed process of this change which occurred in the context of the abandonment of Classical Chinese, and of educational reforms[29]. However, one

[25] The transmitted version is a compilation of earlier works; however, the language of the earlier texts was polished and regularised by the 3rd century mathematician Liu Hui. One can refer to Anicotte (2015 b, 2017, 2019: 17–18) for the date of the expression of fractions in the text and to Chemla & Guo (2004: 71–97) for a detailed history of the manuscript.
[26] Qin Jiushao used a symbolic notation to write down numbers involved in operations; however, all the numbers were repeated in the text using the linguistic numeration.
[27] The pattern "denominator's name + *bun no* + numerator's name" is found in Japanese (Martin 1975: 767) with *bun*, the Sino-Japanese reading of Chinese *fēn*, and *no* the Japanese determination linker. In Korean there are "denominator's name + *pun uy/ci* + numerator's name" with *uy* the indigenous Korean genitive linker or *ci* its Sino-Korean counterpart (Martin 1992: 188).
[28] Notably by Li Zhizao in his *Tóng wén suàn zhǐ* [*Arithmetic Guidance of the Combined Learning*] (1613) and Wu Jiashan in his *Suàn xué èr shí yī zhǒng* [*Twenty-one Books of Arithmetic*] (1863).
[29] The part played by institutions of standardization is in itself an object of research. One can consider the action of the Standardization Office of the Ministry of Education which implemented the change in

should note that free formation – or conservatism – can still be seen in Sinitic languages with, for example, *zhī* non-compulsory in Cantonese fraction names (Matthews & Yip 2011: 453).

3-4 *Bi-dimensional forms as divisible or indivisible semantic units*

Let us now focus on one particular characteristic of bi-dimensional fraction names when they become part of a partitive expression: they can perform as indivisible semantic units or, on the contrary, the sequence of their constituents can be split by nouns or classifiers.

This issue arose with the examples (29), (31) and (33) where the constituents of the bi-dimensional phrases for fractional numbers in Chinese excavated texts of the 3rd and 2nd centuries BCE could be separated, and a measure word would be placed just after the sequence "denominator's name + *fēn*", and therefore before the numerator's name.

On the contrary, the English partitive expressions in (34) and the Modern Greek one in (35) both involve fraction names which are indivisible units connected to the quantified item with a genitive construction (a preposition in English and the genitive case in Greek).

(34)	two	fifth-s	of	a	litre	of	water
	2.CARD	5.ORD-PL	of	ART-SG	litre	of	water

(35)	*t-a*	*dyo*	*pempt-a*	*t-on*	*Ellen-on*
	ART.DEF-NOM.PL	2.CARD	5.CARD-NOM.PL	ART.DEF-GEN.PL	Greek-GEN.PL
	'2/5 of the Greeks'				

Modern Standard Arabic partitive expressions in (36) and (37) also involve indivisible fraction names; an article is required on the nouns.

(36)	*ḫumusay*	*aš-ša ʿb*
	5.FRAC.DU.ADJ	ART.DEF-people
	'2/5 of the people'	

official corpora and textbooks, and notably the work of Yan Fu (see Wang Shuhuai, 1969). There was also the Educational Association of China – also known as the Text-Books Series Committee – organized by the Protestants in 1877 (see Wang Yangzong1991). Kaske (2007) gives an extensive survey – notwithstanding the current restrictions to access to the records of the late Qing administration and the Republic of China, provided that the relevant ones were not destroyed – but it does not include the standardization of mathematical expressions.

(37) *ṯālṯat ʾaḥmās aš-šaʿb*
3.CARD.ADJ 5.FRAC.PL ART.DEF-people
‘3/5 of the people’

In Contemporary Chinese, the sequences “denominator’s name + *fēn zhī* + numerator’s name” are also indivisible semantic units. They can occur before or after what they quantify, directly juxtaposed or linked with the determination particle *de*. Examples of juxtaposition are given in (38) with a measure word and in (39) with a noun:

(38) *wǔ fēn zhī èr shēng*
5.CARD *fēn zhī* 2.CARD litre
‘2/5 [of a] litre’

(39) *wǔ fēn zhī èr rénkǒu*
5.CARD *fēn zhī* 2.CARD population
‘2/5 [of the] population’

Examples of the use of the determination particle *de* are given in (40) and (41). The order “fraction name + *de* + noun” in (40) leaves the noun in the head position while the order “noun + *de* + fraction name” in (41) puts the fraction in the head position, slightly changing the focus of the noun phrase.

(40) *wǔ fēn zhī èr de rénkǒu*
5.CARD *fēn zhī* 2.CARD DET population
‘2/5 of the population’

(41) *rénkǒu de wǔ fēn zhī èr*
population DET 5.CARD *fēn zhī* 2.CARD
‘2/5 of the population’

In Chinese, the shift from divisible to indivisible bi-dimensional expressions made it possible for fraction names to be used in the same manner as integers when they occurred in quantification phrases.

Actually, separable numerical expressions are known in English and Latin; however, but they concern mixed numbers, not proper fraction names. For example, English in (42) is a

specific situation which does not rely on an indivisible expression for the mixed number 1½; but note the common usage ‘one and a half hours’ with a slightly shifted emphasis.

(42)	*an*	*hour*	*and*	*a*	*half*
	ART.SG	hour[SG]	and	ART.SG	{1/2}

Latin, in (43) and (44) displays mixed numbers and sums of fractions with repetition of the measure words.

(43)	hab-et	diametr-i
	have-IND.PRS.3SG	diameter-GEN.M.SG

‘have a diameter of

digit-um	un-um	et	trientem	digit-i
digit-ACC.M.SG	1.CARD-ACC.M.SG	and	{1/3}.ACC.M.SG	digit-GEN.M.SG

1 1/3 digits of diameter’ (or ‘have a diameter of one digit and digit’s third’)
(*De aquae ductu urbis Romae* [*Water management of the city of Rome*], 1.26)

(44)	hab-et	diametr-i
	have-IND.PRS.3SG	diameter-GEN.M.SG

‘have a diameter of 1 1/24 1/72 digits’ (or ‘have a diameter of

digit-um	un-um	et
digit-ACC.M.SG	1.CARD-ACC.M.SG	and

1 digit and a digit’s 1/24 plus 1/72’)

digit-i	sescunci-am	sextul-am
digit-GEN.M.SG	{1/24}-ACC.F.SG	{1/72}-ACC.F.SG

(*De aquae ductu urbis Romae* [*Water management of the city of Rome*], 26.6)

4. CONCLUSION

The words or phrases expressing fractions in one language were rarely considered in the field of linguistics. They can take their origin in contacts and loans from a literary language. They can be standardized or freely made up on the spot. At first glance, the variety of the means used to shape mono- and bi-dimensional phrases seems to defy attempts of classification. Therefore, we

propose a typology of fractional numbers which is precisely independent from the morpho-syntactic intricacies specific to each language: suppletive forms vs analytical forms (further divided into mono- or bi-dimensional).

In a given language, a finite list of fractions can be expressed with suppletive forms. Analytic mono-dimensional phrases stating only the denominator or the numerator can also be observed, this scheme is limited to a few series of fractions. For example, the infinite sets of unit fractions 1/*n* in Sanskrit, Latin, and in the Chinese language of the 3rd and 2nd centuries BCE, all state the denominators only. Also, the phrases *n chéng* in Contemporary Chinese expressing the finite sets of *n*/10 with *n* an integer from 1 through 9 mention only the numerators, and the fractions 2/3 up to 2/9 in Arabic.

However, a *generic* pattern to express fractional numbers is achieved with analytical bi-dimensional phrases accounting lexically for both the numerator and the denominator; these bi-dimensional patterns of formation can potentially express any fraction of two integers.

This typology is resistant to the cross-linguistic variety of morpho-syntactic data. We describe in this paper that not all languages rely on the ordinal form or other special form of the denominator; cardinals and a given structure or order for the constituents can suffice. Moreover, the order of the constituents can put the name of the numerator in first position (this is the case in Arabic, English, French, German, Modern Greek, Hebrew, Latin, Spanish, Swedish, Thai, etc.), or in second position (e.g. in Chinese, Ancient Greek, Japanese, Japhug, Korean, Tibetan, etc.). Sanskrit had instances of both orders.

The cross-linguistic variety of the partitive expressions with fractions is due to the cross-linguistic diversity of the partitive expressions themselves, and this is not relevant to this paper. However, whether bi-dimensional phrases for fractional numbers are indivisible semantic units or not is a characteristic which becomes apparent only when they are part of partitive expressions. In Chinese for example, they are indivisible units today, but they were divisible expressions in former times, with the measure words positioned between the denominator and the numerator. For that reason, whenever enough data is accessible, not only the details of the formation of fractional numbers, but also their syntactic insertion in partitive expressions deserve to accounted for.

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