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Active vibration control of axisymmetric membrane through partial pole placement

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Abstract: This paper deals with the problem of active vibration control for a flexible axisymmetric membrane. This mechanical system is equipped with two piezoelectric circular chips where one of them works as an actuator, whereas the other is used as a sensor. Both are glued on the membrane, one on each side, and centered according to its axis of symmetry. The model of this system is obtained from a finite element modelling, leading to a linear state-space model. The design of the proposed controller is based on delayed proportional actions. We exploit a property called *Coexisting Real Roots inducing Dominancy* to an assignment of spectral values in an appropriate sector corresponding to a desired damping. The aim of this work is to examine the performances of the proposed output feedback controller in terms of vibration damping of the main observable and controllable vibrating modes.

Keywords: Time-delay, infinite dimension system, quasipolynomial, partial pole placement, spectral abscissa, active vibration damping.

1. INTRODUCTION

In recent studies Boussaada et al. [2019], a property called *Multiplicity-Induced-Dominancy* has been introduced in the framework of pole placement for delayed systems. It is worth mentioning that multiple spectral values for time-delay systems can be characterized by using a Birkhoff/Vandermonde-based approach, refer for instance to Boussaada and Niculescu [2016b,a, 2014], Boussaada et al. [2016]. Moreover, it has been emphasized that the admissible multiplicity of the real spectral values is bounded by the generic *Polya and Szegő bound* (denoted B_{PS}), which corresponds to the *degree* of the associated quasipolynomial, *i.e.* the number of the involved polynomials plus their degree minus one, refer for instance to [Pólya and Szegő, 1972, Problem 206.2, page 144 and page 347]. Such a bound has been recovered using structured matrices in Boussaada and Niculescu [2016a] rather than the *Principle Argument* as done in Pólya and Szegő [1972].

One can note that the multiplicity of a root does not really matter, but its relation with the dominancy of this root may be a useful tool for feedback control design. An analytical proof of the dominancy of a spectral value for the scalar equation with a single delay has been proposed and discussed in the 50s, see Hayes [1950]. Then, the dominancy property has been deeply studied and analytically proven for scalar delay equations in Boussaada et al. [2016]. Next, for second-order systems controlled by a delayed proportional controller, this property has been exploited in Boussaada et al. [2018b, 2017] for the active damping of the vibrations in a flexible beam. In Boussaada and Niculescu [2018], Boussaada et al. [2018a], the dominancy property has been parametrically characterized and proven using the *Argument Principle* in the special case of the delayed proportional-derivative controller.

Finally, in Boussaada et al. [2018a], Boussaada and Niculescu [2018], an analytical proof for the dominancy of the spectral value with maximal multiplicity for second-order systems controlled via a delayed proportional-derivative controller has been discussed. Recently, in Amrane et al. [2018] it is shown that under appropriate conditions the coexistence of exactly B_{PS} distinct negative zeros of quasipolynomial of reduced degree guarantees the exponential stability of the zero solution of the corresponding time-delay system. The dominancy of such real spectral values is shown using an extended factorization technique which generalizes the one proposed in Amrane et al. [2018]. More precisely, in Bedouhene et al. [2020], the framework of partial pole placement has been extended to real spectral values which are not necessarily multiple. The effect of the coexistence of such non oscillatory modes on the asymptotic stability of the obvious solution has then been investigated. In reality, the coexistence of B_{PS} real spectral values makes them the rightmost-roots of the associated quasipolynomial. As a matter of fact, the negativity of such assigned real roots guarantees the asymptotic stability of the obvious solution. These results has given rise to a new control approach for the design of an output feedback controller, leading to a control law composed with proportional and delayed proportional terms. Such a controller will be called in the sequel a *Partial Pole Placement (PPP)* controller.

Furthermore, if the number of coexistent real spectral values reaches the B_{PS} , then a necessary and sufficient condition for the asymptotic stability is provided (which is equivalent to the exponential stability [Kolmanovskii and Nosov, 1986, p79]), see also Mori et al. [1982] for an estimate of the exponential decay rate for stable linear delay systems. Notice also that the proposed constructive approach, which consists in providing an appropriate factorization of a given quasipolynomial func-

tion and then to focus on the location of zeros of one of its factors, gives further insights on such a qualitative property. Namely, it furnishes the exact exponential decay rate rather than just counting the number of the quasipolynomial roots on the left-half plane as may be done by using the principle argument, refer for instance to Stépán [1989].

Moreover, it was stressed in Niculescu et al. [2010] that, in some cases, time-delay has a *stabilizing effect* on the closed-loop system. Indeed, the closed-loop stability is guaranteed by the existence of the delay. Also, in Niculescu and Michiels [2004], it has been proven that a chain of n integrators can be stabilized using n distinct delays. The noticeable interest of considering delayed control laws lies in the simplicity of the controller as well as in its easy practical implementation.

The problem under consideration and the proposed approach give rise to an *exponential decay assignment* method using only two parameters: a "gain" and a "delay". Note that the idea of using roots assignment for a controller design of time-delay system is not a novelty. For instance, in Manitius and Olbrot [1979] a feedback law has lead to a finite spectrum assignment for the closed-loop system. Such an assignment could be located at an arbitrary preassigned set of points in the complex plane. For the systems with delayed input, a necessary and sufficient condition for finite spectrum assignment has been obtained. One should note that the resulting feedback law involves integrals over the past control. In case of delays in state variables, it has been shown that a technique based on the finite Laplace transform leads to a constructive design procedure. The resulting feedback consists in a proportional and (finite interval) integral terms over present and past values of state variables. In Loiseau [2001], a similar finite pole placement for time-delay systems with commensurate delays has been proposed. Feedback laws defined in terms of Volterra equations have been obtained thanks to the properties of the Bezout ring of operators including derivatives, localized and distributed delays. Other analytical/numerical placement methods for retarded time-delay systems are proposed in Michiels et al. [2002], Mondie and Loiseau [2001], Zitek et al. [2013].

In this work, the methodology described in Amrane et al. [2018] is applied in order to reduce the peaks of resonance of the first three vibration modes for a flexible axisymmetric membrane, firstly introduced for control purposes in Tliba [2011].

The remaining paper is organized as follows: in Section 2, the axisymmetric membrane under consideration is presented and its modelling is briefly explained. Section 3 provides a description of the PPP-delayed controller design. Some numerical simulations illustrating the efficiency of the proposed method are presented in Section 4. Finally, the paper ends with some concluding remarks.

2. SYSTEM DESCRIPTION AND ITS MODELLING

The system under consideration is depicted in Fig. 1. It is a thin metallic membrane with a clamped circular edge. This disc is embedded into a mobile support that only moves along the z axis. The support is submitted to an unknown acceleration, noted $d(t)$ in the sequel. This flexible membrane is equipped with two ceramic-based piezoelectric chips. One of them is used as an actuator and the other is used as a

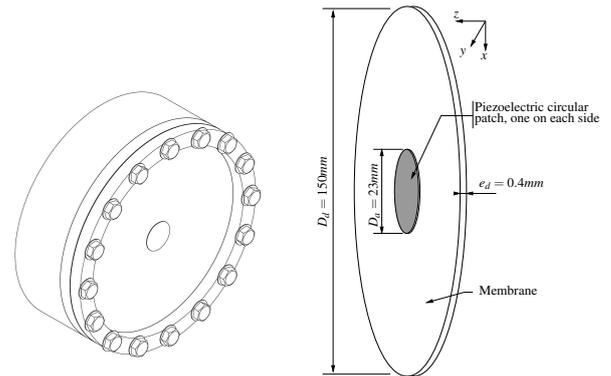


Fig. 1. Axisymmetric flexible membrane (dimensions on the right) inserted in the device which is subjected to vibrations (Computer Aided Design figure on the left).

sensor. The actuator's thickness is $0.4mm$. It is lower than the sensor's thickness which is $0.7mm$. The reason why should the thickness of a piezoelectric actuator must be lower than the sensor's one is given in Tliba [2011] and references therein. These circular chips are assumed to be perfectly bounded on the disc, one on each side, and centered according to the axis of symmetry of the disc. All the physical parameters of the materials used here can be found in Tliba [2011] and references therein. The dimensions of the used elements are indicated in Fig. 1.

The control signal (input), noted $u(t)$, is the voltage applied across the piezoelectric actuator. The measured output, noted $y(t)$, is the electric voltage across the piezoelectric sensor. The disturbance signal $d(t)$ is the total acceleration applied to the clamped circular edge of the structure. The controlled output that is considered in this paper, noted $z(t)$, is the z component (see Figure 1) of the acceleration for a particular point on the structure. This point is located at the center of the disc and on the upper side of the piezoelectric chip used as a sensor.

The mathematical modeling of the dynamical behavior for such mechanical structures is based on several coupled Partial Differential Equations (PDEs) in space and time-dependant variables. See for example Tliba [2011] and references therein, for more details that are out of the scope of this paper. Such equations, that are of infinite dimension, are usually addressed through numerical methods like the Finite Element Modeling (FEM) Petyt [2010]. This method leads to a set of Ordinary Differential Equations (ODEs) that are linear but with a very great number of variables. Because of the numerical feature of this method, it is worth at mentioning that the physical parameters of the PDEs are dissolved within numerical coefficients of the corresponding ODEs.

An accurate model in state-space form is derived from the set of ODEs thanks to a model reduction method called *modal analysis*. It describes the inputs-to-outputs dynamical behavior in the bandwidth of interest, say $[0 - 4000Hz]$ for our system. This model, called the *analysis model*, is of order 12 and includes only controllable and observable modes. A reduced-order model, dedicated to the controller design, is obtained from the analysis one thanks to a reduction method based on state-variable truncation. It is called the *synthesis model*. It is of order 6 and includes the first three vibration modes that are controllable and observable. Let $\eta_p \in \mathbb{R}^{n_p}$ be the state vector of the system, with $n_p = 12$ in the case of the analysis model

and with $n_p = 6$ for the synthesis model. Both models can be described by the linear state-space model in (1), where the details of the matrices ($\mathcal{A}_p, \mathcal{B}_{p,d}, \dots$) are voluntarily omitted in this text since they are not of special interest here (see Tliba [2011] and references therein for more details). Note that, in the case of the synthesis model, there are non-zero feedthrough terms between the inputs d and u and the outputs z and y , whereas these feedthrough terms are equal to zero in the case of the analysis model. Figure 2 shows the geometrical shapes of the vibration modes in the synthesis model.

$$\mathcal{S} \begin{cases} \dot{\eta}_p(t) = \mathcal{A}_p \eta_p(t) + \mathcal{B}_{p,d} d(t) + \mathcal{B}_{p,u} u(t) \\ z(t) = \mathcal{C}_{p,z} \eta_p(t) + \mathcal{D}_{p,zd} d(t) + \mathcal{D}_{p,zu} u(t) \\ y(t) = \mathcal{C}_{p,y} \eta_p(t) + \mathcal{D}_{p,yd} d(t) + \mathcal{D}_{p,yu} u(t) \end{cases} \quad (1)$$

The control input u is a scalar signal, as well as the measured output signal. Then, the considered flexible membrane is a single-input-single-output system. The disturbance input d is the total acceleration applied to the clamped edge of the structure. The controlled output z is the vertical total acceleration of the membrane center. The frequency responses of the full and reduced order models, from the input d to the output $d + z$, are shown in Fig. 3. In order to comply with the framework of models used in our controller design procedure, the state-space model in (1) is converted into a model based on transfer functions. This is achieved by applying the Laplace Transform to the equations in (1). By denoting the Laplace variable by s , the model based on transfer functions is given by

$$\mathcal{S} \begin{cases} z(s) = \frac{N_{dz}(s)}{\Psi(s)} d(s) + \frac{N_{uz}(s)}{\Psi(s)} u(s), \\ y(s) = \frac{N_{dy}(s)}{\Psi(s)} d(s) + \frac{N_{uy}(s)}{\Psi(s)} u(s). \end{cases} \quad (2)$$

These polynomials are written as

$$N_{dz}(s) := \sum_{k=0}^{n_p} n_{dz_k} s^k,$$

$$N_{uz}(s) := \sum_{k=0}^{n_p} n_{uz_k} s^k,$$

$$N_{dy}(s) := \sum_{k=0}^{n_p} n_{dy_k} s^k,$$

$$N_{uy}(s) := \sum_{k=0}^{n_p} n_{uy_k} s^k$$

$$\text{and } \Psi(s) := \sum_{k=0}^{n_p} a_k s^k,$$

where $a_{n_p} := 1$ is imposed for simplicity. The numerical values of the polynomials' coefficients for the synthesis model in (2) are given in Table 1.

3. CONTROL DESIGN

The main goal of the sought controller is to reduce the peaks of resonance for the first three controllable/observable modes, by using an *output feedback controller*, without making unstable the vibration modes that are not in the synthesis model. By using the same notations as in Tliba et al. [2019], the considered system is inserted in the output feedback control structure of Fig. 4, where the reference signal is always equal to zero. A rectangular impulse signal is used for the disturbance input

n_{dz_6}	$-1.14828 \cdot 10^0$	n_{uz_6}	$-1.0868 \cdot 10^0$
n_{dz_5}	$-1.2958 \cdot 10^2$	n_{uz_5}	$-1.2264 \cdot 10^2$
n_{dz_4}	$-5.9240 \cdot 10^7$	n_{uz_4}	$-4.1250 \cdot 10^7$
n_{dz_3}	$-3.0537 \cdot 10^9$	n_{uz_3}	$-2.0051 \cdot 10^9$
n_{dz_2}	$-7.5305 \cdot 10^{14}$	n_{uz_2}	$-2.2105 \cdot 10^{14}$
n_{dz_1}	$-5.3167 \cdot 10^{15}$	n_{uz_1}	$-2.3803 \cdot 10^{15}$
n_{dz_0}	$5.3049 \cdot 10^{14}$	n_{uz_0}	$-1.0173 \cdot 10^{20}$
n_{dy_6}	$-1.4896 \cdot 10^{-3}$	n_{uy_6}	$8.3825 \cdot 10^{-2}$
n_{dy_5}	$-1.6809 \cdot 10^{-1}$	n_{uy_5}	$9.4592 \cdot 10^0$
n_{dy_4}	$7.5073 \cdot 10^4$	n_{uy_4}	$5.8545 \cdot 10^6$
n_{dy_3}	$-1.1243 \cdot 10^6$	n_{uy_3}	$2.6728 \cdot 10^8$
n_{dy_2}	$-3.2099 \cdot 10^{12}$	n_{uy_2}	$5.4039 \cdot 10^{13}$
n_{dy_1}	$4.6902 \cdot 10^{13}$	n_{uy_1}	$5.0870 \cdot 10^{13}$
n_{dy_0}	$1.7174 \cdot 10^{19}$	n_{uy_0}	$2.9349 \cdot 10^{19}$
a_5	$1.1284 \cdot 10^2$	a_4	$6.5167 \cdot 10^7$
a_3	$2.9844 \cdot 10^9$	a_2	$5.7253 \cdot 10^{14}$
a_1	$5.4259 \cdot 10^{15}$	a_0	$3.0464 \cdot 10^{20}$

Table 1. Numerical values for the polynomials' coefficients in the case of the synthesis model.

d . The control problem consists in reducing the vibrations generated by the first three modes, when the mobile support imposes a shock to the whole flexible membrane. Let us define the output feedback control law in frequency domain by

$$u(s) := K(s, \tau) y(s), \quad (3)$$

where $K(s, \tau)$ stands for the PPP controller. This last is given in Laplace domain by

$$K(s, \tau) := \frac{Q(s, \tau)}{P(s, \tau)} \quad (4)$$

where

$$Q(s, \tau) := q_0 + q_{r_0} e^{-\tau s},$$

$$\text{and } P(s, \tau) := p_0 + p_{r_0} e^{-\tau s}.$$

After applying the inverse Laplace transform, this control law given in time-domain is

$$u(t) = -\frac{p_{r_0}}{p_0} u(t - \tau) + \frac{q_0}{p_0} y(t) - \frac{q_{r_0}}{p_0} y(t - \tau) \quad (5)$$

which is an output feedback control law based on proportional actions plus delayed proportional actions.

Proposition 1. *There exists a set of parameters $q_0, q_{r_0}, p_0, p_{r_0}$ and $\tau > 0$ such that the controller (4) allows to assign three negative dominant poles (spectral values) λ_1, λ_2 and λ_3 , for the system (2) in closed-loop with the control law (3).*

Sketch of the proof. *The first assertion of the proposition is related to a problem of quasipolynomial interpolation, that is, one has four free parameters and three interpolating points, this guarantees that the interpolation problem is poised. Next, the dominancy of the assigned poles is guaranteed by establishing an integral factorization of the quasipolynomial in the spirit of the ones given for instance in Amrane et al. [2018], Bedouhene et al. [2020], see also Tliba et al. [2019].*

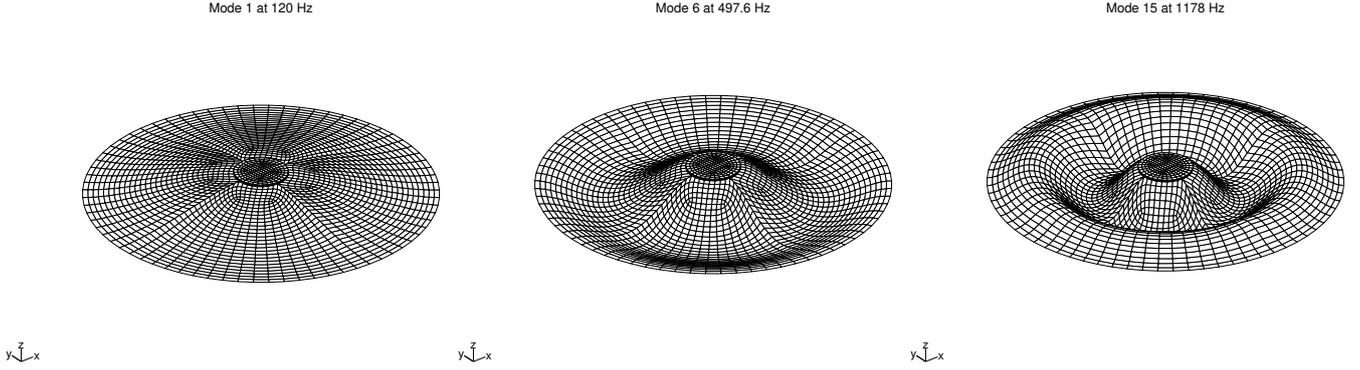


Fig. 2. First three controllable and observable vibration modes.

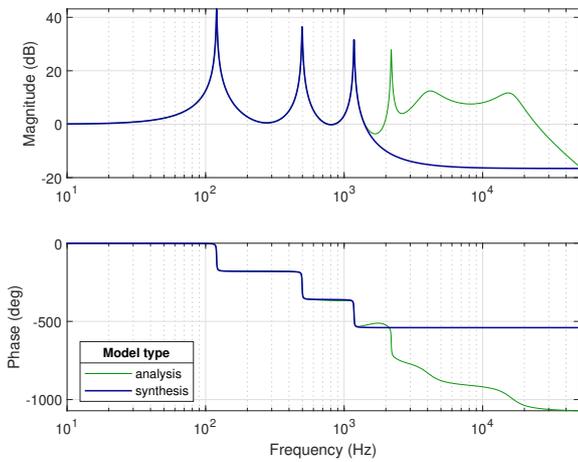


Fig. 3. Frequency responses of the analysis (thin-green) and synthesis (thick-dark blue) models in the case of the transfer function from d to z .

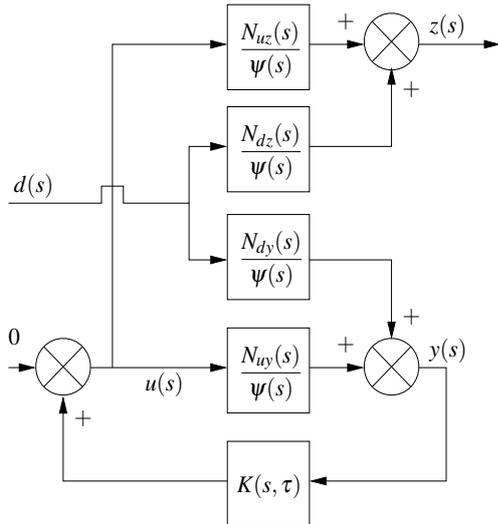


Fig. 4. Feedback control structure.

4. SIMULATION RESULTS

The main issue of the proposed method relies on the choice of the closed-loop dominant poles. As for the pole placement in the linear, but finite dimensional framework, this is often

empirical. One clever way is to keep the magnitude of the open-loop dominant pole, in order to get a closed-loop control signal with a reasonable amplitude. It reduces the risk of input saturation due to actuator's limitation while ensuring an efficient damping.

The PPP method gives the following numerical values for the parameters of the controller in (4) that assigns $\lambda_1 = -500$, $\lambda_2 = -550$ and $\lambda_3 = -600$ as the dominant roots of the characteristic polynomial for $p_0 \simeq 4.4611$, $p_{r0} \simeq 3.0515$, $q_0 \simeq 41.290$, $q_{r0} \simeq 36.403$ and $\tau \simeq 3.360 \cdot 10^{-4}$.

One should pay attention to the value of the delay. This last should be compatible with the sampling capabilities of some real-time digital standard facilities. In the case of the considered system, the last kept mode in the synthesis model, with a resonant frequency close to 1 kHz, imposes a sampling time lower than 0.1 millisecond. In consequence, the value of the delay can then be considered acceptable with respect to this specification.

Figure 5 shows the distribution of the closed-loop poles, with a focus around the three assigned poles λ_1 , λ_2 and λ_3 . To

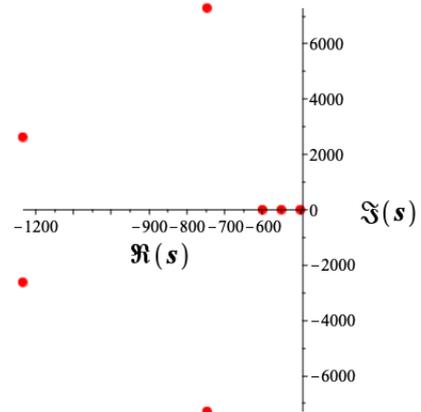


Fig. 5. Distribution of the closed-loop poles when $p_0 \simeq 4.4611$, $p_{r0} \simeq 3.0515$, $q_0 \simeq 41.290$, $q_{r0} \simeq 36.403$, $\tau \simeq 3.360 \cdot 10^{-4}$.

show the performances of the proposed PPP-controller, the time responses of both output signals in open-loop (blue) and in closed-loop (orange) are depicted in Fig. 6, when the disturbance d is a rectangular impulse (black), modelling a very brief shock imposed to the whole flexible structure. The time response of the control signal u is also plotted in Fig. 6.

A peak of magnitude roughly equal to 12 V can be observed, a value that is quite realistic for this kind of application.

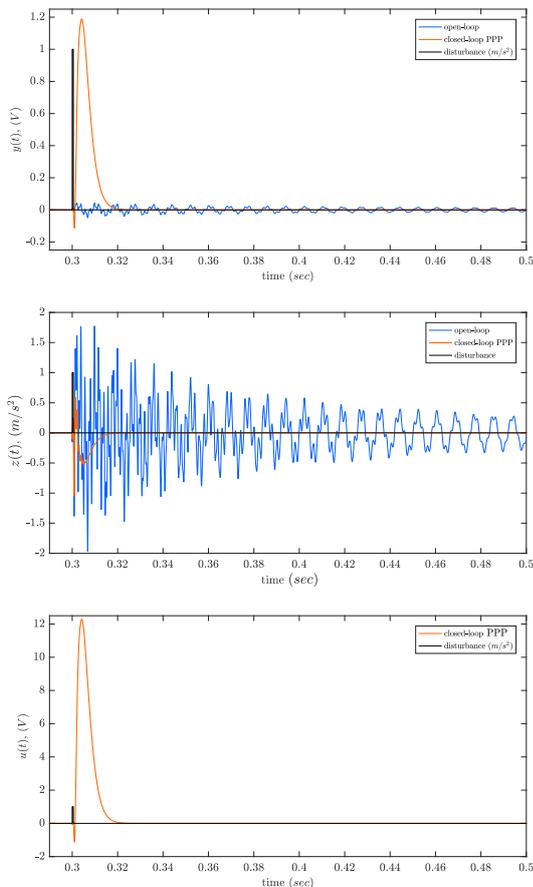


Fig. 6. Time responses of the measured output y at the top, of the controlled output z at the middle and of the closed-loop control signal u at the bottom.

5. CONCLUSIONS

This work has shown the interest to use time-delay in a controller as a design parameter. The practical application that we have considered is a flexible axisymmetric membrane equipped with a piezoelectric sensor and a piezoelectric actuator in a collocated configuration. For this system, it was expected to achieve an active vibration control thanks to an output feedback controller. This controller has been designed by using the partial pole placement approach in order to ensure the damping of the first three observable and controllable vibrating modes. The output feedback feature opens the possibility to implement such a controller in an experimental test bench. An extended version of this note will be available soon with the complete proofs.

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