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On the number of singular points of plane curves *

S. Orevkov and M. Zaidenberg

Abstract

This is an extended, renovated and updated report on our joint work [OZ]. The main result is an inequality for the numerical type of singularities of a plane curve, which involves the degree of the curve, the multiplicities and the Milnor numbers of its singular points. It is a corollary of the logarithmic Bogomolov-Miyaoka-Yau's type inequality due to Miyaoka. It was first proven by F. Sakai at 1990 and rediscovered by the authors independently in the particular case of an irreducible cuspidal curve at 1992. Our proof is based on the localization, the local Zariski-Fujita decomposition and uses a graph discriminant calculus. The key point is a local analog of the BMY-inequality for a plane curve germ. As a corollary, a boundedness criterium for a family of plane curves has been obtained. Another application of our methods is the following fact: a rigid rational cuspidal plane curve cannot have more than 9 cusps.

This is an extended, renovated and updated report on our joint work [OZ] which the second named author presented at the Conferencè on Algebraic Geometry held at Saitama University, 15-17 of March, 1995. It is his pleasure to thank Professor F. Sakai who invited him to take part in this nice conference.

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1 Asymptotics of the number of ordinary cusps

We start with a brief survey of known results in the simplest case of ordinary cusps.

It is well known that for a nodal plane curve $D \subset \mathbf{P}^2$ of degree d the number of nodes can be an arbitrary non-negative integer allowed by the genus formula, i.e. any integer from the interval $[0, \binom{d-1}{2}]$. If D is a Plücker curve with only ordinary cusps as singularities, which has κ cusps, then still

$$\kappa \leq \binom{d-1}{2},$$

but this time the inequality is strict starting with $d = 5$. Indeed, by Plücker formulas

$$0 < d^* = d(d-1) - 3\kappa$$

where d^* is the class of D , and

$$0 \leq f = 3d(d-2) - 8\kappa$$

where f is the number of inflexion points of D . Thus, we have

$$\kappa < \frac{1}{3}d(d-1) \tag{1}$$

and

$$\kappa \leq \frac{3}{8}d(d-2), \tag{2}$$

which is strictly less than $\binom{d-1}{2}$ for $d \geq 5$.

Note that $d \leq 4$ for a rational cuspidal Plücker curve D , due to (2) and the genus formula. Therefore, up to projective equivalence there exists only two such curves, namely the cuspidal cubic and the Steiner three-cuspidal quartic (we suppose here that at least one cusp really occurs, otherwise we have to add also the line and the smooth conic).

From (2) it follows that

$$\limsup_{d \rightarrow \infty} \frac{\kappa}{d^2} \leq \frac{3}{8}. \tag{3}$$

Using the spectrum of singularity (or, equivalently, the Mixed Hodge Structures) A. Varchenko [Va] found an estimate

$$\limsup_{d \rightarrow \infty} \frac{\kappa}{d^2} \leq \frac{23}{72}, \quad (4)$$

which is better than (3) by $\frac{1}{18}$. Another $\frac{1}{144}$ was gained in the work of F. Hirzebruch and T. Ivinskis [H, Iv] by applying Miyaoka's logarithmic form of the Bogomolov-Miyaoka-Yau (BMY) inequality:

$$\limsup_{d \rightarrow \infty} \frac{\kappa}{d^2} \leq \frac{5}{16}. \quad (5)$$

Furthermore, in this work an elegant example was given which shows that

$$\limsup_{d \rightarrow \infty} \frac{\kappa}{d^2} \geq \frac{1}{4}, \quad (6)$$

where c means now the maximal number of cusps among all the cuspidal Plücker curves of degree d .

Example [H, Iv]. Starting with a generic smooth cubic C , consider its dual curve $D = C^*$, which is an elliptic sextic with nine ordinary cusps as the only singularities. Let $F = 0$ be the defining equation of D . Set $D_k = \{F(x^k : y^k : z^k) = 0\}$. Then D_k is again a cuspidal Plücker curve. It has degree $d_k = 6k$ and $\kappa_k = 9k^2$ cusps (indeed, $(x : y : z) \mapsto (x^k : y^k : z^k)$ is a branched covering $\mathbf{P}^2 \rightarrow \mathbf{P}^2$ of degree k^2 ramified along the coordinate axes which meet D normally). Thus, here $\kappa_k = d_k^2/4$.

In fact, the lower bound $1/4$ can be improved, by a similar method, at least by $1/32$ (A. Hirano [Hi]). Together with (5) this yields

$$\frac{10}{32} \geq \limsup_{d \rightarrow \infty} \frac{\kappa}{d^2} \geq \frac{9}{32}, \quad (7)$$

which is still far away from giving the exact asymptotic. See also [Sa] for a discussion on what is known for small values of d .

2 The main inequality. Bounded families of plane curves

Next we consider, more generally, plane curves with arbitrary singularities. By a *cuspidal* we mean below a locally irreducible singular point. We say that

$D \subset \mathbf{P}^2$ is a *cuspidal curve* if all its singular points are cusps. The following theorem, which is the main result presented in the talk, was first proven by F. Sakai [Sa]. Independently and later it was also found by the authors in the special case of cuspidal curves [OZ] (actually, the proof in [OZ] goes through without changes for nodal-cuspidal curves, i.e. plane curves with nodes as the only reducible singular points). Both proofs are based on the logarithmic version of the BMY-inequality due to Miyaoka [Miy], but technically they are different.

Theorem 1. *Let D be a plane curve of degree d with the singular points P_1, \dots, P_s . Let μ_i resp. m_i be the Milnor number resp. the multiplicity of $P_i \in D$. If $\mathbf{P}^2 \setminus D$ has a non-negative logarithmic Kodaira dimension, then*

$$\sum_{i=1}^s \left(1 + \frac{1}{2m_i}\right) \mu_i \leq d^2 - \frac{3}{2}d. \quad (8)$$

In particular,

$$\sum_{i=1}^s \mu_i \leq \frac{2m}{2m+1} \left(d^2 - \frac{3}{2}d\right), \quad (9)$$

where $m = \max_{1 \leq i \leq s} \{m_i\}$.

Remarks. a) For $m \leq 3$ and D irreducible Theorem 1 had been proved by Yoshihara [Yo1,2], whose work stimulated the later progress.

b) For an irreducible plane curve D of degree $d \geq 4$ the logarithmic Kodaira dimension $\bar{\kappa}(\mathbf{P}^2 - D)$ is non-negative besides the case when D is a rational cuspidal curve with one cusp; see [Wak].

Corollary 1. *If $D \subset \mathbf{P}^2$ is an irreducible cuspidal curve of geometric genus g , then under the assumptions of Theorem 1 one has*

$$g \geq \frac{d^2 - 3(m+1)d}{2(2m+1)} + 1. \quad (10)$$

In particular, a family of such curves is bounded iff g and m are bounded throughout the family.

In general, the latter conclusion does not hold for non-cuspidal curves (indeed, the family of all the irreducible rational nodal plane curves is unbounded). However, it becomes true if one replaces the geometric genus $g = g(D)$ by the Euler characteristic $e(D)$ (thus, involving not only the topology of the normalization, but the topology of the plane curve itself). Moreover, in this form it works even for reducible curves.

Corollary 2. *Under the assumptions of Theorem 1 one has*

$$d(d - 3(m + 1)) \leq (2m + 1)(-e(D)) . \quad (11)$$

Therefore, a family of (reduced) plane curves is bounded iff the absolute value of the Euler characteristic and the maximal multiplicity of the singular points are bounded throughout the family.

The Corollary easily follows from Theorem 1 and the formula (see [BK])

$$\sum_{i=1}^s \mu_i = d(d - 3) + e(D) .$$

In the case of irreducible curves, in the estimate (11) it is convenient to use the first Betti number $b_1(D) = 2 - e(D)$ instead of the Euler characteristic. In particular, for irreducible nodal curves it is the only parameter involved.

An immediate consequence of (11) is that $d \leq 3m + 3$ if $e(D) \geq 0$. Furthermore, $d \leq 3m + 2$ if $e(D) > 0$; this is so, for instance, if D is a rational cuspidal curve. In fact, in the latter case $d < 3m$ [MaSa], and also by the genus formula

$$\sum_{i=1}^s \frac{\mu_i}{m_i} \leq 3d - 4$$

[OZ].

3 BMY-inequalities

These inequalities provide the basic tool in the proof of Theorem 1. Let $\sigma : X \rightarrow \mathbf{P}^2$ be the minimal embedded resolution of singularities of D , and let $\tilde{D} \subset X$ be the reduced total preimage of D . Thus, \tilde{D} is a reduced divisor of simple normal crossing type, and $\tilde{D} = \sigma^{-1}(D)$. Let $K = K_X$ be the

canonical divisor of X . If $\bar{k}(\mathbf{P}^2 \setminus D) = \bar{k}(X \setminus \tilde{D}) \geq 0$, i.e. if $|m(K + \tilde{D})| \neq 0$ for m sufficiently large, then (see [Fu]) there exists the Zariski decomposition $K + \tilde{D} = H + N$, where H, N are \mathbf{Q} -divisors in X such that

- i) the intersection form of X is negatively definite on the subspace $V_N \subset \text{Pic}X \otimes \mathbf{Q}$ generated by the irreducible components of N ;
- ii) H is nef, i.e. $HC \geq 0$ for any complete irreducible curve $C \subset X$;
- iii) H is orthogonal to the subspace V_N .

By (iii) we have

$$(K + \tilde{D})^2 = H^2 + N^2,$$

where $N^2 \leq 0$. Thus, $H^2 \geq (K + \tilde{D})^2$.

Theorem (Y. Miyaoka [Miy]; R. Kobayashi–S. Nakamura–F. Sakai [KoNaSa]).

a) If $\bar{k}(\mathbf{P}^2 \setminus D) \geq 0$, then

$$(K + \tilde{D})^2 \leq 3e(\mathbf{P}^2 \setminus D). \quad (12)$$

b) If $\bar{k}(\mathbf{P}^2 \setminus D) = 2$, then

$$H^2 \leq 3e(\mathbf{P}^2 \setminus D). \quad (13)$$

Remark. (13) holds, for instance, in the case when D is an irreducible curve with at least three cusps [Wak].

Next we describe an approach to the proof of Theorem 1, mainly following [OZ]. An advantage of this approach is that, in the particular case of irreducible cuspidal curves, we obtain formulas which express all the ingredients of the above BMY-inequalities in terms of the Puiseux characteristic sequences of the cusps. In fact, we prove a local version of Theorem 1 for the case of a cusp (see Theorem 2 in Section 11 below). Together with the BMY-inequality (12) this provides a proof of Theorem 1 in the cuspidal case. A similar local estimate participates in the proof in [Sa], which is, by the way, much shorter. Instead of the Puiseux data it deals with the multiplicity sequences of the singular points. Combining both approaches, we give in

Section 12 a proof of the local estimate for arbitrary singularity (Theorem 3), thus proving Theorem 1 in general case. Actually, this is the proof of F. Sakai, with more emphasize separately on the local and the global aspects.

In the final Section 13 we apply the methods developed in the previous sections for pushing forward in the rigidity problem for rational cuspidal plane curves (see [FZ1,2]).

4 Localization

Let, as above, $D \subset \mathbf{P}^2$ be a plane curve of degree d and let $\sigma : X \rightarrow \mathbf{P}^2$ be the minimal embedded resolution of the singular points P_1, \dots, P_s of D . Let D' be the proper preimage of D in X , $\tilde{D} = D' \cup E$ be the reduced total preimage of D and $E = E_1 \cup \dots \cup E_k$, $E_i = \sigma^{-1}(P_i)$, be the exceptional divisor of σ . Let $E_i = \sum_{j=1}^{k_i} E_{ij}$ be the decomposition of E_i into irreducible components. Fix also a line $L \subset \mathbf{P}^2$ which meets D normally; denote by L' the proper preimage of L in X . Then, clearly, $\{E_{ij}\}$ and L' form a basis of the vector space $\text{Pic}X \otimes \mathbf{Q}$. Let $V_i = V_{E_i}$ be the subspace generated by the irreducible components E_{ij} of E_i and $V_{L'}$ be the one-dimensional subspace generated by L' in $\text{Pic}X \otimes \mathbf{Q}$. Since the intersection form of X is non-degenerate, we have the orthogonal decomposition

$$\text{Pic}X \otimes \mathbf{Q} = V_{L'} \oplus \left(\bigoplus_{i=1}^s V_i \right).$$

Therefore, for each $i = 1, \dots, s$ there exists the unique orthogonal projection $\text{Pic}X \otimes \mathbf{Q} \rightarrow V_i$, and also such a projection onto the line $V_{L'}$. For any \mathbf{Q} -divisor Z denote by $Z_{L'}$ resp. Z_i its projection into $V_{L'}$ resp. into V_i . Then we have

$$Z^2 = Z_{L'}^2 + \sum_{i=1}^s Z_i^2.$$

In particular, since $K_{L'} + \tilde{D}_{L'} = (d-3)L'$ we have

$$(K + \tilde{D})^2 = (d-3)^2 + \sum_{i=1}^s (K_i + \tilde{D}_i)^2,$$

where the summands in the last sum are all negative (indeed, E_i being an exceptional divisor, the intersection form of X is negatively definite on the

subspace V_i). It is easily seen that (8) follows from (12) and the local estimates

$$-(K_i + \tilde{D}_i)^2 \leq (1 - \frac{1}{m_i})\mu_i. \quad (14)$$

In what follows we trace a way of proving (14). This is done in particular case of an irreducible singularity in Section 11 (Theorem 2), and in general in Section 12 (Theorem 3). Note that the assumption of local irreducibility is important only in Sections 9, 10, 11.

5 Weighted dual graph

Let $E = E_1 \cup \dots \cup E_k$ be a curve with simple normal crossings in a smooth compact complex surface X . Assume, for simplicity, that all the irreducible components E_i of E are rational curves and that their classes in $\text{Pic}X \otimes \mathbf{Q}$ are linearly independent. Let A_E be the matrix of the intersection form of X on the subspace $V_E = \text{span}(E_1, \dots, E_k) \subset \text{Pic}X \otimes \mathbf{Q}$ in the natural basis E_1, \dots, E_k (we denote by the same letter a curve and its class in $\text{Pic}X \otimes \mathbf{Q}$). Then A_E is at the same time the incidence matrix of the dual graph Γ_E of E , which is defined as follows. The vertices of Γ_E correspond to the irreducible components E_i of E ; two vertices E_i and E_j , where $i \neq j$, are joint by a link $[E_i, E_j]$ iff $E_i \cdot E_j > 0$. The weight of the vertex E_i is defined to be the self-intersection index E_i^2 .

Let C be another curve in X which meets E normally. Then we consider also the dual graph $\Gamma_{E,C}$ of E near C ; it is the graph obtained from Γ_E by attaching $E_i \cdot C$ arrowheads to the vertex E_i , $i = 1, \dots, k$. We denote by ν_i resp. $\tilde{\nu}_i$ the valency of E_i in Γ_E resp. in $\Gamma_{E,C}$.

By a twig of a graph Γ one means an extremal linear branch of Γ ; its end point is called the tip of the twig.

6 Local Zariski–Fujita decomposition

Since in the sequel we are working only locally over a fixed singular point $P = P_i$ of D , we change the notation. Omitting subindex i , from now on we denote by E the corresponding exceptional divisor E_i and by V_E the corresponding subspace V_i . Thus, K_E, \tilde{D}_E, D'_E etc. mean the projections

$K_i, \tilde{D}_i, D'_i \dots$ of the corresponding divisors into $V_E = V_i$. Set also $\mu = \mu_i$ and $m = m_i$. Note that in this case the dual graph Γ_E is a tree.

By the local Zariski-Fujita decomposition we mean the decomposition

$$K_E + \tilde{D}_E = H_E + N_E,$$

where $H_E, N_E \in V_E$ are effective \mathbf{Q} -divisors such that

i) the support of N_E coincides with the union of all the twigs of Γ_E which are not incident with the proper preimage D' of D in X , i.e. all the twigs of $\Gamma_{E, D'}$ without arrowheads, and

ii) H_E is orthogonal to each irreducible component of $\text{supp } N_E$.

Note that all the twigs in the $\text{supp } N_E$ are *admissible*, i.e. all their weights are ≤ -2 . Using non-degeneracy of the intersection form on an admissible twig, T. Fujita [Fu, (6.12)] proved that there exists the unique such decomposition. Moreover, he proved that up to certain exceptions the global Zariski decomposition $K + \tilde{D} = H + N$ provides the local one via the projection (see [Fu, (6.20-6.24); OZ, Theorem 1.2]). Here we do not use this result, and so we do not give its precise formulation.

What we actually use is the equality

$$(K_E + \tilde{D}_E)^2 = H_E^2 + N_E^2.$$

According to [Fu, (6.16); OZ, 1.1, 2.4], the latter summands can be computed in terms of the weighted graph $\Gamma_{E, D'}$. This is done in the next section.

7 Graph discriminants and inductances

By definition, the *discriminant* $d(\Gamma)$ of a weighted graph Γ is $\det(-A)$, where A is the incidence matrix of Γ (or the intersection matrix of E , if $\Gamma = \Gamma_E$). It is easily seen that $d(\Gamma_E) = 1$, because in our case E is a contractible divisor.

The *inductance* of a twig T of Γ is defined as

$$\text{ind}(T) = \frac{d(T - \text{tip}(T))}{d(T)}.$$

Denote by T_1, \dots, T_k the twigs of Γ_E which are not incident with D' , i.e. the twigs of $\Gamma_{E, D'}$. Then we have

Lemma 1 [Fu, (6.16)].

a)

$$-N_E^2 = \sum_{i=1}^k \text{ind}(T_i). \quad (15)$$

b) Let v_T be the first vertex of $T = T_i$ ($1 \leq i \leq k$), i.e. the vertex of T opposite to the tip of T . Then the coefficient of v_T in the decomposition of the divisor N_E is equal to $1/d(T)$.

Since the graph Γ_E is a tree, for given vertices E_i and E_j (not necessary distinct) there is the unique shortest path in Γ_E which joins them. Denote by Γ_{ij} the weighted graph obtained from Γ_E by deleting of this path together with the vertices E_i and E_j themselves (and, of course, with all their incident links). So, in general the graph Γ_{ij} is disconnected.

Let $B_E = (b_{ij}) = A_E^{-1}$ be the inverse of the intersection matrix A_E . The following formula can be easily obtained by applying the Cramer rule.

Lemma 2 [OZ, (2.1)].

$$b_{ij} = -d(\Gamma_{ij}). \quad (16)$$

Recall that $\bar{\nu}_i$ resp. ν_i denotes the valency of the vertex E_i of the graph $\Gamma_{E, D'}$ resp. Γ_E and $\mu = \mu_i$ denotes the Milnor number of the singular point $P = P_i \in D$. We have

Lemma 3 [OZ, (2.4), (2.7), (4.1)]. *In the notation as above*

$$H_E^2 = \sum_{\bar{\nu}_i > 2, \bar{\nu}_j > 2} b_{ij} c_i c_j, \quad (17)$$

where

$$c_i = (\bar{\nu}_i - 2) - \sum \frac{1}{d(T_j)}$$

and the last sum is taken over all the twigs T_j which are incident with the vertex E_i ;

$$(K_E + E)^2 = -2 - \sum_{i=1}^n b_{ii}(\nu_i - 2), \quad (18)$$

and ¹

$$\mu = 1 - \sum_{i,j} b_{ij}(\bar{\nu}_i - 2)(\bar{\nu}_j - \nu_j). \quad (19)$$

The proof of (17) is based on the Adjunction Formula and Lemmas 1,2; (18) is proven by induction on the number of blow-ups; (19) follows from the adjunction formula and the formula

$$\mu = 1 - D'_E(K_E + \tilde{D}_E). \quad (19')$$

8 Calculus of graph discriminants

To use the formulas from the preceding section we have to compute the entries b_{ij} of the inverse $B_E = A_E^{-1}$, where A_E is the intersection matrix of the exceptional divisor E ; that means to compute corresponding graph discriminants (see Lemma 2). This section provides us with the necessary tools. They were developed in the work of Drücker-Goldschmidt [DG] (cf. also [Ra, Ne, Fu, (3.6)]) and afterwards interpreted by S. Orevkov [OZ] in the following way.

Let Γ be a weighted graph and let $A = A_\Gamma$ be its intersection form. Recall that $d(\Gamma) = \det(-A)$. For a vertex v of Γ let $\partial_v \Gamma$ denotes the graph obtained from Γ by deleting v together with all its incident links. If X is a subgraph of Γ such that $v \notin X$, then $\partial_v X$ denotes the subgraph of Γ which is obtained from X by deleting all the vertices in X closest to v together with their links. In what follows we suppose that Γ is a tree and X is a subtree; in this case there is always the unique vertex in X closest to v . Let X_1, \dots, X_N be all the non-empty subtrees of Γ , and put $P = \mathbf{Z}[X_1, \dots, X_N]$, where we identify 1 with the empty subtree and regard the disjoint union of subtrees as their product. Then the discriminant d extends to a ring homomorphism

$$d : P \rightarrow \mathbf{Z}$$

and ∂_v generates a ring derivation

$$\partial_v : P \rightarrow P.$$

¹Whereas (17) is valid for any rational SNC-tree E with non-degenerate intersection form and admissible twigs on a smooth surface, (18) and (19) are true only when E is the exceptional divisor of the resolution of singularity of a plane curve germ.

Denote also $d_v(\Gamma) = d(\partial_v \Gamma)$ and $d_{vv}(\Gamma) = d(\partial_v \partial_v \Gamma)$. Let a_v be the weight of a vertex $v \in \Gamma$.

Proposition 1. *Let the notation be as above, and let Γ be a weighted tree. Then*

a) *For any vertex $v \in \Gamma$ we have*

$$d(\Gamma) = -a(v)d_v(\Gamma) - d_{vv}(\Gamma). \quad (20)$$

b) *If Γ is a linear tree with the end vertices v and w , then*

$$d_v(\Gamma)d_w(\Gamma) - d(\Gamma)d_{vw}(\Gamma) = 1. \quad (21)$$

c) *Let $[v, w]$ be a link of Γ . Put $\Gamma \setminus]v, w[= \Gamma_1 \cup \Gamma_2$, where $v \in \Gamma_1$ and $w \in \Gamma_2$. Then*

$$d(\Gamma) = d(\Gamma_1)d(\Gamma_2) - d_v(\Gamma_1)d_w(\Gamma_2). \quad (22)$$

d) *Let T be a twig of Γ incident with a branch vertex v_0 of Γ , and let v be the tip of T . Put $d_T(\Gamma) = d(\Gamma - T - v_0)$. Then*

$$d_T(\Gamma) = d_v(\Gamma)d(T) - d(\Gamma)d_v(T). \quad (23)$$

Corollary. *Let T be a twig of Γ such that $d(T) \neq 0$. Assume that $d(\Gamma) = 1$. Denote $a = d_T(\Gamma) / d(T)$. Let $[a]$ be the least integer bigger than or equal to a and $]a[= [a] - a$ be the upper fractional part of a . Then, in the notation of (d) above, we have*

$$d_v(\Gamma) = [a] \quad \text{and} \quad \text{ind}(T) =]a[. \quad (24)$$

9 Puiseux data as graph discriminants (after Eisenbud and Neumann)

Let $(C, 0)$ be a germ of an irreducible analytic curve, and let

$$x = t^m, \quad y = a_n t^n + a_{n+1} t^{n+1} + \dots, \quad a_n \neq 0,$$

be its analytic parametrization. We may assume that $m < n$ and m does not divide n . Following [A] set $d_1 = m$, $m_1 = n$;

$$d_i = \text{gcd}(d_{i-1}, m_{i-1}), \quad m_i = \min\{j \mid a_j \neq 0 \text{ and } j \not\equiv 0 \pmod{d_i}\}, \quad i > 1.$$

Let h be such that $d_h \neq 1$, $d_{h+1} = 1$. Thus, m_i resp. d_i are defined for $i = 1, \dots, h$ resp. for $i = 1, \dots, h+1$, and

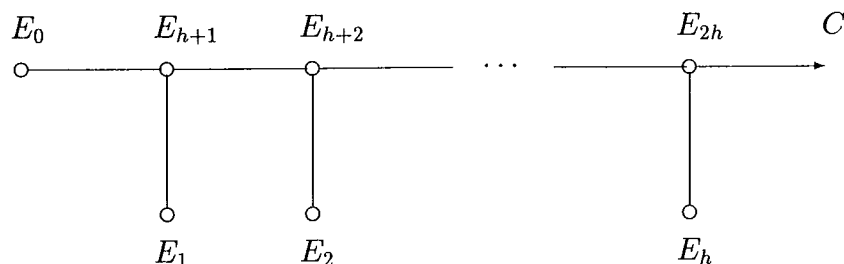
$$0 < n = m_1 < m_2 < \dots < m_h, \quad m = d_1 > d_2 > \dots > d_{h+1} = 1.$$

Set $q_1 = m_1$, $q_i = m_i - m_{i-1}$ for $i = 2, \dots, h$, and

$$r_i = (q_1 d_1 + \dots + q_i d_i) / d_i, \quad i = 1, \dots, h. \quad (25)$$

The sequence $(m; m_1, m_2, \dots, m_h)$ is called the *Puiseux characteristic sequence of the singularity $(C, 0)$* [A, Mil]. The whole collection (m_i) , (d_i) , (q_i) , (r_i) we call the *Puiseux data*. We have the following

Proposition 2 [EN]. a) Let $X \rightarrow \mathbb{C}^2$ be the embedded minimal resolution of the singularity $(C, 0)$ with the exceptional divisor $E = \cup E_i$. The proper preimage of C in X we denote by the same letter. Then the dual graph $\Gamma_{E,C}$ of E near C looks like



where the edges mean linear chains of vertices of valency two, which are not shown.

b) Denote by R_i , D_i and S_i the connected components of $\Gamma_{E,C} - E_{h+i}$ which are to the left, to the bottom and to the right of the node E_{h+i} , respectively. Denote by Q_i the linear chain between E_{h+i-1} and E_{h+i} (excluding E_{h+i-1} and E_{h+i}). Then

$$d(R_i) = \frac{r_i}{d_{i+1}}, \quad d(D_i) = \frac{d_i}{d_{i+1}}, \quad d(S_i) = 1, \quad d(Q_i) = \frac{q_i}{d_{i+1}}.$$

This graph is usually called a *comb* (see e.g. [FZ1]); M. Miyanishi suggested more pleasant name a *Christmas tree* (in this case it is drawn in a slightly different manner).

10 Expressions of the local BMY-ingredients via the Puiseux data

Proposition 3 [OZ, (5.2), (5.4)]. *Let $(C, 0)$ be the local branch of D at a cusp $P = P_i$ of D . Then in the notation of Sections 6 and 9 we have*

$$\mu = 1 - d_1 + \sum_{i=1}^h r_i \left(\frac{d_i}{d_{i+1}} - 1 \right) = 1 - d_1 + \sum_{i=1}^h q_i (d_i - 1); \quad (26)$$

$$2\mu + H_E^2 = -\frac{d_1}{r_1} + \sum_{i=1}^h \frac{r_i}{d_{i+1}} \left(\frac{d_i}{d_{i+1}} - \frac{d_{i+1}}{d_i} \right) = -\frac{d_1}{q_1} + \sum_{i=1}^h q_i \left(d_i - \frac{1}{d_i} \right); \quad (27)$$

$$-N_E^2 = \left\lceil \frac{d_1}{r_1} \right\rceil + \sum_{i=1}^h \left\lceil \frac{r_i}{d_i} \right\rceil = \left\lceil \frac{d_1}{r_1} \right\rceil - \frac{d_1}{r_1} + \sum_{i=1}^h \left(\left\lceil \frac{r_i}{d_i} \right\rceil - \frac{r_i}{d_i} \right); \quad (28)$$

$$2\mu + (K_E + \tilde{D}_E)^2 = -\left\lceil \frac{d_1}{r_1} \right\rceil + \sum_{i=1}^h \left(\frac{r_i d_i}{d_{i+1}^2} - \left\lceil \frac{r_i}{d_i} \right\rceil \right). \quad (29)$$

In particular, if m and n are coprime (i.e. there is the only one Puiseux characteristic pair), then (cf. [Mil, p. 95]) $\mu = (m-1)(n-1)$ and

$$-H_E^2 = (m-2)(n-2) + (m-n)^2/mn, \quad -N_E^2 = \left\lceil \frac{m}{n} \right\rceil + \left\lceil \frac{n}{m} \right\rceil. \quad (30)$$

The proof is based on the formulas in Lemma 3, where the corresponding entries b_{ij} have been expressed in terms of the Puiseux data as it is done in Proposition 2 above, by using the graph discriminant calculus from Section 8.

11 Local inequality for irreducible singularities

Now we can prove the local inequality (14) for a cusp $P = P_i \in D$. As above, we regard the local branch of D at $P = P_i$ as a germ $(C, 0)$ of an analytic curve.

Theorem 2 [OZ, (6.2)]. *In the notation as above, for an irreducible plane curve germ $(C, 0)$ one has*

$$-(K_E + \tilde{D}_E)^2 \leq \left(1 - \frac{1}{m}\right)\mu, \quad (31)$$

where m is the multiplicity and μ is the Milnor number of $(C, 0)$. The equality in (31) holds iff $m = 2$.

The proof proceeds as follows. (31) is equivalent to the inequality

$$\mu + (K_E + \tilde{D}_E)^2 - \frac{\mu}{m} \geq 0. \quad (31')$$

Using (25) and (26–29) we can express the quantity at the left hand side as

$$\mu + (K_E + \tilde{D}_E)^2 - \frac{\mu}{m} = d_1 \left(1 - \frac{1}{q_1}\right) - \frac{1}{d_1} + N_E^2 + \sum_{j=1}^h q_j \left(1 - \frac{d_j}{d_1}\right) \left(1 - \frac{1}{d_j}\right). \quad (32)$$

It is easily verified that this quantity vanishes when $m = 2$. The last sum in (32) is always positive. Let us show that for $m > 2$ the rest at the right hand side of (32) is also positive. Indeed, by (28) we have

$$\frac{d_1}{q_1} - N_E^2 = \left\lceil \frac{d_1}{q_1} \right\rceil + \sum_{i=1}^h \left\lceil \frac{r_i}{d_i} \right\rceil \left\lceil \frac{m}{n} \right\rceil + h = 1 + h.$$

Thus, it is enough to show that $d_1 - \frac{1}{d_1} - (1 + h) = m - \frac{1}{m} - (1 + h) > 0$. It is true for $m \geq 4$ because $h \leq \log_2 m$; it is also true for $m = 3$ because then $h = 1$, and we are done.

Remark. The estimate in Theorem 2 is asymptotically sharp in the following sense. For any positive integer m and for any $\epsilon > 0$ there exists an irreducible curve germ $(C, 0)$ of multiplicity m such that

$$\mu + (K_E + \tilde{D}_E)^2 < \mu + H_E^2 < (1 + \epsilon)\mu/m.$$

Indeed, consider the curve $x^m = y^n$, where $\gcd(m, n) = 1$ and $n \gg m$.

12 Local inequality for arbitrary singularities

Here we prove (31) in general case when $(C, 0)$ is not supposed to be irreducible, combining our approach with those of F. Sakai [Sa] (see the discussion at the end of Section 3). Let r be the number of local branches of C , and let

$$(m_1 = m, m_2, \dots, m_n, \underbrace{1, \dots, 1}_r)$$

be the multiplicity sequence of $(C, 0)$, i.e. the sequence of multiplicities of $(C, 0)$ in all its infinitely near points (where n is the total number of blow-ups in the resolution process). Recall [Mil] that

$$\mu + r - 1 = \sum_{j=1}^n m_j(m_j - 1). \quad (33)$$

Remind also that the blow-up at an infinitely near point which belongs to only one irreducible component of the exceptional divisor is called *sprouting* or *outer* blow-up, and the other blow-ups are called *subdivisional* or *inner* [MaSa, FZ1]. Following [Sa] denote by ω the number of subdivisional blow-ups, and set

$$\eta = \sum_{j=1}^n (m_j - 1).$$

Lemma 4. *In the notation as above, the following identities hold:*

$$-E^2 = \omega, \quad EK_E = \omega - 2, \quad E^2 + EK_E = -2, \quad -K_E^2 = n \quad (34)$$

$$ED'_E = r, \quad K_ED'_E = \sum_{j=1}^n m_j, \quad K_E(K_E + D'_E) = \eta \quad (35)$$

$$-D_E'^2 = \sum_{j=1}^n m_j^2, \quad -D'_E(K_E + D'_E) = \sum_{j=1}^n m_j(m_j - 1) = 2\delta \quad (36)$$

$$\mu + (K_E + \tilde{D}_E)^2 = (\eta - 1) + (\omega - 1) + (r - 1) \quad (37)$$

Proof. The third equality in (34) resp. the second one in (36) immediately follows from the preceding ones. The first equality in (35) is evident. The other formulas in (34) - (36) are proven by an easy induction by the number of

steps in the resolution process (cf. [MaSa, Lemma 2] for the first equality in (34)). To prove (37), transform its left hand side by using (19') and (34)–(36) as follows:

$$\begin{aligned}\mu + (K_E + \tilde{D}_E)^2 &= 1 + (K_E + \tilde{D}_E)(K_E + E) = 1 + (K_E + D'_E + E)(K_E + E) \\ &= 1 + K_E(K_E + D'_E) + 2EK_E + E^2 + ED'_E = \eta + \omega + r - 3.\end{aligned}$$

□

The next theorem is a generalization of Theorem 2 to the case when $(C, 0)$ is not necessarily irreducible.

Theorem 3. *The inequality (31) is valid for any singular plane curve germ $(C, 0)$, with the equality sign only for an irreducible singularity of multiplicity two.*

Proof. Replace (31) by the equivalent inequality (31'). Applying (37) we obtain one more equivalent form of (31):

$$\eta + \omega + r - 3 \geq \frac{\mu}{m}. \quad (38)$$

This inequality was proven in [Sa]. For the sake of completeness we remind here the proof. From (33) it follows that

$$\eta \geq \frac{\mu + r - 1}{m}, \quad \text{or} \quad \eta - \frac{\mu}{m} \geq \frac{r - 1}{m}. \quad (39)$$

Therefore, it is enough to proof the inequality

$$\omega + r - 3 + \frac{r - 1}{m} \geq 0, \quad (40)$$

which is in turn a consequence of the following one

$$\omega + r \geq 3. \quad (41)$$

Notice, following [Sa], that $\omega \geq 2$ as soon as at least one irreducible branch of C at 0 is singular, and $\omega = 1$ otherwise. But in the latter case $r \geq 2$, because C is assumed being singular. This proves (41), and thus also (31). Due to (41) the inequality (40), and hence also (31), is strict if $r > 1$. In the case when $r = 1$ by Theorem 2 the equality sign in (31) corresponds to $m = 2$. This completes the proof. □

13 On the rigidity problem for rational cuspidal plane curves

Let Y be a smooth affine algebraic surface $/\mathbf{C}$. Assume that Y is \mathbf{Q} -acyclic, i.e. $H_i(Y; \mathbf{Q}) = 0$ for all $i > 0$, and that Y is of log-general type, i.e. $\bar{k}(Y) = 2$. In [FZ1] the problem was posed whether such a surface should be rigid. The latter means that $h^1(\Theta_X\langle \tilde{D} \rangle) = 0$, where X is a minimal smooth completion of Y by a simple normal crossing divisor \tilde{D} and $\Theta_X\langle \tilde{D} \rangle$ is the logarithmic tangent bundle of X along \tilde{D} . The rigidity holds in all known examples of \mathbf{Q} -acyclic surfaces of log-general type [FZ1]. Moreover, in all those examples Y (or, more precisely, the logarithmic deformations of Y , see [FZ1]) is unobstructed, i.e. $h^2(\Theta_X\langle \tilde{D} \rangle) = 0$, and therefore also the holomorphic Euler characteristic $\chi(\Theta_X\langle \tilde{D} \rangle)$ vanishes (indeed, by Iitaka's Theorem [Ii, Theorem 6] $h^0(\Theta_X\langle \tilde{D} \rangle) = 0$ as soon as Y is of log-general type). We have the identity $\chi(\Theta_X\langle \tilde{D} \rangle) = K(K + \tilde{D})$ [FZ1, Lemma 1.3(5)], where $K = K_X$. Since \tilde{D} is a curve of arithmetic genus zero [FZ1, Lemma 1.2], the equality $\chi(\Theta_X\langle \tilde{D} \rangle) = K(K + \tilde{D}) = 0$ is equivalent to the following one

$$(K + \tilde{D})^2 = -2. \tag{42}$$

Thus, if Y is unobstructed, then it is rigid iff (42) holds.

Consider now an irreducible plane curve D . It is easily seen (cf. [Ra]) that $Y = \mathbf{P}^2 \setminus D$ is a \mathbf{Q} -acyclic surface iff D is a rational cuspidal curve. Furthermore, if D has at least three cusps, then Y is of log-general type [Wak]. The rigidity of Y is equivalent to D being projectively rigid in the following sense: any small deformation of D , which is a plane rational cuspidal curve with the same types of cusps (i.e. an equisingular embedded deformation), is projectively equivalent to D [FZ2, (2.1)]. Once again, the rigidity holds in all known examples [FZ2, (3.3)], as well as the equality in (42) [FZ2, (2.1)]. Here we prove the following

Proposition 4. *A projectively rigid rational cuspidal plane curve cannot have more than 9 cusps.*

Before giving the proof we remind the notation. Let $\sigma : X \rightarrow \mathbf{P}^2$ be the minimal embedded resolution of singularities of D , $K = K_X$ be the canonical divisor, $\tilde{D} = \sigma^{-1}(D)$ and $K + \tilde{D} = H + N$ be the Zariski decomposition. For

a fixed cusp $P \in \text{Sing } D$ let $K_E + \tilde{D}_E = H_E + N_E$ be the local Zariski-Fujita decomposition, where $E = \sigma^{-1}(P)$ is the exceptional divisor.

The proof of Proposition 4 is based on the following two observations.

Lemma 5. *For the negative part N_E of the local Zariski-Fujita decomposition over a cusp $P \in D$ the inequality $-N_E^2 > 1/2$ holds.*

Proof. From (28) it follows that ²

$$-N_E^2 = \lfloor \frac{d_1}{r_1} \lfloor + \sum_{i=1}^h \frac{r_i}{d_i} \rfloor \rfloor \leq \lfloor \frac{d_1}{r_1} \lfloor + \frac{r_1}{d_1} \rfloor = \lfloor \frac{m}{n} \lfloor + \frac{n}{m} \rfloor \rfloor. \quad (43)$$

By the definition of the Puiseux sequence (see section 9) we have $0 < \frac{m}{n} < 1$ and $\frac{m}{n} \neq \frac{1}{2}$. Thus, the desired inequality follows from (43) and the next estimate, which is an easy exercise. \square

Claim. *If $0 < x < 1$, then $\lfloor x \lfloor + \rfloor \frac{1}{x} \rfloor \geq \frac{1}{2}$, where the equality holds only for $x = \frac{1}{2}$.*

Lemma 6. *Let D be a rational cuspidal plane curve with at least three cusps. Then in the notation as above we have*

$$H = (d-3)L' + \sum_{P \in \text{Sing } D} H_E \text{ and } N = \sum_{P \in \text{Sing } D} N_E, \quad (44)$$

i.e. the global Zariski decomposition agrees with the local Zariski-Fujita ones.

Proof. By [Wak] we have $\bar{k}(Y) = 2$, where $Y = \mathbf{P}^2 \setminus D = X \setminus \tilde{D}$. Thus, being a smooth \mathbf{Q} -acyclic surface of log-general type, Y does not contain any simply connected curve (this was first proven in [Za] for acyclic surfaces and then generalized in [MT] to \mathbf{Q} -acyclic ones). In particular, X does not contain any (-1) -curve C with $C \cdot \tilde{D} = 1$. Since D has at least three cusps, the dual graph $\Gamma_{\tilde{D}}$ of \tilde{D} has at least three branching points. Under these conditions the lemma follows from the results in [Fu, (6.20-6.24)] (see also [OZ, Theorem 1.2]). \square

²Recall that $\lfloor a \rfloor$ denotes $\lfloor a \rfloor - a$, where $\lfloor a \rfloor := \min\{n \in \mathbf{Z} \mid n \geq a\}$.

Proof of Proposition 4. Let κ be the number of cusps of D . Evidently, we may suppose that $\kappa \geq 3$. It follows from Lemmas 5 and 6 that

$$(K + \tilde{D})^2 = H^2 + N^2 = H^2 + \sum_{P \in \text{Sing } D} N_P^2 < H^2 - \frac{1}{2}\kappa.$$

Due to BMY-inequality (13) we also have $H^2 \leq 3$, and hence

$$(K + \tilde{D})^2 < 3 - \frac{1}{2}\kappa.$$

Set $h^i = h^i(\Theta_X\langle \tilde{D} \rangle)$, $i = 0, 1, 2$. The surface $Y = \mathbf{P}^2 \setminus D$ being of log-general type [Wak], by Iitaka's Theorem [Ii, Theorem 6] we have $h^0 = 0$. Since D is assumed to be rigid, i.e. $h^1 = 0$, we also have $\chi(\Theta_X\langle \tilde{D} \rangle) = h^2 = K(K + \tilde{D}) \geq 0$, i.e. $(K + \tilde{D})^2 \geq -2$. It follows that

$$\kappa < 6 - 2(K + \tilde{D})^2 \leq 10, \quad (45)$$

which completes the proof. \square

Remark. Actually, for a rational cuspidal plane curve with at least three cusps we have proved the inequality

$$\kappa < 6 - 2(K + \tilde{D})^2 = 10 - 2K(K + \tilde{D}). \quad (46)$$

Therefore, $\kappa < 10$ as soon as $K(K + \tilde{D}) \geq 0$, which is the case if D is rigid.

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埼玉大学

SAITAMA UNIVERSITY

URAWA, March 16-18, 1995

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会場 埼玉大学理学部 9 番教室 (理学部 2 号館 2 階)

3 月 16 日 (木)

15:00 - 16:00 岡睦雄 (東工大理)

Geometry of plane curves via Tschirnhausen resolution tower

16:15 - 17:15 宮西正宜 (大阪大理)

Minimized embeddings of algebraic curves into the affine plane

3 月 17 日 (金)

10:00 - 11:00 M.Zaidenberg (Grenoble 大)

Some estimates for plane cuspidal curves I

11:15 - 12:15 鈴木昌和 (九大数理)

Immersion of lines into the plane

13:45 - 14:45 宮西正宜 (大阪大理)

An algebro-topological proof of the Lin-Zaidenberg theorem

15:00 - 16:00 酒井文雄 (埼玉大理)

The irregularity of cyclic coverings and singularities of plane curves

16:15 - 17:15 飯高茂 (学習院大理)

Noether inequality and minimal model of plane curves

3 月 18 日 (土)

10:00 - 11:00 M.Zaidenberg (Grenoble 大)

Some estimates for plane cuspidal curves II

11:15 - 12:15 岡睦雄 (東工大理)

Two basic transforms of plane curves and their fundamental groups

13:45 - 14:45 足利正 (東北学院大)

Surface singularities on cyclic coverings and the signature of smoothings

15:00 - 16:00 吉原久夫 (新潟大理)

Degrees of irrationality of some algebraic surfaces

16:15 - 17:15 宮岡洋一 (数理研)

Deformation of morphisms under certain constraints and applications