

ProPac model: opinion-action dynamics over online social networks

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1 Introduction

The study of Opinion Dynamics (OD) gains a growing deal of attention from numerous research communities, in particular from complex networks community [1, 2]. There is specifically a need for models that can be used to study large-scale networks such as Online Social Networks (OSN). Nevertheless, the main part of the literature dealing with OD does not consider explicitly the choice of the agent. Opinion and choice models evolve separately, especially since from the game-theoretic point of view, the notion of opinion is quite unorthodox. Nevertheless, as highlighted by the authors of [7], a large body of literature in psychosocial studies fosters to propose a mathematical framework coupling opinion and actions. There are some papers dealing with choices and opinions co-evolution [3], [7]. The latter is very close to our model but defines a discrete time deterministic OD. The goal of the present paper is to propose a model that takes into account the coupled opinion-action dynamics in a stochastic framework. The Markov Process frame is especially suitable for modeling large-scale networks such as OSN. Moreover, the aim is to represent agent's choice as a result of a strategic and complex process in connection with her opinion. First, let us mention that we assume opinions to be *private*, but actions are displayed and *public* in the sense that action can be observed. There are many situations in which people make decisions without revealing their opinions, even though their opinions influence these choices. This distinction justifies the choice to build a model with **Private Opinions** and **Public actions** (ProPac). The formalism is inspired by discrete choice model (DCM) [5] but coupled with OD systems. For sake of simplicity, only two admissible actions (or choices, or goods) are possible. Notice that opinion evolution does not depend on the past chosen actions, but only on other neighbours action. The underlying actions can represent credence goods [6] where there is no or few experiential feedback loop.

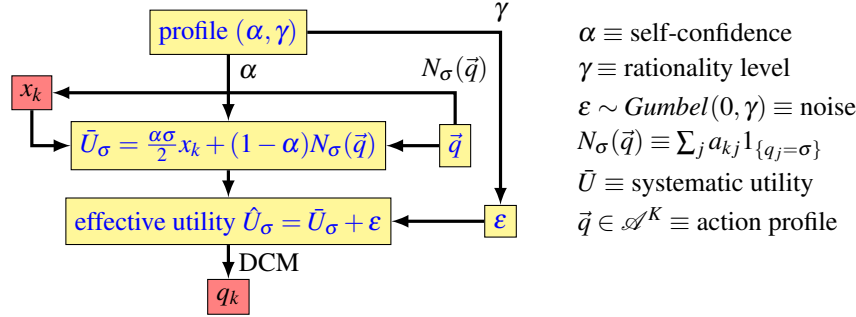
The main objective of this work is to establish the existence of remarkable social events such as *consensus* and *polarization*, especially at equilibrium. We investigate the influence of both agent's preference and the network structure on the convergence properties of OD.

2 Model

Let consider a finite population of K agents interacting through a static weighted directed graph represented through the adjacency matrix $A = (a_{ij})$. The finite opinion set

is $\Theta = \{\theta_{min} < \dots < 0 < \dots < \theta_{max}\}$ and the binary action set is $\mathcal{A} := \{+1, -1\}$. Positive opinion levels represent preferences for action +1, while negative opinion levels represent preferences for action -1. At any time t_0 , the state of each agent k is represented by two variables: her opinion $X_k(t_0) \in \Theta$ and her action $Q_k(t_0) \in \mathcal{A}$. Each vector $\{(X_k, Q_k)(t)\}_{t \in \mathbb{R}_+}$ is actually a stochastic process whose state space is $\Theta \times \mathcal{A}$, and the overall system $(X, Q) = ((X_k)_k(t), (Q_k)_k(t))_{t \in \mathbb{R}_+}$ forms a jump Markov Process whose state space is $\Theta^K \times \mathcal{A}^K$. We assume opinions to be private, but agents indeed influence one another throughout their actions: the action taken by any agent is an observable information to all its neighbors until the agent takes another action. Thus, agent k sees the actions $\{q_j : a_{kj} > 0\}$.

Opinion $x_k = \theta$ of agent k can shift to the right $\theta^+ = \theta + 1$ or to the left $\theta^- = \theta - 1$ at rates based on the *Network effect* terms N_σ . Action of agent k q_k can flip to $-q_k$ according to a rate based on the effective utility maps $\hat{U}_\sigma, \sigma \in \mathcal{A}$. The systematic utility map $\bar{U}_\sigma : (x_k, \vec{q}) \in \Theta \times \mathcal{A}^K \rightarrow \mathbb{R}$ is a deterministic function which gathers the self-opinion term and the *Network effect* $N_\sigma : \mathcal{A}^K \rightarrow \mathbb{R}_+$. N_σ quantifies the pressure exerted on agent k to follow the neighbours playing σ . The weight α is the *self-confidence* coefficient, $0 \leq \alpha \leq 1$, while $\gamma > 0$ represents the *rationality level*. Variables, model parameters and their interactions are summarized in the diagram below (viewed from agent k).



3 Preliminary results and Conjectures

Numerical simulations of opinion trajectories for any agent are represented for a system of 10 agents, hence 10 piece-wise constant curves. The parameter γ is taken constant: $\gamma = 2$. The impact of the self-confidence value α is illustrated. On figure 1, two values are compared: $\alpha = 1$ (left) and $\alpha = 0.1$ (right). In both cases the network topology is relatively sparse: an Erdős-Renyi model $\text{ER}(p = 0.2)$. It can be observed that *high α enforces synchronicity of the trajectories and convergence to a consensus*.

Alternatively, in a denser network topology $\text{ER}(p = 0.9)$, the same effect occurs and it is even more accentuated: higher values of the self-confidence α stabilizes trajectories (see figure 2).

Conjecture 1. When α grows, equilibrium measure puts more mass around extreme opinions.

In addition, the system seems to exhibit metastable points [4]:

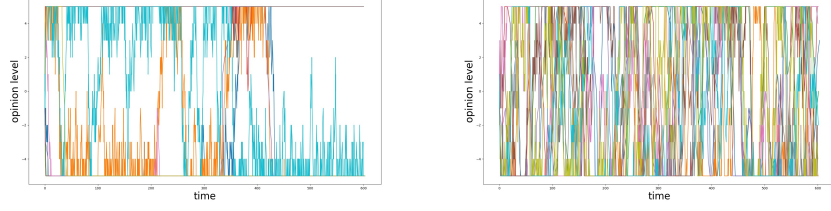


Fig. (1) Sparse network topology with two values of α : $\alpha = 1$ (left) and $\alpha = 0.1$ (right).

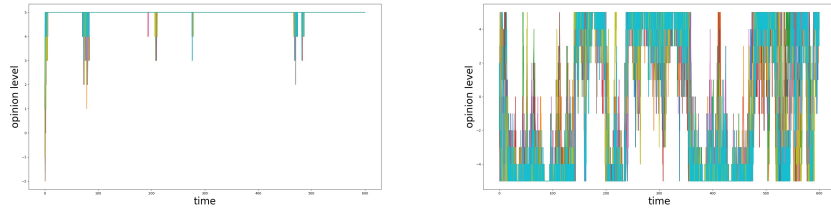


Fig. (2) Dense network topology with two values of α : $\alpha = 1$ (left) and $\alpha = 0.1$ (right).

Conjecture 2. For a certain class of well connected interaction graphs, the social landscape possesses two valleys separated by a no-go region: one valley around the point $(\theta_{\max} \mathbb{1}_K, \mathbb{1}_K)$ and the other on the opposite $(\theta_{\min} \mathbb{1}_K, -\mathbb{1}_K)$.

Finally, we advise that the rich notion of consensus can be weakened in several directions: for instance, $|X_k - \theta_{\text{cons}}| < \varepsilon$ for some small ε and for a majority of $k \in [K]$, where θ_{cons} may be time-varying.

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