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UPSCALED MODEL FOR NON-STEADY INERTIAL FLOW IN HOMOGENEOUS POROUS MEDIA

Didier Lasseux*[1], Francisco J. Valdés-Parada[2], and Fabien Bellet[3]
[1]*CNRS, UMR 5295, Univ. Bordeaux, Talence, France*
[2]*Universidad Autónoma Metropolitana-Iztapalapa, Mexico City, Mexico*
[3]*CNRS, CentraleSupélec, Univ. Paris-Saclay, France*

Summary A macroscopic model for non-steady and inertial one-phase incompressible flow in rigid porous media is presented using an upscaling technique. The resulting model is non-local in time and involves two effective coefficients, namely, a dynamic apparent permeability tensor and a vector accounting for the time-decaying influence of the flow initial condition. Ancillary closure problems are provided, which allow computing the effective coefficients. Symmetry and positiveness analyses of the apparent permeability are carried out. The performance of the macroscopic model for different macroscopic forcing is examined along with validation with direct numerical simulations. It is shown that the purely heuristic classical model, consisting in a Darcy-like model complemented with an accumulation term on the filtration velocity, is not appropriate.

MICROSCALE EQUATIONS AND UPSCALING

The commonly employed model to described non-steady one phase incompressible and creeping flow in homogeneous porous media is a heuristic Darcy equation to which an accumulation term on the filtration velocity is added [1, 2]. This empirical macroscale momentum equation has never been established on a theoretical basis. The purpose of this work is hence to formally derive a macroscopic non-steady model for one-phase flow (including inertia) in rigid and periodic porous media. The procedure starts with the classical Navier-Stokes equations at the pore-scale considering a no-slip boundary condition at the solid-fluid interface together with the velocity initial condition. An upscaling procedure, based on a short-cut version of the volume averaging method [3], is then applied to the pore-scale equations. Due to the non-linear character of the Navier-Stokes equation, a pseudo-linearization is applied considering Reynolds numbers smaller than the critical value corresponding to the first Hopf bifurcation. Nevertheles, this pseudo-linearization does not introduce any approximation [4].

MACROSCOPIC MODEL

The macroscopic mass and momentum equations, valid in the homogeneous part of the porous medium and obtained from upscaling, are given by

$$\nabla \cdot \langle \mathbf{v} \rangle = 0, \quad t > 0 \tag{1a}$$

$$\langle \mathbf{v} \rangle = -\frac{1}{\mu} \int_{t_0=0}^{t_0=t} \left. \frac{\partial \mathbf{H}_t}{\partial t} \right|_{t-t_0} \cdot \left(\nabla \langle p \rangle^\beta - \rho \mathbf{b} \right)\Big|_{t_0} + \boldsymbol{\alpha}, \quad t > 0 \tag{1b}$$

Here, $\langle \psi \rangle$ and $\langle \psi \rangle^\beta$ denote the superficial and fluid-phase (β) intrinsic averages of ψ. In addition, $\mathbf{v}$, p and t are the velocity, pressure and time whereas μ and ρ are the dynamic viscosity and density, both being constant. In the macroscopic momentum equation, $\rho\mathbf{b}$ is the body force per unit volume, $\mathbf{b}$ being space-independent (but eventually time-varying). In equation (1b), which is non-local in time, $\mathbf{H}_t$ is the dynamic apparent permeability tensor (it is apparent due to the inertial effects) whereas $\boldsymbol{\alpha}$ is an effective coefficient which only contains a source due to the initial velocity field, $\mathbf{v}_0$, and is zero when $\mathbf{v}_0 = \mathbf{0}$. The tensor $\mathbf{H}_t$ depends on $\mathbf{v}_0$ and on the macroscopic forcing. These two effective coefficients are determined from the solution of two closure problems [4]. In the long time limit, $\mathbf{H}_t \to \mathbf{H}$, $\mathbf{H}$ being the steady apparent permeability tensor introduced earlier [5]. The final value theorem applied to equation (1b) conveniently yields the steady form of the macroscopic inertial momentum equation also reported in [5]. Moreover, using a formalism reported in [6], it can be shown that $\mathbf{H}_t$ is non-symmetric, the skew symmetric part being only due to inertial effects. In the frequency domain, the dynamic apparent permeability is shown to be a positive definite tensor. Clearly, the macroscopic momentum equation (1b) contrasts with the classical heuristic model given by $\frac{\rho}{\varepsilon}\rho\frac{\partial\langle\mathbf{v}\rangle}{\partial t} = -\left(\nabla\langle p\rangle^\beta - \rho\mathbf{b}\right) - \mu\mathbf{H}^{-1}\cdot\langle\mathbf{v}\rangle, \quad t > 0.$

RESULTS

To illustrate the relevance and accuracy of the macroscopic model reported above, its predictions are compared to the results of direct numerical simulations (DNS) of the initial pore-scale equations. To do so, a two-dimensional model pore-structure made of a square pattern of parallel cylinders of square cross section is considered. Periodicity is applied in the vertical (y-) direction allowing to consider a single row of n cylinders ($n = 21$) in the horizontal (x-) direction.

*Corresponding author. E-mail: didier.lasseux@u-bordeaux.fr

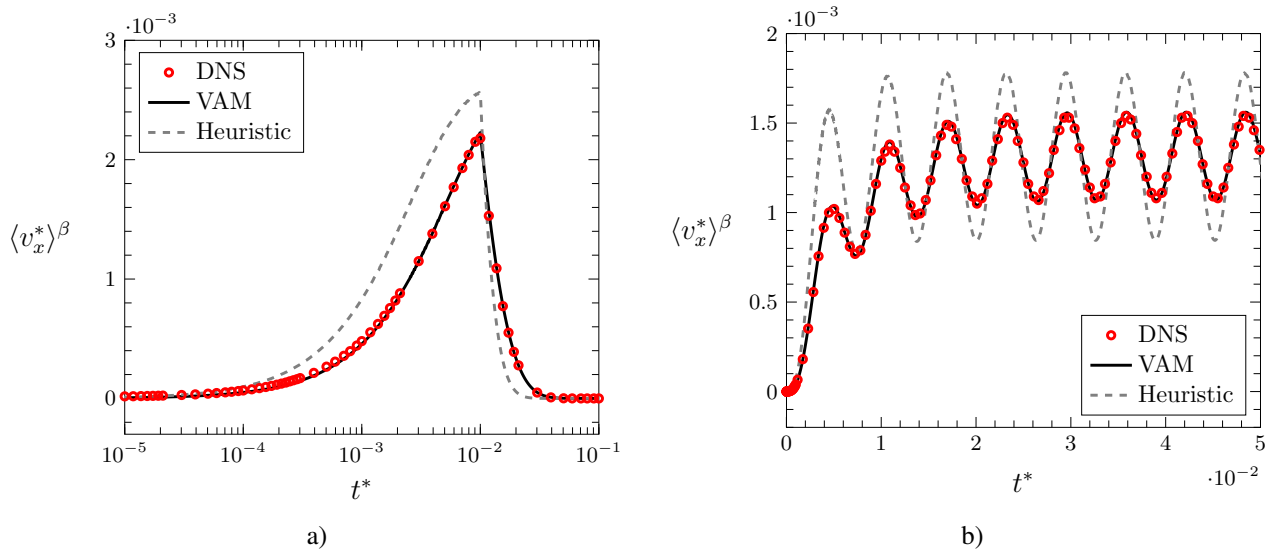

Figure 1: Dynamics of the x-component of the macroscale velocity as a response to a) a macroscopic pressure gradient pulse and b) an oscillatory macroscopic pressure gradient (see text for their expressions). Results from direct numerical simulations, from the solution of the macroscale model obtained by the volume averaging method (VAM) and from the heuristic model. In both cases, fluid is at rest at $t^\star = 0$. The porosity of the medium is 0.4 and the Reynolds number is $Re = 10^3$.

For the DNS, a pressure signal is imposed at the entrance of the medium ($x = 0$) while, at the exit, a zero pressure is maintained. For comparison with the macroscopic model, results are averaged over the central unit cell (*i.e.* far from the macroscopic boundaries where the medium is reasonnably homogeneous). For the macroscopic model, which reduces to 1D, a macroscopic pressure gradient, corresponding to that at the microscale, is applied in the x-direction.

Two different forms of the dynamic macroscopic pressure gradient were considered. The first one is a pulse given by $-\partial\langle p^\star\rangle^\beta/\partial x^\star = a$ with $a = 0,\ t^\star \leq 0;\ a = 1,\ 0 < t^\star < 10^{-2};\ a = 0,\ t^\star \geq 10^{-2}$. The second one corresponds to an oscillatory dimensionless pressure gradient given by $-\partial\langle p^\star\rangle^\beta/\partial x^\star = 0.5\,(1 - \cos(1000t^\star))$. In both cases, the initial velocity is $\mathbf{v}_0 = \mathbf{0}$.

Results on the dynamics of the dimensionless x-component of the intrinsic average velocity, $\langle v_x^\star\rangle^\beta$, are represented for the two cases in figure 1. The results from the macroscpoic model obtained with the volume averaging method (VAM) are compared to the DNS results and to the prediction of the heuristic model. This figure clearly shows that, in both cases, the dynamics of the velocity is perfectly predicted by the macroscopic model as it coincides with the DNS results. Moreover, the results obtained from the heuristic model poorly perform, proving the inadequacy of this model.

CONCLUSIONS

A macroscopic model for non-steady (inertial) one-phase Newtonian flow in a rigid homogeneous porous medium was formally derived in this work by upscaling the pore-scale mass conservation and Navier-Stokes equations. This macroscopic model is non local in time and involves a dynamic apparent permeability tensor together with an effective coefficient which accounts for the initial condition effects. Both effective parameters can be computed from the solution of two time-dependent closure problems. The macroscopic model contrasts with the heuristic model classically employed in the literature for which the macroscopic momentum equation is a Darcy-like equation to which an accumulation term on the filtration velocity is added. Numerical simulations on model test cases show that the macroscopic model derived here accurately predicts the velocity dynamics and is validated by comparison with direct numerical simulation of the pore-scale problem. The comparison also indicates that the prediction of the heuristic model poorly performs, making this model inadequate in the general case.

Apart from the fundamental interest in this formal macroscopic model, this represents an important step for further upscaling. Among many applications, it is also of major concern for the interpretation of non-steady experiments in which inertial effects and the influence of the initial condition are to be considered.

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