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A PRIORI, AN EXTENSION OF THE SSG MODEL ON COMPRESSIBLE TURBULENT FLOWS

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Abstract:

Several studies of compressible flows show that the pressure-strain is the main indicator of the structural compressibility effects. This term controls the change in the Reynolds stress anisotropy. The incompressible model of the pressure strain correlation proposed by Speziale, Sarkar and Gatski (SSG) shows a great success in simulating important incompressible flows. Thus, extension of this model on compressible flows is the main focus of this work. The standard coefficients SSG model become function of the turbulent Mach number. Applications of the compressibility correction SSG model in compressible homogeneous shear flow, show in general, that predictions of turbulence characteristic variables agree with the DNS results.

Mots clefs : compressible, pressure-strain, models.

1 Introduction

Compressible turbulence modelling have played a central role in the understanding compressibility effects. This is highly relevant to important applications in the design of advanced aerospace and engineering problems. The techniques and methodologies that have been proved useful in the incompressible flows may represent a reasonable way to develop compressible turbulence models. In this context, the direct extension of incompressible models was used in simulating different compressible flows. The Reynolds stress closures were employed with the addition of the dissipation and pressure-dilatation models for the prediction of compressible turbulent flows. It has been shown that this practice of modelling, called compressibility correction models, may be able to reproduce compressibility phenomenon at small values of Mach number. But, when the compressibility effects are more significant, the extended models do neither predict correctly the decrease in spreading rate of mixing layers, as it is observed in the experiments of Goebel and al.[6] nor the reduction in the growth rate of turbulent kinetic energy Sarkar[2]. The deficiencies of this closure is principally due to the use of the incompressible models of the pressure strain correlation which controls the level of Reynolds stress anisotropy. However, new models taking into account structural compressibility effects are needed for the pressure strain correlation. The present work focuses on this major issue. In the present study, we concentrate on the extension of the Speziale Sarkar and Gatski model[3] on compressible flow. Two compressible models du to Park and al.[4] and Fujiwira and al.[7] are used to modify the standard
coefficient of the SSG model which become dependant on the turbulent Mach number. The modified SSG model is applied to the compressible homogeneous shear flow showing an acceptable success to capture compressibility effects on turbulence.

2 Turbulence models of the pressure-strain correlation

2.1 Model of Speziale Sarkar and Gatski[3]

Speziale Sarkar and Gatski (1991) developed a model for the pressure-strain correlation namely:

$$\phi_y = (C_3 p e_y + C_4 p P)y_j + (C_3 - C_4 H^{1/2}) p K (\delta_j^1 + \frac{1}{3} \delta_j^2) + C_3 p D_i (b_{ij} b_{ij} - \frac{1}{3} b_{im} b_{jm} \delta_{ij})$$

$$+ \frac{\overline{p p}}{K} [b_{ij} \delta_{ij} + b_{ij} \delta_{ij} + \frac{1}{3} b_{ij} \delta_{ij}], 1 + \frac{\overline{p p}}{K} (b_{ij} \delta_{ij} + b_{ij} \delta_{ij} + \frac{4}{3} d_{ij} \delta_{ij} b_{ij})$$

(1)

where $C_1$, $C_2$, $C_3$, $C_4$ and $C_5$ are: $C_1 = 3.4$, $C_2 = 1.8$, $C_3 = 4.2$, $C_4 = 0.8$, $C_5 = 1.25$, $C_6 = 0.4$. $b_{ij} = R_{ij}/2K - (1/3) \delta_{ij}$, $H = b_{ij} b_{ij}$ and $P = -R_{ij} \tilde{U}_{ij}$.

2.2 Model of Park and al.[4]

The authors used the concept of moving equilibrium in homogeneous shear flow to modify the linear pressure strain term part as follows:

$$\phi_y = -C_1 \overline{p p}_{e_i} F(M_r) - (B_4 + 1.2 F(M_r)) \overline{p p} K \delta_{ij} - (B_2 + \alpha F(M_r) \overline{p p} K (P_{ij} - \frac{2}{3} P_k \delta_{ij}) - (B_3 + F(M_r) \overline{p p} \delta_{ij})$$

$$+ \overline{p p} K (D_{ij} - \frac{2}{3} P_k \overline{p p} \delta_{ij}) - B_{ij} \delta_{ij} \tilde{S}_{kk}.$$  

$$F(M_r) = F_{max} (1 - \exp(-\beta \beta_{ij}^2))$$

$$, F_{max} = \frac{\overline{p p}}{2 + \alpha},$$

$$P_{ij} = -\overline{p p} R_{jm} \tilde{U}_{ij} - \overline{p p} R_{jm} \tilde{U}_{ij}$$

$$D_{ij} = -\overline{p p} R_{jm} \tilde{U}_{ij} - \overline{p p} R_{jm} \tilde{U}_{ij}.$$  

For the function $F(M_r)$, $\beta = 4$ and $\alpha$ between 1 and 4 are the suited values by Park and al. One can see that, when $F(M_r) = 0$, the model like the original Launder Reece and Rodi (LRR) model[5] and the coefficients $C_1$, $B_2$, $B_3$, $B_4$ and $B_5$ are: $B_2 = \frac{8 + C}{11}, B_3 = \frac{8C - 2}{11}, B_4 = \frac{60C - 4}{55}, B_5 = \frac{6C + 4}{11}$. $C_1 = 1.5$ and $C = 0.4$.

2.3 Model of Fujiwara and al.[7]

Fujiwara and al. developed a correction to the incompressible model for the pressure-strain correlation. The model is written as:

$$\phi_y = (1 - F(M_r)) \Pi_{ij}$$

(3)

where $F(M_r)$ is an arbitrary function of the turbulent Mach number and $\Pi_{ij}$ is the incompressible strain correlation.

3 A priori a compressibility correction of the SSG model

- The compressible SSGP model

The pressure strain ($\phi_{ij}^*$)Ssg can be split in two parts : ($\phi_{ij}^*$)lin is linear of the mean strain and Reynolds stress which uses a formulation like that of Launder Reece and Rodi[5]. Moreover, the non-linear part ($\phi_{ij}^*$)non-lin is quadratic in the Reynolds stress. Thus, we can write:
\( (\phi^*_y)_{SSG} = (\phi^*_y)_{lin} + (\phi^*_y)_{non-lin} \) \hspace{1cm} (4)

where

\[
(\phi^*_y)_{lin} = -C_i\rho\delta_{ij} + C_i\rho K (\delta_{ij} - \frac{1}{3} \delta_k \delta_{ij}) + \rho K C_i \left[ b_\alpha \delta_{ij} + \alpha b_\delta \delta_{ij} - \frac{2}{3} \delta_m \delta_{ij} \right] + \rho K C_i \left[ b_\delta \delta_{ij} + \alpha b_\delta \delta_{ij} - \frac{4}{3} d_2 \delta_{ij} \right]
\]

\[
(\phi^*_y)_{non-lin} = -C_i\rho P b_{ij} - C_i\rho \frac{1}{2} \rho K (\delta_{ij} - \frac{1}{3} \delta_k \delta_{ij}) + C_i\rho \rho e_{ij} (b_\alpha b_{ij} - \frac{1}{3} b_\alpha b_{ij} \delta_{ij})
\]

\hspace{1cm} (5)

The starting point of the proposed compressibility modification of this model is from the several analysis and development in which Pope\[8\] establishes correspondence between some second order closure models. Here, we concentrate on the modification of \( (\phi^*_y)_{lin} \) and we propose to write :

\[
(\phi^*_y)_{SSG} = (\phi^*_y)_{lin} + (\phi^*_y)_{non-lin}
\]

Different compressible models can be used for \( (\phi^*_y)_{lin} \) which are essentially extension of the LRR model \[5\] using different extra compressibility parameters like pressure variance, gradient Mach number and turbulent Mach number. The compressible model due to Park and al.\[4\] is used to express \( (\phi^*_y)_{lin} \). Thus, the SSG coefficients model \( C_3 \), \( C_4 \) and \( C_5 \) become dependent on the turbulent Mach turbulent number as

\[
C_3 = 0.8 + \lambda_2 F, \quad C_4 = 1.25 + \lambda_4 F, \quad C_5 = 0.4 - \lambda_3 F, \quad \lambda_2 = \frac{4}{3} (1-\alpha)/3-6/5, \quad \lambda_4 = 2(1-\alpha), \quad \lambda_3 = 2(1+\alpha)
\]

\[
F = \frac{0.54}{2+\alpha} (1-e^{-\rho M_t^2}).
\]

- The compressible SSGF model

The model due to Fujiwara and al. is used to derive a simplest-compressibility correction of the SSG model as :

\[
\phi^*_y = (1-f(M_t))(\Pi_{ij})_{SSG}, \quad f(M_t) = \beta e^{(-\alpha/M_t^2)}
\]

we use for the coefficients \( \alpha \) and \( \beta \) the values 0.02 and 1 respectively.

4 Simulation of compressible homogeneous shear flow

The compressible homogeneous shear flow is defined by the mean gradient velocity as:

\[
\overline{U}_{i,j} = S \delta_{ij} \delta_{j2}
\]

where \( S \) is the constant mean shear. The Favre averaged Reynolds stress should be solution to the transport equation:

\[
\frac{\partial}{\partial t} (\bar{\rho} R_{ij}) = P_{ij} + \phi^*_y - \frac{2}{3} \rho e \delta_{ij} + \frac{2}{3} \rho \bar{\eta} \bar{d} \delta_{ij}
\]

where the symbols \( P_{ij} \), \( \phi^*_y \), \( \rho e \), \( \rho \bar{\eta} \) represent turbulent production, pressure strain correlation and the turbulent dissipation.

\[
P_{ij} = -\overline{\rho R_{jm} \bar{U}_{i,m} - \rho R_{im} \bar{U}_{j,m}}
\]

\[
\phi^*_y = \rho' (u''_{i,j} + u''_{j,i}) = \phi^*_y + \frac{2}{3} \rho' u''_{k,k} \delta_{ij}
\]
The turbulent Mach number $M_t = \sqrt{2K/\bar{a}}$, ($K$ is the turbulent kinetic energy and $\bar{a}$ is the mean velocity of sound) is described by the equation, namely  

$$
\frac{dM_t}{dt} = M_t \left( \frac{P}{2k} \right) + \frac{M_t}{2}\left( 1 - \frac{1}{2}(\gamma - 1)M_t^2 \right) \left( \rho u_i \rho u_j - \bar{\rho} \bar{u}_i \bar{u}_j \right) 
$$

(10)

where $P = -\bar{p}R_{ij} \bar{U}_{i,j}$ is the turbulent production and $\gamma = \frac{c_p}{c_v}$.

The turbulent dissipation is obtained by resolving the following equation:

$$
\frac{d\varepsilon_t}{dt} = C_{\varepsilon 1} \frac{\varepsilon_t}{k} P - C_{\varepsilon 2} \frac{\varepsilon_t^2}{k} 
$$

(11)

The values of the constants models used in the present simulation are: $C_{\varepsilon 1} = 1.4, C_{\varepsilon 2} = 1.8$.

In this work, the Sarkar and al. model [1] is used to express the pressure dilatation correlation and the turbulent dilatation of dissipation rate:

$$
\bar{p}\bar{d}' = 0.15M_t \rho (R_{ij} - \frac{2}{3}K\delta_{ij}) + 0.2\rho \varepsilon_t^2 \varepsilon_t \text{ and } \varepsilon_t = 0.5M_t^2 \varepsilon_t 
$$

(12)

5 Results and discussion

In this study, we consider predictions of two versions of the SSG. Model: the original version (the coefficient are as in the incompressible version) and the corrected version where the proposed $M_t$ correction is included. The ability of the model to predict the anisotropy of the compressible homogeneous turbulent shear flow will now be considered. The model predictions are compared with the DNS data [2] for four cases corresponding to different specific initial conditions (see Table 1).

From all of the figures, it is clear that the incompressible SSG model [3] is still unable to predict the dramatic changes in the magnitude of the Reynolds-stress anisotropy that arise from compressibility. The proposed extension of the SSG model involves the turbulent Mach number. It provides an acceptable performance in compressible homogeneous shear flow.

<table>
<thead>
<tr>
<th>case</th>
<th>$M_{10}$</th>
<th>$(SK/\varepsilon)_0$</th>
<th>$M_{g0}$</th>
<th>$b_{11}$</th>
<th>$b_{22}$</th>
<th>$b_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.4</td>
<td>1.8</td>
<td>0.22</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A2</td>
<td>0.4</td>
<td>3.6</td>
<td>0.44</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A3</td>
<td>0.4</td>
<td>5.4</td>
<td>0.66</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A4</td>
<td>0.4</td>
<td>10.8</td>
<td>1.32</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1 Initial conditions for the DNS results of Sarkar [2]
FIG. 1 Time evolution of the Reynolds stress: symbol: DNS, solid lines: present compressible model (SSGF), dashed lines: standard SSG model. Arrow show the trend with increasing the gradient Mach number.

FIG. 2 Time evolution of the Reynolds stress in case A4: (a): $b_{11}$, (b): $b_{22}$, (c): $b_{12}$.

FIG. 3 Time evolution in case A4: (a): $esk = \varepsilon_s / SK$, (b): $((dK/dt)/SK)$, (c): $M_t$.

Figures 1 and 2 show that the proposed compressibility correction models: SSGF and SSGP appear to be able to predict correctly the significant decrease in the magnitude of the normalized production term $-2b_{12}$ and the increase in the magnitude of the streamwise $b_{11}$ and the transverse $b_{22}$ Reynolds-stress anisotropy. Figure 1 show that the proposed (SSGF) model yields results that are in good agreement with the DNS results[2]. From figures (2-a,b,c), an acceptable improvement of the results especially at high gradient Mach number $M_g = St/\ell$, where $\ell$ is an integral length scale, (see Sarkar[2]) and at high turbulent Mach number (case A4) can be noticed with the use of the SSGP model. Figure 3-a present the behavior of the normalized dissipation $\varepsilon_s / SK$, $(-2b_{12}\varepsilon_s / P)$ for case A4. It can be seen that there is a decrease
in $\varepsilon_s / SK$ when $M_{g0}$ increases, since the compressibility effects cause significant reduction in the Reynolds turbulent shear stress $b_{12}$. It is clear that the proposed (SSGp) model is in accordance with the DNS results. Figure 3-b show the predicted growth rates of the turbulent kinetic energy $\Lambda$ , ($\Lambda = (dK/dt)/SK$) with the standard model SSG and the compressibility correction SSGp model. It is clearly seen that all of the two models appear to be able to predict accurately the trend of reduced growth rate with increasing the compressibility effects. This phenomenon has often been observed in DNS results of compressible homogeneous shear flow. From figure 3-c, one can see that the SSGP model appears to be able to predict better agreement with the DNS data of the turbulent Mach number.

6 Conclusion

In this study, the second order closure has been used for the prediction of compressible homogeneous shear flow. The standard–stress closure with the addition of the dilatational terms: the pressure dilatation correlation and the turbulent dissipation of the dilatation yields very poor predictions of the changes in the Reynolds stress anisotropy magnitude. The deficiencies of this closure are due to the use of the incompressible models of the pressure-strain correlation. This term controls the structural compressibility effects on the turbulence. A modification of the standard SSG model of the pressure strain correlation has been made, the usual coefficients become depend on the turbulent Mach number $M_t$. In general, this model successfully predict the compressibility effects on the homogeneous turbulent shear flow. This leads to think that compressibility correction of the SSG model of the pressure strain correlation is a prominent way for the compressible second order closures.

References