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# Advertising, Goodwill, and the Veblen Effect

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## Abstract

The increase of demand in price, an exception to the law of demand, is known as the Veblen effect. In this work, we consider a profit maximizing monopoly which by means of advertising impacts the price-demand relationship. We show that advertising and goodwill play an important role in making the Veblen effect more prevalent than expected. By employing optimal control theory we capture the evolution of the variables over time which may exhibit the Veblen effect where price and demand move in the same direction. Incorporating this dynamics into firms' decisions has a promising impact on long-term profit. Consequently, it may even trigger a slew of studies on product line extension, competition and pricing by allowing firms to control their status.

**Key words:** OR in marketing; Dynamic pricing; dynamic advertising; Veblen effect; conspicuous consumption

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# 1 Introduction

Luxury-good retailers benefit from price augmentations to increase sales, by taking advantage of the Veblen effect (Kapner and Passariello, 2014) which is based on conspicuous consumption, and is best explained by Bagwell and Bernheim (1996, p. 349):

Thorstein Veblen (1899) argued that wealthy individuals often consume highly conspicuous goods and services in order to advertise their wealth, thereby achieving greater social status. Veblen’s writing have spawned a significant body of research on “prestige” or “status” goods. In the context of this literature, “Veblen effects” are said to exist when consumers exhibit a willingness to pay a higher price for a functionally equivalent good.

Consequently, Wood (1993) concluded that Veblen goods’ price and demand move in the same direction. Therefore, they represent a theoretical curiosity, since they seem to violate the classical law of demand, which posits an inverse relationship between price and demand (Leibenstein 1950, Corneo and Jeanne 1997, Bagwell and Bernheim 1996). This pattern raises a related question regarding Veblen products: Why luxury firms do not set infinite prices, or at least high enough to the point that the demand starts decreasing with price? Indeed, with higher prices luxury firms would gain from both greater markup (price) and sales (demand), boosting the profit.

In this article, we develop an optimal control model, which extends the model of Nerlove and Arrow (1962), in order to examine the ‘total’ effect of price on demand by incorporating the firm’s advertising schedule, which has a positive impact on its goodwill. The model relies on the classical law of demand where the direct relationship between price and demand is inverse, however, the indirect relationship which is influenced by advertising and goodwill may elicit the Veblen effect.

This research is related to formal analyses of the Veblen effect. Leibenstein (1950) integrates conspicuous consumption in the theory of consumer demand, pointing to the importance of social factors. Signaling models of exclusivity and conformity have been used to model Veblen products. Bernheim (1994), Bagwell and Bernheim (1996), Corneo and Jeanne (1997), and Hopkins and Kornienko (2004) examines how consumption of conspicuous products signals higher income, and thereby augments social status. Veblen effects have also been modeled as network effects. Amaldoss and Jain (2005a), Amaldoss and Jain (2005b), Deb (2009), and Wang et al. (2017) posit that consumption exerts externalities,

snobs and followers seeking privileged and standard products. Kort et al. (2006) investigate conspicuous consumption, assuming that price depends on the brand image of the firm, instead of being directly a firm decision. Moldovanu et al. (2007) model a principal-agent relationship, in which the principal designs an organization in which agents mind about their relative position. Aoyagi et al. (2016) study a duopoly with buyers whose product preference is affected by extrinsic valuation and may be characterized as conspicuous. Caulkins et al. (2011), Huschto et al. (2011), and Huschto and Sager (2014) explore the management of conspicuous goods during recessions through pricing. The first attempt to link advertising to the Veblen effect is made by Pepall and Reiff (2016). Our article builds on this attempt.

This research also refers to literature on the Nerlove and Arrow (1962) model. Several articles survey the extensive literature (Sethi 1977a, Feichtinger et al. 1994, Huang et al. 2012). The model has quickly been extended to diverse situations, such as uncertainty (Sethi, 1979), oligopoly (Tapiero 1979 and Rao 1982), and non-durable goods (Chintagunta et al., 1993). The work of Nerlove and Arrow (1962) has also been extended within the supply chain (distribution channel) context where a manufacturer and retailer(s) determine their optimal prices as well as advertising investment in non-cooperative as well as cooperative fashions (Jørgensen and Zaccour, 1999, 2003; Jørgensen et al., 2001; He et al., 2009; Chutani and Sethi, 2012). More recently, Karray and Zaccour (2005) and Karray and Martín-Herrán (2009) study retailers, which offer national as well as store brands. Helmes et al. (2013) investigate general models of new product adoption; Schlosser (2016) considers a time-dependent demand within a stochastic model. Artzrouni and Cassagnard (2017) provide an explicit solution (a recursive process) by discretizing the original problem of Nerlove and Arrow (1962). Chenavaz et al. (2020) investigate the price-quality-advertising relationship. From a modeling view point, our article is most closely related to Chenavaz (2017), Chenavaz and Jasimuddin (2017), Vörös (2019), and Ni and Li (2019), which investigate the price- and advertising-quality relationships. These articles, however, do not examine the dynamics of demand.

This article contributes to the literature on the Veblen effect by (1) examining the intermediating role of goodwill on the price-demand relationship, and (2) showing that the Veblen effect is consistent with a standard downward-sloping demand curve. It contributes to the literature on dynamic pricing and advertising by (1) using general non-linear demand and goodwill functions, and (2) obtaining structural (opposing parametric) results. The main results are as follows. Generalizing the condition of Nerlove and

Arrow (1962), we provide a new condition giving the optimal marketing mix between price and advertising. Building on this generalized condition, we explicitly express the circumstances under which demand moves in the same direction as price. Our analysis explains the mechanism that brings forth the Veblen effect and consequently clarifies the “mystery” behind this pattern. The role of goodwill, whose importance is recognized for luxury products, is pivotal in explaining the Veblen effect. Of theoretical interest, the results offer novel insights on the relationship between price and demand. Of practical interest, this study provides straightforward managerial implications regarding pricing and advertising of luxury goods whose sellers benefit from the Veblen effect.

## 2 The Model

In this work, we consider a profit maximizing monopoly, which sells a good to fulfill customers’ demand. The product’s demand depends on its price, as well as the firms’ goodwill. Thus, the firm seeks to jointly determine the products’ price and its investment in advertising (which boosts goodwill) so that its profit will be maximized over time. Employing optimal control theory we explore the relationships among the various variables and assess the price-demand dynamics to determine conditions for the viability of the Veblen effect.

The demand,  $D(p, G)$  ( $\mathbb{R}^{2+} \rightarrow \mathbb{R}^+$ ), depends on the price,  $p$ , and goodwill,  $G$ , and is assumed twice continuously differentiable. Furthermore, the demand is fully satisfied and there is no excess inventory either. The *direct price effect on demand*, the *direct goodwill effect on demand*, and the *cross effect of price and goodwill on demand* are given by  $\frac{\partial D}{\partial p}$ ,  $\frac{\partial D}{\partial G}$ , and  $\frac{\partial^2 D}{\partial p \partial G}$ . Furthermore, the demand falls with price and rises with goodwill when either one is solely considered, and it is assumed not “too” convex in price:

$$\frac{\partial D}{\partial p} < 0, \quad \frac{\partial D}{\partial G} > 0, \quad 2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}} > 0. \quad (1)$$

Note that the technical assumption  $2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}} > 0$  is required for profit optimization, ensuring a unique maxima of the profit function. Such assumption is standard in the dynamic pricing literature using structural demand functions (see for instance Kalish 1983, Dockner et al. 2000, Chenavaz 2012, 2017).

Table 1: Main Notations

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$r$	= interest rate,
$p(t)$	= product price at time $t$ (control variable),
$a(t)$	= advertising expense at time $t$ (control variable),
$G(t)$	= goodwill at time $t$ (state variable),
$\frac{dG}{dt}$	= $A(a, G)$ = goodwill dynamics,
$\lambda(t)$	= shadow price of goodwill at time $t$ (co-state variable),
$D(p, G)$	= current demand,
$\pi(p, a, G)$	= $pD - a$ = current profit,
$H(p, a, G, \lambda)$	= $\pi + \lambda A$ = current-value Hamiltonian,
$e_x$	= $\left  \frac{\partial D}{\partial x} \frac{x}{D} \right $ = elasticity of demand $D$ with respect to $x$ ,
$e_{A/a}$	= $\frac{\partial A}{\partial a} \frac{a}{A}$ = elasticity of goodwill dynamics with respect to advertising.

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The firm investment in advertising,  $a(t) \in \mathbb{R}^+$ , has a positive impact on goodwill (called brand image in Kort et al. 2006, Caulkins et al. 2011, Huschto et al. 2011, and Huschto and Sager 2014),  $G(t) \in \mathbb{R}^+$ , which is used as a state variable to capture the system dynamics that is given by

$$\frac{dG(t)}{dt} = A(a(t), G(t)), \text{ with } G(0) = G_0, \quad (2)$$

where  $A : \mathbb{R}^{2+} \rightarrow \mathbb{R}$  is twice continuously differentiable. Hereafter and when no confusion arises, we omit any argument for simplicity.

Advertising investment,  $a$ , boosts goodwill,  $G$ , with diminishing returns. Furthermore, goodwill decays autonomously:

$$\frac{\partial A}{\partial a} > 0, \quad \frac{\partial^2 A}{\partial a^2} < 0, \quad \frac{\partial A}{\partial G} \leq 0. \quad (3)$$

The firm's profit is determined by the net sales (margin times demand) minus advertising cost,  $\pi(p, a, G) = (p - c)D(p, G) - a$  ( $\pi : \mathbb{R}^{3+} \rightarrow \mathbb{R}$ ), which is assumed twice continuously differentiable. Furthermore, similarly to Karray and Martín-Herrán (2009), we assume without loss of generality that the unit production cost  $c$  is normalized to zero. Consequently, the profit can be expressed as:

$$\pi(p, a, G) = pD(p, G) - a. \quad (4)$$

The firm maximizes its intertemporal profit (or present value of the profit stream) by simultaneously choosing the advertising and pricing policies over the planning horizon, while accounting for the goodwill

dynamics. Given interest rate  $r \in \mathbb{R}$ , the objective function of the firm is

$$\begin{aligned} & \max_{a(t), p(t) \geq 0, \forall t \in [0, \infty]} \int_0^\infty e^{-rt} \pi(p(t), a(t), G(t)) dt, \\ & \text{subject to } \frac{dG(t)}{dt} = A(a(t), G(t)), \text{ with } G(0) = G_0. \end{aligned}$$

We solve the problem using the current-value Hamiltonian

$$H(p, a, G, \lambda) = pD(p, G) - a + \lambda A(a, G),$$

which measures the intertemporal profit, summing current profit  $pD - a$  and future profit  $\lambda A$ . We confine our interest to interior solutions for  $a$  and  $p$ , assuming their existence. The Hamiltonian  $H$  is assumed strictly concave in advertising  $a$  and price  $p$ . The necessary and sufficient first- and second-order conditions and the maximum principle for  $H$  maximization yield the followings for all  $t \in (0, \infty)$

$$\frac{\partial H}{\partial a} = 0 \implies \frac{\partial A}{\partial a} = \frac{1}{\lambda}, \quad (5a)$$

$$\frac{\partial H}{\partial p} = 0 \implies p = -\frac{D}{\frac{\partial D}{\partial p}}, \quad (5b)$$

$$\frac{\partial^2 H}{\partial a^2} < 0 \implies \lambda \frac{\partial^2 A}{\partial a^2} < 0, \quad (5c)$$

$$\frac{\partial^2 H}{\partial p^2} < 0 \implies 2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}} > 0, \quad (5d)$$

$$\frac{\partial^2 H}{\partial a^2} \frac{\partial^2 H}{\partial p^2} - \left( \frac{\partial^2 H}{\partial a \partial p} \right)^2 > 0 \implies \lambda \frac{\partial^2 A}{\partial a^2} \left( -2 + D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}} \right) > 0, \quad (5e)$$

$$\frac{d\lambda}{dt} = r\lambda - \frac{\partial H}{\partial G} \implies \frac{d\lambda}{dt} = \left( r - \frac{\partial A}{\partial G} \right) \lambda - p \frac{\partial D}{\partial G}, \quad (5f)$$

with the transversality condition for a free terminal state and infinite terminal time  $\lim_{t \rightarrow \infty} e^{-rt} \lambda(t) = 0$ .

The first-order condition for advertising (5a) is an interior following (3). In addition, the higher the shadow price of goodwill,  $\lambda$ , the larger are the advertising expenses,  $a$ . Due to the diminishing returns of advertising as stipulated in (3), the lower the impact of advertising on goodwill,  $\frac{\partial A}{\partial a}$  is. The first-order condition for price (5b) yields an interior solution as the price is positive ( $D \geq 0$  and  $\frac{\partial D}{\partial p} < 0$  imply  $-\frac{D}{\frac{\partial D}{\partial p}} \geq 0$ ).

The second-order condition (5c) together with the diminishing returns of advertising (3) impose<sup>1</sup>

$$\lambda(t) > 0, \forall t \in [0, \infty), \quad (6)$$

suggesting that greater goodwill always augments the intertemporal profit. The second-order condition (5d), verified with assumption (1), corresponds to the strict concavity of the Hamiltonian  $H$  with respect to price  $p$ . The second-order condition (5e) is technical and is required for a maximum (as opposed to a saddle) point. This condition is satisfied because  $\frac{\partial^2 H}{\partial a^2} < 0$ ,  $\frac{\partial^2 H}{\partial p^2} < 0$ , and  $\frac{\partial^2 H}{\partial a \partial p} = 0$ .

Let  $a^*(p)$  denote the optimal advertising level, for a given price,  $p$ , corresponding to (5a). Similarly, let  $p^*(a)$  denote the price satisfying (5b), which maximizes the intertemporal profit for advertising expense,  $a$ . The intertemporal profit is maximized with the advertising and pricing pair such that  $(a^*, p^*) = (a^*(p^*), p^*(a^*))$ . In the following, the advertising and pricing levels are referred as *optimal* in the sense that they maximize the intertemporal profit, and for simplicity, we omit the \* superscript notation. Note that the advertising elasticity of goodwill dynamics  $e_{A/a} = \frac{\partial A}{\partial a} \frac{a}{A}$  and the price elasticity of demand  $e_p = -\frac{\partial D}{\partial p} \frac{p}{D}$ .

**Proposition 1.** *The optimal relationship between price and advertising levels can be expressed as*

$$\frac{e_{A/a}}{e_p} = \frac{a}{A\lambda}. \quad (7)$$

*Proof.* Follows from (5a) and (5b). □

Proposition 1 originates from the joint optimal price and advertising policies, which are required for intertemporal profit maximization. Proposition 1 stipulates that the ratio of advertising elasticity of goodwill dynamics to price elasticity of demand  $\frac{e_{A/a}}{e_p}$  should be equal to the advertising intensity, measured by the amount of advertising deflated by goodwill dynamics times goodwill shadow price  $\frac{a}{A\lambda}$ . Note that according to (5b),  $e_p = 1$ , and (7) simplifies to  $e_{A/a} = \frac{a}{A\lambda}$ . However, the structure of (7) allows for an immediate comparison of our result with the literature (which also include the price elasticity of demand) as suggested in the following remarks.

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<sup>1</sup>Condition  $\frac{\partial A}{\partial a} > 0$  from (3) combined with (5a) lead to the same conclusion.

**Remark 1.** *Proposition 1 is a dynamic (opposing static) counterpart of the condition of Dorfman and Steiner (1954, Section 1, p. 828).*

**Remark 2.** *Proposition 1 is a structural (opposing parametric) counterpart of the condition of Nerlove and Arrow (1962, Equation (17), p. 134).*

**Remark 3.** *Proposition 1 (and also Equation (5b)) implies that the price, demand, and profit are finite (rather than infinite).*

Proposition 1 provides the relationship between the optimal values of the two control variables, price and advertising expense. In section 3 we explore the static setting which suggests the rationale for the mechanism that may potentially trigger the Veblen effect. However, in order to experience the change in demand with respect to price due to the effect of advertising and goodwill over time we explore the dynamic setting in Section 4. As Proposition 1 equates the marginal revenues to the marginal costs at all time it has to hold throughout the entire planning period. For a maximal profit, changes in marginal revenue must balance changes in marginal cost. Changes in goodwill, in turn, drive changes in pricing and advertising. The causality in changes anchors the price-goodwill and advertising-goodwill relationships.

### 3 The Price-Demand Relationship: Static Setting

In order to demonstrate the mechanism which triggers the Veblen effect, we first consider the static setting of the problem.

Noting that as  $D = D(p(t), G(t))$ , the relationship between the price and demand can be expressed as

$$\frac{dD}{dt} = \frac{\partial D}{\partial p} \frac{dp}{dt} + \frac{\partial D}{\partial G} \frac{dG}{dt}, \quad (8)$$

which suggests that the dynamics of the demand  $\frac{dD}{dt}$  is based on the direct price effect,  $\frac{\partial D}{\partial p}$ , the dynamics of price,  $\frac{dp}{dt}$ , the direct goodwill effect,  $\frac{\partial D}{\partial G}$ , and the dynamics of goodwill  $\frac{dG}{dt}$ . Furthermore, using the time elimination method, we obtain

**Proposition 2.**

$$\frac{dD}{dp} = \underbrace{\frac{\partial D}{\partial p}}_{-} + \underbrace{\frac{\partial D}{\partial G}}_{+} \underbrace{\frac{dG}{dp}}_{\pm}. \quad (9)$$

*Proof.* Assuming that state variable  $G$  depends on control variable  $p$  (regardless of the specific relationship) enables the application of the time elimination method (Mulligan and Sala-i Martin, 1991). Furthermore,  $G$  is assumed to be continuously differentiable  $G : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  in  $p$ , and  $\frac{dp}{dt} \neq 0$ . Dividing (8) by  $\frac{dp}{dt}$  completes the proof.  $\square$

Following time elimination, the total effect of price on demand,  $\frac{dD}{dp}$ , is comprised of: 1) the direct effect of price on demand,  $\frac{\partial D}{\partial p}$ , which is negative, 2) the direct effect of goodwill on demand,  $\frac{\partial D}{\partial G}$ , which is positive, and 3) the effect of price on goodwill,  $\frac{dG}{dp}$ , which for generality purpose we allow to be positive or negative. Thus, even though  $\frac{\partial D}{\partial p}$  is negative, the sign of  $\frac{dD}{dp}$  is yet to be determined as it is based on the comprehensive effect of price on demand including the role of goodwill as illustrated in Figure 1.

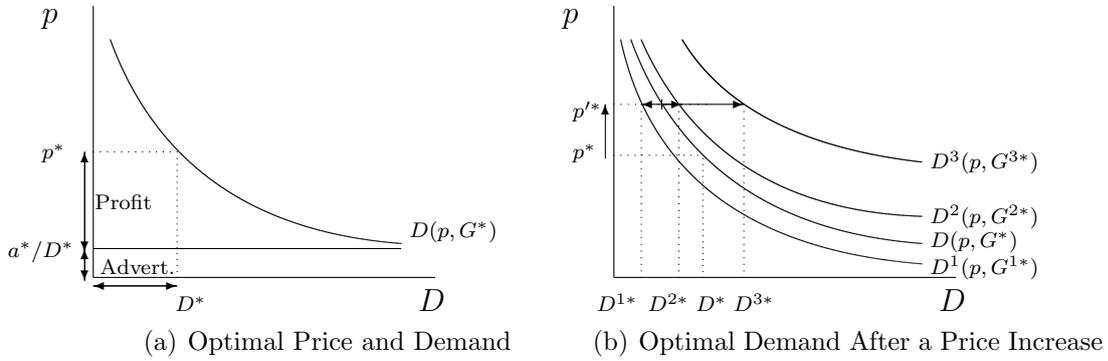


Figure 1: Possible Relationships between Price and Demand for a Given Goodwill

Figure 1(a) depicts the “traditional” relationship between price and demand. As a reference point we consider the optimal price  $p^*$  and demand  $D^*$  (though other points could have been chosen). According to the pricing rule (5b), the optimal price  $p^*$  is equal to the markup  $-\frac{D}{\partial D}$ . Recalling that sales equal demand, substituting  $p = p^*$  in  $D(p, G^*)$  yields the optimal sales  $D^* = D(p^*, G^*)$ . The revenue area corresponds to the rectangle of price length and sales width. The advertising expenses area is depicted by the rectangle of advertising per unit sales length and sales width rectangle. According to the advertising rule (5a), the optimal advertising expenses  $a^*$  must maintain  $\frac{\partial A}{\partial a} = \frac{1}{\lambda}$ . The profit area equals the revenue minus the advertising expenses. Profit maximization is achieved by maximizing the surface of the profit area. Figure 1(a) conforms with profit function (4), verifying that profit boosts with the price and the demand, whereas it falls with advertising expenses.

Beyond the direct effect of price on demand, the total effect of price on demand also depends on the impact of price on the other decision variable, advertising. Figure 1(b) illustrates the possible cascading

effect of a price increase from  $p^*$  to  $p'^*$ . First, due to the direct price effect on demand,  $\frac{\partial D}{\partial p} < 0$ , a higher price leads to diminishing demand along curve  $D$ . Second, a higher price may trigger the profit maximizing firm to modify its spending on advertising in order to maintain an optimal advertising level following the new price and to conform to Proposition 1. Hence, advertising may either increase or decrease and lead to the following potential consequences:

1. If higher price triggers a decrease in advertising expenses, it will also lead to a decrease in goodwill. Consequently, the demand curve shifts to the left, indicating a reduction in demand. In this case, in addition to the direct price effect, the goodwill decreases, which reduces demand further, as illustrated by curve  $D^1$ .
2. Alternatively, if a higher price triggers an increase in advertising, it will result in an increase in goodwill as well, shifting the demand curve to the right (augmenting the demand). However, if the direct price effect dominates the goodwill effect, then demand will still decrease in price, as illustrated by curve  $D^2$ . Whereas, if the direct price effect is dominated by the goodwill effect, then the Veblen effect, in which demand increases in price (as illustrated by curve  $D^3$ ) takes place.

Built on a static setting, Figure 1 offers insights regarding the potential viability of the Veblen effect. Indeed, a higher price may imply greater revenue, allowing the firm to advertise more in order to enhance goodwill, which may attract consumers more than what a higher price discourages them.

## 4 The Price-Demand Relationship: Dynamic Setting

In order to capture the dynamics of the change in price, advertising and demand over time, in this section we employ the dynamic setting which allows to characterize the conditions under which a higher price leads to larger demand.

### 4.1 Value of $\lambda(t)$

Costate variable  $\lambda(t)$  represents the potential increase in future profit if goodwill is increased by one unit. Define  $e_G = \frac{\partial D}{\partial G} \frac{G}{D}$ , the goodwill elasticity of demand, and recall  $e_p = -\frac{\partial D}{\partial p} \frac{p}{D}$ , the price elasticity of

demand. Hence,  $\frac{\partial D}{\partial G} = -\frac{e_G}{e_p} \frac{p}{G}$ . Substituting  $e_G$ ,  $e_p$  and (5b) in (5f) implies

$$\frac{d\lambda}{dt} - \left( r - \frac{\partial A}{\partial G} \right) \lambda = -D \left( \frac{e_G}{e_p} \frac{p}{G} \right) \text{ with } \lim_{t \rightarrow \infty} e^{-rt} \lambda(t) = 0,$$

which has the solution

$$\lambda(t) = \int_t^\infty D \frac{e_G}{e_p} \frac{p}{G} e^{-(r - \frac{\partial A}{\partial G})(s-t)} ds, \quad (10)$$

suggesting that the shadow price of goodwill  $\lambda$  is directly linked to the demand  $D$  and the markup effect  $\frac{e_G}{e_p} \frac{p}{G}$ .

The markup effect  $\frac{e_G}{e_p} \frac{p}{G}$  captures the price increase that the consumer is willing to pay following a rise in goodwill. The markup effect increases with the relative demand sensitivity to goodwill and to price  $\frac{e_p}{e_G}$ , and the goodwill-deflated price  $\frac{p}{G}$ .

Equations (6) and (10) together ensure that the markup effect is strictly positive at any point in time.

$$\frac{e_G}{e_p} \frac{p}{G} > 0, \quad \forall t \in [0, \infty). \quad (11)$$

The markup effect exerts a positive influence on  $\lambda$  ( $\frac{e_G}{e_p} \frac{p}{G} > 0$  as all terms are positive), because goodwill promotes willingness to pay, and consequently boosts profits.

## 4.2 Variations of $p(t)$

In order to gain comprehensive insights regarding the price-demand relationship we extend condition (5b) to the dynamic case where the variables are linked through their evolution (path) over time.

Condition (5b) should hold at any point in time, so that at the optimum, variations in revenue due to changes in price must equal the corresponding variations in advertising expenses. The formal link among these variations as well as variations in goodwill manifests with the differentiation of the first-order pricing condition (5b) with respect to time:

$$\frac{dp}{dt} = \frac{d}{dt} \left( -\frac{D}{\frac{\partial D}{\partial p}} \right) = -\frac{\left( \frac{\partial D}{\partial p} \frac{dp}{dt} + \frac{\partial D}{\partial G} \frac{dG}{dt} \right) \frac{\partial D}{\partial p} - D \left( \frac{\partial^2 D}{\partial p^2} \frac{dp}{dt} + \frac{\partial^2 D}{\partial p \partial G} \frac{dG}{dt} \right)}{\frac{\partial D^2}{\partial p}},$$

that is

$$\frac{dp}{dt} \left( 1 + 1 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D}{\partial p}} \right) = \frac{dG}{dt} \left( - \frac{\frac{\partial D}{\partial G} \frac{\partial D}{\partial p}}{\frac{\partial D^2}{\partial p}} + D \frac{\frac{\partial^2 D}{\partial p \partial G}}{\frac{\partial D^2}{\partial p}} \right).$$

Substituting  $-\frac{\frac{\partial D}{\partial p} \frac{\partial D}{\partial G}}{\frac{\partial D^2}{\partial p}} = \frac{e_G}{e_p} \frac{p}{G}$  and rearranging yields

$$\frac{dG}{dt} \left( \underbrace{\frac{e_G}{e_p} \frac{p}{G}}_{(+)} + \underbrace{D \frac{\frac{\partial^2 D}{\partial p \partial G}}{\frac{\partial D^2}{\partial p}}}_{(\pm)} \right) = \frac{dp}{dt} \underbrace{\left( 2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D^2}{\partial p}} \right)}_{(+)}, \quad (12)$$

which clearly identifies the link between the dynamics of goodwill  $\frac{dG}{dt}$  and the dynamics of price  $\frac{dp}{dt}$ .

Recall the markup effect  $\frac{e_G}{e_p} \frac{p}{G}$  and define the sales effect as  $D \frac{\frac{\partial^2 D}{\partial p \partial G}}{\frac{\partial D^2}{\partial p}}$ , it can be claimed that

**Proposition 3.** *The price and goodwill evolve in the same direction if the sum of the markup and sales effects is positive.*

*Proof.* Immediate from (12). □

Proposition 3 provides the managerial condition for the shift of the demand curve to the right as illustrated though curves  $D^2$  and  $D^3$  in Figure 1(b).

### 4.3 Variations of $D(t)$

Rearranging (12) let us obtain an expression for the dynamics of goodwill:

$$\frac{dG}{dt} = \frac{dp}{dt} \frac{2 - D \frac{\frac{\partial^2 D}{\partial p^2}}{\frac{\partial D^2}{\partial p}}}{\frac{e_G}{e_p} \frac{p}{G} + D \frac{\frac{\partial^2 D}{\partial p \partial G}}{\frac{\partial D^2}{\partial p}}}, \quad (13)$$

and substituting (13) in (8) yields:

$$\frac{dD}{dt} = \frac{dp}{dt} \left( \underbrace{\frac{\partial D}{\partial p}}_{-} + \underbrace{\frac{\partial D}{\partial G}}_{+} \frac{\overbrace{2 - D \frac{\partial^2 D}{\partial p^2}}^{+}}{\underbrace{\frac{e_G p}{e_p G} + D \frac{\partial^2 D}{\partial p \partial G}}_{\pm}} \right) \quad (14)$$

Defining the *intermediating goodwill effect* as  $\frac{\partial D}{\partial G} \frac{2 - D \frac{\partial^2 D}{\partial p^2}}{\frac{e_G p}{e_p G} + D \frac{\partial^2 D}{\partial p \partial G}}$ , it follows that

**Proposition 4.** *The price and demand evolve in the same direction if the sum of the direct price effect and intermediating goodwill effect is positive.*

*Proof.* Immediate from (14). □

Proposition 4 specifies the conditions when the seller can experience the Veblen effect and further benefit from it by aligning its advertising and pricing schedule. [It may be noted that the development of this result has been prompted by the incorporation of the advertising-goodwill relationship.](#)

Propositions 3 and 4 provide the structure for the viability of the Veblen effect. When the firm invests in advertising it may lead to an increase in its goodwill if its impact is stronger than the reduction associated with goodwill's autonomous decay. An increase in goodwill boosts the demand, however, the firm may respond by adjusting the price accordingly. Proposition 3 provides the condition when the firm reacts by increasing the price. At this point the demand is subject to two opposing “forces”: the increase in goodwill which pushes it up and the increase in price which pushes it down. The firm will seek the price-demand combination that will maximize its profit. Proposition 4 provides the condition when both, price and demand, move in the same direction.

## 5 Examples

In order to illustrate the dynamics that leads to the Veblen effect we consider a simple, though common, additive separable demand function in the form  $D(p, G) = h(p) + l(G)$ , where  $h(p) = b - cp^d$  (and

consequently  $\frac{\partial D}{\partial p} = -cdp^{d-1}$  and  $\frac{\partial^2 D}{\partial p^2} = -cd(d-1)p^{d-2}$ , where  $b \geq 0$ ,  $c > 0$ ,  $d > 0$ ). Evaluating (14) and also substituting  $D$  with  $-p\frac{\partial D}{\partial p}$  (according to (5b)) we get  $\frac{dD}{dt} = \frac{dp}{dt}cd^2p^{d-1} > 0$ . This result indicates the viability of the Veblen effect, as demand and price move in the same direction ( $\text{sgn } \frac{dD}{dt} = \text{sgn } \frac{dp}{dt}$ ).

To further demonstrate the mechanics of the relationships among the variables we study their evolution with finite time horizon  $T$ . We incorporate a goodwill dynamics  $A = \sqrt{a} - \delta G$  (as in Karray and Martín-Herrán 2009) so the Hamiltonian is:

$$H = p[h(p) + l(G)] - a + \lambda(\sqrt{a} - \delta G).$$

As  $\frac{\partial H}{\partial a} = -1 + \frac{\lambda}{2\sqrt{a}} = 0$  we get  $a = \frac{\lambda^2}{4}$ ,  $\frac{d\lambda}{dt} = (r + \delta)\lambda - pl'(G)$  which has the solution  $\lambda(t) = \int_{s=t}^T pl'(G)e^{-(r+\delta)(s-t)} ds = pl'(G)\frac{1-e^{-(r+\delta)(T-t)}}{r+\delta}$  and  $\frac{d\lambda(t)}{dt} = -pl'(G)e^{-(r+\delta)(s-t)} < 0$ .

For ease of exposition we use  $h(p) = b - p$  (i. e.,  $c = 1$ ,  $d = 1$ ) and  $l(G) = G$ , that is  $D = b - p + G$ . According to (5b)  $p = \frac{b+G}{2}$  and combined with  $l'(G) = 1$  and  $a = \frac{\lambda^2}{4}$  we obtain  $A = \frac{\lambda}{2} - \delta G = \frac{b+G}{4} \frac{1-e^{-(r+\delta)(T-t)}}{r+\delta} - \delta G$ .

As  $1 - e^{-(r+\delta)(T-t)}$  decreases in  $t$ , its largest value is achieved at  $t = 0$  and reaches zero at  $t = T$ . Consequently, based on the values of  $r$  and  $\delta$ ,  $A$  may be positive in the beginning of the period and then turn negative or be negative throughout the entire planning period. Thus, goodwill will increase and later decrease or decrease all the time. As depicted in Figure 2, with  $T = 1$  and  $r = 0.1$ , when  $\delta = 0.1$  goodwill increases during 79% of the period and decreases afterwards. However, when  $\delta = 0.4$  it decreases throughout the entire period. Finally, since  $p = \frac{b+G}{2}$ , then  $D = b - p + G = b - p + 2p - b = p$ , indicating that demand will move in the same direction as the price and consequently exhibiting the viability of the Veblen effect.

## 6 Concluding Remarks

While the classical explanation for the Veblen effect is based on conspicuous consumption, which promotes exclusivity, prestige, and status, this work offers an alternative explanation. Through the exploitation of the time dimension we are able to capture the relationship among advertising, price, and demand, and how they evolve over time. In addition we provided conditions when the Veblen effect will transpire.

Although the paper focuses on the Veblen effect it also advises firms of all products (including “stan-

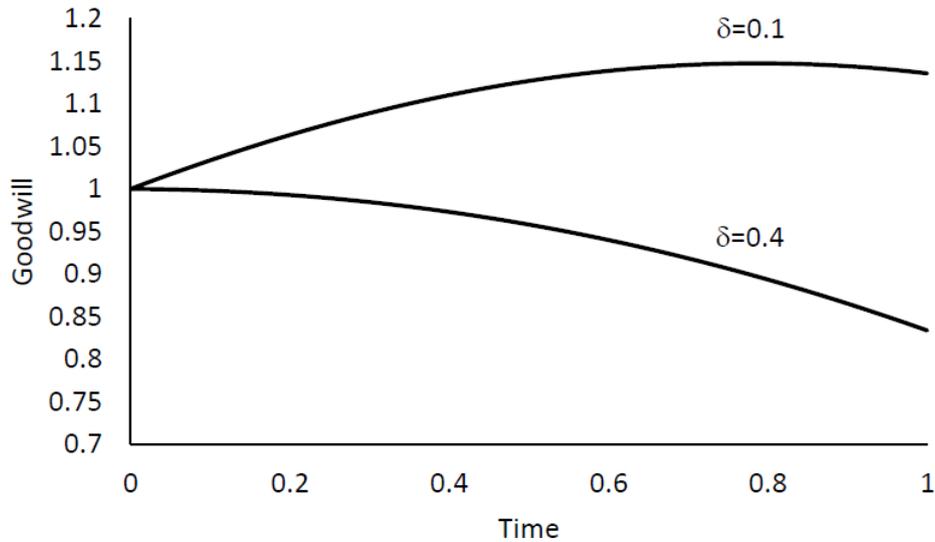


Figure 2: Evolution of Goodwill over Time

dard”) how much to invest in advertising at each point in time in order to maximize profit by benefiting the most from the goodwill-price-demand dynamics.

This paper may promote future research in several promising directions, as investment in advertising plays a major role in the enactment of the Veblen effect. Previous studies on conspicuous demand products such as Rao and Schaefer (2013), Gao et al. (2017), and Li (2019) assumed a given consumers’ sensitivity to status. By treating sensitivity as a variable which is subject to investment in advertising as shown in this paper such studies may be advanced further. Similarly, incorporating advertising policy during challenging recession periods as described in Caulkins et al. (2011), Huschto et al. (2011), and Huschto and Sager (2014) will allow sellers to further leverage on conspicuous demand. Another development would be to provide a micro-foundation to the demand function used. A demand function resulting from a consumers’ utility-maximization program would provide a sound normative basis. Moreover, it will be intriguing to determine how spending over time should change and the impact it will have on the Veblen effect when firms face budget constraints as in Sethi (1977b). Particularly, facing a budget constraint when selling multiple products (for example, national brand and store brand as in Karray and Martín-Herrán 2009) may be even more challenging as the different products compete on the limited available budget. Furthermore, how such limited budget should be distributed among the products when some are Veblen goods? Will the limited budget augment the Veblen effect or eliminate it?

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