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Profit maximizing coalitions with shared capacities in distribution networks

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Abstract

We study the distribution network structure of multiple firms in the context of demand sensitivity to market offers. The problem consists in determining the profitability of horizontal collaboration between firms in a collaborative distribution schema. It considers the case of a set of regional distribution centers (DCs) where each DC is initially dedicated solely to one firm's distribution activities and studies when it is beneficial that the DC owners collaborate through sharing their storage-throughput capacity. Such strategic decisions are made in order to improve the distribution capabilities of firms in terms of response time and cost-efficiency compared to the stand-alone situation. The problem is modeled as a coalition formation game in a cooperative framework, and we propose a collaborative distribution game with profit maximization. Three sharing mechanisms are modeled and tested: egalitarian allocation, proportional allocation, and Shapley value. The collaboration decision conditions for a given firm are analytically derived according to the sharing method considered and used to enhance the solution approach. Our numerical results clearly highlight the impact of this innovative collaboration opportunity on the firms' performance in terms of distribution cost savings and revenue increases. An observed behavior is that the formation of several sub-coalitions prevails over the formation of a grand coalition, and that different cost sharing methods can lead to different sub-coalitions. We also provide managerial insights on the appropriate size of a coalition in various business instances tested, and on the key drivers that foster horizontal collaborative behavior among firms.

Keywords:

Supply chain management, Game theory, Coalition formation, Cost allocation, Distribution networks, Shared capacities.

1. Introduction

Today's shoppers expect large product assortments in the stores they visit, and to receive products ordered online after a couple of hours at their preferred location. For a given company, the efficiency and agility of its distribution system is crucial to remaining competitive in such an on-demand business context. A distribution system enables moving products from one or several storage locations to a demanding customer base in a responsive and cost-effective way (Martel & Klibi, 2016). As known, the effective positioning of inventories helps firms hedge against demand uncertainty and achieve good service levels at minimum costs. However, the increasing expectations of customers in terms of service level, and the high uncertainty in demand, seriously challenge current distribution systems. At the strategic level, distribution network design has tended to be oriented toward opening new distribution centers (DCs) subject to maximum response time constraints and

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limited investments. Capital expenditure refers to a company’s acquisition of fixed assets or long-term resources, such as facilities, equipment, etc. Such strategic acquisition decisions involve an initial investment, and generate monetary outflows (property taxes, insurance, etc.) throughout the assets’ useful life. At this level, a major preoccupation is the long-term financing of the capital required and the expected return on investments. When companies are globally deployed and have to serve a large customers base, installing a set of distribution centers requires large investments to purchase, build, or renovate these assets. However, in practice, only few companies (e.g., Walmart, Amazon, jd.com) have the scale and means to maintain such expenses in the long run. For most companies, constrained capital expenditure will limit the expansion of their DC network in the territory, thus affecting their performance in terms of service level and the provision of large product assortments.

On the other hand, with the rise of the sharing economy, companies perceive horizontal collaboration as an interesting approach to deal with their distribution challenges. For instance, a study by Deloitte (2016) reported how the sharing economy is impacting the transportation ecosystem with new collaborative opportunities such as coordination of warehouse space. In the same way, Matzler et al. (2015) underlined that a way to profit from the sharing economy is for companies to share existing assets and capacities, especially underutilized resources. In this context, the main question is how a closed distribution network (i.e., when a company relies on only its own resources) can be enhanced through the shared distribution capabilities of external entities. The opportunity to exploit companies underutilized capacity is a key principle in the Physical Internet initiative toward a service-driven logistics system with assets sharing (Montreuil (2011); Montreuil et al. (2013)). The issue of capacity imbalance in non-cooperative contexts was recently tackled in (Roels & Tang, 2017) and is nowadays well managed for instance in the airline alliances. Conversely, in a multi-industry context, companies managing the distribution of their own brands in a given region does not necessary compete for demand. In this case, their competitive advantage cannot be altered by horizontal collaborations and thus there is no reason to be reluctant to share when there is a financial benefit in doing so. Consequently, several distribution networks now consider sharing their DC capacities as well as the joint replenishment and delivery of goods. Mutual inbound and/or outbound transportation schemas provide benefits mainly in terms of transportation costs. The Henkel case (Henkel, 2010) provides a clear example that sharing warehouses can positively affect performance. Henkel uses storage and transportation pooling of manufacturers flows that have compatible finished goods. The advantages include increasing the frequency of deliveries and service level, as well as reducing stock in the retail warehousing stage. By pooling inventories, Henkel reduced retailer stocks by 13%, increased the number of deliveries, and reduced carbon dioxide emissions (Henkel, 2010). Another example is Kuehne-Nagel, that joined the distribution alliance of Nestle and Danone. Kuehne-Nagel located a shared warehouse in Toulouse (France) and joined Nestle and Danone’s outbound transportation. All three companies benefited not only from the better use of warehousing capacity and the logistics service provider’s truck fleet, but also reducing warehousing and transportation costs for the manufacturers (Logistics Manager, 2010). Moreover, the emergence of on-demand warehousing startups, such as Flexe (www.flexe.com) and Warehouse-anywhere (warehouseanywhere.com), is a clear indication of the need for shareable distribution capacity among different companies. With this in mind, this paper investigates how a company can design its distribution network through horizontal collaboration and form a profitable coalition. This is done under a deterministic setting, characterized by a demand that is sensitive to the market offer in terms of response time. To this end, we describe and model the distribution network design problem with shareable capacities as a coalition formation game.

Alongside these practical cases, the sharing resources concept has received increasing attention in the distribution systems literature. For instance, Kukreja & Schmidt (2005), Kranenburg

& Van Houtum (2009), and Guajardo et al. (2015) tackle warehousing-product pooling problems where different warehouses may supply each other to fulfil demand requirements for the same product. In other cases, collaboration may occur through warehousing-capacity pooling, that is, sharing capacity at a single site to fulfil demand from different geographic markets, which has also proven beneficial. However, most of the literature has focused on problems where the different warehouses or different markets relate to a single firm (i.e., the firm individually makes the decision on whether or not to pool resources). When sharing potentially involves different firms, other important challenges arise, such as which coalitions to form and how the costs should be allocated among the firms within a given coalition. Recently, Roels & Tang (2017) introduced the concept of co-distribution alliances where the authors investigate the benefit of the ex-ante capacity reservation contract in bidirectional alliances using non-cooperative game theory. In the same vein, our work studies how profitable coalitions should first be formed at the strategic level to share capacity based on complementary resources prior to turning to designing the bidirectional alliances contract.

Furthermore, Elomri et al. (2012) focus on a joint replenishment system involving a set of independent and freely interacting retailers purchasing an item from one supplier to meet a deterministic demand. The authors investigate this issue as a coalition formation and cost allocation problem where they provide an iterative procedure to form the so-called efficient coalition structure. In the same context, Ben Jouida et al. (2017) study a coalition-formation problem for cooperative replenishment with a single supplier and multiple firms. The authors also investigate the profitability of horizontal cooperation between firms when designing collaborative ordering and transportation contracts. Their analytical study shows the potential benefit of horizontal collaboration between firms in terms of profit maximization compared to the stand-alone situation. Yu et al. (2015) study production/service capacity sharing in a supply chain network with multiple independent firms using cooperative games. The objective of such collaboration is minimizing the delay and capacity investment costs. Their results show that there are certain conditions where capacity sharing can still be beneficial for a set of firms. However, the authors also show that capacity sharing is not always beneficial, particularly when firms have heterogeneous work contents and service variabilities.

The aforementioned literature highlights a gap in considering horizontal collaboration in the design of a supply chain network with multiple firms. In addition, the literature on collaboration in storage capacity in the context of independent firms is limited, and most studies deal with only one cost sharing method to manage the collaboration decision. Accordingly, this study aims to fill this gap by considering the possibility of sharing storage capacities taking into account multiple suppliers, multiples firms, and multiple customer zones. We study the problem as a coalition formation problem based on cooperative game theory, and investigate alternative cost sharing methods. As Cachon & Netessine (2006) note, most supply chain models based on game theory use a non-cooperative approach. However, cooperative game theory would seem more appropriate to analyze a supply chain network at its design level, as it is characterized by numerous possibilities for enterprise coalitions and allocation patterns. A general characteristic of cooperative games is that the players understand they can obtain a larger global benefit from pooling their resources than by acting separately. Furthermore, few studies incorporate the concepts from cooperative game theory in warehousing problems (Elomri et al., 2012; Guajardo & Rönnqvist, 2016a; Ben Jouida et al., 2017). Coalitional game theory provides a formal analytical framework with a set of mathematical tools to study the complex horizontal interactions among rational firms. The two central questions in coalitional games are: (1) what coalitions (also called alliances) of players are likely to form, and (2) how will the players share the benefits of collaboration? It is generally assumed that a *grand coalition* involving all players forms, thus reducing the problem to only dealing with the second question (Guajardo & Rönnqvist, 2016b). Partitioning the set of players into disjoint coalitions is part of the *coalition structure* problem (Aumann & Dreze, 1974). This is more general than assuming

a grand coalition and is more suitable in situations where, for example, capacity limitations do not allow establishing coalitions that are too large.

In this paper, we assume that each firm deals with a set of customer zones, geographically scattered in a given region, and where each customer zone is characterized by demand that is sensitive to the market offer in terms of service level (i.e., response time). In order to meet its customers' needs along the planning horizon, each firm has the option of either continuing to operate only with its own DC or forming a coalition with other distribution partners to access their DCs. In the latter case, the members of the coalition will have access to the joint DC network and then optimize the positioning of their inventories in the shared DCs, as well as their inbound and outbound flows in the network. In this context, several questions arise: Is collaboration always beneficial for all firms? Which profitable coalition should each firm join? And if a subset of firms decides to collaborate and share capacity, how will the joint costs of the shared DC be allocated?. To answer these questions, we model the coalitional game of the distribution problem in the stand-alone and collaborative scenarios. For the collaborative scenario, we use cooperative game principles and develop an algorithm to find a coalition structure and a cost allocation for each, such that coalitions of maximum profitability are formed, and none is incentivized to deviate from the structure. We also undertake an analytical study of the profitability conditions based on sharing methods to enhance the collaboration decisions. According to different notions of fairness, many benefits sharing methods have been proposed in the literature. Using one or another method may heavily affect the outcome of different players and the size of the coalitions (see, e.g., Basso et al. (2020); Guajardo et al. (2016); Le Cadre et al. (2019)). We focus here on three well-known methods: Shapley value, egalitarian allocation, and proportional allocation, which as reviewed by Guajardo & Rönnqvist (2016b) are among the most used allocation methods in the literature on collaborative logistics. We address several problem instances to assess the efficiency of collaboration in generating appropriate coalition structures that fulfill all firms' requirements. We show through numerical experiments the profit savings of this collaborative approach, and analyze the impact of capacity-sharing on service level.

The remainder of the paper is structured as follows. Section 2 describes the problem context and models the distribution network under the stand-alone and collaborative configurations. Section 3 presents the coalitional game and develops an algorithm to identify a coalition structure. The computational results and managerial insights are presented in Section 4. Section 5 concludes the paper.

2. Description of the Distribution Problem with Shareable Capacities

2.1. Notation

I The set of firms, indexed by $i \in I$

J The set of distribution centers, indexed by $j \in J$

Z The set of customer zones, indexed by $z \in Z$

$j_{(i)}$ The index of the distribution center operated by firm i

$k_{(i)}$ The index of the supplier associated with the firm i

p_i The sold unit price of products of firm i

cap_j The available capacity at the distribution center j

\hat{d}_z^i The minimum estimated demand of customer zone z for firm i

d_{zj}^i The demand of customer zone z of firm i assigned to DC j

δ_1, δ_2 The maximum distance threshold for response time levels one and two, respectively

ρ_1, ρ_2 Positive parameters for increasing demand per coverage level

q_i The ordered quantity of firm i

a_i The fixed order cost associated with firm i

- β_0, β_1 The fixed and variable transportation function's parameters, respectively
- α_0^j, α_1 The fixed and variable warehousing function's parameters for each DC j , respectively
- n_i The number of orders for firm i along the planning horizon
- $n_{j,s}^i$ The number of orders for firm i when joining coalition s from DC j
- $\delta_{k(i)j}$ The distance separating supplier k supplying products to firms i from DC j
- δ_{jz} The distance separating DC j from the location of customer zone z
- P_i The expected profit associated with firm i in a stand-alone scenario
- R_i The expected revenues reached by firm i in a stand-alone scenario
- C_i^X The expected cost $\{X = O, R, H, T\}$ for ordering, replenishment, warehousing holding, and distribution activities
- Ω The set of all the universe of coalitions indexed by s
- Ω' The subset of all profitable coalitions in Ω
- Ω^* A coalition structure that includes the coalitions of maximum profitability.
- I_s The set of firms in a given coalition s
- J_s The set of collaborative distribution centers under coalition s
- $J_{(i)}$ The set of distribution centers operated by firm i under coalition s
- $Z_{j,s}^i$ The set of customer zones assigned by firm i to a collaborative distribution center j in coalition s
- $d_{j,s}^i$ The demand of firm i assigned to the collaborative distribution center $j \in J_s$ when joining coalition s
- d_s^i The total demand of firm i when joining coalition s
- $q_{j,s}^i$ The ordered quantity of firm i stored in the collaborative distribution center $j \in J_s$
- q_s^i The total ordered quantity of firm i when joining coalition s
- P_s The total profit of all firm forming coalition s
- $R_{i,s}$ The revenues reached by firm i when joining coalition s
- $C_{i,s}^X$ The cost $\{X = O, R, H, T\}$ for ordering, replenishment, warehousing holding, and distribution activities for firm i when joining coalition s
- m The index of the sharing method $m = 1, 2, 3$
- $C_{i,s}^{H(m)}$ The warehousing holding cost supported by firm $i \in I_s$ and shared using method m
- $P_{i,s}^m$ The individual profit allocated to a firm $i \in I_s$ under the sharing method m .

2.2. The problem context

The distribution context illustrated in Figure 1 describes a given market/sales territory, partitioned into six customer zones (CZ), where a set of firms must regularly serve a customer base with non-substitutable products. In this context, each firm i operates its own regional DC $j_{(i)}$ and decides its mission in terms of replenishment, inventory level, and delivery to customers. We consider here that the products are already in stock at the DC when the orders are received. Figure 1 illustrates the case of three firms serving the customer zones where each firm acts alone and hence operates only its own DC and does not share the structural part of its network. From firm's 1 perspective, the accessible part of the network is depicted in the figure, which underlines various coverage levels to the customer zones based on the positioning of the firm's DC in the region. The figure also underlines the utilized capacity and the remaining unused one at the DC of firm 1, and the non accessibility to the other firm's DCs when no collaboration. Identical network flows mapping could be appended for firm 2 and firm 3, with similar behaviors of coverage, unused capacity and non accessibility to other DCs in a dedicated distribution context. During its business period, each firm must minimize its operational expenses by optimizing its replenishments, its inventory level at the DC, and its deliveries to customer zones. It must also maximize its revenues by serving the demand with an adequate service level. It is assumed here that the firms are independent, purchase non-substitutable products, and are supplied from different locations.

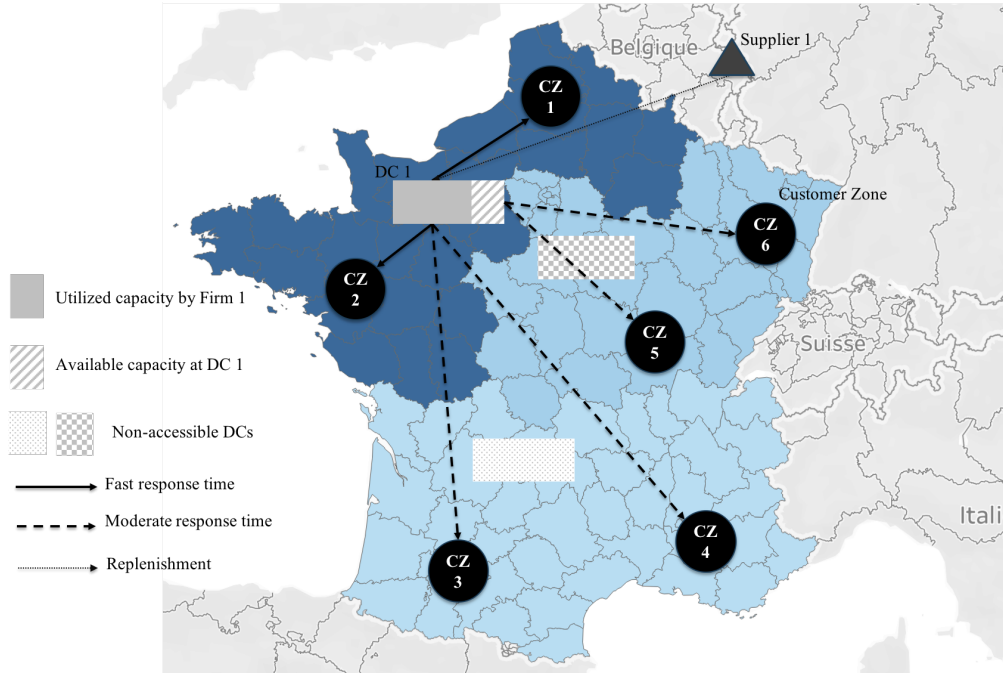


Figure 1: Firm 1 network in a dedicated distribution problem

Moreover, the business context considers a planning horizon of one year and relies on the fact that firms have no opportunity to open/build additional DCs in the area covered. It is also characterized by the increasing importance of offering a higher service level to customers through the modeling of the sensitivity of demand to the response time offered. This is a contemporary and crucial issue in the retail and FMCG sectors for instance where the firm's competitiveness relies on the effectiveness and speed of its distribution system. Accordingly, firms have the opportunity to collaborate by sharing their DC capacities, which means giving access to their DC to other firms in the coalition in order to operate their distribution. Figure 2 presents for instance how the network of firm 1 would be designed when the three firms are forming a profitable coalition and thus firm 1 is accessing to the three DCs to serve its customers. Here the firm would benefit from the available DCs capacity in the shared distribution network to offer a faster response time for all its customer zones. In contrast with Figure 1, the coalitional distribution network in Figure 2 is optimized differently with respect to sourcing flows, inventory level per DC, and shipping flows to customers to maximize the profit of each firm in the coalition. The figure underlines also how firm 2 and firm 3 would also [gain](#) access to firm 1 DC and operate it for their distribution. This latter description gives rise to the Distribution Problem with Shareable Capacities (DPSC) between multiple firms, serving a set of customer zones in a given region.

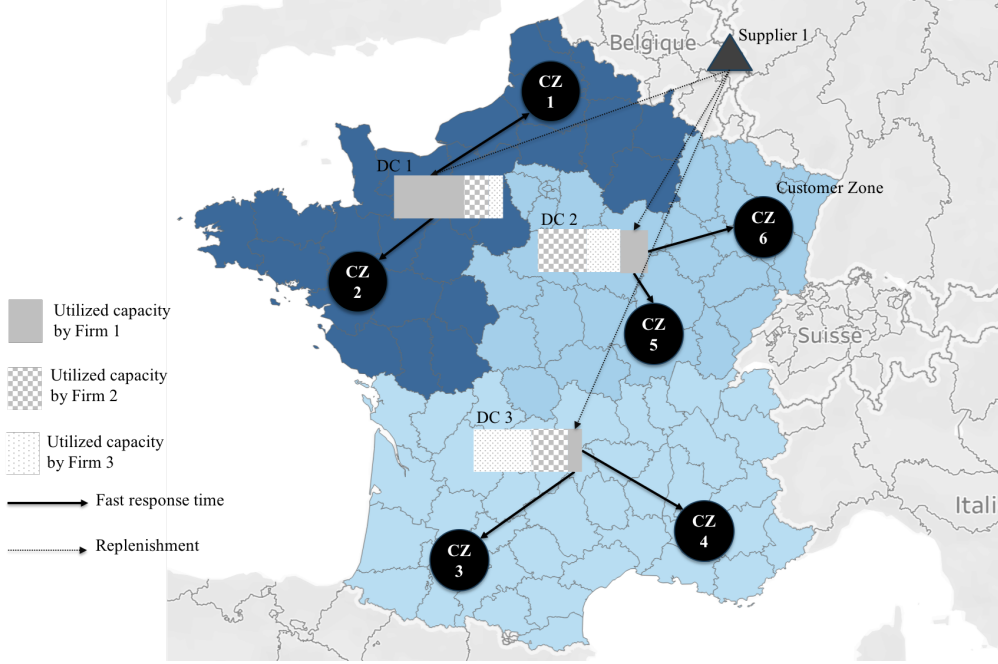


Figure 2: Firm 1 distribution in a shared network

When a firm i acts alone, the allocation to each customer zone z to the firm DC j is implicit, and thus, the allocated demand d_{zj}^i is fixed a priori based on the possible coverage from $j_{(i)}$. However, when firm i is in a given coalition s , then the allocation of customer zones could be optimized from a set of DCs $J_{(i)}$ pertaining to the firms in I_s . Such optimization is based on the best tradeoffs along the coalitional distribution network: on one side, using the DC that reduces the sourcing costs with regard to the supplier base, and the warehousing costs, given the economies of scale; on the other side, access to closer DCs in their customer zones to increase their revenues and reduce their outbound transportation costs.

Next, we propose characterizing the demand for each customer zone and modeling its sensitivity to the firm offer in terms of response time. First, when looking at a given firm, the demand level in a given customers zone is an aggregation over the set of customers in that zone and the set of operational periods over the planning year. Using historical data the demand could be bounded by an estimated minimum demand level \hat{d}_z^i , which corresponds to the minimum market penetration target of firm i in zone z . [We consider here that the model and results rely on the assumption that minimum demand is perfectly forecasted.](#) Second, we consider that the demand (d_{zj}^i) for a given customer zone z would depend on the estimated response time from the assigned DC j , which reflects its sensitivity to the firm i offer in terms of service level. The strategic optimization of a firm market offer is discussed and modeled in Martel & Klibi (2016). It builds on a *value added maximization* framework where revenues are affected by order-winners, such as response time, that are provided by the capabilities of the physical deployment of the network resources on the geographical territory covered by the firm. Under this framework, the delivery response time is clearly dependent on the distance between the assigned DC j location to the customer zone z , denoted by δ_{jz} . The demand behavior described above underlines an explicit relationship between the demand value, and two explanatory variables: the minimum demand level, and the DC coverage level. This dependency is characterised conceptually in equation (1) for each firm i based on the distance matrix between customer zones and the set of available DCs (denoted by $\delta_{J_{(i)}}$). More explicitly, we consider here three coverage levels based on the delivery distance δ_{jz} , that are delineated by maximum-distance parameters δ_1 and δ_2 , with $0 < \delta_1 < \delta_2$. These distance-based coverage levels of the territory

provide three potential levels of demand d_{zj}^i expressed by an increase of the minimum demand level \hat{d}_z^i , with positive parameters ρ^1 and ρ^2 , with $0 < \rho^1 < \rho^2$. Equation (2) expresses the piecewise linear demand function modeled in this work. Such relation assumes that a step decrease in the delivery distance to cover δ_{jz} leads to an increase in the customer zone's demand d_{zj}^i . For instance, based on Figure 2 example, less than a one hundred mile distance guarantees a same day delivery service; up to a three hundred mile distance allows a next day delivery service; and at a least a two-day delivery service is required for higher distances.

$$d^i = \phi(\hat{d}^i, \delta_{J(i)}), \forall i \in I, \quad (1)$$

$$d_{zj}^i = \begin{cases} \hat{d}_z^i(1 + \rho^2) & \text{if } \delta_{jz} < \delta_1, \\ \hat{d}_z^i(1 + \rho^1) & \text{if } \delta_1 \leq \delta_{jz} < \delta_2, \\ \hat{d}_z^i & \text{if } \delta_{jz} > \delta_2, \end{cases} \quad \forall i \in I, \forall j \in J(i), \forall z \in Z, \quad (2)$$

With this demand modeling approach, it is clear that the demand level (d_{zj}^i) of firm i , for demand zone z , allocated to DC j is deterministic. This setting is considered, at this decision level, as sufficient to anticipate at the beginning of the planning horizon the future demand level of each zone and at each DC. In this decision-making process, we assume that the computation of the ordering, warehousing and replenishment costs for a given firm i is mainly based on the optimal ordering quantity (q_i). It is worthy to notice that our modeling approach works straightforwardly with any lot-sizing approach that computes the ordering quantity. In this work we considered the economic ordering quantity (EOQ) formula which is often considered in the inventory literature since it provides good approximations when determining the (s, S) policy parameters under a sequential optimisation framework (Porteus, 1985). In addition, from a practical perspective, Ferguson et al. (2007) and Dobson et al. (2017) claimed that EOQ is well suitable for perishable goods, which is among the most favorable practical contexts for collaborative distribution.

2.3. The dedicated distribution problem

In the stand-alone case, once demand is anticipated and the quantities are determined, each firm $i \in I$ individually optimizes its expected profits for the planning horizon considered by finding the best compromise between revenues and costs. Thus, for each firm, the expected profit is calculated as the expected sales revenues minus the sum of the expected operating costs (ordering, replenishment, inventory, and transportation costs) as follows:

$$P_i = R_i - C_i^O - C_i^R - C_i^T - C_i^H, \quad (3)$$

where

- $R_i = \sum_{z \in Z} p_i d_{zj}^i$,
- $C_i^O(q_i) = a_i n_i$,
- $C_i^R(q_i) = (\beta_0 + \beta_1 \delta_{k(i)j}) q_i n_i$,
- $C_i^T(d_{zj}^i) = \sum_{z \in Z} (\beta_0 + \beta_1 \delta_{jz}) d_{zj}^i$,
- $C_i^H(q_i) = \alpha_0^j(q_i)^{\alpha_1} n_i$.

From equation (3), it is established that the ordering costs C_i^O depend mainly on a fixed cost per order and the number of orders made to the supply side. The frequency of orders is implicitly obtained from the fraction of the annual demand and the ordering quantity. The replenishment costs C_i^R depend on variable and fixed parts of the transportation fees using a distance-based linear

function that estimates the unit arc-flow costs from the source location to the DC of firm i . In the same way, the computation of the outbound transportation costs C_i^T is based on a distance-based linear function, estimating the costs of delivery from firm i 's DC to the customer zones. Moreover, an elaborated warehousing cost function is proposed, which reflects warehousing concentration requirements leading to economies of scale (Shapiro & Wagner, 2009). More specifically, higher storage quantities promote higher efficiency with warehousing technologies, which lead to lower average warehousing levels and costs. Thus, the computation of the warehousing costs C_i^H for a given firm i is based on the optimal ordering quantity (q_i), and two estimated parameters. The first parameter, denoted with α_0^j , is a positive factor estimating warehousing and inventory holding costs, and the second parameter denoted with α_1 reflects warehousing concentration requirements. The latter generally falls in the range $[0.5, 0.8]$ in case of economies of scale, taking value 1 otherwise, as discussed in Shapiro & Wagner (2009).

2.4. The collaborative distribution problem

As shown in Figure 2, horizontal collaboration offers the opportunity for firms to access additional distribution capabilities and share their own capacities when underutilized. When looking closely at the demand sensitivity to the response time (equation (2)), it is clear that forming coalitions could be beneficial for several firms, and this opportunity needs to be investigated. To do so, several questions are raised here: Can the firms improve their individual economic performance when they collaborate? If so, what is the best coalition to form for each firm so that the profit of each collaborating firm is increased? And how will the firms react to collaboration according to the cost sharing method proposed? More specifically, the two main issues are whether the firms could increase their individual profit when they collaborate in terms of capacity sharing, and whether the selection of the suitable cost sharing method in terms of warehousing costs incentivizes the firms' decision to collaborate. In addressing these issues, we adopt what Smirnov & Sheremetov (2012) refer as *centralized coalition formation*. This assumes that all data is available for a central planner that plays the role of a *solver* to find the collaborative solution. In practice, this could be an organization such as *TRI-VIZOR*, a so-called impartial orchestrator for transport and logistics that prepares, designs and operates horizontal partnerships (Creemers et al., 2017). This centralized approach is also well established in related literature: as pointed out in a review by Gansterer & Hartl (2018), a large share of the literature on collaborative vehicle routing adopts a central planning perspective.

With this in mind, let us now explicitly characterize the coalition formation problem for the DPSC. We note that in the current problem setting only warehousing costs are subject to sharing among the collaborating firms. The argument behind this is that collaborating firms would initially prefer to maintain their distribution system management independent (i.e., no joint ordering, replenishment, and/or transportation processes). Let s denote a given coalition composed of a collection of firms that accept sharing their DC capacities.

For a given coalition s , we now have a set of DCs accessible for each firm $i \in I_s$. Thus, the allocation of customer zones $z \in Z$ is no more unique (i.e., $Z_{j(i)}$) and could be reoptimised taking into account the opportunity access to the coalitional DCs $j \in J_s$. Accordingly, the allocation rule to update, denoted by $Z_{j,s}^i$, is given by Equation (4). It looks for the nearest re-assignment of each customer zone $z \in Z$ among the set of collaborative DCs J_s based on δ_{jz} , the distance to cover from DC $j \in J_s$ to a given customer zone z . This allocation is made such that the total demand assigned to DC j related to all the firms in the coalition, verifies the DC capacity constraint cap_j as given by Equation (5). The capacity constraint of each DC limits the number of firms that can form coalitions and share their capacities. For a given s , the update of the allocation rule $Z_{j,s}^i$, $i \in I_s$, is done by prior inspection, i.e., by solving (4) with respect to the capacity constraint (5).

$$j(z) = \arg(\min_{j \in J_s} \delta_{jz}), \quad \forall z \in Z. \quad (4)$$

$$\sum_{i \in I_s} \sum_{z \in Z_{j,s}^i} d_{zj}^i \leq \text{cap}_j, \quad \forall j \in J_s. \quad (5)$$

Once a coalition s is formed, the optimal ordering quantity for each firm $i \in I_s$ at DC $j \in J_s$, denoted with $q_{j,s}^i$, is based on its demand $d_{j,s}^i$ assigned to DC j , and determined according to the EOQ formula as follows:

$$q_{j,s}^i = \sqrt{\frac{2a_i d_{j,s}^i}{h_j}}. \quad (6)$$

Once the reassignment and ordering quantity are optimized for a given coalition, the expected profit for each firm pertaining to the coalition can be computed with respect to the revenue and cost components. The revenues are obtained by:

$$R_{i,s} = \sum_{j \in J_s} \sum_{z \in Z_{j,s}^i} p_i d_{zj}^i. \quad (7)$$

where d_{zj}^i is computed with Equation (2) according to the new assignments set $Z_{j,s}^i$. Next, the ordering costs $C_{i,s}^O$ of firm i joining coalition s are computed, similarly to the stand-alone case, based on the number of orders.

$$C_{i,s}^O(q_{j,s}^i) = \sum_{j \in J_s} a_i n_{j,s}^i. \quad (8)$$

The replenishment costs $C_{i,s}^R$ must be computed also with respect to the re-assignment decisions for each customer zone from the newly assigned collaborative DC. In this case, single sourcing is replaced by multi-sourcing management of the replenishments based on the ordered quantity $q_{j,s}^i$ among the accessible set of collaborative DCs $j \in J_s$ (Figure 2). Hence, the equation becomes:

$$C_{i,s}^R(q_{j,s}^i) = \sum_{j \in J_s} (\beta_0 + \beta_1 \delta_{k(i)j}) q_{j,s}^i n_{j,s}^i. \quad (9)$$

In the same way, the distribution from the shared DCs impacts the deliveries schema that turns into multiple arc-flows from the set of coalitional DCs to the set of customer zones. The total transportation cost $C_{i,s}^T$ associated with each firm $i \in I_s$ is then summed on all the DCs of the coalition, and is calculated as follows:

$$C_{i,s}^T(d_{zj}^i) = \sum_{j \in J_s} \sum_{z \in Z_{j,s}^i} (\beta_0 + \beta_1 \delta_{jz}) d_{zj}^i. \quad (10)$$

When firms collaborate, they share DC capacities to optimize their warehousing and inventory holding activities, and thus, once a coalition is formed, a cumulative warehousing cost is explicitly assigned to the coalition's members. Let C_s^H be the collaborative warehousing cost that can be determined through equation (11):

$$C_s^H(q_{j,s}^i) = \sum_{j \in J_s} \alpha_0^j \left(\sum_{i \in I_s} q_{j,s}^i \right)^{\alpha_1} n_{j,s}^i. \quad (11)$$

The above discussion stands for an implicit cost sharing method and implies that the collaborative warehousing cost $C_s^H(q_{j,s}^i)$ is split and replaced by a set of individual warehousing costs $C_{i,s}^{H(m)}$ associated with each firm $i \in I_s$, and a cost sharing method m .

Once a coalition is formed, the question to ask is how much profit each firm gains. The decision to join a coalition is made according to the realized profit of each firm within the coalition. Given all the revenues and costs previously defined, the profit $P_{i,s}^m$ of each firm i in coalition s is as follows:

$$P_{i,s}^m = R_{i,s} - (C_{i,s}^O + C_{i,s}^R + C_{i,s}^T) + C_{i,s}^{H(m)}. \quad (12)$$

Such profit can also be expressed for the entire coalition s with the following collaborative profit:

$$P_s^m = \sum_{i \in I_s} (R_{i,s} - (C_{i,s}^O + C_{i,s}^R + C_{i,s}^T)) + C_s^H. \quad (13)$$

Given the sharing methods and the profit structure of the distribution network problem described above, the coalition formation problem will be modeled and solved hereafter.

3. Game modelling of the DPSC

To deal with the question concerning the coalitions that are likely to form, a completely centralized approach enables identifying the coalition structure that minimizes total warehousing costs. For such coalition-formation problems, achieving optimal coalitions requires full agreement between all firms. This arrangement seems obvious if the system contains only one company. In this paper, we deal with different interacting firms. Therefore, there are good reasons to expect that forming a grand coalition is not straightforward. Hence, the main purpose is to generate the best coalitions that maximize profit for all collaborating firms. In addition, enumerating all the possible coalitions and solving their underlying optimization problem would be too time-consuming, if not impossible. In fact, one can prove that this problem is NP-hard, and that for n potential firms, the number of alternatives is exponential. For this reason, in this section, we first characterize the coalition structure we seek and then derive the conditions based on the individual rationality principle that helps identify the non-profitable coalitions that can be discarded.

3.1. The coalitional game

Given limited capacities and the location of the distribution centers, the resulting game is not superadditive. Thus, there is no guarantee that any two or more disjoint coalitions, when merging into one coalition, increase the profit of the collaborating firms. Consequently, the firms are expected to form several separate coalitions instead of a grand coalition, and hence the relevance of the coalition structure. Given a set of firms $I = \{1, \dots, n\}$, a coalition structure $\mathcal{L}(I)$ is a partition of I , that is, a collection of disjoint sets whose union is I . When a coalition S is formed, the firms in S accept sharing their storage capacities and split the warehousing holding cost C_S^H according to some sharing method. Given a specific sharing method, $P_{i,S}^m$ is the profit reached by firm $i \in I_S$ after sharing costs using method m . Let $\bar{P}_i^m(I) = \max_{S \subseteq I: i \in S} P_{i,S}^m$, i.e., $\bar{P}_i^m(I)$, be the maximum profit that can be achieved by firm i among all subsets of I that include firm i when the sharing method is m . Given a fixed sharing method m , a coalition of maximum profitability is a set of firms S ($S \subseteq I$) providing each of them with its corresponding maximum profit $\bar{P}_i^m(I)$. We are interested in finding a coalition structure $\mathcal{L}(I) = \{S_1, \dots, S_u\}$, where for all $r \in \{1, \dots, u\}$, subset S_r is a coalition of maximum profitability within set $I \setminus \bigcup_{p=1}^{r-1} S_p$. Since the allocations to each player are predetermined by the prescribed sharing method m , it is important to note that the resulting coalitions here do not necessarily fulfil coalitional structure stability in the sense of Aumann & Dreze (1974). However, when a coalition S_r becomes part of the structure, each of its players achieves the maximum profitability among the coalitions it can form with the remaining universe of players, given the prescribed sharing method.

3.2. Conditions for specific sharing methods

As mentioned in the introduction, different sharing methods may conduce to different coalitions and payoffs for the players. The literature offers a great variety of methods, constructed according to different notions of fairness. Accordingly, we consider three cost sharing methods that affect the warehousing costs, and thus the profit of each firm in the coalition. These methods are: 1) egalitarian allocation, 2) proportional allocation, and 3) Shapley value, which are among the most used in the collaborative logistics literature (Guajardo & Rönnqvist, 2016b).

Definition 1. Egalitarian allocation is the simplest sharing method (Berger & Bierwirth, 2010; Massol & Tchong-Ming, 2010; Lehoux et al., 2011). It regards as fair to assign equal warehousing costs to all collaborating firms, that is,

$$C_{i,s}^{H(1)} = \frac{C_s^H}{|I_s|}. \quad (14)$$

Definition 2. Based on a proportional fairness principle (Ben Jouida et al., 2017; Krichen et al., 2011), the collaborative warehousing cost is shared between firms proportionally to their ordered quantities, that is,

$$C_{i,s}^{H(2)} = \frac{C_s^H \times q_s^i}{\sum_{i \in I_s} q_s^i}, \quad (15)$$

where $q_s^i = \sum_{j \in J_s} q_{j,s}^i$, derived from equation (6).

Definition 3. The Shapley (1953) cost allocation is one of the most used in cooperative games. This method regards as fair to allocate to each collaborating firm $i \in I_s$ an average of the marginal warehousing costs implied by the firm over all possible coalitions that contains it, as follows:

$$C_{i,s}^{H(3)} = \sum_{\bar{I} \subseteq I_s: i \in \bar{I}} \frac{(|I_s| - |\bar{I}|)! (|\bar{I}| - 1)!}{|I_s|!} \left(C_{\bar{I}}^H - C_{\bar{I} \setminus \{i\}}^H \right). \quad (16)$$

As described above, the cost sharing method influences the decision of firms to join coalitions. An essential requirement is that the resulting allocations satisfy the individual rationality condition for all firms, that is, $P_{i,s}^m \geq P_i$. We now present conditions that filters which coalitions are acceptable according to the individual rationality condition. Since collaborating firms share only the warehousing costs, which depend mainly on the α^1 parameter, ordered quantity $q_{j,s}^i$ and sharing methods $m \in \{Egal, Prop, Shap\}$, it suffices for firms to consider the conditions that depend on α^1 and m . For each of the three methods, in what follows we derive conditions for values $\alpha^1 = 0.5$ and $\alpha^1 = 1$. The intermediary steps to derive these conditions are given in **Appendix A**.

Intensive economics of scale: Suppose that $\alpha^1 = 0.5$. Then each firm i using sharing method m has interest in accepting coalition s (i.e., individual rationality holds) if one of the following relations (17), (18) or (19) is satisfied:

$$C_{i,s}^{H(m)} = M_s^i - L_s^i, \quad \text{with } M_s^i = \left(\frac{-(\alpha_0^i n_i)^2}{4(\beta_0 + \beta_1 \delta_{k(i)j(i)}) n_i} \right) \quad (17)$$

$$C_{i,s}^{H(m)} > M_s^i - L_s^i \quad \text{and} \quad q_i < X_1^m. \quad (18)$$

$$C_{i,s}^{H(m)} > M_s^i - L_s^i \quad \text{and} \quad q_i > X_2^m. \quad (19)$$

$$X_1^m = \frac{(-(\alpha_0^i n_i) - \sqrt{\Delta^m})^2}{4(\beta_0 + \beta_1 \delta_{k(i)j(i)})^2 n_i^2}. \quad \text{and} \quad X_2^m = \frac{(-(\alpha_0^i n_i) + \sqrt{\Delta^m})^2}{4(\beta_0 + \beta_1 \delta_{k(i)j(i)})^2 n_i^2}. \quad (20)$$

Where,

$$L_s^i = R_i - R_{i,s} + C_{i,s}^{O+} + C_{i,s}^R + C_{i,s}^T - C_i^T \quad (21)$$

with the new term $C_{i,s}^{O+}$ expressing the difference between $C_{i,s}^O$ and C_i^O when isolating the term $(a_i n_i)$ from the equation. This term would be zero when the same ordering frequency is used by the firm in both scenarios (i.e., in $n_{j,s}^i$ and n_i). In what follows, we develop these expressions for $C_{i,s}^{H(m)}$ and Δ^m , as they depend on the cost sharing mechanism.

1. *Egalitarian allocation:*

$$C_{i,s}^{H(Egal)} = \frac{C_s^H}{|I_s|}; \quad \Delta^{Egal} = (\alpha_0^i n_i)^2 + 4 \left(\beta_0 + \beta_1 \delta_{k(i)j(i)} \right) \left(\frac{C_s^H}{|I_s|} + L_s^i \right) n_i. \quad (22)$$

2. *Proportional allocation:*

$$C_{i,s}^{H(Prop)} = \frac{C_s^H \sum_{i' \in I_s} \sum_{j \in J_s} q_{j,s}^{i'}}{\sum_{j \in J_s} q_{j,s}^i}; \quad \Delta^{Prop} = (\alpha_0^i n_i)^2 + 4 \left(\beta_0 + \beta_1 \delta_{k(i)j(i)} \right) \left(\frac{C_s^H \sum_{i' \in I_s} \sum_{j \in J_s} q_{j,s}^{i'}}{\sum_{j \in J_s} q_{j,s}^i} + L_s^i \right) n_i. \quad (23)$$

3. *Shapley value:* for the Shapley value method, we propose conditioning the firms' decisions to accept or reject a coalition s in a particular case with two firms (i.e., $|I_s| = 2$).

$$C_{i,s}^{H(Shap)} = \frac{C_s^H}{2}; \quad \Delta^{Shap} = (\alpha_0^i n_i)^2 + 4 \left(\beta_0 + \beta_1 \delta_{k(i)j(i)} \right) \left(\frac{C_s^H}{2} + L_s^i \right) n_i. \quad (24)$$

No economies of scale: Suppose that $\alpha^1 = 1$. Then each firm i using sharing method m has interest in accepting coalition s if relation (25) is satisfied:

$$C_{i,s}^{H(Shap)} > N_s^i + L_s^i \quad \text{with} \quad N_s^i = (-q_i n_i ((\beta_0 + \beta_1 \delta_{k(i)j(i)}) + \alpha_0^i)). \quad (25)$$

From the expressions (22-25), the values $C_{i,s}^{H(m)}$ and Δ^m are now defined specifically for each sharing mechanism m . These expressions are then inserted in conditions (17)-(21) to check when the individual rationality condition is guaranteed for a given firm i in coalition s (i.e., if the shared profit for $i \in I_s$ exceeds its individual profit: $P_{i,s}^m \geq P_i$). Figure 3 illustrates the decision line based on the $C_{i,s}^{H(m)}$ and the collaboration conditions (17)-(21) elaborated for a given firm i according to the sharing method m . These conditions are explicitly included in the resolution algorithm hereafter to avoid generating and evaluating a number of non-profitable coalitions, and thus enhance the efficiency of the solution approach.

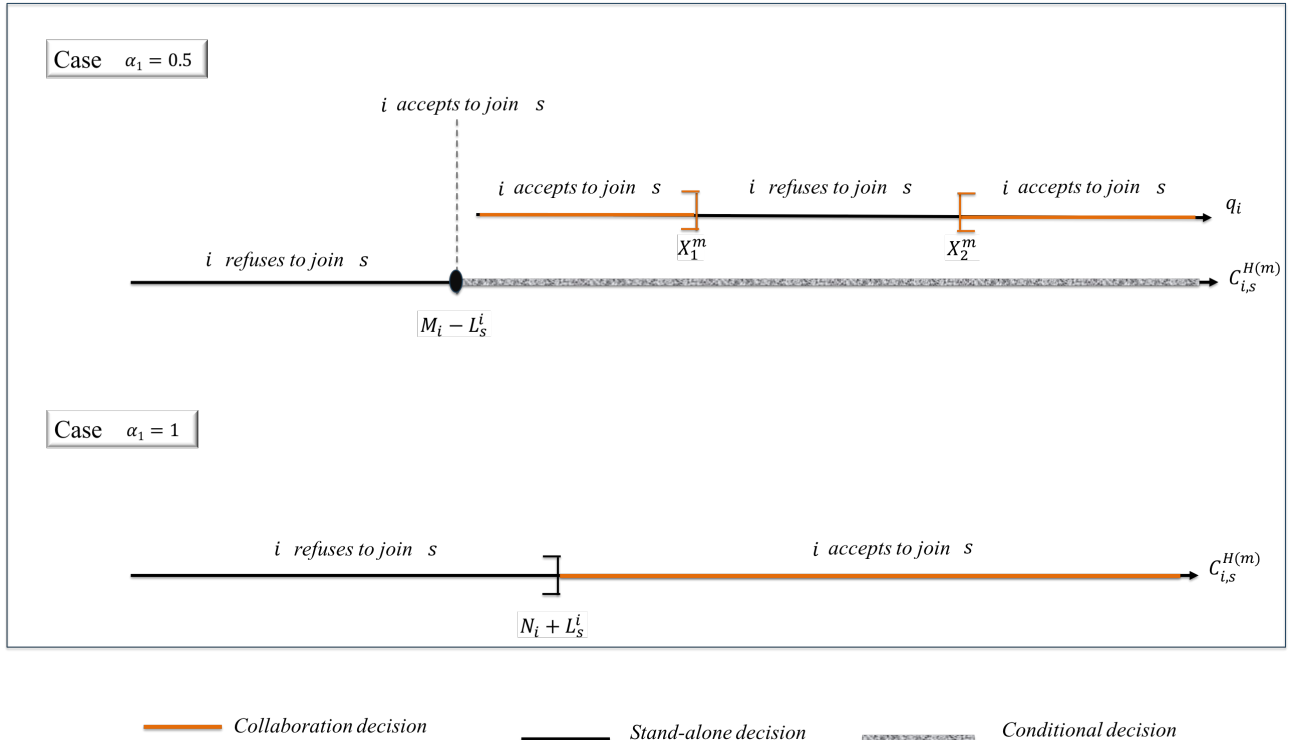


Figure 3: Decision-line conditions for collaboration with $\alpha_1 = 0.5$ and $\alpha_1 = 1$

3.3. Solution algorithm

In this subsection, the collaborative warehousing algorithm (CWA) steps are presented in detail. Figure 4 presents the CWA composed of three steps. The first step determines the individual profit for all firms in the stand-alone context. The second step selects only profitable coalitions based on the conditions given in subsection 3.2. It proceeds by enumerating for each firm the subset of firms that can be beneficial to cooperate with. Subsequently, all non-profitable coalitions are initially discarded when performing the preliminary calculations of thresholds with $\alpha_1 = 0.5$ or $\alpha_1 = 1$, respectively, in order to compare coalition-based and stand-alone profits. The third step finds coalitions ensuring that all players achieve maximum profitability among the remaining set of profitable coalitions. That is, whenever a subset $I_{s''}$ is found to improve the profit of a player in coalition $I_{s'}$, then s'' is ruled out from the candidate subsets for the final structure. On the other hand, when there is no subset improving the profit of any player in $I_{s'}$, then s' becomes part of Ω^* , while the sets in Ω' that contain players of $I_{s'}$ are removed from the remaining possibilities of collaboration. The convergence of the algorithm to a feasible solution is guaranteed since in the worst case, it ends up with a structure where all players stand alone.

```

1. For all  $i \in I$  then
  Compute the individual profit  $P_i$  of each firm  $i$  using equation (3)
End for
2.  $\Omega = \{1 \dots 2^{|I|} - 1\}$ 
 $\Omega' = \emptyset$ 
For all  $s \in \Omega$  then
  While  $i \in I_s$  then
    2.1. If  $(\alpha_1 = 0.5)$  then
      Compute  $C_{i,s}^{H(m)}$ ,  $M_i$ ,  $L_s^i$ ,  $q_i$ ,  $(q_{j,s}^i, j \in J_s)$ ,  $X_1^m$  and  $X_2^m$ 
      If  $(C_{i,s}^{H(m)} = M_i - L_s^i)$  then
         $\Omega' = \Omega' \cup \{s\}$ 
      ElseIf  $(C_{i,s}^{H(m)} > M_i - L_s^i)$  then
        If  $(q_i < X_1^m)$  Or  $(q_i > X_1^m)$  then
           $\Omega' = \Omega' \cup \{s\}$ 
        End if
      End if
    End if
    2.2. If  $(\alpha_1 = 1)$  then
      Compute  $C_{i,s}^{H(m)}$ ,  $N_i$ ,  $L_s^i$  and  $(q_{j,s}^i, j \in J_s)$ 
      If  $(C_{i,s}^{H(m)} > N_i + L_s^i)$  then
         $\Omega' = \Omega' \cup \{s\}$ 
      End if
    End if
  End while
End for
3.  $\Omega^* = \emptyset$ 
For all  $s' \in \Omega'$  do
  MaxProfit=true
    3.1 While  $s'' \in \Omega'$  and MaxProfit=True
      If  $(P_{i,s''}^m < P_{i,s'}^m$  for some  $i)$  then
        MaxProfit = false
      End if
    End while
    3.2 If MaxProfit=true do
       $\Omega^* = \Omega^* \cup \{s'\}$ 
      Remove from  $\Omega'$  sets that contain players in  $I_{s'}$ 
    End if
  End for

```

Figure 4: Schema of the Collaborative Warehousing Algorithm (CWA) for a given m

4. Numerical analysis

4.1. Experimental design

To test the modeling and the proposed solution approach to solve the coalition-formation problem, we generate several instances varying market size, network configuration, cost structure, and capacity. In this study, we consider a fixed one-year horizon and one product family. Considering a population density and customer demand size, we repartition the set of firms to calibrate their market share ψ . Accordingly, a yearly demand for each firm i is derived, so that we generate, proportionally to ψ , small-sized firms (SF), medium-sized firms (MF) and large-sized firms (LF). We assume that all firms have the same shipment frequency n_i for their demand. Next, we define the problem instances according to the four dimensions below.

1. Problem size: Market size is determined according to the combination $(|J|, |Z|)$ where $|J|$ designates the number of DCs of all firms and $|Z|$ the number of customer zones. Recall that in these instances $|I|$ is equal to $|J|$ since we assume that each firm i operates initially one DC $j_{(i)}$. Let $\{4, 6, 20\}$ and $\{8, 12, 24\}$ be the set values of $|J|$ and $|Z|$, respectively. Then, three types of problems arise according to the area size:
 - (a) Small-sized problem (SP) with $|J| = 4$ and $|Z| = 8$
 - (b) Medium-sized problem (MP) with $|J| = 6$ and $|Z| = 12$
 - (c) Large-sized problem (LP) with $|J| = 12$ and $|Z| = 24$
2. Network configuration: given the predefined firm classes, two types of networks ensue:
 - A large network denoted by LN , composed of (20%SF; 30%MF; 50%LF)
 - A small network denoted by SN , composed of (30%SF; 50%MF; 20%LF)
3. Warehousing costs: (a) low-cost (LC) level, $\alpha_1 = 0.5$; (b) high-cost (HC) level, $\alpha_1 = 1$.
4. DCs capacity: We express the capacity of each DC j operated by firm i to its stored quantity q_i with $cap_j = \mu \times q_i$, and use two different values of μ to define DCs size:
 - (a) Small-sized DC (SDC) with 50% more storage, ($\mu = 1.5$)
 - (b) Large-sized DC (LDC) with 80% more storage, ($\mu = 1.8$)

Consequently, each problem instance is denoted by the combination (x, y, z, w) where x represents the problem size ($x \in \{SP, MP, LP\}$), y the network configuration ($y \in \{LN, SN\}$), z the α_1 value ($z \in \{LC, HC\}$), and w the DC capacities ($w \in \{LDC, SDC\}$). Recall that each of these 24 instances (x, y, z, w) must be solved for the stand-alone situation and for the cooperative situation with the three cost sharing methods. This combination yields 144 problems to solve with the collaborative warehousing algorithm (CWA). Furthermore, the ordering, replenishment, and distribution parameters are fixed, while holding and purchasing costs are varied. These values are defined according to Ben Jouda et al. (2017) and Elomri et al. (2012) for the replenishment and transportation component. All the computational experiments are performed in Java language on a 32-bit computer with Pentium (R) CPU, 2.13 GHz, and 4GB of RAM. The next section presents the numerical results of the tested instances and discusses their managerial implications.

4.2. Numerical analysis

Our aim is to conduct a sensitivity analysis related to the main parameters and the appropriate sharing method that yields the best collaboration decisions. We then analyze the results and obtain insights on the collaboration decisions for all firm types.

4.2.1. Coalition condition analysis

In previous studies, Shapiro & Wagner (2009) set $\alpha_0 = 10$, and Elomri et al. (2012) set $\beta_0 = 10$ and $\beta_1 = 0.07$. In this study, we set $\alpha_0 \in \{3, 5, 10, 20, 30, 40, 50, 60\}$ to illustrate diverse values of α_0 below and above the value fixed in Shapiro & Wagner (2009). Following the same logic in Elomri et al. (2012), the parameters β_0 and β_1 are selected from the sets $\{0, 10, 20, 30, 40, 50, 60, 70, 80, 500, 800, 900, 1000\}$, $\{0, 0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35\}$, respectively. All sensitivity tests are conducted for the three sharing methods, using the small-sized instance with attributes (SP, SN, HC, LDC). Therefore, for each parameter and anticipating the cost sharing method to any given number of firms, we determine the gap when deciding to collaborate (see columns 5, 9 and 13 in Tables 1, 2 and 3). Gap values are reported in percentage terms in all tables and computed as follow:

$$Gap = \frac{\sum_{s \in \Omega^*} \sum_{i \in I_s} P_{i,s}^m - \sum_{i \in I} P_i}{\sum_{i \in I} P_i} \times 100. \quad (26)$$

On the other hand, to examine the impact of the α_0 , β_0 and β_1 parameters on the collaboration decisions, we generate the results related to the number of collaborative firms and degree of collaboration (denoted respectively with *No.col.firms* and *Coll.(%)* in Tables 1, 2, and 3).

α_0	J	Shapley			Egalitarian allocation			Proportional allocation		
		No.col.firms	Coll.(%)	Gap	No.col.firms	Coll.(%)	Gap	No.col.firms	Coll.(%)	Gap
0	4	0	0	0.00	3	75	6.23	3	75	6.23
3	4	2	50	2.22	3	75	2.64	3	75	6.74
5	4	2	50	2.60	3	75	2.76	3	75	7.08
10	4	2	50	7.89	3	75	3.07	3	75	9.19
20	4	0	0	0.00	3	75	3.71	0	0	0.00
30	4	0	0	0.00	3	75	4.38	0	0	0.00
40	4	0	0	0.00	0	0	0.00	0	0	0.00
50	4	0	0	0.00	0	0	0.00	0	0	0.00
60	4	0	0	0.00	0	0	0.00	0	0	0.00
Avg				1.41%			2.53%			3.24%

Table 1: Sensitivity analysis for α_0 parameter and (SP, SN, HC, LDC) instance

We first find that for values of α_0 greater than 10, there is no collaboration except with the egalitarian allocation where this threshold increases to 30. This phenomenon can be explained by the fact that beyond a certain α_0 -threshold, the shared fixed costs, which are proportional to the quantities stored, are greater for firms in the coalition than in the individual situation (see Figure 5). In addition, we note that the grand coalition is obtained for the Shapley value and proportional allocation when $\alpha_0 = 10$. The grand coalition is formed when all firms in the network belong to the same coalition. The average gap values shown in the last row of Table 1 reveal that the egalitarian and proportional allocation methods lead to the formation of more coalitions than the Shapley value method.

β_0	$ J $	Shapley			Egalitarian allocation			Proportional allocation		
		No.col.firms	Coll.(%)	Gap	No.col.firms	Coll.(%)	Gap	No.col.firms	Coll.(%)	Gap
0	4	2	50	2.54	3	75	3.05	3	75	7.01
10	4	2	50	2.55	3	75	3.06	3	75	5.87
20	4	2	50	2.55	3	75	3.07	3	75	4.14
30	4	2	50	2.56	3	75	3.08	3	75	4.10
40	4	2	50	2.57	3	75	3.09	3	75	3.93
50	4	2	50	2.58	3	75	3.10	3	75	3.84
60	4	2	50	2.59	3	75	3.12	3	75	3.27
70	4	2	50	2.60	3	75	3.13	3	75	3.18
80	4	2	50	2.61	3	75	3.14	2	50	3.15
500	4	2	50	3.98	2	50	2.65	2	50	3.31
800	4	2	50	3.03	2	50	1.12	2	50	1.62
900	4	2	50	1.84	2	50	0.68	2	50	1.36
10000	4	2	50	2.71	3	75	0.49	2	50	0.74
Avg			2.67%				2.57%			3.45%

Table 2: Sensitivity analysis for β_0 parameter and (SP, SN, HC, LDC) instance

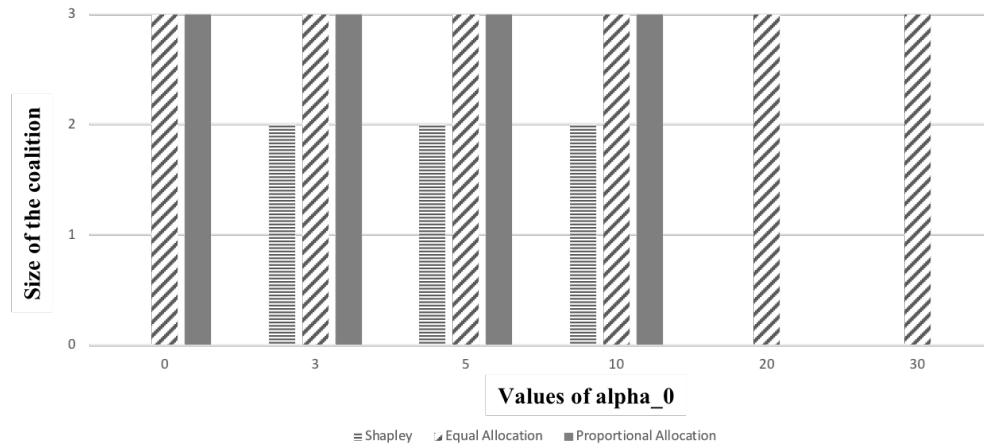


Figure 5: Number of collaborative firms when varying α_0 parameter

Second, we examine the impact of parameter β_0 on the collaboration decision. Table 2 shows that for the three cost sharing methods, the firms are interested in collaborating when $\beta_0 = 0$ and for all other values. As for the replenishment cost parameter β_1 , the results in Table 3 reveal that coalitions of two or three players form under all cost sharing methods (see Figure 7). Increasing the value of β_1 above 0.1 does not imply changes in the size of the coalition and, as can be seen in Figure 6, the gap remains relatively close. Thus, we use $\beta_1 = 0.1$ for the remaining experiments.

β_1	$ J $	Shapley			Egalitarian allocation			Proportional allocation		
		No.col.firms	Coll.(%)	Gap	No.col.firms	Coll.(%)	Gap	No.col.firms	Coll.(%)	Gap
0	4	0	0	0.00	0	0	0.00	0	0	0.00
0.01	4	0	0	0.00	0	0	0.00	0	0	0.00
0.05	4	0	0	0.00	0	0	0.00	0	0	0.00
0.1	4	2	50	2.57	3	75	3.09	3	75	4.63
0.15	4	2	50	2.59	3	75	3.12	2	50	4.64
0.2	4	2	50	2.62	3	75	3.15	3	75	8.12
0.25	4	2	50	2.65	3	75	3.18	3	75	8.19
0.3	4	2	50	2.68	3	75	3.21	3	75	8.25
0.35	4	2	50	2.70	3	75	3.24	3	75	8.32
Avg				1.75%			2.11%			4.68%

Table 3: Sensitivity analysis for β_1 parameter and (SP, SN, HC, LDC) instance

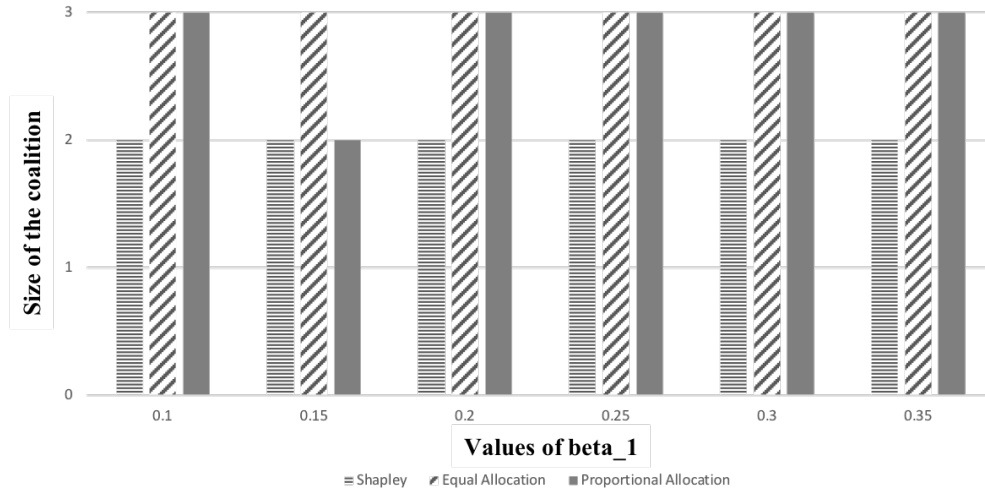


Figure 7: Number of collaborative firms when varying β_1 parameter

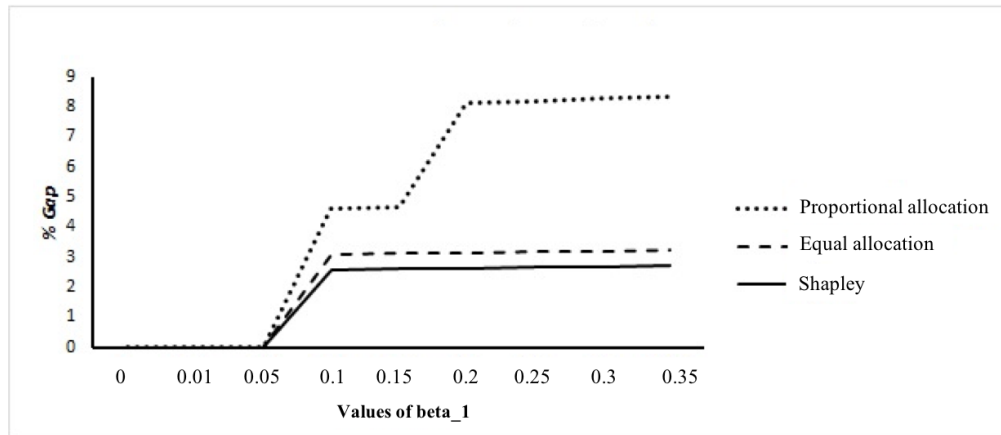


Figure 6: Gap values when varying β_1 parameter

4.2.2. Solvability

Our CWA algorithm considerably decreases the total number of potential coalitions, as it takes into account the individual rationality conditions expressed in equations (17)-(25) that discard non-profitable coalitions. Column 2 in Table 4 represents the number of possible coalition structures examined for each market size. Columns 3, 5, and 7 of Table 4 represent the number of discarded coalition structures explored when using our algorithm. This remarkable result is a consequence of the algorithm filtration phase. Thus, the number of coalition structures formed tends to be smaller with respect to the total number of coalition structures. To be noted is that over 50% of coalitions were eliminated for the three sharing methods (see last row in Table 4). The largest average percentage of discarded coalitions occurs for the Shapley values, equivalent to 84.19%. This translates into a reduction of the solution time as illustrated in Figure 8, where the CWA is contrasted with a classic enumeration-based algorithm (denoted EA).

$ J $	Tot.no.col.	Shapley		Egalitarian allocation		Proportional allocation	
		No.dis.coal.	(%)of dis.coal.	No.dis.coal.	(%)of dis.coal.	No.dis.coal.	(%)of dis.coal.
4	15	9	60.00	8	53.33	6	40.00
6	63	56	88.89	55	87.30	53	84.13
8	255	238	93.33	239	93.73	235	92.16
10	1023	967	94.53	969	94.72	964	94.23
12	4095	4016	98.07	4018	98.12	4026	98.32
<i>Avg</i>			84.19		82.27		77.63

Table 4: Filtration phase for $(., SN, HC, LDC)$ instances with varying $|J|$

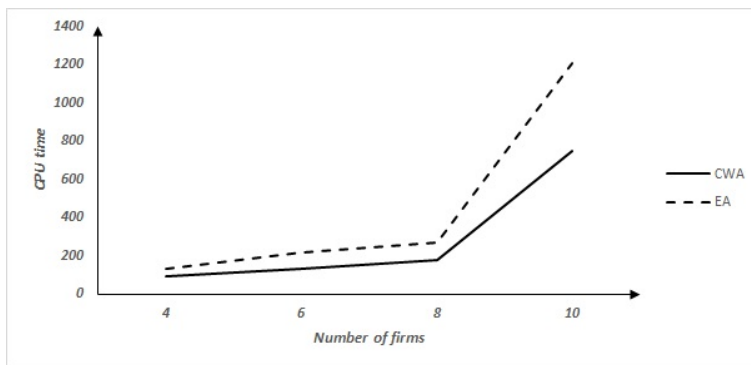


Figure 8: CPU time (in seconds) of the CWA algorithm and an enumeration-based algorithm (EA) with Shapley value.

4.3. Numerical results

In the following, we set $\alpha_0 = 10$, $\beta_0 = 20$, and $\beta_1 = 0.1$, and present a summary of the results according to several criteria.

- *Gap quality.* Figure 9 illustrates the gaps for four instances with opposite attributes where high and low warehousing cost values are compared in each market size and in terms of the three sharing methods. The computational results show that the collaborative profit is always greater than the dedicated profit. In addition, the highest gap is found when the warehousing cost is high and the network is composed mainly of small firms. These results underline first that the sharing method plays an important role in collaboration effectiveness, and second, collaboration is particularly suitable in the presence of many firms and high warehousing costs.
- *Degree of collaboration.* Tables 5, 6, and 7 report the number of profitable coalitions in all tested instances for the three sharing methods. All tables confirm that the cardinality of the coalition structure, for all instances, is less than the number of firms, showing the tendency to form coalitions in the collaborative scenario.

Tables 5, 6, and 7 highlight the lowest number of alliance of firms choosing to share their warehousing resources. We observed that instances with high cost attributes, and for the three profit sharing methods, lead to more collaboration. This shows that we do not obtain the grand coalition for any instance, which again confirms the DCs' capacity constraints in forming large coalitions. In addition, the last row in Tables 5, 6, and 7 provides the average proportion of distinct entities in the coalition structure in all instances inspected (i.e., the average proportion of the number of firms divided by the number of coalitions per market size). It can be seen that the lowest average coalition-formation proportion is obtained with the Shapley value, for example, under $(., SN, HC, LDC)$ instances with 58.33%. We conclude that collaboration is more efficient with high-cost attributes for cost savings. However, how does the sharing method used influence the final collaboration decisions? To study the impact of the cost sharing method used, we checked in Tables 5, 6, and 7 the appropriate sharing

method regarding each instance (i.e., the method giving the lowest average proportion of the number of firms divided by the number of coalitions per market size). In almost all cases, the Shapley value is the best method to share profit and to achieve savings. This implies that the Shapley value function allowed enough flexibility to collaborate when the collective interest demands it. Although we assumed that the DC capacities, the warehousing costs, and the customer service level with the three cost sharing methods have the same weight in the final decision, the profit sharing methods used determine the effective weight in the final decision.

- *Impact of collaboration on service level.* For a better understanding of the impact of collaborative warehousing on the customer service level, we present in Figure 10, for each customer instance, the total traveled distances (in kilometers) to distribute the product from the assigned DCs to all customer zones. This shows the efficiency of the collaborative scenario in comparison to the stand-alone scenario in reducing the total shipment distance and thus increasing the customer service level. Finally, based on our experimental results, we note that forming a coalition on average reduced the traveled distance by 34.20%.

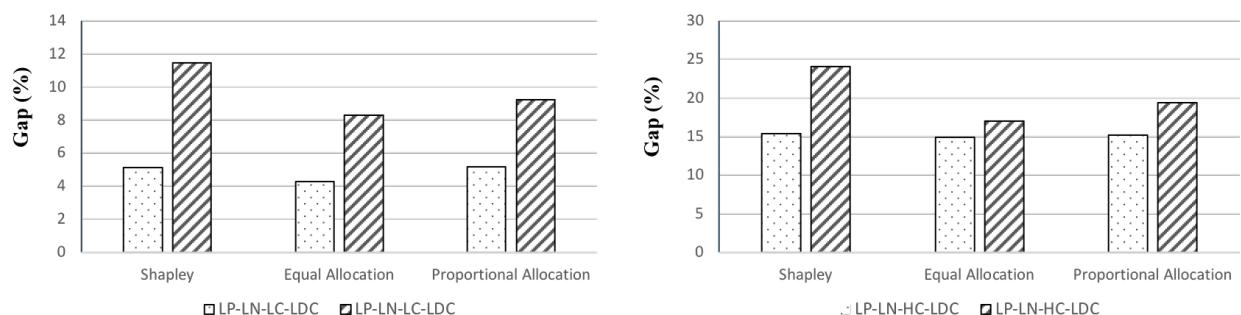


Figure 9: Gaps comparison between stand-alone and collaborative scenarios for the three sharing methods

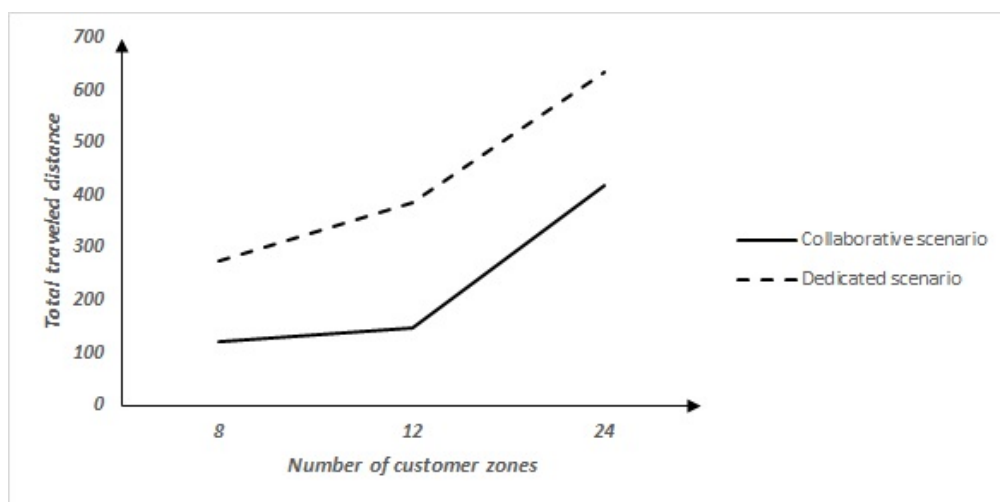


Figure 10: Dedicated vs Collaborative distribution network distances

(J , Z)	Small network				Large network			
	Low-cost		High-cost		Low-cost		High-cost	
	SDC	LDC	SDC	LDC	SDC	LDC	SDC	LDC
(4,8)	2	2	2	2	2	2	2	2
(6,12)	5	4	4	3	4	5	4	3
(12,24)	6	8	7	7	7	6	8	8
<i>Avg</i> (%)	61.11	61.11	58.33	52.77	58.33	61.11	61.11	55.55

Table 5: Number of coalitions in collaborative situations using egalitarian allocation

(J , Z)	Small network				Large network			
	Low-cost		High-cost		Low-cost		High-cost	
	SDC	LDC	SDC	LDC	SDC	LDC	SDC	LDC
(4,8)	2	2	2	2	2	2	2	2
(6,12)	5	4	3	4	4	4	3	4
(12,24)	8	6	8	7	7	6	7	7
<i>Avg</i> (%)	66.67	55.55	55.55	58.33	58.33	55.55	52.77	58.33

Table 6: Number of coalitions in collaborative situations using proportional allocation

(J , Z)	Small network				Large network			
	Low-cost		High-cost		Low-cost		High-cost	
	SDC	LDC	SDC	LDC	SDC	LDC	SDC	LDC
(4,8)	3	3	3	3	3	3	3	3
(6,12)	3	3	3	2	4	3	3	3
(12,24)	6	6	6	5	6	6	7	6
<i>Avg</i> (%)	58.33	58.33	58.33	50	63.88	58.33	61.11	58.33

Table 7: Number of coalitions in collaborative situations using the Shapley value

- *Analysis of the impact of firms' demand level on collaboration decisions.* We observe that the size of all coalitions formed varies from one to two with market size. The last row in Table 8 represents, in percentages, the average number of times that the current firm type forms a coalition according to each instance. To note is that for almost all instances, the coalitions including medium firms exceed their counterparts for small and large firms according to the three cost sharing methods. This underlines the flexibility of this type of firm in forming coalitions with the other types of firms to reduce warehousing costs. These results well explain that collaboration, in general, is more pronounced in a small network dominated by firms with small or medium demand than large demand, given the potential economies of scale they can achieve.

Instances	Size of the formed coalitions involving each type of firm								
	Shapley			Equal allocation			Proportional allocation		
	SF	MF	LF	SF	MF	LF	SF	MF	LF
(., SN, LC, SDC)	2	2	1	3	3	1	3	3	1
(., SN, LC, LDC)	1	2	2	1	3	3	3	3	1
(., SN, HC, SDC)	2	1	2	3	3	1	3	3	1
(., SN, HC, LDC)	1	2	2	1	3	3	1	3	3
(., LN, LC, SDC)	2	2	1	3	1	3	3	1	3
(., LN, LC, LDC)	1	1	2	1	3	3	1	3	3
(., LN, HC, SDC)	2	2	1	3	3	1	3	3	1
(., LN, HC, LDC)	2	1	2	1	3	3	3	3	1
<i>Avg</i> (%)	62.5	62.5	62.5	50	87.5	62.5	75	87.5	37.5

Table 8: Firms' behaviors in a collaborative scenario with $|J| = 4$ instances.

5. Conclusion

The distribution problem studied in this paper considers the positioning of firms' inventory level at owned DCs or those of collaborating firms to efficiently serve a number of service-prone customer

zones. Collaboration is viewed as a concept aiming to pool multiple warehousing capacities and reduce the warehousing and distribution costs. In this article, we developed a game theoretical approach for multiple firms with multiple DCs that act jointly to maximize their profits. We applied three cost sharing methods to sharing warehousing costs: Shapley value, egalitarian allocation, and proportional allocation. The coalition-formation problem we investigate in this article is difficult to solve, and we thus developed an efficient CWA exact method. The method allowed us to find all those coalitions assuring its members maximum profitability under a prescribed cost sharing method.

This experimental study highlights the importance of horizontal collaboration in firms' pursuit of profit growth. It also underlines the broad range of cooperation opportunities in service level and warehousing costs. These experiments confirm that the presence of a capacity sharing possibility is profitable for all types of firms in terms of their demand level. Another observed behavior is that collaboration depends on the cost sharing method used. Indeed, using various cost sharing methods, we observed in all instances that the formation of several sub-coalitions prevails over the formation of a grand coalition, and that different methods can lead to different sub-coalitions. This is an important observation since to a great extent prior literature takes the sharing method as given, or compares different methods assuming the formation of a grand coalition.

Finally, this article assumed the business case of a stationary-customer demand process. Future works could address various stationary and non-stationary demand processes and extend the coalitional game to stochastic and dynamic settings. Recent work on this line has been provided in Le Cadre et al. (2019). Also, the business case could be extended to cover the modeling of non-cooperative gaming framework. An interesting avenue would be to study alternative inventory policies with the aim to have an order-by-order analysis of the service level impact thanks to the collaboration. In addition, while we focused on finding coalitions of maximum profitability, there might be opportunities to form coalitions that do not provide its members with maximum profitability but improve their stand-alone situation. Devising an efficient method to find these coalitions and studying their stability are also topics of further interest. Set partitioning approaches in this regard have been provided in Guajardo & Rönnqvist (2016a). When additional features are added, the solvability of the coalition-formation problem is challenged, and the development of meta-heuristics could be a promising research avenue to generate coalition structures in a reasonable running time.

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Appendix A.

This appendix details the conditions presented in Section 3.

Individual rationality conditions: The convergence to individual rationality is guaranteed for a given firm i in coalition s if the shared profit for $i \in I_s$ exceeds its individual profit. The cases of convergence to individual rationality with regard to the main parameters of the problem can be derived as follows. Let us assume a given coalition s . Under profitability assumption of firm $i \in I_s$, the individual rationality condition implies inequality (i.1). Using (i.1), inequalities (i.2) and (i.3) are derived by substituting the profit with the terms in equations (3) and (12). Expanding (i.3) by isolating key parameters on the right-hand side and eliminating the similar terms in the ordering cost ($a_i n_i$) produces the inequality (i.4). Note that this is done by removing the term ($a_i n_i$) from $C_{i,s}^O$ and replace it in (i.4) by $C_{i,s}^{O+}$. This latter could be null when the same ordering frequency is imposed to $n_{j,s}^i$ to get n_i .

$$P_i \leq P_{i,s}^m \quad (i.1)$$

$$R_i - C_i^O - C_i^R - C_i^H - C_i^T \leq R_{i,s} - C_{i,s}^O - C_{i,s}^R - C_{i,s}^{H(m)} - C_{i,s}^T \quad (i.2)$$

$$R_i - \alpha_i n_i - (\beta_0 + \beta_1 \delta_{k(i)j(i)}) q_i n_i - \alpha_0^i (q_i)^{\alpha_1} n_i - C_i^T \leq R_{i,s} - C_{i,s}^O - C_{i,s}^R - C_{i,s}^{H(m)} - C_{i,s}^T \quad (i.3)$$

$$-(\beta_0 + \beta_1 \delta_{k(i)j(i)}) q_i n_i - \alpha_0^i (q_i)^{\alpha_1} n_i + C_{i,s}^{H(m)} + (R_i - R_{i,s} + C_{i,s}^{O+} + C_{i,s}^R + C_{i,s}^T - C_i^T) \leq 0 \quad (i.4)$$

Let $L_s^i = R_i - R_{i,s} + C_{i,s}^{O+} + C_{i,s}^R + C_{i,s}^T - C_i^T$, equation (i.4) becomes:

$$(\beta_0 + \beta_1 \delta_{k(i)j(i)}) q_i n_i + \alpha_0^i (q_i)^{\alpha_1} n_i - C_{i,s}^{H(m)} - L_s^i \geq 0 \quad (i.5)$$

Case with $\alpha_1 = 0.5$

Assuming that $\alpha_1 = 0.5$ and defining that $X = \sqrt{(q_i)}$, relation (i.5) can be written as follows:

$$(\beta_0 + \beta_1 \delta_{k(i)j(i)}) n_i X^2 + \alpha_0^i n_i X - (C_{i,s}^{H(m)} + L_s^i) \geq 0 \quad (i.6)$$

The resolution of the polynomial, expressed in relation (i.6), is based on the value of a new parameter Δ^m , where:

$$\Delta^m = (\alpha_0^i n_i)^2 + 4((\beta_0 + \beta_1 \delta_{k(i)j(i)}) n_i)(C_{i,s}^{H(m)} + L_s^i)$$

Three cases arise:

1. if $\Delta^m = 0$:

\Rightarrow Relation (i.6) is true and any firm $i \in I_s$ has interest in accepting the coalition s , if condition (i.7) is satisfied

$$C_{i,s}^{H(m)} = \frac{-(\alpha_0^i n_i)^2}{4(\beta_0 + \beta_1 \delta_{k(i)j(i)}) n_i} - L_s^i \quad (i.7)$$

2. if $\Delta^m > 0$:

\Rightarrow Relation (i.6) is true if:

$$\begin{cases} X > \frac{-(\alpha_0^i n_i) + \sqrt{\Delta^m}}{2(\beta_0 + \beta_1 \delta_{k(i)j(i)}) n_i} \\ X < \frac{-(\alpha_0^i n_i) - \sqrt{\Delta^m}}{2(\beta_0 + \beta_1 \delta_{k(i)j(i)}) n_i} \end{cases}$$

\Rightarrow Given the definition that $X = \sqrt{q_i}$, the decision of each firm $i \in I_s$ to accept or not coalition s is as follows:

$$\begin{cases} \text{if } q_i > \frac{(-(\alpha_0^i n_i) + \sqrt{\Delta^m})^2}{4(\beta_0 + \beta_1 \delta_{k(i)j(i)})^2 n_i^2} & i \text{ accepts to join } s \\ \text{if } q_i < \frac{(-(\alpha_0^i n_i) - \sqrt{\Delta^m})^2}{4(\beta_0 + \beta_1 \delta_{k(i)j(i)})^2 n_i^2} & i \text{ accepts to join } s \\ \text{otherwise} & i \text{ refuses to join } s \end{cases} \quad (i.8)$$

3. if $\Delta^m < 0$, there is no real solution for relation (i.6). In such case, any firm $i \in I_s$ has no interest in accepting coalition s

Case with $\alpha_1 = 1$

Assuming that $\alpha_1 = 1$ equation (i.5) becomes:

$$(\beta_0 + \beta_1 \delta_{k(i)j(i)}) q_i n_i + \alpha_0^i (q_i) n_i - C_{i,s}^{H(m)} - L_s^i \geq 0 \quad (i.9)$$

In such case, each firm $i \in I_s$ has interest in accepting coalition s only when condition (i.10) is satisfied:

$$C_{i,s}^{H(m)} \leq (\beta_0 + \beta_1 \delta_{k(i)j(i)}) q_i n_i + \alpha_0^i q_i n_i - L_s^i \quad (i.10)$$