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Flow-induced self-sustained oscillations in a straight channel with rigid walls and elastic supports

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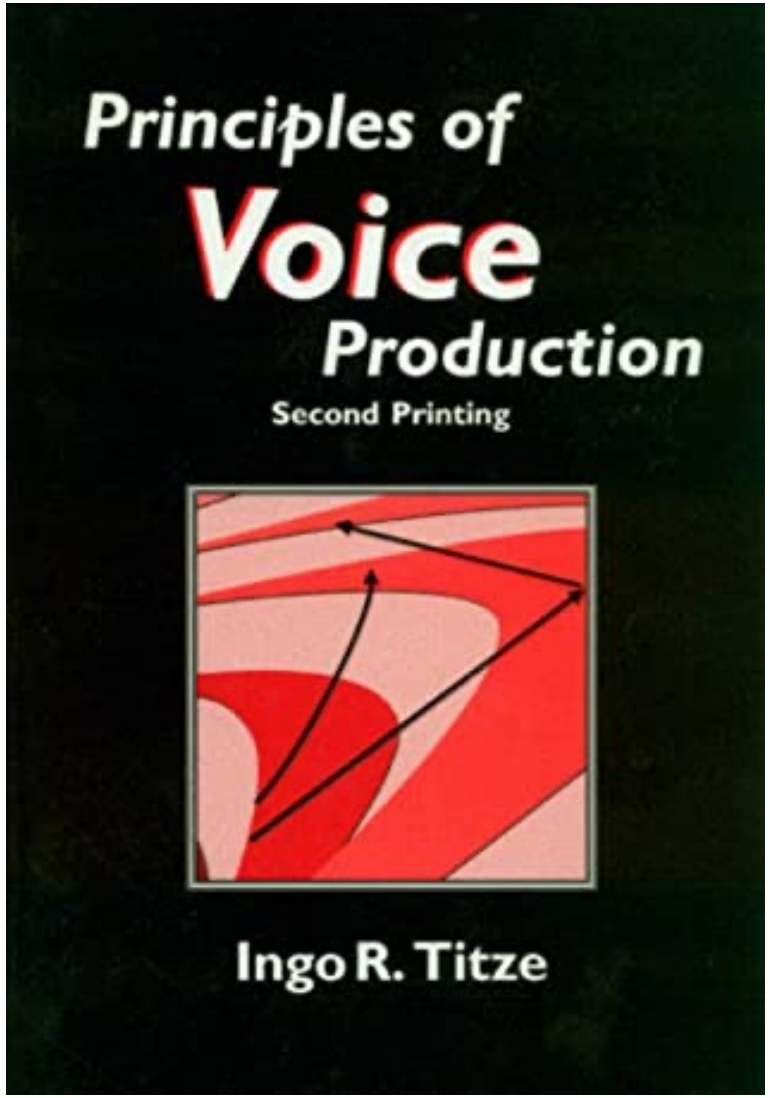
FLOWS BETWEEN CHANNELS w/ OSCILLATING WALLS

Flows between parallel-sided channels with oscillating walls are encountered in numerous and diverse problems such as lubrication, peristaltic pumps, valves, pulsating diaphragms or in aerodynamic particle focusing devices.

In physiological systems, some paradigmatic examples of this kind of flow are found in the respiratory system of some insects, such as ground beetle and Lethocerus uhleri or in the myoelastic-aerodynamic mechanism responsible for voice production in songbirds and mammals.

Flow-induced oscillations of a parallel sided channel have also been largely discussed in the context of voice production. In this context, the idealized vocal-fold models normally consider a symmetric motion of both folds, except when trying to model vocal fold unilateral paralysis, in which only one of both folds moves.

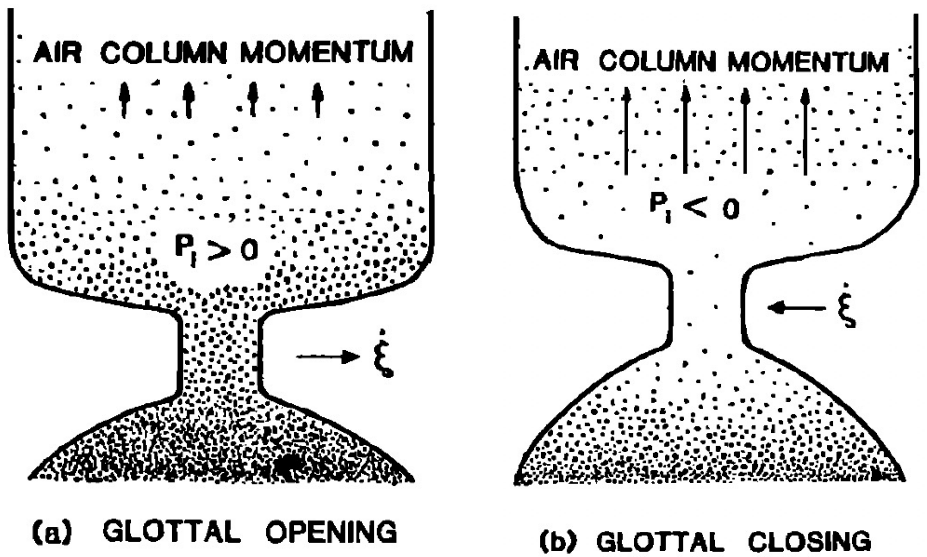


BASIC PRINCIPLES OF VOCAL FOLD VIBRATION

The glottal airstream and the yielding duct wall (vocal folds) form a mechanical system that may demonstrate instability under specific flow conditions. If these conditions are met, a continual transfer of energy from the glottal airstream to the tissue will overcome frictional energy losses in the vocal folds.

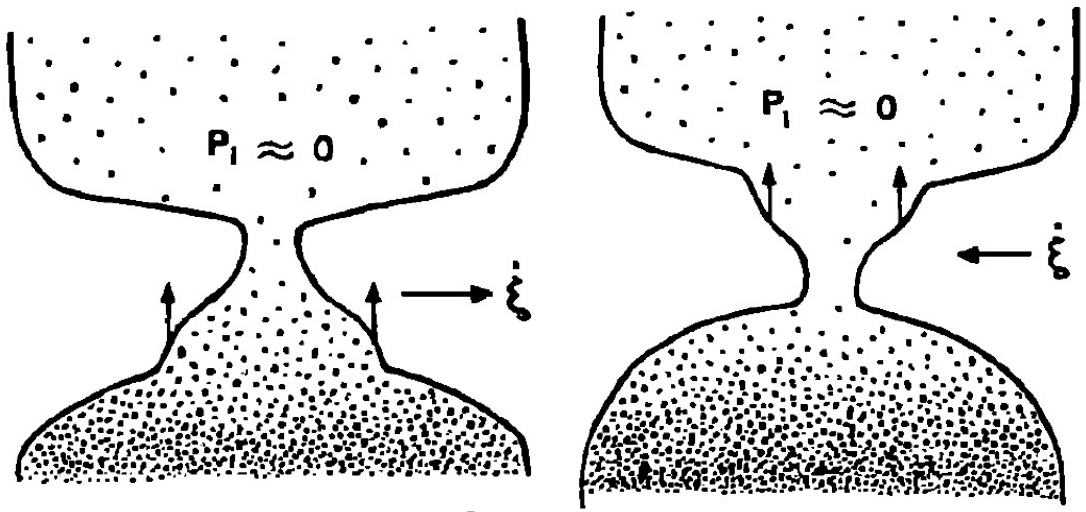
Which is the main mechanism by which flow-induced oscillations are self-sustained?

ONE-MASS MODELS



(a) GLOTTAL OPENING (b) GLOTTAL CLOSING

TWO-MASS MODELS



(a) GLOTTAL OPENING (b) GLOTTAL CLOSING

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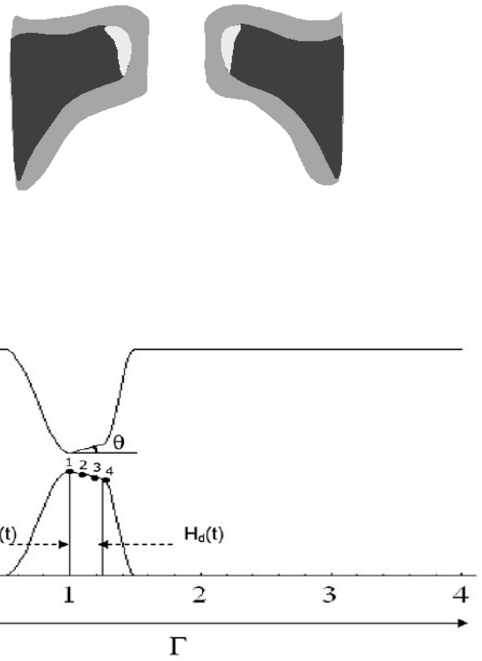
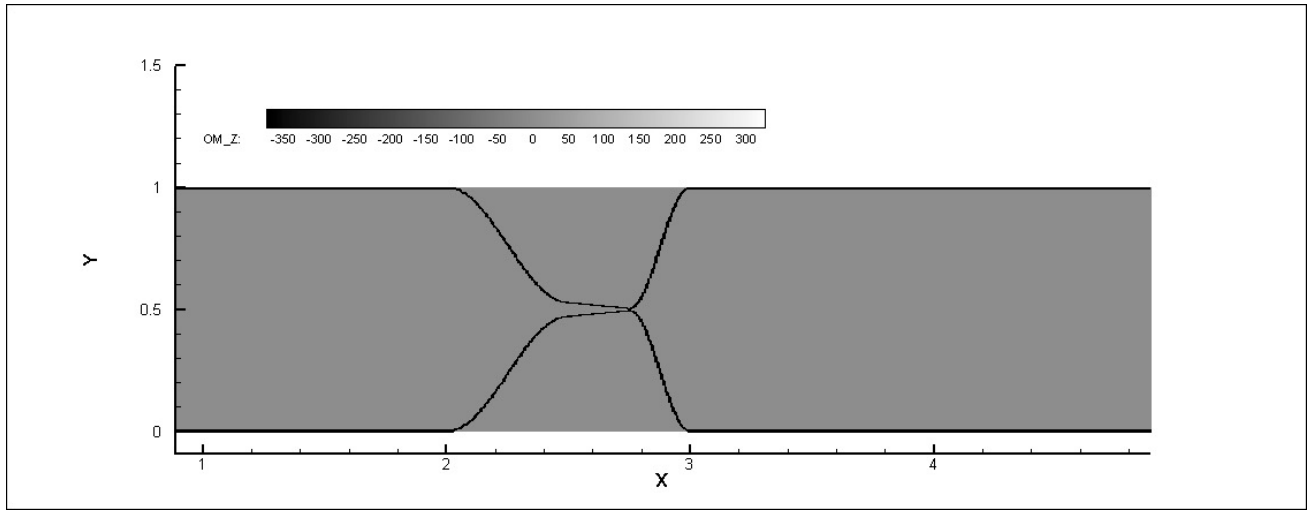
BASIC PRINCIPLES OF VOCAL FOLD VIBRATION

Non-uniform tissue modes are observed in normal vocal fold oscillation, and therefore two-mass models have been much more studied than the one-mass counterparts. Intra-glottal flow separation is also supposed to help in sustaining the oscillations.

TWO-MASS MODELS





D. Sciamarella, P. Le Quéré / European Journal of Mechanics B/Fluids 27 (2008) 42–53

BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

How important is the geometry of the channel? Can self-sustained oscillations be achieved in straight channels with rigid walls?

EXPERIMENTS WITH DIFFERENT CHANNEL GEOMETRIES

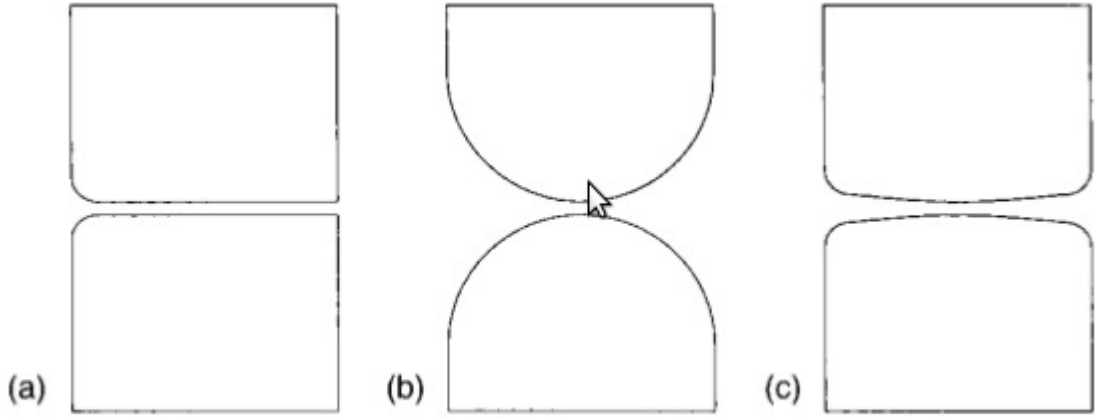


FIG. 1. Mechanical models of the vocal folds: (a) straight uniform channel, (b) rounded vocal folds, and (c) Gaussian vocal folds.

VI. CONCLUSION

From this study it was observed that upon collision the flow unsteadiness due to the wall movement appears to be only significant in the case of the straight uniform replica. In such a case the unsteady theory of Reynolds (Sec. III E) predicts qualitatively the measurements. In the case of more physiological vocal folds shapes, a transition between a boundary-layer behavior towards a friction dominated behavior was clearly observed without significant effect of the flow unsteadiness.

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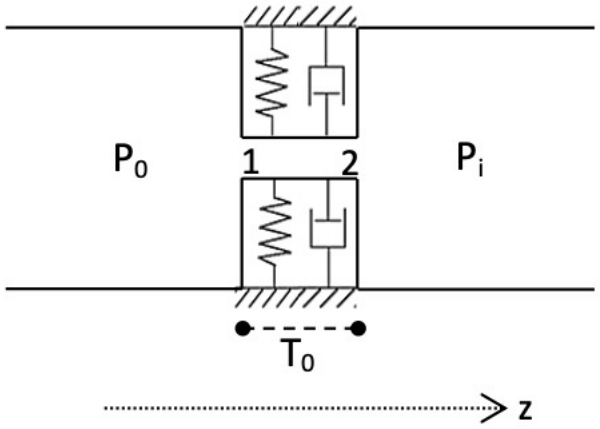
BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

How important is the geometry of the channel in the simple case of a one-mass model? Can flow-induced oscillations be achieved and sustained in straight channels with rigid walls?

In the one-mass model, the phenomenon of flow separation from a rounded geometry or non-uniform tissue modes are not present.

But the inclusion of glottal flow unsteadiness in the modeling equations brings a fluid-dynamic degree of freedom into the system, which can be used to account for self-sustained oscillations in this scheme.



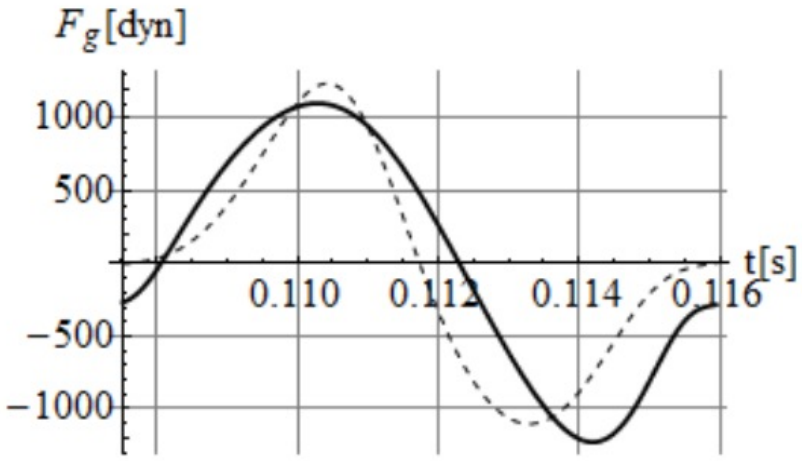
$$a_1 = a_2$$
$$P(z_2, t) = P_i$$

$$F(t) = L_g \int_0^{T_0} P(z,t)\,\mathrm{d}z$$

$$P(z,t) = P_0 - \rho\left(\frac{1}{2}U^2(z,t) + \int_0^z \partial_t U(z,t)\,\mathrm{d}z\right)$$

$$\int_z^{T_0} \partial_t a(z,t)\,\mathrm{d}z + U_2 a_2 - U_1 a_1 = 0$$

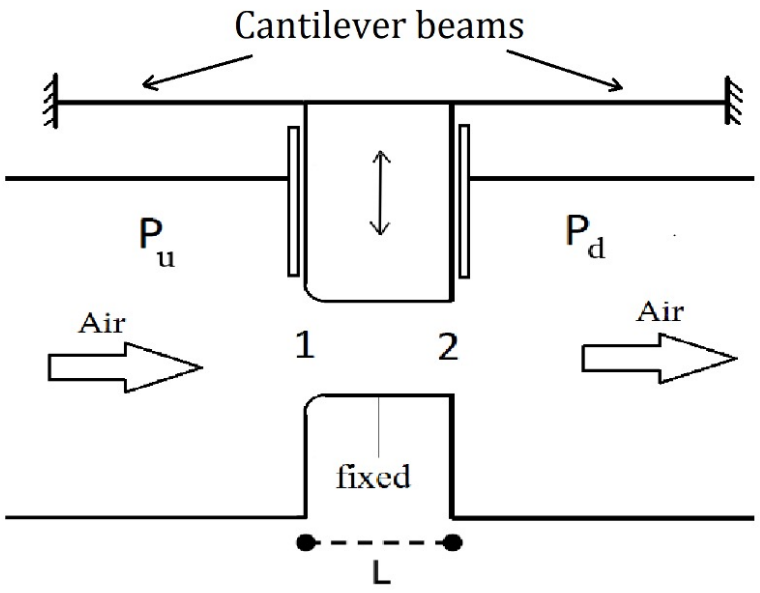
$$U(z,t) = U_2(t) + \frac{(T_0 - z)\,x'}{x_0 + x}$$

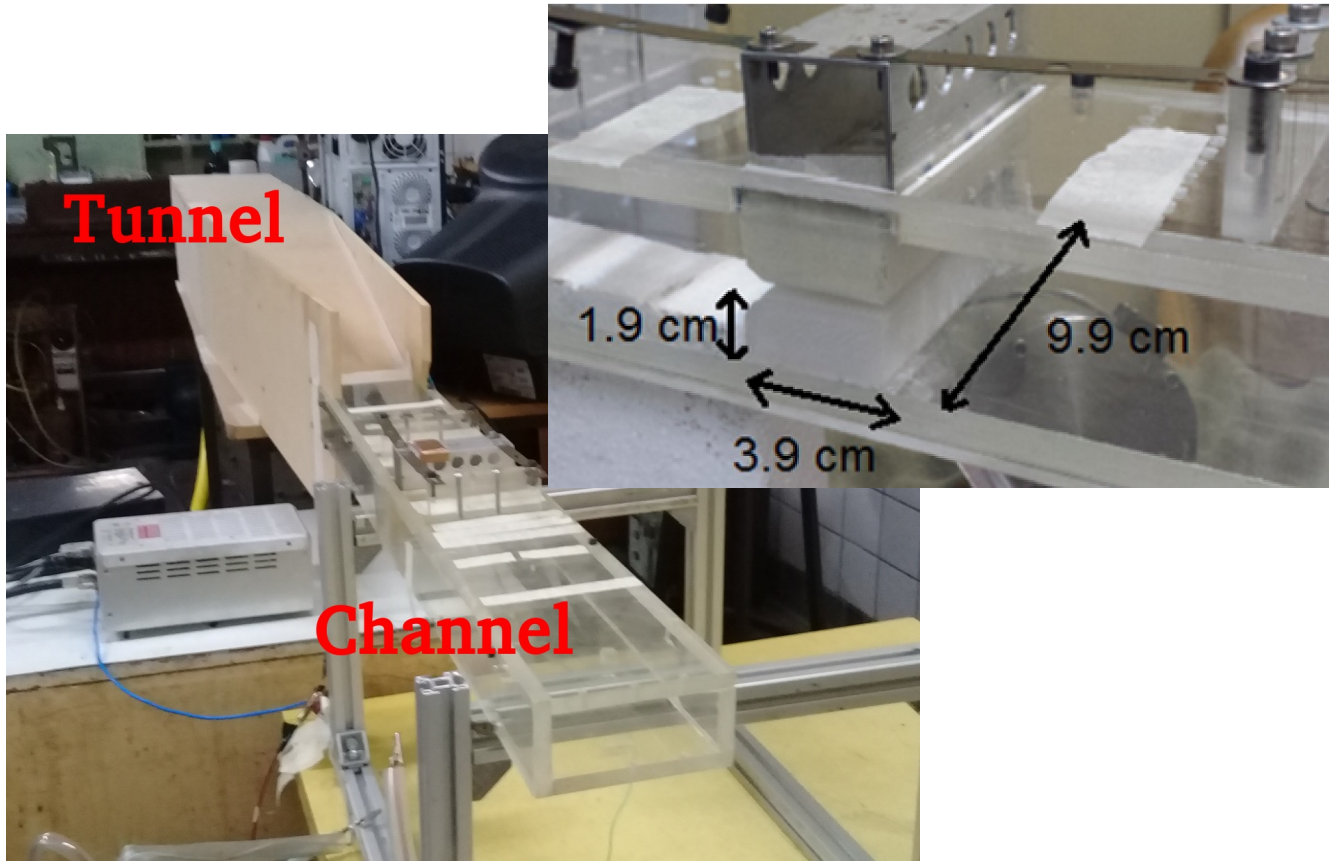


BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

Can flow-induced oscillations be achieved with a single one-mass model, as in a case vocal-fold paralysis?

An experimental setup is mounted with one elastically mounted mass, and the other fixed.

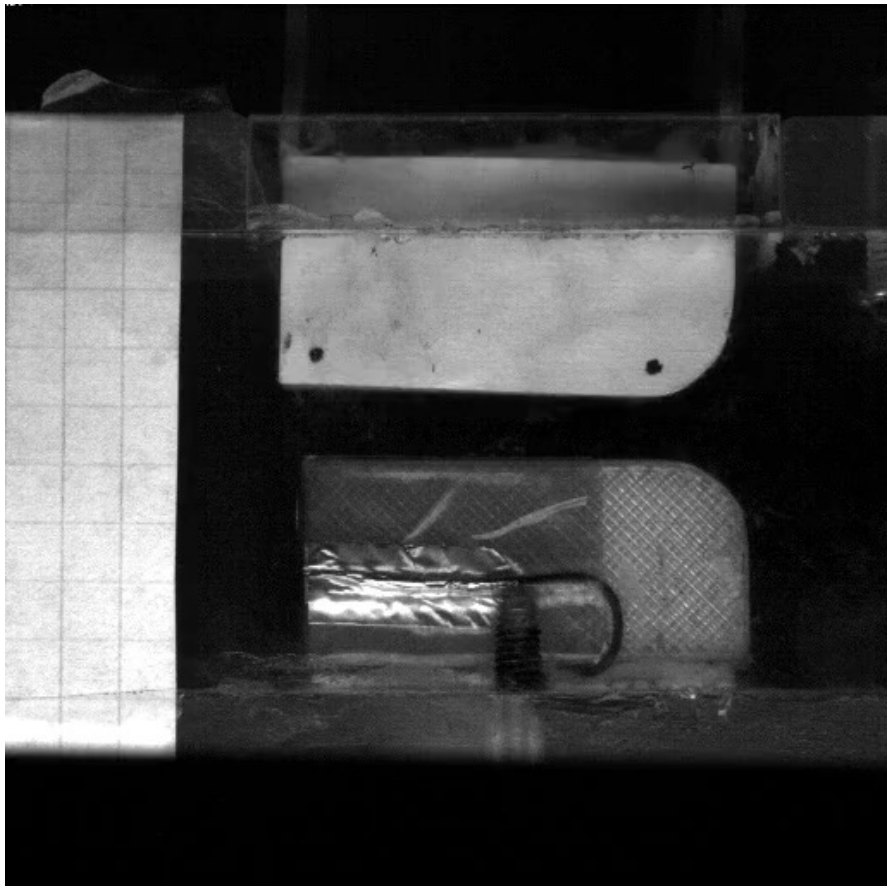


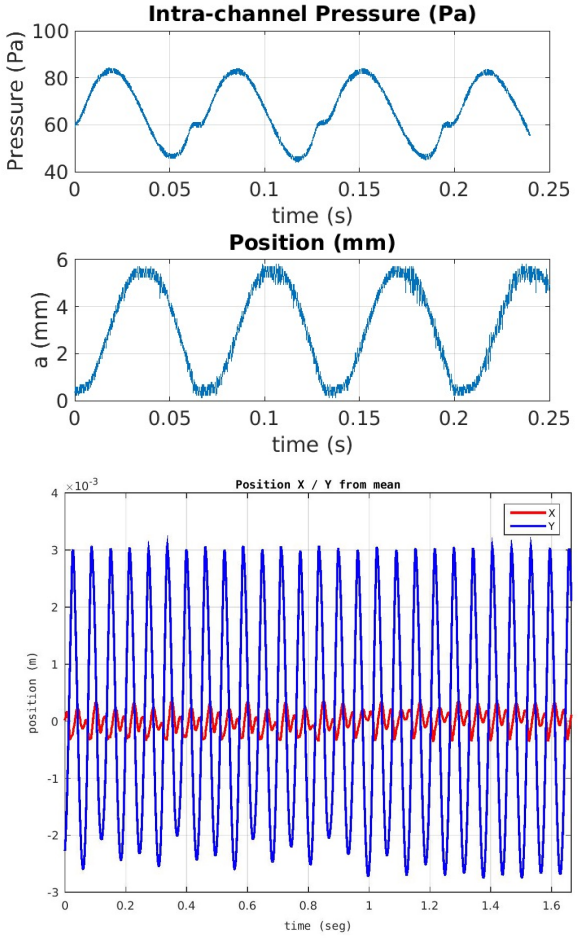


BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

Can flow-induced oscillations be achieved with a single one-mass model, as in a case unilateral fold paralysis?

YES.





BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

Accounting for these oscillations with an analytic model.

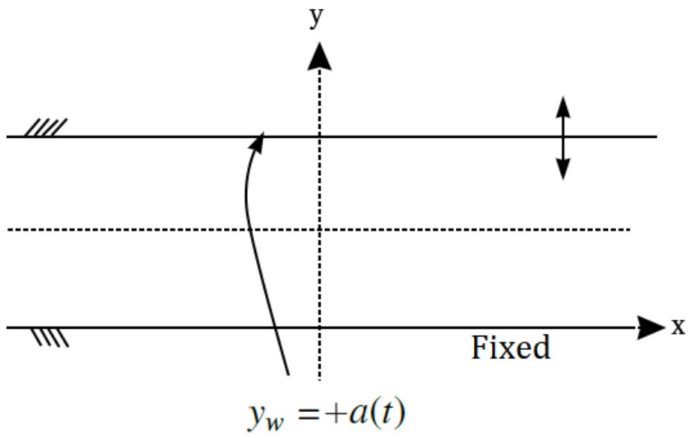
When the wall channels are rigid and large enough, the flow can be elegantly described in terms of an exact self-similar solution of the Navier-Stokes equation.

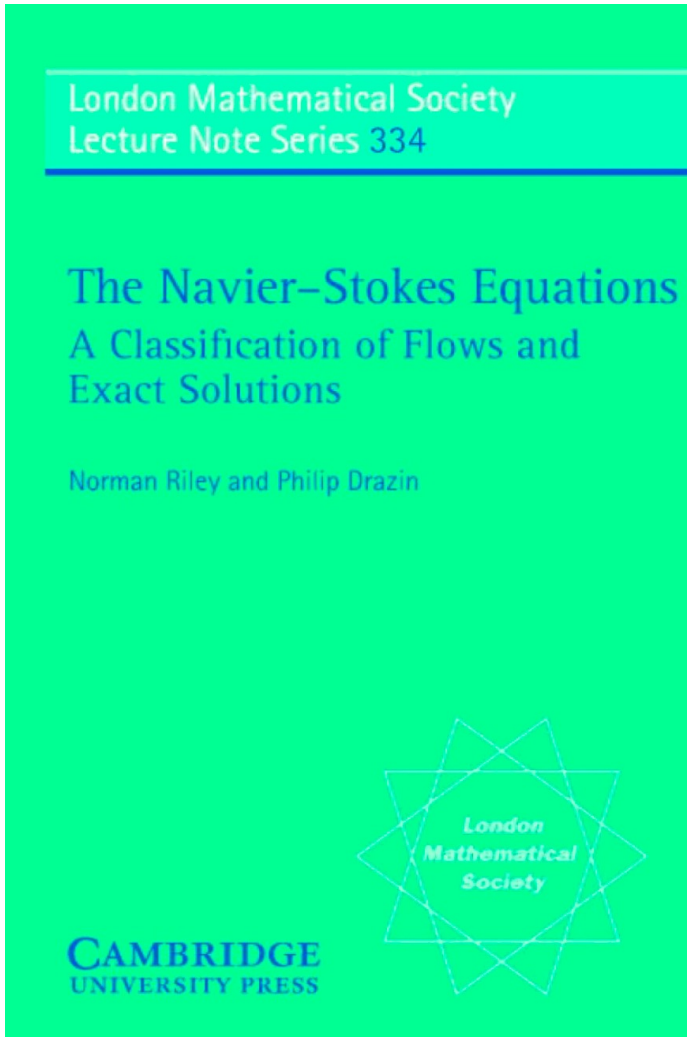
This derivation involves an assumption concerning a form of the flow field that enables reducing the equations of motion to an ordinary differential equation for the similarity function.

<u>Assumption</u>:

The vertical component of the velocity v_y does not depend on the horizontal coordinate x.

Incompressibility leads to a linear dependence of the horizontal velocity component v_x with x.





BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

In the case of channel flows with oscillating walls, one of the prominent studies is reported in Secomb (1978), using a perturbation technique.

- This work analyzes both walls moving periodically and symmetrically along the normal direction.
- The author finds asymptotic solutions for the part of the longitudinal velocity that presents a dependence with x.
- The solution is valid even when an external gradient of pressure is imposed to the channel.
- The perturbation technique was also considered by other researchers, who analyzed flows with non-periodic squeeze rates.
- An initial transient may exist in which such solutions fail, but the regular perturbation solution provides accurate results in a larger Reynolds number range, that extends up to Re $\sim$ 10.

J. Fluid Mech. (1978), *vol.* 88, *part* 2, *pp.* 273–288
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Flow in a channel with pulsating walls

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(Received 18 November 1977)

In this paper calculations are made of the two-dimensional flow field of an incompressible viscous fluid in a long parallel-sided channel whose walls pulsate in a prescribed way. The study covers all values of the unsteadiness parameter α and the steady-streaming Reynolds number. The wall motion is, in general, assumed to be of small amplitude and sinusoidal. Particular attention is given to the steady component of the flow at second order in the amplitude parameter ϵ. The results for the corresponding problem in axisymmetric geometry are given in an appendix.

Next the following problem is considered: the calculation of the wall motion which will result, in response to prescribed unsteady pressures imposed at the ends of the channel and outside its walls, if the walls are assumed to respond elastically to variations in transmural pressure. It is found that the system has a natural frequency of oscillation, and that resonance will occur if this frequency is close to a multiple of the frequency of the external pressure fluctuations. Finally the preceding work is applied in a discussion of blood flow in the coronary arteries of large mammals.

1. Introduction

A central feature of many physiological flow problems is the distensibility of vessel walls and the wall motion which results under pulsatile flow conditions. The usual approach to such problems is in terms of wave propagation, with the wall displacement dependent on both time and axial distance. Here, however, we consider two model problems in which the wall position depends only on time: two-dimensional flow in a long channel of width $2a(t)$ and axisymmetric flow in a long tube of radius $a(t)$. An important reason for studying these two problems is that the velocity field in each depends linearly on axial distance. This simplifies the equation of motion and makes it possible to find solutions over a wide range of flow parameters (§§ 2–5).

Under physiological conditions the wall motion of a vessel is not usually directly determined by external factors. It is more appropriate to consider the motion as being driven by external *pressures* applied to the ends of a length of vessel and outside its walls. A model problem of this type is investigated in § 6, and the results are applied in § 7 to the flow in the coronary arteries.

Uchida & Aoki (1977) have considered the same problem (in the axisymmetric case) but with a different choice for the function $a(t)$. They showed that if $a(t)$ is a particular *monotonic* function of time then time can be used as a similarity variable, and a solution can be obtained to the full nonlinear equations. The results of the present paper are complementary to theirs.

BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

Let us analyze the dynamics of this system combining the experimental study with the analytical approach.

The channel's half length b is much larger than its mean width a_0. We assume that the channel ends are connected to large fluid reservoirs, both at constant but different pressure. Our interest is laid upon those regions where the flow is fully developed and is not affected by the exit/entrance conditions of the fluid.

Let us focus on the case in which one of the rigid walls experiences a periodic motion along the normal direction expressed as:

$$y_w = a(t)$$

The equations of motion of this flow are:

$$u_t + uu_x + vu_y = -\phi_x + \nu(u_{xx} + u_{yy}) \qquad \phi = \frac{p}{\rho}$$

$$v_t + uv_x + vv_y = -\phi_y + \nu(v_{xx} + v_{yy}) \qquad u_x + v_y = 0$$

BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

Self-similar solutions, supposing that the component of the velocity normal to the wall does not depend on the longitudinal coordinate. The subindex indicates derivation with respect to the indicated variable

$$\eta = \frac{y}{a(t)} \qquad v(\eta,t) = \dot{a} f(\eta)$$

Substitution into the Navier-Stokes equation of these expressions yields an equation which translates into one of three possible conditions.

<u>Condition 1</u>

$$\frac{a\dot{a}}{\nu} = C_1$$

$$\frac{a^2\ddot{a}}{\nu\dot{a}} = C_2$$

↓

$a(t)$ scales as $t^{\frac{1}{2}}$

<u>Condition 2</u>

$$f_{\eta\eta} = 0$$

$$\frac{a\ddot{a}}{\dot{a}^2} = C_0$$

↓

$a(t)$ is sinusoidal.

<u>Condition 3</u>

$$f_{\eta\eta} = 0$$

↓

No restriction on $a(t)$

BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

One possibility is to consider asymptotic solutions. The price to pay is that the solutions may be restricted to a limited range of the non-dimensional control parameters. The following scaling is proposed:

$$\zeta = \frac{x}{b}; t^* = t \times \omega; u^* = \frac{u}{U_0}; \upsilon^* = \frac{v}{a_0 \Delta\omega}; \phi^* = \frac{\phi}{U_0^2} \quad \text{with} \quad \delta = \frac{a_0}{b} << 1$$

Depending on the case under study, the value of U_0 can be associated to a mean flow velocity produced by the gradient of pressure imposed to the channel or to the longitudinal velocity associated to the wall movement.

$$U_0 \approx \sqrt{\frac{p_u - p_d}{\rho}}) \qquad U_0 \approx b\omega$$

The height of the channel can be expressed as:

$$a(t) = a_0 H(\Delta, \omega, t)$$

BASIC PRINCIPLES OF FLOW-INDUCED SELF-SUSTAINED OSCILLATIONS

This scaling allows rewriting the Navier-Stokes equations in terms of the non-dimensional variables:

$$\left(\frac{b\omega}{U_0}\right)u^*_{t^*} + u^* u^*_\zeta + \left(\frac{b\omega\Delta}{U_0}\right)(\upsilon^* - \dot{H}\eta)\frac{u^*_\eta}{H} = -\phi^*_\zeta + \left(\frac{\nu b}{U_0 a_0^2}\right)\left(u^*_{\zeta\zeta}\delta^2 + \frac{u^*_{\eta\eta}}{H^2}\right)$$

$$\left(\frac{b\omega}{U_0}\right)\upsilon^*_{t^*} + u^* v^*_\zeta + \left(\frac{b\omega\Delta}{U_0}\right)(\upsilon^* - \dot{H}\eta)\frac{v^*_\eta}{H} = -\left(\frac{U_0}{a_0\omega\Delta}\right)\frac{\phi^*_\eta}{H} + \left(\frac{\nu b}{U_0 a_0^2}\right)\left(\upsilon^*_{\zeta\zeta}\delta^2 + \frac{\upsilon^*_{\eta\eta}}{H^2}\right)$$

$$\left(\frac{U_0}{b\omega\Delta}\right)u^*_\zeta + \frac{\upsilon^*_\eta}{H} = 0$$

$$\frac{1}{Re\delta}$$

at $\eta = 1$ and $\eta = 0$ $u^* = 0$,

at $\eta = 1$ $\upsilon^* = \dot{H}$,

at $\eta = 0$ $\upsilon^* = 0$.

$$Re = \frac{U_0 a_0}{\nu}$$

To find a self-similar solution:

$$u^*(\zeta, \eta, t^*) = u^*_0(\eta, t^*) + u^*_1(\eta, t^*)\zeta$$

Inviscid solution

The inviscid flow assumption is obtained when $\frac{1}{Re\delta} \approx 0$. The solution of the problem is assumed to be self-similar and to satisfy Condition 3. In our case, this determines that:

$$v(\eta,t) = \eta\dot{a}$$

$$u(x,t) = U(t) - \frac{\dot{a}}{a}x$$

$$\Phi(x,\eta,t) = \Phi_{0(\eta,t)} + \Phi_{1(t)}x + \Phi_{2(t)}x^2$$

where

$$\Phi_{0(\eta,t)} = \phi_c(t) - \frac{\ddot{a}a}{2}\eta^2$$

$$\Phi_{1(t)} = \frac{U\dot{a}}{a} - \dot{U}$$

$$\Phi_{2(t)} = \frac{1}{2}(\frac{\ddot{a}}{a} - 2(\frac{\dot{a}}{a})^2)$$

$U(t)$ and $\phi_c(t)$ depend on conditions at the channel ends, but not on x.

Viscous solution

Let us now consider the case $\frac{b\omega}{U_0} << 1$, and neglect all the terms that are affected by this coefficient. Let us also recall that ϕ_1^* and ϕ_2^* are functions of time only. The solution of the problem is assumed to be self-similar and to satisfy Condition 2. The solution can be expressed as :

$$u(x,\eta,t) = u_0(\eta,t) + u_1(\eta,t)x$$

$$\phi(x,\eta,t) = \phi_0(\eta,t) + \phi_1(t)x + \phi_2(t)x^2$$

Incorporating the boundary conditions of the problem, we have:

$$\upsilon(\eta,t) = \dot{a}(3\eta^2 - 2\eta^3)$$

$$u_1(\eta,t) = 6(\dot{a}/a)(-\eta^2 + \eta)$$

$$\Phi_0(\eta,t) = 6b\nu\frac{\dot{a}}{a}(\eta - \eta^2) + \phi_c(t)$$

$$\Phi_2(t) = 6\nu\frac{\dot{a}}{a^3} - 18\frac{\dot{a}^2}{a^2}(\eta^4 - 2\eta^3 + \eta^2)$$

DISCUSSION

The dynamics of the mass is governed by: $\ddot{y} + 2\gamma\dot{y} - \omega_0^2 y = g + F_{aer}(t) \longrightarrow F_{aer}(t) = \int_{-b}^{b} \phi(t)dx$

- In the experiments, we detected that the movement of the mass in the y-direction is accompanied by a small rotation. The linear term $\Phi_1(t)$ contributes to explaining the slight rotation observed in the experiments.
- The quadratic term $\Phi_2(t)$ with this hypothesis prevails when $a(t)$ attains a minimum value, indicating that the forcing and trend of the mass to rotate relatively diminishes as the gap is narrowing.
- With the viscous approach, obtaining the value of $\Phi_1(t)$ requires determining the point at which the flow is fully developed. This amounts to finding an entrance length that corresponds to the position at which the boundary layers of both walls merge.
- A correct estimation of this entry length is not a simple task as it varies with time. The boundary layer of the lower mass must be calculated considering a pulsating flow. For the upper one, the situation complexifies as the wall is moving.

DISCUSSION

In flow-induced vibrations, self-sustained oscillation is assured when the energy supplied to the mass by the flow during a cycle of period T compensates the dissipation in the same time interval. This requires that:

$$\int_0^T F_{aer(t)} \dot{a} dt > 0$$

- In the inviscid approach and considering a sinusoidal law for the wall motion, the only term that can contribute to the flow-induced oscillation is the one corresponding to $\phi_c(t)$ (all the others give no net energy along a cycle). The value of $\phi_c(t)$ can be easily calculated if we neglect the non steady terms outside the channel.

- With the viscous analysis, there are contributions of Φ_0 and Φ_2 to the flow-induced oscillation. The viscous case solution has however some restrictions. It is suitable to analyze mass motions in which:

$$\frac{a\ddot{a}^2}{\dot{a}} = C_0$$

- This condition is satisfied for instance when: $a(t) = a_0(1 + \Delta sin(\omega t))$. This law agrees quite well with the recorded mass motion of our experiments.

CONCLUSIONS

- We presents for the first time an experimental and analytical study of a long parallel-sided channel with one wall pulsating periodically.

- We observe that the oscillations induced by the flow of the mass that limits one side of the channel occurs at a frequency close to the natural frequency of the system. The gap varies following a sinusoidal wall of motion.

- The mean velocity in the channel attains a relatively high value compared to the horizontal squeeze flow produced by the oscillating mass.

- The analytical study assumes a self-similar behavior of the flow inside the channel. This study allows us to analyze the nature of flow-induced oscillation in the context of both, inviscid and viscous flow.

- The inviscid approach is not restricted to cases of simple periodic wall motion: it also enables incorporating arbitrary periodic laws. Here, we find that the flow-induced movement can be explained only in terms of flow states that depend on the exit and entrance conditions.

- In the viscous approach, the solution obtained is restricted to sinusoidal wall motions, the oscillation does not rely only on the exit and entrance conditions since the energy dissipation are compensated by pressure forces that depend on the non steady viscous terms.