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▶ **To cite this version:**

James Hines Jr., Michael J. Keen. Certain effects of random taxes. Journal of Public Economics, 2021, 203, pp.104412. 10.1016/j.jpubeco.2021.104412 . hal-03420151

HAL Id: hal-03420151
https://hal.science/hal-03420151

Submitted on 14 Jan 2022

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Certain Effects of Random Taxes*

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March 2021

* We thank Nathan Hendren, Andreas Pieschl, Joel Slemrod, Stefanie Stantcheva, three anonymous referees, and seminar participants at the University of Kyoto, University of Michigan, ETH Zurich, and the National Tax Association and American Law and Economic Association annual conferences for valuable comments on earlier drafts. Saida Khamidova provided excellent research assistance. Views and opinions are ours alone, and should not be attributed to the management or Executive Board of the International Monetary Fund.

Certain Effects of Random Taxes

ABSTRACT

This paper explores the implications of tax rate randomness, identifying circumstances in which revenue-neutral rate variability increases profitability, economic activity, and the efficiency of resource allocation. Furthermore, with heterogeneous taxpayers, tax rate variability is shown to perform an efficiency-enhancing screening function, imposing heavier expected tax burdens on less responsive taxpayers. And while efficient tax randomness enables governments to reduce average costs of taxation, it necessarily increases the marginal cost of taxation over some ranges of expected revenue, so may reduce efficient levels of government spending.

JEL Classifications: H21, H22.

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1. Introduction

Thoughtful observers have long criticized uncertainty in tax matters. One of Adam Smith's (1802, p. 256) canons of taxation is that "the tax which each individual is bound to pay ought to be certain," while for Alexander Hamilton (Hamilton, 1865, p. 163), "The genius of liberty…exacts that every man, by a definite and general rule, should know what proportion of his property the State demands." Such concerns are with us still. Perceptions of an increasingly random tax environment[1] prompted the G20 (2016) to stress "the benefits of tax certainty to promote investment and trade," and to put the IMF and OECD (2017) to work on how to enhance it,[2] with the European Union also addressing these issues.[3] Tax uncertainties loom large as an element of policy uncertainty more widely; in constructing their influential index of economic policy uncertainty, Baker, Bloom and Davis (2016, p. 1602) find that "Fiscal matters, especially tax policy, stand out…as the largest source of policy uncertainty, especially in recent years." Against the background of this long-lasting and increasingly high profile concern, the purpose of this paper is to explore one aspect of uncertainty in taxation—tax randomness—and its impact on economic activity and economic welfare.

By 'random taxation' we mean a situation in which tax parameters are uncertain ex ante but economic actors make decisions, without costs of delay, once tax parameters are revealed.[4] This contrasts with situations in which irreversible decisions must be made before tax outcomes are known.[5] In practice, both types of unpredictability are commonly at work. For example, firms may need to decide where to locate new plants before knowing the tax treatment that will apply to their investments, workers and sales; afterwards, however, they have flexibility in varying operational decisions in light of the treatment they find themselves facing. The concern of this paper is with uncertainty of the latter kind, randomness in taxation that is an omnipresent

[1] Largely, though not only, in relation to implementation of the outcomes of the G20-OECD project on Base Erosion and Profit Shifting (BEPS), intended to curb tax avoidance by multinationals.
[2] See also IMF and OECD (2018).
[3] See Zangari, Caiumi and Hemmelgarn (2017).
[4] This is the kind of uncertainty analyzed in Weiss (1976), Stiglitz (1982a,b), Alm (1988), Skinner (1988), and Pestieau, Possen, and Slutsky (1998) in terms of individual welfare and behavior, and by Hartman (1972), Pindyck (1982), Abel (1983), and Auerbach and Hines (1988) in relation to investment behavior.
[5] This is the kind of uncertainty analyzed, in relation to investment, by Pindyck (1982), Abel (1983), Alvarez, Kanniainen, and Sodersten (1998), Hassett and Metcalf (1999), Niemann (2004, 2011), Bloom, Bond and van Reenen (2007) and Bloom (2014).

aspect of business life, as a result of which tax burdens can differ widely between firms in otherwise very similar business situations, or even for the same firm at different times.

Tax randomness can have an aspect of capriciousness. Favorable tax treatment might be afforded firms that happen to be located in certain regions, have certain ownership or organizational structures, export their output, or are simply politically well-connected.[6] Capricious though that may seem, it might also possibly serve some useful purpose. Concentrating tax benefits in a subset of firms, for example, may stimulate greater economic activity than spreading them broadly. It is notable too that in recent years a variety of administrative measures that create a form of tax randomness have become more common. Advance rulings, for instance, enable businesses to resolve uncertainty over the tax consequences of potential transactions before they are undertaken, and allow firms to adjust their actions accordingly.[7] Furthermore, the haphazardness of electoral politics and other shocks mean that tax rates can vary over time in ways that for practical business purposes are random. The tax uncertainties that have troubled Hamilton and the G20 include much more than randomness. Nonetheless, the randomness that we focus on here is an inescapable and critical element of uncertainty.

Interest in tax randomness goes back many years, and it has long been recognized that while random taxation poses challenges to consumers and firms it need not be harmful to their interests. Indeed, one of the fundamental propositions of price theory is that a competitive firm's profits are convex in input and output prices (Mas-Colell, Whinston and Green, 1995, pp. 138, 141), implying that its expected profits increase with mean-preserving spreads in these prices—and hence too with corresponding randomization of input or output tax rates. Input price variation, for instance, permits firms to economize on expensive input purchases while prices are high, substituting with other inputs and reducing output; and they can expand the use of inexpensive inputs when prices are low. By adjusting its purchases and sales, a firm can thus effectively use price variability to reduce the average unit cost of its inputs, thereby improving after-tax profits.

[6] So-called 'rifle-shot' provisions in U.S. tax laws that provide tax benefits tailored to specific taxpayers are extreme examples of this kind of tax randomness.

[7] Other administrative approaches that also have much current support—such as 'cooperative compliance' programs, which aim to foster the identification and resolution of 'risky' tax positions—have similar effects (see for example IMF and OECD (2017), pp. 48-52).

Remarkable though these results are, however, they offer incomplete guides to policy. This is because firm responses mean that randomizing a tax rate, holding its expected value unchanged, generally reduces expected tax revenue.[8] A more appropriate assessment of the effects of tax randomness would hold constant not expected tax rates but expected tax revenue. The point is of considerable practical importance. In the context of the G20's current policy concerns, it is important—but rarely attempted—to disentangle business worries over tax uncertainty from those related to expected tax liabilities. Empirically too, while such evidence as is currently available suggests that tax uncertainty may discourage investment, it generally does little to distinguish the effects of uncertainty as such from the effects of expected levels of taxation.[9]

A further limitation of the standard price theory result is that it speaks only to the effect of tax randomness on after-tax profits, whereas the effect of tax randomness on input use—as with the G20's concern for investment—and economic output are also important for policy assessment. There are of course some results on this, notably those presented by Hartman (1972) and Abel (1983), who show that a mean preserving spread in an output tax rate leads a competitive firm to increase its capital stock, so long as the marginal value product of capital is convex in that output price. Again, however, this does not condition on expected tax revenue—as is also true of more recent results on the effect of tax uncertainty with irreversible investment.[10]

This paper takes up these and related issues, examining the effect of random taxation on profitability, economic activity, and the efficiency cost of raising government revenue, all while holding constant expected tax revenue. The context of this analysis is what appears to be a widely held presumption, shared by the G20, that random taxes inefficiently reduce economic activity and depress profits; some see this view as simply 'common sense.'[11] And there might

[8] This is true not only of previous models of tax randomization, such as Hartman (1972), but also of models with irreversible investment, such as Hassett and Metcalf (1999).
[9] Edmiston (2004), for example, uses the deviation of tax rates from trend as an indicator of tax randomness, without reference to the expected level of revenue. Baker, Bloom and Davis (2016) use as their indicator of tax uncertainty the dollar value of tax provisions set to expire, which holds expected revenue constant only under limited circumstances. And much of the empirical literature reviewed in IMF and OECD (2017) and Zangari, Caiumi and Hemmelgarn (2017) does not distinguish the effects of tax randomness from those of uncertainty more generally.
[10] Such as those of Alvarez, Kanniainen, and Sodersten (1998) and Hassett and Metcalf (1999).
[11] As for instance do Zangari, Caiumi and Hemmelgarn (2017).

indeed appear to be some principled reason to suppose this to be the case. Higher tax rates discourage taxed activity, and the resulting economic losses are commonly presumed to rise with the amount of tax revenue collected at an increasing rate, making it ever more expensive—in an efficiency sense—to collect additional tax revenue. The familiar Harberger triangle approximation, that deadweight loss is proportional to the square of the tax rate (Harberger, 1964, 1971; Auerbach, 1985; Hines, 1998), lends support to this intuition. It implies that tax rate variability increases the deadweight loss associated with raising a given level of expected tax revenue; the cost of this deadweight loss is borne by consumers in the form of higher prices and business in the form of lower profits, and these discourage economic activity. This intuition is in many cases correct: in contrast to the standard result that a mean-preserving spread in tax rates increases a firm's after-tax profits, for instance, randomization that holds expected tax revenue constant may well reduce profits. But this intuition can also be plain wrong.

There are two reasons why tax rate variability might instead improve the efficiency of the tax system. The first is that the discouraging effects of higher increments to tax rates can become more moderated at high rates, causing the marginal deadweight loss of collecting additional tax revenue to decline with tax revenue. This possibility reflects an important inaccuracy in the intuitive approximation to the size of the deadweight loss triangle, and is the basis of earlier normative arguments (e.g., Stiglitz, 1982b) made in favor of random taxation. The analysis below identifies the circumstances—looking first at the impact on a single firm—in which, conditional on expected tax revenue, tax rate variability does indeed reduce expected profits, output and input use, as well as those in which, counter to 'common sense,' tax randomness actually increases all or some of these.

A second reason why randomness can improve the efficiency of the tax system—previously unnoticed but commonly more powerful—emerges from the analysis below. This is that random taxation serves as a screening device, effectively imposing higher tax rates on firms and activities that are relatively unresponsive to taxation, and lower tax rates on firms and activities that are more responsive; and that, by standard intuition, improves efficiency. If there is any type of taxpayer heterogeneity, then the fraction of taxed activity undertaken by taxpayers that are most responsive to taxation will decline as tax rates rise. Consequently, as tax rates rise the high tax rates increasingly bear on activities that are relatively unresponsive to taxation. The

opposite happens when tax rates decline: the resulting low tax rates apply to a population that is relatively dominated by taxpayers and activities that are highly responsive to taxation. Random taxation therefore implements a subtle version of the Ramsey rule, imposing higher tax rates on the activities of less-responsive taxpayers, and lower tax rates on the activities of more-responsive taxpayers—which has the effect of reducing the total cost of taxation. If the government cannot distinguish firms and activities any other way, it can do so indirectly with tax rate variability.

All of this carries implications for efficient tax policy that are also subtle and to some degree surprising. In the absence of randomness, and with a single tax instrument, tax policy is dictated by the government's revenue needs. Tax rate variability expands the range of possibilities, and may allow the government to reduce the total economic cost of taxation by randomizing between disparate tax rates, each associated with relatively low deadweight loss per dollar of tax revenue collection. While over some ranges of expected tax revenue such policies will reduce the marginal cost of public funds, it is also the case that over some ranges they will increase the marginal cost of public funds. It follows that there are many circumstances in which the adoption of efficiency-enhancing tax randomization will reduce the efficient level of government spending even though it reduces the average cost of taxation.

The paper is structured as follows. Section 2 sets out the basic framework of the analysis, and Section 3 analyzes the effects of random taxes on firm profits and economic activity in settings with identical taxpayers. Section 4 introduces taxpayer heterogeneity, showing it to have a marked effect on the efficiency properties of tax randomization by performing the screening function described above. Section 5 considers the efficient design of random taxes and its implications for the marginal cost of public funds. Section 6 concludes, noting additional implications and generalizations.

2. Preliminaries

A perhaps surprising degree of theoretical richness is available from considering a simple setting in which a fixed number of identical price-taking competitive firms each purchases a

taxed input in amount *x*, paying suppliers a unit price, and also paying a specific tax of τ per unit purchased;[12] it is assumed that $\tau > 0$, but will be seen later that the results extend readily to cases in which τ is strictly negative. Each firm has a production function[13] $q(x)$ that is increasing, continuous and continuously differentiable as needed in *x*, and exhibits decreasing returns, so that $q'(x) > 0$ and $q''(x) < 0$. There may be other untaxed inputs, in which case $q(x)$ represents output net of the cost of these optimally chosen additional inputs.[14] Firms sell their output at constant unit price (and untaxed) on the world market. The profits of the representative firm (π), arising from the presence of some fixed factor(s), are therefore given by

$$\pi = q(x) - (1+\tau)x. \tag{1}$$

The first order condition[15] for profit maximization is $q'(x) = (1+\tau)$, which generates a derived demand for *x* as a function of τ, $x(\tau)$, that is continuous and continuously differentiable, with $x'(\tau) < 0$.

Tax revenue is $R \equiv \tau x(\tau)$. The analysis confines attention to ranges of tax rates over which tax revenue monotonically increases with the tax rate, which requires that

$$x(\tau) + \tau x'(\tau) > 0. \tag{2}$$

[12] Specific and ad valorem taxes have equivalent effects in competitive markets in the absence of uncertainty in the tax-exclusive price of the taxed input, as noted by Suits and Musgrave (1953). The effects of (non-stochastic) specific and ad valorem taxes will, however, differ in competitive markets with price uncertainty (Goerke, 2011; Kotsogiannis and Serfes, 2014; Goerke, Herzberg, and Upmann, 2014) and in imperfectly competitive markets (Delipalla and Keen, 1992; Weyl and Fabinger, 2013). When, for either of these reasons, equivalence fails, the distinction between specific and ad valorem taxes will be material for the impact of random taxes; this paper's analysis, however, abstracts from these issues.

[13] It is an important feature of the model that production functions are exogenously given. If firms could choose among possible production functions then their choices, and therefore output, would be affected by the anticipated distribution of tax rates. Furthermore, the production functions considered here exhibit no adjustment costs or other forms of intertemporal dependence, so input choices and output are determined only by contemporaneous prices and taxes.

[14] Appendix A of Hines and Keen (2018) extends the analysis to allow for other variable inputs taxed at fixed rates.

[15] The assumption that $q''(x) < 0$ ensures that the firm's second order condition is satisfied.

Of course, (2) is not always satisfied, particularly at high tax rates. It is nonetheless constructive to restrict attention to ranges of tax rates over which (2) is satisfied, since there are predictably strange consequences of randomizing tax rates into ranges in which higher tax rates are associated with reduced tax collections. Thus, it is assumed throughout that (2) holds, or, equivalently, that

$$1+\varepsilon(\tau)>0, \tag{3}$$

where

$$\varepsilon(\tau)\equiv\tau\frac{x'(\tau)}{x(\tau)}<0 \tag{4}$$

denotes the elasticity of input demand with respect to the tax rate. This elasticity (strictly negative at any positive tax rate), and its properties, plays a central role in the analysis that follows.

From the implied one-to-one relationship between the tax rate and tax revenue, it is possible to express the tax rate as an implicit function of tax revenue, $\tau(R)$, with

$$\tau'(R)=\frac{1}{x(\tau)+\tau x'(\tau)}>0. \tag{5}$$

Input demand is then also an implicit function of tax revenue, $x(\tau(R))$, and so too are both output $q\left[x(\tau(R))\right]$ and profits $\pi(R)$. The device of characterizing firm behavior and profits as a function of tax revenue, used repeatedly in the next section, makes it possible to infer the effect of revenue-neutral tax rate randomization from their convexity or concavity in revenue. For example, a local randomization of tax rates that leaves expected revenue unchanged[16] increases the expected value of profits if and only if $\pi(R)$ is convex in *R*.

[16] For brevity, this is referred to simply as a revenue neutral reform.

3. The impact of random taxes with identical firms

To identify clearly the considerations that influence the effects of random taxes on expected profits, input use and output, it is helpful to start by considering randomness that applies to a single firm or to a set of identical firms.

3.1 Expected profits

Consider first the impact of a revenue neutral tax rate randomization on the firm's expected profits. Shephard's lemma implies, from (1), that $\pi'(R) = -x(\tau)\tau'(R)$ and so, using (4) and (5), that:

$$\pi'(R) = \frac{-1}{1+\varepsilon(\tau)}. \tag{6}$$

Recalling (3), it follows that at any positive tax rate $\pi'(R) < -1$: profits decline by more than any increment to tax revenue, reflecting that the marginal deadweight loss associated with raising an additional dollar is positive—a point returned to in Section 5. It also follows from inspection of (6) that if $\varepsilon'(\tau) < 0$, so that the elasticity of input demand has a larger negative magnitude as the tax rate increases, then increments of tax revenue will be associated with ever larger reductions in firm profits, making the profit function concave in revenue. More formally, differentiating (6) produces:

$$\pi''(R) = \frac{\varepsilon'(\tau)}{\left[1+\varepsilon(\tau)\right]^2}\tau'(R). \tag{7}$$

Hence given (5), profits are locally strictly concave in tax revenue if and only if $\varepsilon'(\tau) < 0$ and locally strictly convex in tax revenue if and only if $\varepsilon'(\tau) > 0$. Therefore:[17]

[17] Equation (7) readily generalizes to cases in which there are multiple taxed inputs, as elaborated in Appendix A of Hines and Keen (2018).

PROPOSITION 1: *A revenue neutral local tax rate randomization reduces expected profits if and only if* $\varepsilon'(\tau) < 0$, *and increases expected profits if and only if* $\varepsilon'(\tau) > 0$.

The impact of tax rate randomization thus turns on whether the elasticity of derived input demand increases or decreases with the tax rate.

Figure 1 provides some intuition for this. It displays the relationship between tax rates and firm profits, with the solid locus in the figure plotting firm profit as a decreasing convex function of the tax rate. This captures the notion that a mean-preserving price spread gives firms more options than they would have under price stability; formally, convexity is guaranteed by the combination of Shephard's Lemma, which implies that the slope of the profit function is $-x(\tau)$, and the input demand implication that $x'(\tau) < 0$. The figure describes a setting with random tax rates: the tax rate takes the high value τ_H with probability 0.5, and the low value τ_L with probability 0.5.

Consider then a revenue-neutral reform that increases the scope of tax randomness by increasing τ_H by an amount that would raise an additional \$1 in tax revenue while lowering τ_L by an amount that would lose \$1 in tax revenue. Denoting input demand at τ_H by $x_H \equiv x(\tau_H)$, and analogously the input demand elasticity at τ_H by $\varepsilon_H \equiv \varepsilon(\tau_H)$, it follows from (5) that the new high value of the tax rate is $\tau_H + \tau'(R_H) = \tau_H + \left[\frac{1}{x_H(1+\varepsilon_H)}\right]$. Since the slope of the profit function at τ_H is given by $-x_H$, the change (a reduction) in profit if this small tax increase were to arise is $\frac{-x_H}{x_H(1+\varepsilon_H)} = \frac{-1}{(1+\varepsilon_H)}$. By a similar calculation, the increase in profit if the tax were reduced is $\frac{1}{(1+\varepsilon_L)}$. Consequently, with equal probabilities of high and low tax rates, the net increase in expected profits from widening the spread of tax rates in this expected revenue-neutral way is given by $0.5\left[\frac{1}{(1+\varepsilon_L)} - \frac{1}{(1+\varepsilon_H)}\right]$. Expected profits therefore fall if and only if $\varepsilon_H < \varepsilon_L$, which is the case if and only if the tax elasticity of input demand

declines (is more negative) at higher tax rates—which is precisely the condition that $\varepsilon'(\tau)<0$, just as in Proposition 1.[18]

Much thus turns on the sign of the derivative $\varepsilon'(\tau)$. This, as a general matter, is ambiguous. To see the forces at work, denote by $\eta(\tau)\equiv(1+\tau)\frac{x'(\tau)}{x(\tau)}<0$ the price elasticity of demand for the taxed input, so that $\varepsilon(\tau)=\left(\frac{\tau}{1+\tau}\right)\eta(\tau)$ and hence $\varepsilon'(\tau)=\frac{1}{(1+\tau)^2}\eta(\tau)+\frac{\tau}{(1+\tau)}\eta'(\tau)$. Since the first term on the right points toward $\varepsilon'(\tau)<0$, it is sufficient, but not necessary, for $\varepsilon'(\tau)<0$ that the price elasticity of the input demand be decreasing or constant in the input price—which will be the case, for instance, if input demand is linear or (a case explored further below) Cobb-Douglas. Conversely, it can be the case that $\varepsilon'(\tau)>0$ – so that revenue-neutral randomization of tax rates raises expected profits – only if the price elasticity $\eta(\tau)$ not only increases with the input price but does so sufficiently rapidly to offset the mechanical effect of a higher tax rate. That this is a real possibility is illustrated by considering firms with production functions

$$q(x)=q(\bar{x})+b^{\frac{1}{c}}\int_{\bar{x}}^{x}\left[\ln\left(\frac{z}{a}\right)\right]^{\frac{-1}{c}}dz, \tag{8}$$

with *a*, *b* and *c* all strictly positive constants. The production function (8) generates input demands

$$x=ae^{b(1+\tau)^{-c}}, \tag{9}$$

which imply that $\varepsilon(\tau)=\frac{-bc\tau}{(1+\tau)^{1+c}}$, and for *b* sufficiently small, $0>\varepsilon(\tau)>-1$. It follows that

[18] While this exercise starts from a situation with random taxes, its implications also apply to introducing a small amount of randomness into a setting with tax certainty.

(10) $$\varepsilon'(\tau) = \frac{-bc(1-\tau c)}{(1+\tau)^{2+c}}.$$

Equation (10) then implies that that $\varepsilon'(\tau) > 0$ whenever $\tau > 1/c$. [19]

While theory thus leaves the sign of $\varepsilon'(\tau)$ open, and there are clearly instances in which $\varepsilon'(\tau) > 0$, there is perhaps a cautious presumption that in many cases $\varepsilon'(\tau) < 0$, at least in the sense that this can be shown to be case for the specific functional forms often found convenient to work with and used to guide intuition. The implication is a similarly guarded presumption that, with identical firms, revenue neutral tax rate randomization will commonly lower expected profits. This, of course, stands in sharp contrast to the established result that randomization of tax rates holding the expected tax rate constant increases expected profits: controlling for the expected impact on revenue makes a great deal of difference to the likely consequences of tax randomness for firm profitability.

3.2 Expected input use

The impact of revenue neutral tax rate randomization on input use turns on the convexity or concavity of demand for input *x* expressed as a function of *R*. With $x'(R) = x'(\tau)\tau'(R)$, imposing (5) gives:

(11) $$x'(R) = \frac{\varepsilon(\tau)}{\tau\left[1+\varepsilon(\tau)\right]}.$$

Equation (11) then implies:

[19] More generally it is readily shown that the necessary and sufficient condition for $\varepsilon'(\tau) < 0$ is that the price elasticity of the price elasticity of input demand be greater than $-1/\tau$.

PROPOSITION 2: *Expected input use is reduced by revenue neutral local tax rate randomization if and only if* $\varepsilon'(\tau) < \frac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau} < 0$. *Conversely, expected input use is increased by revenue neutral local tax rate randomization if and only if* $\varepsilon'(\tau) > \frac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau}$.

Proof: Differentiating both sides of (11) with respect to *R* produces:

$$x''(R) = \frac{1}{\tau\left[1+\varepsilon(\tau)\right]^2}\tau'(R)\left\{\varepsilon'(\tau) - \frac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau}\right\}. \tag{12}$$

Since the first two terms on the right side of (12) are positive, the sign of $x''(R)$ depends on the sign of the term in braces, from which the proposition follows. □

Proposition 2 implies that $\varepsilon'(\tau) > 0$ is a sufficient condition for expected input use to increase with tax randomness. So, from Proposition 1, if tax randomness increases expected profits then it will also increase expected input use. The central implication of Proposition 2, however, is that the converse is not true: if $\varepsilon'(\tau) < 0$, so that tax randomness reduces expected profits, it may nonetheless increase expected input use if $\varepsilon'(\tau)$ is sufficiently small in magnitude. When $\varepsilon'(\tau)$ is negative but vanishingly small, for instance, a revenue neutral tax rate randomization reduces expected profits but increases expected input use.

3.3 Expected output

The effect of tax randomness on expected output can be analyzed in the same way. Expressing output as a function of tax revenue, $q(R)$, the output effect of greater tax revenue is $q'(R) = q'(x)x'(\tau)\tau'(R)$. Imposing the firm's first order condition that $q'(x) = (1+\tau)$, together with (5), gives:

$$q'(R) = \frac{(1+\tau)\varepsilon(\tau)}{\tau\left[1+\varepsilon(\tau)\right]}. \tag{13}$$

Differentiating equation (13) then gives:

PROPOSITION 3: *Expected output declines with revenue neutral local tax rate randomization if and only if* $\varepsilon'(\tau) < \dfrac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau(1+\tau)}$. *Conversely, expected output increases with revenue neutral local tax rate randomization if and only if* $\varepsilon'(\tau) > \dfrac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau(1+\tau)}$.

Proof: Differentiating both sides of (13) with respect to *R* produces:

$$q''(R) = \frac{(1+\tau)}{\tau\left[1+\varepsilon(\tau)\right]^2}\tau'(R)\left\{\varepsilon'(\tau) - \frac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau(1+\tau)}\right\}. \tag{14}$$

Since the first two terms on the right side of (14) are positive, the sign of $x''(R)$ depends on the sign of the term in braces, from which the proposition follows. □

The results in Propositions 2 and 3 can be visualized in a fashion similar to that of Proposition 1,[20] though with the important difference that, while profits, input demand and output are all decreasing in the tax rate, only profit is guaranteed to be convex in the tax rate.

Tax randomness thus exerts many of the same effects on expected output as it does on expected input use. In particular, if tax randomness increases expected profits (because $\varepsilon'(\tau) > 0$) then it also increases expected output. Again, however, the converse is not true: tax randomness reduces expected profits but increases expected output if $0 > \varepsilon'(\tau) > \dfrac{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]}{\tau(1+\tau)}$. Comparing the critical values in Propositions 2 and 3, the additional $(1+\tau)$ in the denominator of the latter means that while a reduction in expected input use is a sufficient condition for expected output also to fall, it is not necessary: there is a range of values of $\varepsilon'(\tau)$ over which tax randomness is associated with greater expected input use but reduced expected output.

[20] Appendix B of Hines and Keen (2018) offers an illustration of Proposition 3.

3.4 Interpretation

Figure 2 summarizes the relationships between the value of $\varepsilon'(\tau)$ and the signs of the effects of tax randomness on expected profits, input use, and output implied by Propositions 1, 2 and 3. At sufficiently low (negative) values of $\varepsilon'(\tau)$, for example, revenue neutral tax rate randomization reduces the expected levels of profits, input use, and output. At the other extreme, if (and only if) $\varepsilon'(\tau)$ is strictly positive does a revenue neutral tax rate randomization increases all three. In between, there is a ready intuition for the hierarchy of results shown in Figure 2. Concavity of the production function implies that for any expected level of input use, the expected level of output declines with tax-induced variation in input use. This means, loosely speaking, that output is more likely to be a concave function of revenue than is input use—and so randomness reduces output more readily than it reduces inputs. And since profits are concave in output they are in turn more likely to be reduced by randomness than are outputs.

To illustrate these results, consider the effects of taxation when firms have identical Cobb-Douglas production functions given by:

(15) $$q(x) = kx^{\alpha},$$

with $k > 0$ and $1 > \alpha > 0$. The first order condition for profit maximization implies $x^{1-\alpha} = \dfrac{k\alpha}{(1+\tau)}$; differentiating this with respect to τ yields:

(16) $$\varepsilon(\tau) = \frac{-\tau}{(1+\tau)(1-\alpha)}.$$

Differentiating again produces:

(17) $$\varepsilon'(\tau) = \frac{-1}{(1+\tau)^2(1-\alpha)}.$$

It is clear from (17) that, as claimed earlier, in this case $\varepsilon'(\tau) < 0$, so that, by Proposition 1, tax randomness reduces firm profits. This reflects the fact that in the Cobb-Douglas case the tax elasticity of input demand is zero in the absence of taxation, steadily rising in magnitude with higher tax rates and ultimately converging to $\frac{-1}{(1-\alpha)}$. Furthermore, (16) and (17) together imply:

$$\frac{\varepsilon'(\tau)}{\varepsilon(\tau)\left[1+\varepsilon(\tau)\right]} = \frac{1}{\tau}\frac{(1-\alpha)}{\left[1-\alpha(1+\tau)\right]} > \frac{1}{\tau}, \tag{18}$$

where the inequality in (18) follows from the restriction that $\tau'(R) > 0$, which implies $\varepsilon(\tau) > -1$ and therefore, from (16), that $\alpha(1+\tau) < 1$. Applying Propositions 2 and 3, equation (18) implies that with Cobb-Douglas production functions tax randomness also reduces both expected input use and expected output.

A numerical example for this Cobb-Douglas case, to be exploited further below, is presented in Table 1. The initial tax rate is $\tau = 2$ and the production function has $\alpha = 0.2$, and *k* chosen so that *x* = 10 initially.[21] With these input demand parameters, initial tax revenue is 20, output is 150, and profits are 120. Tax randomness takes the form of reducing the tax rate to 1.6 with probability 0.5, and raising it to 2.86 with probability 0.5, an asymmetric difference that leaves expected tax revenue unchanged at 20. As reflected in the table, this tax rate fluctuation reduces expected profits to 118.5, and similarly reduces expected input use to 9.65 and expected output to 148.15. The rising tax sensitivity of input demand requires a significantly higher tax rate in the state of the world with high taxes to compensate for a lower tax rate in the state of the world with low taxes, distorting production and reducing expected input use and profits.

4. Heterogeneous firms

[21] This entails *k* = 94.64.

The focus so far has been on economies with representative firms, or equivalently, economies in which all firms are identical—and, moreover, are taxed identically ex post. Of course this is not an accurate representation of the world. This section considers the consequences of taxpayer heterogeneity, finding that differences among taxpayers significantly increase the likelihood that tax randomness will lead to higher profits, greater input use, and greater output.

Consider first a setting with a fixed number of heterogeneous firms, indexed by *i*. Letting $x_i(\tau)$ denote firm *i*'s demand for productive input *x*, $\varepsilon_i(\tau)$ its corresponding tax elasticity of input demand, and $\pi_i(\tau)$ its profits, the requirement that aggregate tax revenue increase with the tax rate now implies that $\sum x_i(\tau)\left[1+\varepsilon_i(\tau)\right]>0$. Taking this requirement to be met, the tax rate can again be written as a function of aggregate revenue, $\tau(R)$, with

$$\tau'(R)=\frac{1}{\left[1+\varepsilon_A(\tau)\right]\sum x_i(\tau)}, \tag{19}$$

in which $\varepsilon_A(\tau)$ is the elasticity of aggregate input demand, which in turn is simply a weighted average of the individual demand elasticities:

$$\varepsilon_A(\tau)\equiv\frac{\tau\sum x_i'(\tau)}{\sum x_i(\tau)}=\sum w_i(\tau)\varepsilon_i(\tau), \tag{20}$$

with the weight $w_i(\tau)\equiv\dfrac{x_i(\tau)}{\sum x_i(\tau)}$ being firm *i*'s share of aggregate input demand.

Consider then the impact of revenue neutral local tax rate randomization on aggregate profits $\Pi(R)\equiv\sum\pi_i\left(\tau(R)\right)$. From Shephard's lemma, $\Pi'(R)=-\tau'(R)\sum x_i(R)$; hence from (19), it follows that $\Pi'(R)=\dfrac{-1}{1+\varepsilon_A(\tau)}$. Thus (7) also applies to aggregate profits in the heterogeneous firm case, with $\varepsilon(\tau)$ replaced by $\varepsilon_A(\tau)$, and $\varepsilon'(\tau)$ replaced by $\varepsilon_A'(\tau)$. Differentiating (20), noting that

(21) $$w_i'(\tau) = \frac{1}{\tau} w_i(\tau)\left[\varepsilon_i(\tau) - \varepsilon_A(\tau)\right],$$

and using $\sum w_i'(\tau) = 0$ to replace $\sum w_i'(\tau)\varepsilon_i(\tau)$ with $\sum w_i'(\tau)\left[\varepsilon_i(\tau) - \varepsilon_A(\tau)\right]$, gives

(22) $$\varepsilon_A'(\tau) = \sum w_i(\tau)\varepsilon_i'(\tau) + \frac{1}{\tau}\sum w_i(\tau)\left[\varepsilon_i(\tau) - \varepsilon_A(\tau)\right]^2.$$

Consequently:

PROPOSITION 4: *With an unchanging set of active firms a revenue neutral local tax rate randomization reduces expected aggregate profits if and only if* $\varepsilon_A'(\tau) < 0$, *where* $\varepsilon_A'(\tau)$ *is given by (22). Conversely, a revenue neutral local tax rate randomization increases expected aggregate profits if and only if* $\varepsilon_A'(\tau) > 0$.

Proposition 4 is a straightforward generalization of Proposition 1.[22] The structure of the tax derivative of aggregate demand for the taxed input, however, points to potentially quite different outcomes. For while the first term on the right side of (22) is negative if all of the $\varepsilon_i'(\tau)$s are negative, the second term, which is the variance of the elasticities (weighted by input shares), is unambiguously positive under the very mild condition that the elasticities $\varepsilon_i(\tau)$ differ—and that points to $\varepsilon_A'(\tau) > 0$. In this sense, taxpayer heterogeneity increases the likelihood that revenue neutral tax rate randomization increases aggregate expected profits; and this is so even if $\varepsilon_i'(\tau) > 0$ for each individual firm.[23]

If, for example, all taxpayers have constant input demand elasticities, but those elasticities differ, then (22) implies that the aggregate demand elasticity is unambiguously increasing in the tax rate $\left(\varepsilon_A'(\tau) > 0\right)$. This follows simply from the fact that the variance term in

[22] Appendix B of Hines and Keen (2018) illustrates Proposition 4 by revisiting the analysis in Figure 1 above.
[23] It is perhaps counterintuitive that there are situations in which, from the standpoint of any individual firm, a revenue neutral tax rate randomization would reduce its profits, yet an aggregate revenue neutral tax rate randomization for all firms subject to the same tax rates would increase aggregate profits. The explanation is simply that with heterogeneous firms, a tax rate randomization that keeps aggregate tax payments constant generally will not keep constant the expected tax payments of each firm.

(22) is positive. Intuitively, what happens in this case is that, as the tax rate increases, input demands by firms with relatively inelastic demands decline proportionately less than input demands by firms with more elastic demands, so aggregate input demand $\varepsilon_A(\tau)$ becomes less elastic. Conversely, as the tax rate decreases input demands by firms with relatively elastic demands increase disproportionately, so aggregate demand becomes more elastic. The resulting induced positive correlation between the tax rate and the aggregate demand elasticity gives the aggregate demand elasticity a positive derivative. Profits increase because higher tax rates apply disproportionately to firms with inelastic demands: tax uncertainty serves, in effect, as a screening device for focusing taxation on firms with less elastic input demands.[24]

Proceeding similarly for aggregate output and aggregate input, analogs to Propositions 2 and 3 follow easily, the only difference being that terms in $\varepsilon(\tau)$ are replaced by terms in $\varepsilon_A(\tau)$, and $\varepsilon'(\tau)$ is replaced by $\varepsilon_A'(\tau)$. But this difference evidently matters; and even if there is insufficient taxpayer heterogeneity to make the tax derivative of the aggregate input demand elasticity positive, heterogeneity may so elevate the derivative of the aggregate demand elasticity that tax randomness will stimulate greater expected input use or output.[25]

To illustrate the screening function of random taxes, it is instructive to introduce firm heterogeneity into the example of section 3.4. Suppose that half of input demand comes from firms with identical Cobb-Douglas production functions given by (15), while the other half comes from firms with production functions that give them (over the relevant tax range) perfectly inelastic input demands. Since half of input demand arises from firms with zero demand elasticities, it follows that the aggregate demand elasticity is half its value in (16):

$$\varepsilon_A(\tau) = \frac{-\tau}{2(1+\tau)(1-\alpha)}. \tag{23}$$

[24] There are echoes here of the model of sales of Varian (1980), in which price variability emerges as a device to distinguish between informed and uniformed consumers.

[25] The model takes the firm population to be fixed; but if there is endogenous entry and exit, then firm heterogeneity introduces the possibility that taxes could affect the average characteristics of firms in the market. This additional source of screening changes the properties of the model rather little, and Proposition 5 of Hines and Keen (2018) identifies circumstances in which Propositions 1-4 continue to hold with endogenous entry and exit.

Furthermore, (19) and (23) together imply that, for the half of the firm population with Cobb-Douglas production functions, $\varepsilon'(\tau) = 2\varepsilon_A(\tau)/\left[\tau(1+\tau)\right]$. Consequently, (22) implies:

$$\varepsilon'_A(\tau) = \frac{\varepsilon_A(\tau)}{\tau(1+\tau)} + \frac{\varepsilon_A(\tau)^2}{\tau}. \tag{24}$$

And applying (23), (24) produces:

$$\varepsilon'_A(\tau) = \frac{\tau - 2(1-\alpha)}{4(1-\alpha)^2(1+\tau)^2}. \tag{25}$$

It is clear from (25) that there exist values of τ and α for which $\varepsilon'_A(\tau) > 0$, and for which therefore expected inputs, outputs, and profits all increase with tax randomness; and this is so despite the fact that neither type of individual firm exhibits $\varepsilon'(\tau) > 0$.

Table 2 revisits the random tax scenario examined in Table 1, now adding an equal number of firms with inelastic input demands.[26] Given the assumed parameter values, and as above, at a tax rate $\tau = 2$ firms of both types demand 10 units of *x*, produce output of 150, have profits of 120, and generate tax revenue of 20. A tax rate of 1.6 is associated with greater input use and output by the type 1 firms with Cobb-Douglas production functions, and unchanged input use and output by the type 2 firms; profits rise for both firm types, and tax revenues decline. It is now the case that a high tax rate of only 2.43 is necessary to accompany the low tax rate of 1.6 for the government to collect (in expectation) the same revenue as with a certain tax rate of 2. This high tax rate is of course associated with reduced input demand and output by firms of type 1, but over the two possible tax rates firms of type 1 have expected input demand of 10.25, exceeding their input demand with a certain tax of 2, and an expected output of 150.3, also exceeding output with tax certainty. Since firms of type 2 do not change their inputs and outputs, aggregate expected input demand and output increase with tax randomness. Similarly, tax randomness increases firm 1's expected profits from 120 to 120.25, and since firm 2's expected profits decline from 120 to 119.85, aggregate expected profits rise.

[26] These firms have production functions $q = 0, \forall x < 10$ and $q = 150, \forall x \geq 10$.

5. Efficient tax randomness and the marginal cost of public funds

Any kind of taxation is apt to affect the efficiency of resource allocation. While the analysis in sections 3 and 4 largely concerns the effects of random taxes on incentives and outcomes for firms, it also carries implications for aggregate economic efficiency. This section turns to implications for deadweight loss and efficient tax design, beginning by characterizing the ranges over which tax randomization is efficient and then considering the effect of efficient randomization on the marginal cost of public funds.

5.1 Deadweight loss

Proposition 1 can be interpreted in terms of the deadweight loss associated with input taxation, denoted $DL(R)$ and defined as the difference between the tax-induced loss of profits and the amount of tax revenue collected:[27]

$$DL(R) \equiv \pi(0) - \pi(R) - R \geq 0 . \tag{26}$$

Rearranging gives:

$$\pi(R) = \pi(0) - R - DL(R), \tag{27}$$

from which it follows directly that profit is concave (resp. convex) in tax revenue if and only if deadweight loss is convex (concave) in tax revenue. The effects of an increase in tax revenue on profits and on deadweight loss are thus mirror images: a revenue neutral tax rate randomization reduces expected profit, for instance, if and only if deadweight loss is convex in tax revenue, so that the randomization also increases expected deadweight loss. And convexity of deadweight loss in tax revenue, as stressed at the outset, is not implied by conditions that make deadweight loss convex in the tax rate. Specifically, it follows from (26) and (7) that

[27] This definition of deadweight loss excludes changes in consumer surplus because the model takes consumer demand to be infinitely elastic at unit price.

(28) $$DL''(R) = -\pi''(R) = \frac{-\varepsilon'(\tau)}{\left[1+\varepsilon(\tau)\right]^2}\tau'(R),$$

so that, consistent with Proposition 1, it is the tax elasticity of derived demand that shapes the convexity/concavity of deadweight loss in revenue and so determines the effect of tax rate randomization. Equation (28) implies that:

PROPOSITION 5: *A revenue neutral local tax rate randomization increases expected deadweight loss if and only if* $\varepsilon'(\tau) < 0$. *Conversely, a revenue neutral local tax rate randomization reduces expected deadweight loss if and only if* $\varepsilon'(\tau) > 0$.

5.2 Efficient random tax ranges

In order to understand the effect of tax randomness on the marginal cost of public funds it is necessary to broaden the analysis to incorporate more than local tax rate randomizations. In circumstances when it is efficient to impose random taxes, the resulting tax rates over which the government randomizes will differ substantially, producing a wide range of rates that the government will never find it efficient to impose with certainty. The top panel of Figure 3 depicts the possible scope for and consequences of tax rate randomization that is efficient in the sense of minimizing expected deadweight loss subject to raising some given amount of expected tax revenue. The solid locus plots the deadweight losses produced by tax rates implied by different tax revenue levels; it is possible to reduce expected deadweight loss by randomizing tax collections between any two points on this solid locus for which a line segment connecting them lies below it. While there are many such possibilities between the revenue levels R_1 and R_2, intuition suggests (and it will shortly be proved) that the greatest efficiency gains are to be had by randomizing between these two levels.

It is clear that the region $R_1 - R_2$ includes points over which deadweight loss is concave in tax revenue; indeed, the purpose of randomization is to exploit these ranges of tax revenue to minimize the expected cost of tax collections. But it is also apparent from Figure 3 that the region $R_1 - R_2$ contains points over which deadweight loss is convex in tax revenue, including the endpoints R_1 and R_2 themselves. It may seem paradoxical to randomize tax collections

between points at which deadweight loss is convex in tax revenue, but this follows simply from the nature of the exercise being performed. An efficient program cannot include positive probability of imposing taxes at points where deadweight loss is concave in tax revenue, since randomizing away from these points reduces expected deadweight loss while maintaining expected revenue; hence deadweight loss must be convex in tax revenue at R_1 and R_2. Indeed in order for the deadweight loss minimizing choices of R_1 and R_2 fully to exploit the opportunity created by the concavity of deadweight loss over a region of revenue levels between them, that interval must also contain ranges over which deadweight loss is convex in revenue.

PROPOSITION 6: *For any interval $R_1 - R_2$ of tax revenue over which tax rates are efficiently randomized: (i) The government cannot reduce deadweight loss by randomizing over more than two tax rates; (ii) Deadweight loss is convex in tax revenue at the revenue levels R_1 and R_2; (iii) Marginal deadweight loss is equal at R_1 and R_2, and so marginal deadweight loss under efficient randomization is perfectly certain; and (iv) There are regions over which deadweight loss is concave and convex in tax revenue.*

Proof: Efficient randomization entails minimizing $\sum \gamma_i DL(R_i)$ over the choices of R_i and probability weights γ_i, subject to the constraints that $\sum \gamma_i R_i \geq \bar{R}$ and $1 \geq \gamma_i \geq 0$. The first order condition corresponding to an efficient choice of γ_i is

$$DL(R_i) - \lambda R_i + \mu_{1i} - \mu_{2i} = 0, \tag{29}$$

in which λ is the multiplier corresponding to the tax revenue constraint, μ_{1i} is the multiplier corresponding to the constraint that $\gamma_i \geq 0$, and μ_{2i} is the multiplier corresponding to the constraint that $1 \geq \gamma_i$. Since randomization requires that $\gamma_i < 1, \forall i$, it is appropriate to restrict attention to cases in which the $1 \geq \gamma_i$ constraint does not bind, and therefore $\mu_{2i} = 0$. Since $\mu_{1i} = 0$ for any revenue level R_i with positive probability weight, (29) implies that

$$DL(R_i) = \lambda R_i. \tag{30}$$

From (30), expected deadweight loss is $\lambda \bar{R}$ for any combination of probability weights producing expected revenue $\bar{R}$. Consequently, randomizing over any two revenue levels that satisfy (30) and generate expected revenue $\bar{R}$ minimizes deadweight loss, thereby confirming the first part of the proposition. As a result, it is possible to confine attention to randomizations over just two revenue levels, R_1 and R_2.

The first order condition corresponding to minimizing expected deadweight loss over the choices of R_1 and R_2, subject to the expected revenue constraint, is

(31) $$DL'(R_1) = DL'(R_2) = \lambda .$$

The second part of Proposition 6 comes from the second-order conditions corresponding to the cost-minimizing choices of R_1 and R_2. These conditions are

(32a) $$DL''(R_1) > 0$$

(32b) $$DL''(R_2) > 0,$$

from which this part of the proposition follows directly.

The third part of Proposition 6 follows from (31), which also implies that $DL'(R_2) - DL'(R_1) = 0$. Applying the fundamental theorem of the calculus to this equation,

(33) $$\int_{R_1}^{R_2} DL''(R)\, dR = 0 .$$

And since the second order conditions (32a) and (32b) imply that $DL''(R) > 0$ over at least some parts of the range $R_1 - R_2$, it follows from (33) that $DL''(R) < 0$ over other parts, thereby confirming the fourth part of the proposition. □

5.3 The marginal cost of public funds

The marginal cost of public funds is the cost that government imposes on society in the course of raising an additional dollar of tax revenue. This includes not only the dollar of

resources extracted from the population but also any cost of economic distortions that accompany the use of the tax system. In the setting depicted in Figure 3 the marginal cost of public funds at revenue level R_1, for example, is $\left[1+DL'\left(R_1\right)\right]$, reflecting the additional deadweight loss associated with an incremental dollar of tax revenue. Recall too that efficient randomization implies that marginal deadweight loss is perfectly certain, and equal to its common value at R_1 and R_2.

A key reason for the centrality of concept of the marginal cost of public funds is in shaping the efficient level of public spending. Efficient government policy equates the marginal value of additional public spending to the marginal cost of public funds, so a higher marginal cost of public funds usually entails a lower efficient level of public spending. This may not be the case, however, with efficient randomization of tax rates.

To illustrate this, the bottom panel of Figure 3 depicts the marginal cost of public funds at different revenue levels; this is simply one plus the slope of the corresponding solid locus in the top panel of Figure 3. As stressed above, the marginal cost of public funds at revenue level R_1 must equal that at R_2. Between R_1 and R_2 the marginal cost of public funds first rises, then falls, and then rises again, exceeding its initial level over roughly half of the $R_1 - R_2$ interval, and lying below it over the other half. Since with efficient tax rate randomization the marginal cost of public funds is constant at its R_1 and R_2 levels over the entire $R_1 - R_2$ range, it follows that the effect of tax rate randomization (relative to fully certain levels of tax revenue within this range) is to reduce the marginal cost of public funds at some expected revenue levels and increase it at others. And indeed, equation (33) implies that area A in Figure 3 is equal in size to area B. Proposition 7 brings out a key implication of these observations:

PROPOSITION 7: *Relative to fully certain tax policy, efficient tax rate randomization reduces the marginal cost of public funds at some expected revenue levels and increases it at others.*

Proof: Equations (30) and (31), together with the fundamental theorem of the calculus, collectively imply

$$DL\left(R_2\right)-DL\left(R_1\right)=\int_{R_1}^{R_2}DL'\left(R\right)dR=\left(R_2-R_1\right)DL'\left(R_1\right). \tag{33}$$

It follows from (33) that:

$$\int_{R_1}^{R_2} \left\{ \left[1 + DL'(R) \right] - \left[1 + DL'(R_1) \right] \right\} dR = 0 . \tag{34}$$

Equation (34) indicates that when tax rates are efficiently randomized the additional deadweight loss associated with raising a marginal unit of public funds is given by $\lambda = DL'(R_1)$, so the second bracketed term in (34), $\left[1 + DL'(R_1) \right]$, is the marginal cost of funds with efficient tax rate randomization. The first bracketed term is the marginal cost of public funds at each revenue level without randomization. Since from (32a), $\left[1 + DL'(R) \right]$ will exceed $\left[1 + DL'(R_1) \right]$ at revenue levels just exceeding R_1, and from (32b), $\left[1 + DL'(R) \right]$ will be less than $\left[1 + DL'(R_1) \right]$ at revenue levels just below R_2, it follows that $\left[1 + DL'(R) \right]$ must differ from $\left[1 + DL'(R_1) \right]$ at various points in the $R_1 - R_2$ range. From equation (34) the integral of these differences is zero, implying that since randomization reduces the marginal cost of public funds at some revenue levels it must increase it at others. □

The intuition for Proposition 7 stems from recognizing that randomization does not reduce total deadweight loss at revenue level R_2. Consequently, to the extent that randomization reduces marginal deadweight loss at revenue levels above R_1, it must increase marginal deadweight loss at some revenue levels in the interval $R_1 - R_2$. Indeed, the property that tax rate randomization increases the marginal cost of public funds at some levels of expected tax revenue and reduces it at others does not depend on the randomization being chosen to minimize expected deadweight loss.

The importance of Proposition 7 lies in pointing to the existence of circumstances in which a spending rule equating the marginal value of public expenditure to the marginal cost of public funds will entail reduced public expenditures upon the adoption of efficiency-enhancing tax rate randomization, notwithstanding that this tax rate randomization reduces the deadweight cost of collecting any given level of revenue within the range of the randomization. This spending implication follows from the mixed effect of tax rate randomization on the marginal cost of public funds; Figure 4 offers an illustration. This figure reproduces the bottom panel of

Figure 3, superimposing on this schedules MV_1 and MV_2 representing two alternative specifications of marginal values of public expenditures (which for illustration are taken to be declining in spending levels and unaffected by tax policy choices). In the absence of randomization, equating the marginal value of public spending to the marginal cost of public funds yields an efficient level of revenue (and public expenditure) of S_1 if that marginal valuation is given by MV_1, whereas efficient expenditure is S_4 if marginal valuation is given by MV_2. With efficient randomization, however, the marginal cost of public funds is $(1+\lambda)$ for all levels of revenue between R_1 and R_2. This marginal cost is represented by the horizontal line between R_1 and R_2, with different levels of revenue simply corresponding to different probabilities attached to R_1 and R_2, both of which are associated with the same marginal deadweight loss λ. Equating the marginal valuation of spending to this constant marginal cost of public funds leads to efficient spending of S_2 if the marginal value of public spending is given by MV_1. In this case the adoption of efficient tax rate randomization produces a lower marginal cost of public funds at the efficient allocation and so supports a higher spending level, $S_2 > S_1$. If, however, public expenditures are valued according to MV_2, things are the other way around: efficient tax rate randomization increases the marginal cost of public funds at the efficient allocation and so lowers the efficient spending level from S_4 to S_3. In either case, however, the randomization improves efficiency in taxing and spending.

6. Evaluating random taxes

The proposition that price uncertainty enhances firm profitability is due originally to Oi (1961), which considered output prices and attracted immediate qualifications that the proposition depends on firms' abilities costlessly to adjust production levels (Tisdell, 1963; Oi, 1963) and that uncertainty take the form of mean-preserving price variability (Zucker, 1965). As it happens, an earlier[28] contribution of Waugh (1944), together with comments (Howell, 1945;

[28] And apparently overlooked, at least until the appearance of Waugh (1966), by those analyzing the effect of price uncertainty on firm profits. Samuelson (1972, p. 476) notes that the first version of Samuelson's 1972 critique of

Lovasy, 1945) and reply (Waugh, 1945), considered the symmetric problem from the consumer standpoint, analogously concluding that price instability increases consumer welfare. Samuelson (1972) subjected these lines of inquiry to scorching critiques, noting their inattention to budget constraints and the patent invalidity of their apparent joint implication that random price variations unconnected to fundamentals somehow benefit both firms and consumers. As Samuelson (1972) notes, the contemplated random price variations are infeasible,[29] in that the gains to consumers come from reducing the expected returns of producers, and vice versa, so the only way for both consumers and producers to gain would be with the injection of external resources. Thus "Waugh's result can *never* be applied so as to permit a society to lift its welfare by its own bootstraps through manufactured instability."[30]

Critically, however, this brutal conclusion does not apply to tax randomness: its properties differ from those of the price uncertainty considered by this earlier literature, as taxation creates inefficiencies whether or not tax rates are uncertain.[31] One general lesson from the literature on marginal deadweight loss is that the simple intuition that marginal deadweight loss rises with tax revenue, making total deadweight loss convex in tax revenue, need not describe reality: there can be ranges, such as those depicted in Figure 3, over which marginal deadweight loss declines.[32] There is a common presumption, drawn from implicitly linearizing behavioral response functions, that tax randomness will increase deadweight loss due to the convexity of deadweight loss in tax revenue. This presumption is intuitively appealing for a

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Waugh (1944) was accepted for publication by the *Quarterly Journal of Economics* in 1944/45, but that "when the manuscript was lost in the editorial process, the exigencies of war did not seem to warrant preparing a new copy."

[29] "That is, unless you have a Santa Claus; but then if you do have a Santa Claus available, who needs the Waugh-Oi theorems?" (Samuelson, 1972, p. 488).

[30] Samuelson (1972, p. 476).

[31] This is the basis of the welfare consequences of random taxation analyzed by Balcer and Sadka (1982, 1986) and those of insurance policy differentiation considered by Arnott and Stiglitz (1988).

[32] See, for example, Atkinson and Stern (1974), Ballard (1990), Mayshar (1990), Triest (1990), Fullerton (1991), Kaplow (1996), Snow and Warren (1996), Auerbach and Hines (2002), and Gahvari (2006); Dahlby (2008) offers an interpretive survey of this literature.

world of identical economic agents, and conditioning on expected tax revenue (rather than considering mean preserving spreads of the tax rate) eliminates the unambiguous conclusion that randomness reduces deadweight loss. Indeed the usual convenient functional forms imply, when firms are identical, that tax randomness increases deadweight loss. Nonetheless, the common presumption that tax randomness increases inefficiency is correct only in an unknown portion of cases---and this is considerably less likely to be the case in settings characterized by extensive taxpayer heterogeneity.

Tax stability entails choosing just one point on the deadweight loss-revenue locus in Figure 3, with the choice dictated by government revenue needs. Tax randomness can afford the government the opportunity to extend its taxation into ranges of tax rates over which marginal deadweight loss declines, making it possible to reduce total deadweight loss. The ability to exploit tax ranges over which there are low values of marginal deadweight loss is part of the basis of Stiglitz's (1982b) normative argument favoring random taxation.[33] Interpretation of the Stiglitz model is complicated by its randomization across individuals with differing social welfare weights and marginal propensities to consume taxed goods, but its case for randomization relies at least in part on the ability of governments that randomize to raise tax revenue over ranges in which marginal deadweight loss is particularly low. The analysis here contributes to the understanding of random taxation by identifying the elasticity conditions required for revenue neutral local tax rate randomizations to enhance aggregate profits and economic activity, applying equation (22) to establish that taxpayer heterogeneity increases the

[33] This is implied by conditions (13), (14) and (14´) of Stiglitz (1982b); analogous terms appear in Chang and Wildasin (1986) and Brito et al. (1995), though in all cases without the elasticity formulation of Proposition 1.

likelihood that these conditions are satisfied, and noting that welfare-improving tax randomizations may reduce incentives for government spending.[34]

One notable implication of the possibility that tax randomization can increase expected profits is that firms themselves may seek to increase such variability, even at no expected revenue cost to the government. They may, for instance, consider highly aggressive forms of avoidance in the knowledge that an advance ruling from tax authorities will enable them to go ahead if favorable and simply walk away from the scheme if it is adverse. Importers might claim favorable ex ante classifications of certain items for tariff purposes, rather than simply accept broad classifications at moderate rates, knowing that they can limit the damage if the outcome is unfavorable. Taking risky tax positions can be good business strategy even if it does not reduce expected tax payments. Government policies and practices such as advanced tax rulings and other forms of tax dispute resolution that create opportunities for firms to obtain probabilistic tax treatments thereby effectively introduce an important form of tax randomness.[35]

It is also important to recognize that this tax analysis applies readily and with equal force to subsidies. With $\tau < 0$, the elasticity $\varepsilon(\tau)$ as defined in (4) becomes positive, but it is readily verified that the results above—as summarized for Propositions 1-4 in Figure 2, for example[36]—

[34] Proposition 6 also shows that any efficient randomization must include ranges of tax rates at which local randomizations would be inefficient. Stiglitz (1982b, p. 8, fn. 10) points to a graphical example in which a welfare-improving tax rate randomization includes a point at which a local randomization would be welfare-reducing, but does not address the necessary inclusion of such points in any welfare-improving randomization.

[35] The model introduced in section 2 takes a firm's tax rate to be assigned by the government, which includes cases in which firms can take actions that increase the variability of tax rate assignments. The model assumes that the government does not permit firms facing high tax rates to reorganize themselves (say, by shutting down and restarting in new guises) and thereby obtain more favorable tax treatment, so to the extent that actual policies may permit some firms to do this, the associated resource costs and tax revenue consequences would need to be incorporated in the analysis.

[36] Note that $\varepsilon(\tau)/\tau$ remains negative.

continue to hold as stated.[37] A government eager to stimulate business activity with a limited budget will find that a revenue neutral local subsidy rate randomization increases expected input use if and only if the condition identified in Proposition 2 holds.

Economists have long noted the profit opportunities created by price uncertainty even while extolling the benefits of tax stability in limiting deadweight loss and maintaining incentives for forward-looking economic activities. Since in the framework of this paper firms bear the cost of inefficiencies introduced by the tax system,[38] the effect of tax randomness on profitability is really the effect of tax randomness on efficiency. The second-best nature of resource allocation in the presence of taxation creates the possibility that random taxes can reduce deadweight loss and encourage output by imposing taxes in ranges over which aggregate behavior is relatively unresponsive to taxation. The possibility, counter to 'common sense' though it may be, that random taxation encourages firm operations and enhances profitability is just one aspect of the reality that tax policy adjustments may have very different effects on economies significantly distorted by prior taxation than they do on economies starting from efficient production points in the absence of taxation.

[37] Restricting τ to negative values may, however, change the likelihood of various possibilities; for instance, $\varepsilon'(\tau)$ becomes unambiguously negative for the production function in (8).

[38] If inputs were less than perfectly elastically supplied or outputs less than perfectly elastically demanded then their prices would be affected by tax rates, and firms would share some of the tax burden with other economic actors. Even with firms bearing the full burden of the tax plus deadweight loss, imposition of the tax may well affect the real incomes of economic actors outside the model, since in general equilibrium their returns are functions of supply and demand, notwithstanding the indiscernible effect of the tax on input and output prices (Bradford, 1979; Kotlikoff and Summers, 1987).

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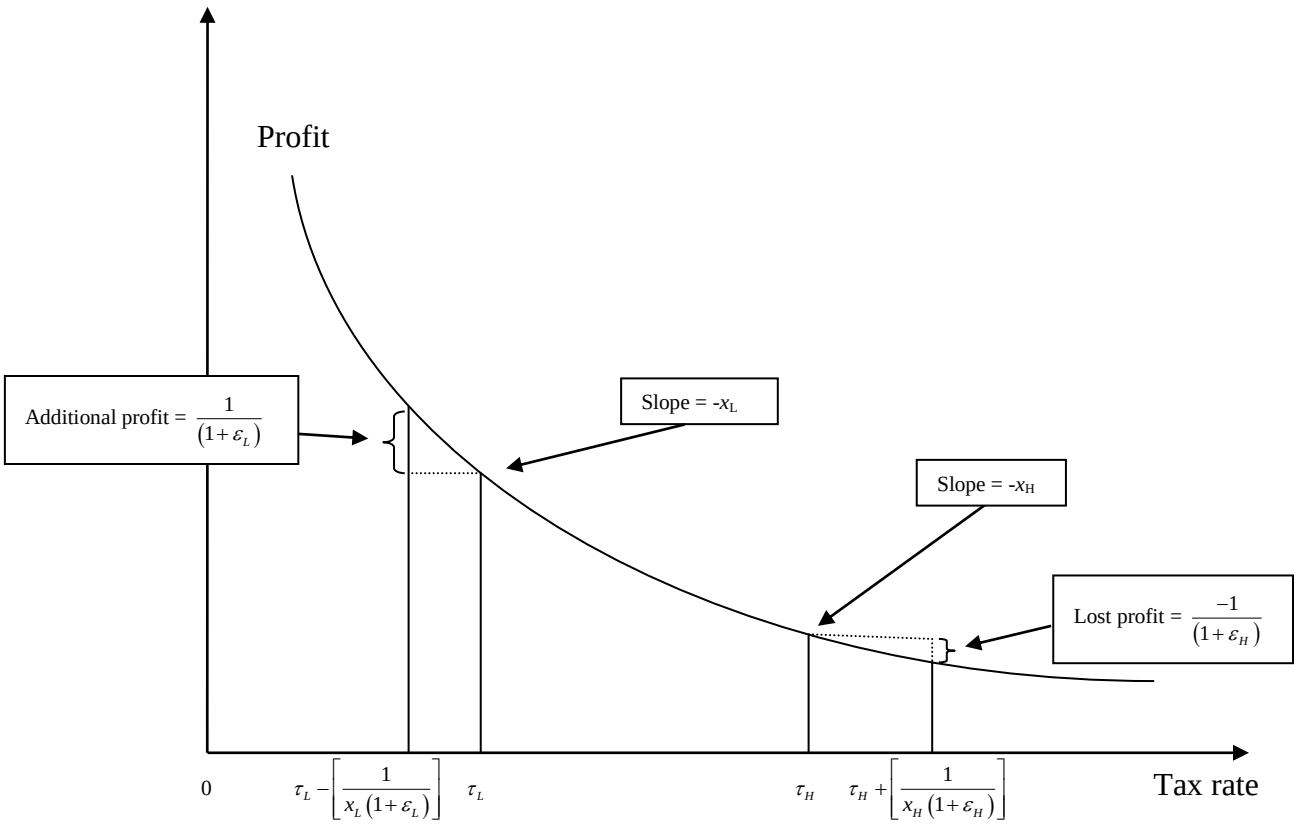
Figure 1

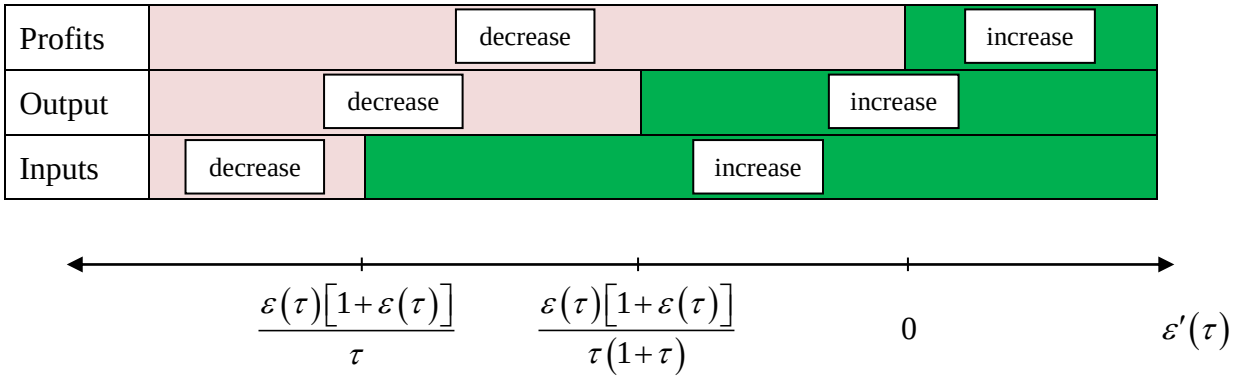
Effect of Tax Randomness on Firm Profitability



Note: Figure 1 depicts the effect on firm profits of broadening the scope of tax randomness. The solid locus is firm profits as a function of the tax rate. Initial taxes are random: the tax rate is τ_H with probability 0.5 and τ_L with probability 0.5. The figure considers the effect of increasing τ_H by an amount that raises \$1 of additional tax revenue while lowering τ_L by an amount that reduces tax revenue by \$1. This variation increases τ_H by $\frac{1}{\left[X_H(1+\varepsilon_H)\right]}$, which reduces profits by $\frac{1}{(1+\varepsilon_H)}$, and reduces τ_L by $\frac{1}{\left[X_L(1+\varepsilon_L)\right]}$, which reduces profits by $\frac{1}{(1+\varepsilon_L)}$. Consequently, the net effect on profits depends on the extent to which $\frac{1}{\left[1+\varepsilon(\tau)\right]}$ increases or decreases with τ.

Figure 2

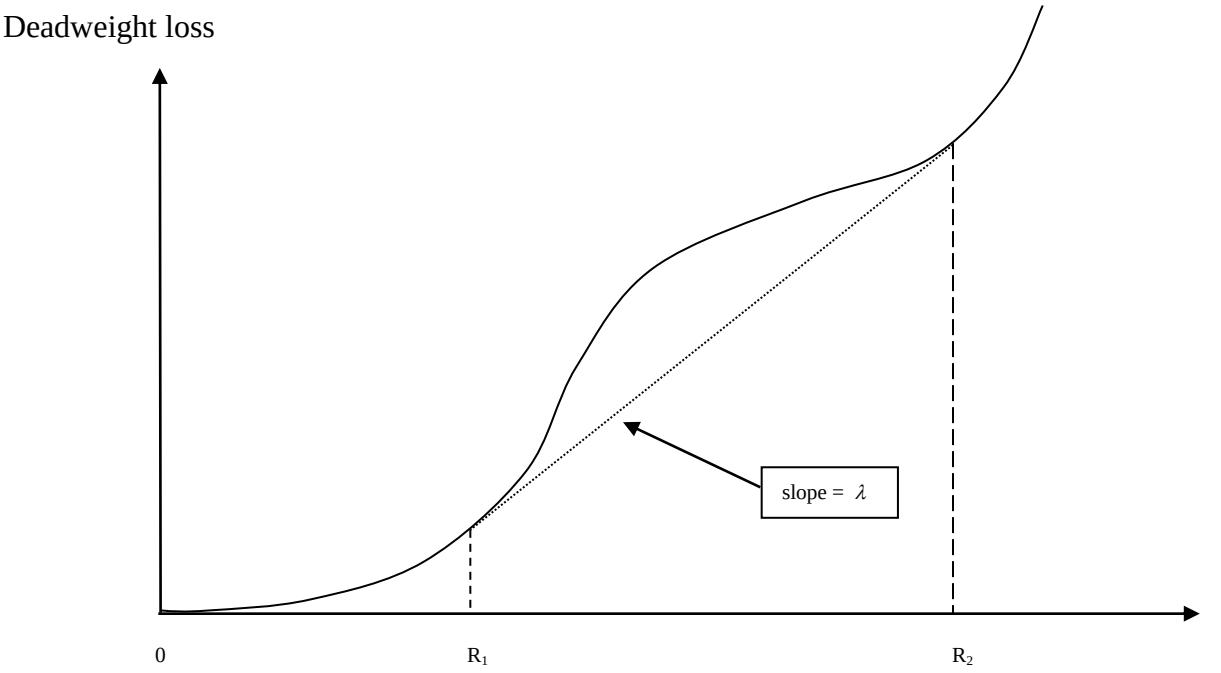
Effects of Tax Randomness on Inputs, Output, and Profit

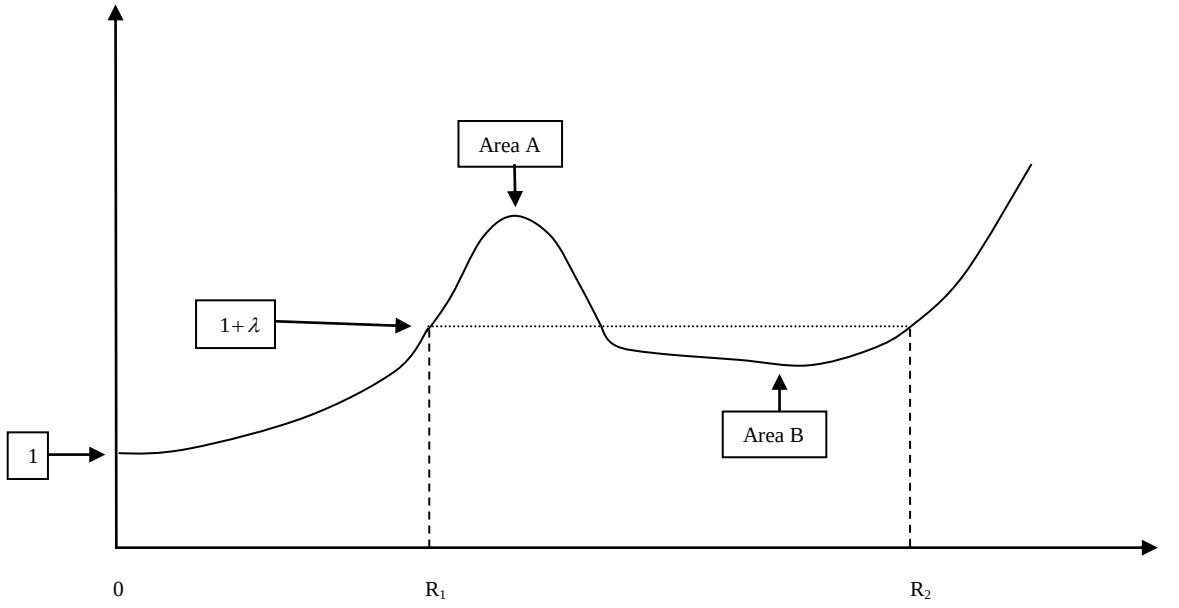


Note: Figure 2 depicts the signs of the effects of tax randomness on expected inputs, expected output, and expected profits for differing values of $\varepsilon'(\tau)$. If $\varepsilon'(\tau) > 0$ then tax randomness increases expected input use, output, and profits. Tax randomness is associated with greater expected output even for some negative values of $\varepsilon'(\tau)$, and is associated with greater expected input use for an even wider range of negative values of $\varepsilon'(\tau)$.

Figure 3

Deadweight Loss and the Marginal Cost of Public Funds



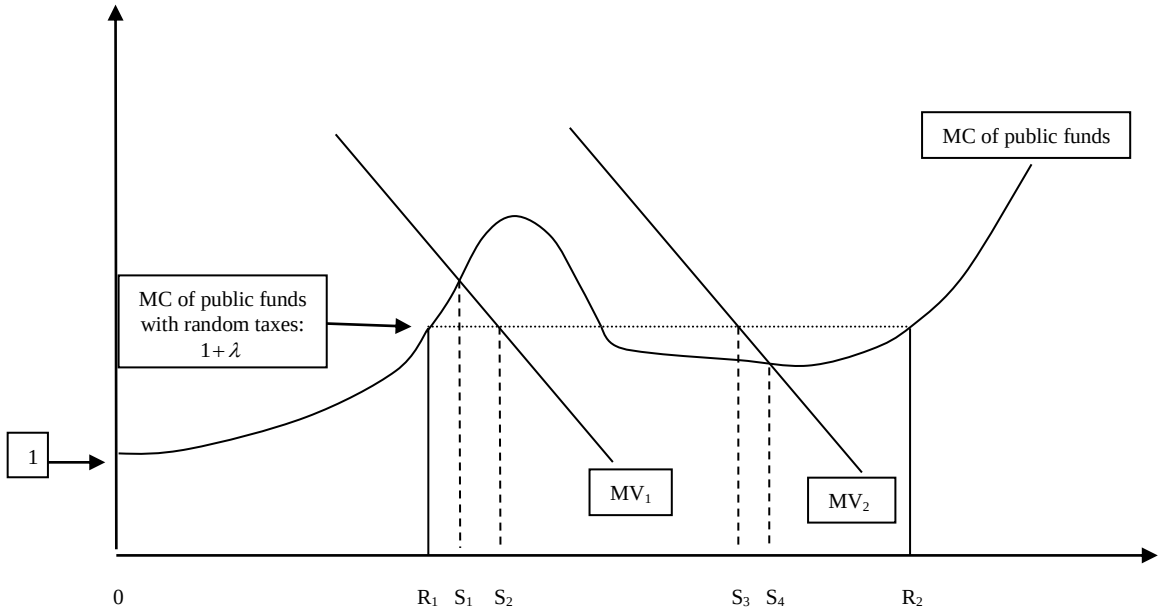


Tax revenue

Note to Figure 3: The solid locus in the top panel depicts deadweight loss as a function of tax revenue. Deadweight loss is increasing in tax revenue, with regions of convexity and concavity. A deadweight-loss-minimizing program does not impose taxes that collect revenues between R_1 and R_2, but instead randomizes tax collections between R_1 and R_2 in order to raise needed revenue in that range. The solid locus in the bottom panel of Figure 3 depicts the marginal cost of public funds, which is one plus marginal deadweight loss (given by the slope of the locus in the top panel). Randomizing tax rates between R_1 and R_2 produces a marginal cost of public funds equal to $1+\lambda$, where λ is the slope of the dotted line in the top panel. Area A is equal in size to Area B.

Figure 4

Efficient Government Spending with and without Random Taxes



Note: Figure 4 presents the marginal cost of public funds schedules that appear in the bottom panel of Figure 3, superimposing two functions representing alternative marginal valuations of public expenditure. If the marginal valuation of public expenditure is given by MV_1, then in the absence of tax rate randomization the efficient spending level is S_1, whereas with efficient tax rate randomization the marginal cost of public funds is $1+\lambda$, and the implied efficient spending level $S_2 > S_1$. If instead the marginal valuation of public expenditure is given by MV_2, then in the absence of tax rate randomization the efficient spending level is S_4, whereas with efficient tax rate randomization the implied efficient spending level is $S_3 < S_4$.

Table 1

Random Taxes with Cobb-Douglas Production Functions

Tax Rate		Inputs	Output	Profits	Tax Revenue
2.0		10	150	120	20
1.6		12	155.5	124.4	19.1
2.86		7.3	140.8	112.6	20.9

Note: the table presents per-firm inputs, outputs, profits and tax revenue produced by firms facing two different tax regimes. The top panel presents outcomes when firms face an input tax rate of 2.0, and have production functions given by $q = (94.64)x^{0.2}$. These firms maximize profits given by $q - (1+\tau)x$, in which x is their input demand; and they generate tax revenue of τx.

The bottom panel of the table presents outcomes for two different input tax rates, 1.6 and 2.86, each with probability of 50 percent, corresponding to a revenue neutral tax rate randomization around a tax rate of 2.0. Production functions are the same as in the specification reported in the top panel.

Table 2

Random Taxes with Heterogeneous Firms

Tax Rate		Firm	Inputs	Output	Profits	Tax Revenue
2.0		1	10	150	120	20
		2	10	150	120	20
1.6		1	12	155.5	124.4	19.1
		2	10	150	124	16
2.43		1	8.5	145.1	116.1	20.6
		2	10	150	115.7	24.3

Note: the table presents per-firm inputs, outputs, profits and tax revenue produced by two types of firms facing two different tax regimes. Firms of type 1 have production functions given by $q = (94.64)x^{0.2}$; firms of type 2 have production functions given by $q = 0, \forall x < 10$ and $q = 150, \forall x \geq 10$. Both types of firms maximize profits given by $q - (1+\tau)x$, in which *x* is their input demand; and each type generates tax revenue of τx.

The top panel presents outcomes when both types of firms face an input tax rate of 2.0. The second and third panels present outcomes for two different input tax rates, 1.6 and 2.43, each with probability of 50 percent, corresponding to a revenue neutral tax rate randomization around a tax rate of 2.0.