Observer-based detection and localization of time shift failures in (max,+) linear systems

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Abstract—This paper addresses a problem of failure detection and localization in production lines modeled, through Timed Event Graphs (TEG), as (max,+) linear systems with disturbances, over which observers can be developed. The state of the observed system is estimated and an indicator that returns true if a time shift failure is detected is defined. The localization step is proposed for elementary structures of TEG through the results of the detection.

I. INTRODUCTION

In the industry, Discrete Event Systems (DES) can be used to model and diagnose faults, malfunctions on production lines. The objective is to detect, locate and identify failures as soon as possible to avoid further equipment unavailability. At STMicroelectronics Crolles 300 plant, we investigate this problem in order to detect production drifts, especially failures that generate time shifts in the production lines. STMicroelectronics is among the world’s largest semiconductor companies, serving all electronics segments. Semiconductor manufacturing is a complex industries and one of its most important challenges is to succeed in detecting production drifts before they have real impact on production plans.

Fault diagnosis involving time in DES has been introduced in [Tri02] using timed automata to refine diagnosis decisions based on timed observations. In [GTY09], time Petri nets allow to easily model concurrency. This paper represents production lines as Timed Event Graphs (TEG) which are a subclass of Petri nets where places are associated with a duration. A TEG can be modeled by (max,+) algebra. A dioid is a set composed of two internal operations \( \oplus \) and \( \otimes \). The addition \( \oplus \) is associative, commutative, idempotent (i.e. \( \forall a \in D, a \oplus a = a \)) and has a neutral element \( \varepsilon \). The multiplication \( \otimes \) is associative, distributive on the right and the left over the addition \( \oplus \) and has a neutral element \( e \). Element \( \varepsilon \) is absorbing by \( \otimes \). When there is no ambiguity, the symbol \( \otimes \) is omitted. A dioid is complete if it is closed for infinite sums and if \( \otimes \) is distributive over infinite sums.

For instance, the set \( \mathbb{Z}_{\text{max}} = (\mathbb{Z} \cup -\infty) \), endowed with the max operation as addition \( \oplus \) and the addition as multiplication \( \otimes \) with neutral element \( \varepsilon = -\infty \) and \( e = 0 \) is a dioid. By adding \( +\infty \) to the dioid \( \mathbb{Z}_{\text{max}} \), we get the complete dioid \( \mathbb{Z}_{\text{max}} \) where \( (-\infty) + (+\infty) = (-\infty) \).

Definition 1: For a dioid \( D \), \( \preceq \) denotes the order relation such that \( \forall a, b \in D, a \preceq b \iff a \otimes b = b \).

Theorem 1 ([BCOQ92]): Let \( \mathcal{D} \) be a complete dioid, \( x = a^*b \) is the solution of \( x = ax \oplus b \) where \( a^* = \bigoplus_{i \geq 0} a^i \) is the Kleene star operator with \( a^0 = e \) and \( a^{i+1} = a \otimes a^i \).

To model TEG as (max,+) linear systems, a specific dioid has to be defined: the dioid \( \mathcal{M}_{\text{max}}^\mathcal{A}[\gamma, \delta] \). It is a particular dioid of formal series with two commutative variables \( \gamma \) and \( \delta \) that will represent event and time shifts of a TEG as explained in the next subsection. This dioid comes from the following \( \mathbb{B}[\gamma, \delta] \) dioid.

The complete dioid \( \mathbb{B}[\gamma, \delta] \) is the set of formal series with two commutative variables \( \gamma \) and \( \delta \) with Boolean coefficients in \( \{\varepsilon, e\} \) and exponents in \( \mathbb{Z} \). A series \( s \in \mathbb{B}[\gamma, \delta] \) is written \( s = \bigoplus_{(n,t) \in \mathbb{Z}^2} s(n,t)\gamma^n\delta^t \) where \( s(n,t) = e \) or \( \varepsilon \) (respectively representing the presence or the absence of the monomial). The neutral elements are \( \varepsilon(\gamma, \delta) = \bigoplus_{(n,t) \in \mathbb{Z}^2} \varepsilon\gamma^n\delta^t \) and \( e(\gamma, \delta) = \gamma^0\delta^0 \).
Graphically, a series of $\mathbb{B}[\gamma, \delta]$ is described by a collection of point of coordinates $(n, t)$ in $\mathbb{Z}^2$ with $\gamma$ as horizontal axis and $\delta$ as vertical axis. For instance, Figure 1 shows series $u_1 = u_2 = \gamma^0\delta^1 + \gamma^1\delta^2 + \gamma^2\delta^3 + \gamma^3\delta^4 + \gamma^4\delta^5 + \gamma^5\delta^6 + \gamma^6\delta^7 + \gamma^7\delta^8 + \gamma^8\delta^9 + \gamma^9\delta^{10} + \gamma^{10}\delta^{11}$ (monomials with $e$ as Boolean coefficient).

The complete dioid $\mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ is the quotient of $\mathbb{B}[\gamma, \delta]$ modulo $\gamma^*(\delta^{-1})^*$ where $\forall a, b \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]: a = b \iff a\gamma^*(\delta^{-1})^* = b\gamma^*(\delta^{-1})^*$. Internal operations and neutral elements are identical to those of $\mathbb{B}[\gamma, \delta]$. All series can be expressed by the following canonical form.

Definition 2: The canonical form of $s \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ is

$$s = \bigoplus_{k=0}^{K} \gamma^{n_k}\delta^k$$

with $K \in \mathbb{N} \cup \{+\infty\}$ and $\{n_0 < n_1 < \ldots, t_0 < t_1 < \ldots\}$

B. Models of $(max,+)$-linear systems

Figure 2 presents a TEG with two inputs $u_1$ and $u_2$ and one output $y_1$. Durations associated to places are expressed here by letters $a, b, c, d, f$ and transition $x_3$ is a synchronization between paths from $u_1$ and $u_2$. The entire structure of such a TEG can be modeled by equations in $\mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ through a set of matrices $A$, $B$, and $C$. Then, relations between input $u$, state $x$ and output $y$ transitions are expressed by its state representation. Let $u \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]^{nx1}$, $x \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]^{nx1}$ and $y \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]^{nx1}$, the state representation of a TEG is:

$$\begin{cases} x = Ax \oplus Bu, \\ y = Cx, \end{cases}$$

where $A \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]^{n \times n}$, $B \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]^{n \times p}$ and $C \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]^{n \times 1}$. Equality $x = Ax \oplus Bu$ can be transformed to $x = A^*Bu$ thanks to Theorem 1 so we have

$$y = CA^*Bu.$$ 

Matrix $H = CA^*B$ represents the transfer function that is the dynamic of the system between inputs and outputs.

Example 1: In the TEG of Figure 2 suppose that the durations are $a = b = d = 1$ for places $p_1, p_3, p_4$, $c = 2$ for $p_2$ and $f = 0$ for $p_5$. The state representation’s matrices are:

$$A = \begin{pmatrix} \gamma^0\delta^1 & \gamma^1\delta^1 & \gamma^2\delta^1 & \gamma^3\delta^1 \\ \gamma^4\delta^1 & \gamma^5\delta^1 & \gamma^6\delta^1 & \gamma^7\delta^1 \\ \gamma^8\delta^1 & \gamma^9\delta^1 & \gamma^{10}\delta^1 & \gamma^{11}\delta^1 \end{pmatrix}, \quad B = \begin{pmatrix} \gamma^0\delta^2 \\ \gamma^1\delta^2 \\ \gamma^2\delta^2 \end{pmatrix},$$

$$C = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

The indices of $\gamma$ represent the backward event shift between transitions and the indices of $\delta$ represent the backward time shift. For instance in $B(1, 1) = \gamma^0\delta^1$, the $n + 1$th firing of $x_1$ depends on the $n$th firing of $u_1$ (no event shift); the firing date of $x_1$ is exactly shifted of duration $a = 1$ from the firing date of $u_1$. A trigger of an input transition $u_i$ represents the occurrence of an event. Then, input flow events of TEG are represented by series of $\mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ as series $u_1 = u_2$ illustrated in Figure 1. In $u_1$ and $u_2$, index 0 of $\gamma$ is the first event occurrence of this input (index 1 is the second, etc.); index 1 of $\delta$ is the date of this first occurrence. The absence of an $s$th event is indicated by $+\infty$ in monomial $\gamma^s\delta^{+\infty}$.

C. Time comparison of series

In the rest of the paper, time comparison of series are made thanks to a particular element called residual.

Definition 3: Let $\Pi : \mathcal{D} \mapsto \mathcal{C}$ be an isometric mapping, where $\mathcal{D}$ and $\mathcal{C}$ are complete dioids. The largest solution of $\Pi(x) = b$, if it exists, is called the residual of $\Pi$ and is denoted $\Pi^*$. When $\Pi$ is residuated, $\Pi^*$ is the unique isometric mapping such that $\Pi \circ \Pi^* \leq I_{\mathcal{C}}$ and $\Pi^* \circ \Pi \geq I_{\mathcal{D}}$ where $I_{\mathcal{D}}$ and $I_{\mathcal{C}}$ are resp. the identity mappings on $\mathcal{C}$ and $\mathcal{D}$.

The right product $R_a : x \mapsto x \oplus a$ defined over a complete dioid $\mathcal{D}$ is residuated. Its residual is $R_a^*(x) = x \ominus a$ and corresponds to the pseudo-inverse of the product.

Now, time comparison between series can then be defined based on residuals. It requires the use of dater functions to get times of $\mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ series.

Definition 4: Let $s \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ be a series, its dater function is the non-decreasing function $D_s(n)$ from $\mathbb{Z} \mapsto \mathbb{Z}$ such that $s = \bigoplus_{n \in \mathbb{Z}} \gamma^n\delta^{D_s(n)}$.

Example 2: Series $u_1$ has for dater function $D_{u_1}(0) = 1$, $D_{u_1}(1) = 2$, $D_{u_1}(2) = 3$, $D_{u_1}(3) = 4$, $D_{u_1}(4) = 5$, $D_{u_1}(5) = 6$ and $D_{u_1}(6) = 7$ that lists all the dates of the event occurrences. As $u_1 = u_2$, $u_2$ has obviously the same dater function.

Definition 5: Let $a, b \in \mathcal{M}_{\infty}^{ax}[\gamma, \delta]$ and their respective dater functions $D_a$ and $D_b$. The time shift function $T_{a,b}(n)$ can be bounded by:

$$\forall n \in \mathbb{Z}, \quad D_{bfa}(0) \leq D_{a,b}(n) \leq -D_{afb}(0),$$

where $D_{bfa}(0)$ is obtained from monomial $\gamma^0\delta^{D_{bfa}(0)}$ of series $bfa$ and $D_{afb}(0)$ is obtained from $\gamma^0\delta^{D_{afb}(0)}$ of $afb$.\footnote{\Pi isosote $\equiv \forall s, s' \in \mathcal{D} : s < s' \Rightarrow \Pi(s) \leq \Pi(s')$.}
Estimated output of the system and $L$ which is given by $\gamma$ Matrix $L_w$ [HMCL10] and shown in Figure 3, aims at estimating the accuracy of the localization information.

Section III-A) and relies on the fact that time shift failures can be characterized by input disturbances (Section III-B). This new indicator is also based on a refinement of the estimated states (Section III-C) that is used to improve the accuracy of the localization information.

### A. Observer of a disturbed (max,+) linear system

An observer of a Timed Event Graph, as defined by [HMCL10] and shown in Figure 3, aims at estimating the internal states of a system based on the measurement of its inputs $u$ and outputs $y_o$ in the presence of unobservable disturbances characterized by specific inputs $w$. Then, the state representation of the system is:

$$ x = Ax \oplus Bu \oplus Rw, $$
$$ y = Cx. $$

where $w \in M^{\infty}_{\infty}[\gamma, \delta]^n \times 1$ with $n$ the number of disturbed internal transitions and matrix $R \in M^{\infty}_{\infty}[\gamma, \delta]^n \times 1$ is filled with $\gamma^0 \delta^0$ monomials where internal transitions are disturbed, with $\varepsilon$ otherwise. The observer’s equations are:

$$ x_e = Ax_e \oplus Bu \oplus L(y_r, y_o) $$
$$ = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw, $$
$$ y_e = Cx_e, $$

where $x_e$ and $y_e$ are the estimated state and the estimated output of the system and $L \in M^{\infty}_{\infty}[\gamma, \delta]^n \times q$ is a new matrix called the observer matrix.

Now, to obtain $x_e$ as close as possible to real state $x$, the observer relies on the largest matrix $L \in M^{\infty}_{\infty}[\gamma, \delta]^n \times q$ such that $x_e \leq x$ meaning:

$$(A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw \preceq A^*Bu \oplus A^*Rw$$

which is given by $L = (A^*B^\daggerCA^*B) \wedge (A^*R^\daggerCA^*R)$. Matrix $L$ actually represents the connections between the observed output $y_o$ of the system and the internal transitions $x_e$ of the observer.

### B. Time shift failure as input disturbance

To take advantage of the state estimation of an observer for the detection and the localization of time shift failures, we need to characterize a time shift failure as an input disturbance. We recall here the characterization of [PCPZ20].

A time shift failure is considered as a permanent phenomena and is formally defined by an unknown delay $\theta$ in a place $p$ of a TEG. The place $p$, with an upstream transition $x_{i-1}$ and a downstream transition $x_i$, is characterized by a number of $o$ tokens and a duration $t$ (see Figure 4). Upstream transition is described by $x_{i-1} = \bigoplus_{n=0}^{K} \gamma^o \delta^n \delta_{i-1}$ (see Definition 3), where $s_n$ is the transition firing number, $h_n$ is the firing date and $K$ the number of firing events. The downstream transition is $x_i = \bigoplus_{n=0}^{K} \gamma^o \delta^n \delta_{i-1}$.

If a time shift failure $\theta > 0$ holds in a place, the downstream transition then becomes: $x_i = \bigoplus_{n=0}^{K} \gamma^o \delta^n + \delta_{i-1}^{\theta}$. The firing dates of $x_i$ will be slowed down by $\theta$ comparing to the firing dates of $x_i$ during a nominal behavior. The same time shift failure over a place $p$ can then be characterized by an input disturbance $w_i$ as shown in Figure 5. If this disturbance is $w_i = \bigoplus_{n=0}^{K} \gamma^o \delta^n + \delta_{i-1}^{\theta}$, with $s_n$ and $h_n$ defined in series $x_{i-1}$, it has the same effect on transition $x_i$. In other words, having an unknown time shift failure $\theta > 0$ in place $p$ is equivalent to say that such an unknown disturbance $w_i$ exists.

Example 5: By taking back Example 3 to characterize a time shift failure in $p_2$ by an offset $\theta = 1$, an input disturbance $w_2$ is added to the transition $x_2$ as illustrated in Figure 5. The transition that takes into account this offset is $x_2 = \gamma^1 \delta^1 + \delta^1 \oplus \gamma^1 \delta^2 + \delta^1 + \gamma^2 \delta^3 + \delta^1 + \gamma^3 \delta^4 + \delta^1 + \gamma^4 \delta^5 + \delta^1 + \gamma^5 \delta^6 + \delta^1 + \gamma^6 \delta^7 + \delta^1 + \gamma^7 \delta^8 + \delta^1 + \gamma^8 \delta^9 + \delta^1 + \gamma^9 \delta^{10} + \delta^1 + \gamma^{10} \delta^{11} + \delta^1$.
the effect of the disturbance can be observed. Then, to copy this time shift failure effect, the disturbance has to be \( w_2 = \gamma^0 \delta_1 + \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \). Then, to prove this property, the disturbance has to be \( \gamma^0 \delta_1 + \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \).

C. Refinement of the estimated state

The indicator proposed in [PLCPV20] is based on the computation of the time shift \( \Delta(x_e, x_s) \) (see Definition 6) where \( x_e \) is the state estimated by the observer and \( x_s \) results from the simulation of a fault-free model of the system based on the real observable input \( u \). As the effect of a time shift failure might not be immediate, the first monomials of \( x_e \) representing the first event occurrences might not be affected by the failure and then are considered as normal occurrences in the indicator of [PLCPV20] while the failure is present. The consequence is that the indicator in [PLCPV20] may generate ambiguous information and we propose here to filter out of \( x_e \) these unaffected monomials and only keep as the estimated state the sensitive-to-disturbance (STD) part of \( x_e \). The way to extract this STD state is through a deeper analysis of the structure of the observer. It depends on values of matrix \( L \) that establishes connections between observed output \( y_o \) of the system and the internal transitions \( x_e \) of the observer. In the following, for a given matrix \( M, M_{ij} \) denotes the element of \( M \) at line \( i \), column \( j \); \( M_{i}^j \) the \( i \)th line of \( M \) and \( M^j_i \) its \( j \)th column. A row \( L_{i}^j \) of matrix \( L \) represents an estimated state \( x_{ei} \) of the system.

**Proposition 1:** Suppose that \( L_{i}^j = (\varepsilon \ldots \varepsilon) \), then \( x_{ei} = A^*_{ei} B_{ib} u \), i.e. the estimated state \( x_{ei} \) does not depend on disturbances.

**Proof sketch.** From [3], estimated state is \( x_{ei} = (A_{ei} \oplus L_{i}^j C_{ei})^*(B_{iu} + L_{i}^j C_{ei} A^*_{ei} B_{ib} u_{ia}) \). Using algebraic operations of \( M_{i}^{\text{out}} \) \( [\gamma, \delta] \) and recalling that \( \varepsilon \) is absorbing by \( \oplus \), it follows that \( x_{ei} = A^*_{ei} B_{ib} u \) does not depend on \( w \).

Let the canonical form \( L_{ij} = \bigoplus_{\gamma} \gamma^n \delta_k \) be an entry of \( L \), in the following \((n_0)_{ij}\) denotes the smallest \( \gamma \) index \( n_0 \) in series \( L_{ij} \). It is possible that \((n_0)_{ij}\) differs from \( 0 \) if tokens are already in the place represented by \( \gamma^n \delta_0 \) in \( L_{ij} \).

Let \( N_i = \min_{1 \leq j \leq q} \{ (n_0)_{ij} \} \) where \( q \) be the number of outputs of the system. The following proposition gives the event occurrence of the estimated state trajectory from which the effect of the disturbance can be observed.

**Proposition 2:** Let \( x_{ei} = \bigoplus_{\gamma} \gamma^n \delta_k \) be the estimated trajectory of \( x_e \) in its canonical form. The first event occurrence of \( x_{ei} \) that takes into account the unknown disturbances from \( w_{ia} \) is the event occurrence of index \( \nu_d \) such that \( \nu_d = N_i + N_{00} + \nu_{w} \) where \( N_{00} \) and \( \nu_{w} \) are the smallest \( \gamma \) indices of series \( b = C_{ei} A^*_{ei} B_{ib} u \).

**Proof sketch.** From [3], estimated state is \( x_{ei} = (A_{ei} \oplus L_{i}^j C_{ei})^*(B_{iu} + L_{i}^j B_{ib} u_{ia}) \) where \( b = C_{ei} A^*_{ei} B_{ib} u_{ia} \) is the only part depending on disturbances \( w \) and \( \nu_d \) is defined as the smallest \( \gamma \) index of series \( L_{i}^j B_{ib} u_{ia} \).

Now, the STD state \( x_{ed1} \) is based only on \( \gamma \) indices from the state \( x_{ei} \) that takes into account disturbances, which are the ones from \( \nu_d \).

**Definition 7:** The sensitive-to-disturbance (STD) estimated state \( x_{ed1} \) is defined from \( x_{ei} \) by

\[
x_{ed1} = \bigoplus_{k=d}^{K} \gamma^k \delta^r_k
\]

where \( \nu_d \) is defined by Proposition 2.

**Example 6:** Generally speaking, if the complete estimate of a state \( x_{e1} = \gamma^0 \delta_1 + \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \) and if \( \nu_d = 1 \), the event occurrence of index \( \nu_d \) from which the disturbance is taken into account is the event numbered by 1. The STD estimated state is then \( x_{ed1} = \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \).

D. Indicator of time shift failure

Based on dater and time shift functions introduced in Section 2, we propose a new time shift indicator that compares the fault-free model with the observed system through \( u \) and \( y_o \), by only using information of the STD estimated state \( x_e \) and the fault-free model state \( x_{si} \). To perform a consistent comparison between \( x_e \) and \( x_{si} \), event occurrences from \( x_{si} \) before \( \nu_d \) must also be filtered out.

**Definition 8:** Let \( x_{si} = A^* B u = \bigoplus_{k=0}^{K} \gamma^k \delta^s_k \) be the complete fault-free model state. Series \( x_{si} \) denotes the fault-free model state comparable with \( x_{ei} \):

\[
x_{si} = \bigoplus_{k=d}^{K} \gamma^k \delta^s_k
\]

**Example 7:** From Example 6 if \( x_{e1} = \gamma^0 \delta_1 + \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \), then \( x_{si} = \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \).

**Definition 9:** Let \( x_{ed1} \) be the STD estimated state of \( x_e \) and \( x_{si} \) its fault-free model state. Indicator \( I_{x_e}(u, y_o) \) of state \( x_e \) is defined as the Boolean function:

\[
I_{x_e}(u, y_o) = \begin{cases} \text{false} & \text{if } \Delta(x_{ed1}, x_{si}) = [0; 0], \\
\text{true} & \text{otherwise},
\end{cases}
\]

with \( \Delta(x_{ed1}, x_{si}) = [D_{x_{ed1}} x_{si}(0); -D_{x_{ed1}} x_{si}(0)] \).

**Proposition 3:** The indicator returns true only when a time shift failure has occurred in the system.

**Proof sketch.** By construction this indicator has the same detection capabilities as the indicator of [PLCPV20], hence the result.

**Example 8:** Back to Examples where a time shift failure of 1 is in place \( p_2 \). The state \( x_{ed1} \) comparable with the state \( x_{ed1} \) is the vector \( [x_{ed1}, v^T] \) where \( v^T \) is the only part depending on disturbances \( w \) and \( \nu_d \) is defined as the smallest \( \gamma \) index of series \( L_{i}^j B_{ib} u_{ia} \).

\[
\begin{bmatrix} \gamma^0 \delta_1 + \gamma^1 \delta_5 + \gamma^2 \delta_6 + \gamma^3 \delta_7 + \gamma^4 \delta_8 + \gamma^5 \delta_9 + \gamma^6 \delta_{10} + \gamma^7 \delta_{\infty} \\
\end{bmatrix}
\]
A failure is then detected by the indicators whose computed intervals are:

\[
\begin{align*}
\Delta(x_{ed1},x_{s1}) &= [D_{x_{ed1}}f_{x_{s1}}(0), -D_{x_{ed1}}f_{x_{s1}}(0)] = [0, 0], \\
\Delta(x_{ed2},x_{s2}) &= [D_{x_{ed2}}f_{x_{s2}}(0), -D_{x_{ed2}}f_{x_{s2}}(0)] = [1, 1], \\
\Delta(x_{ed3},x_{s3}) &= [D_{x_{ed3}}f_{x_{s3}}(0), -D_{x_{ed3}}f_{x_{s3}}(0)] = [1, 1].
\end{align*}
\]

IV. TIME SHIFT FAILURE LOCALIZATION IN (MAX,+) LINEAR SYSTEMS WITH OBSERVER

This section details how to extract information from the computed intervals of the indicators to determine the source of the failure in the TEG. All along this section, we assume that only one permanent time shift failure is present in the TEG, in a place \( p \) with a delay \( \theta \). We propose in this section to analyse how to retrieve the localization for three types of elementary structures that are contained in any TEG: tandem (see Figure 6), parallelism (see Figure 7) and synchronization (see Figure 8). Computing the localization of the failure in a specific TEG then consists in assembling these elementary analyses to get the global conclusion.

A. Tandem and parallel structures

Table I summarizes the localization results that can be expected when a tandem structure is involved given all the possible configurations. The first set of configurations (Rows 1-4) is when \( x_1 \) and \( x_2 \) can be associated with an indicator (i.e. the observer is able to provide a STD estimated state for both \( x_1 \) and \( x_2 \)). The second set of configurations (Rows 5-7) is when only the state \( x_2 \) of the tandem has an indicator. Each row corresponds to a possible configuration of the indicator intervals and the conclusion about where the failure is in the analysed structure. For instance, in Row 3, \( \Delta(x_{ed1}, x_{s1}) = [\theta, \theta] \) and \( \Delta(x_{ed2}, x_{s2}) = [\theta, \theta] \) the time shift failure \( \theta \) is located either in place \( p_1 \) or in a place in the upstream of the tandem. Indeed, \( x_1 = \bigoplus_{k=0}^{K} \gamma_{nk} s^k + a \) and the next transition is \( x_2 = \bigoplus_{k=0}^{K} \gamma_{nk} s^k + a + b \). If the time shift failure \( \theta \) is in place \( p_1 \) (or in a place in its upstream), \( x_{ed1} = \bigoplus_{k=d_1}^{K} \gamma_{nk} s^k + a + \theta \) and \( x_{ed2} = \bigoplus_{k=d_2}^{K} \gamma_{nk} s^k + a + \theta + b \), hence the results in the indicators.

<table>
<thead>
<tr>
<th>( \Delta(x_{ed1}, x_{s1}) )</th>
<th>( \Delta(x_{ed2}, x_{s2}) )</th>
<th>Localisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0,0]</td>
<td>[0,0]</td>
<td>( \varnothing )</td>
</tr>
<tr>
<td>[\theta,\theta]</td>
<td>[\theta,\theta]</td>
<td>( p_1 )</td>
</tr>
<tr>
<td>[0,0]</td>
<td>( \varnothing )</td>
<td>( p_2 )</td>
</tr>
</tbody>
</table>

\[
\Delta(x_{ed2}, x_{s2}) = [\theta, \theta], \quad \text{Localisation:} \quad \varnothing, \quad (p_1) \text{ or } p_2
\]

TABLE I
SUMMARY OF INTERVAL INTERPRETATIONS FOR TANDEM STRUCTURE

Similarly, Table III summarizes the results for the parallel structure (see Figure 7). Parallelism is an association of multiple tandems which shares a common transition. The results for parallelism are a direct consequence of the ones for the tandem structure.

B. Synchronization structure

Figure 9 describes the synchronization structure. This one has many input places \( p_1 \) and \( p_2 \) and the localization results then depend on how tokens arrive in \( p_1 \) and \( p_2 \). To perform the exhaustive analysis, we suppose without loss of generality that the synchronization structure is governed by two inputs \( u_1 = \bigoplus_{n \in \mathbb{Z}} \gamma_{n} D_{u_1}(n) \) and \( u_2 = \bigoplus_{n \in \mathbb{Z}} \gamma_{n} D_{u_2}(n) \), where \( D_{u_1}(n) \) is a dater function (see Definition 4). Table III details all the results into 3 blocks defining the three estimation configurations: in the first one, all the \( x_1 \)'s are estimated; in the second one, \( x_3 \) and only one among \( x_1 \) and \( x_2 \) are estimated; and finally in the last one, only \( x_3 \) is estimated. Consider for instance Row 5. First, as \( \Delta(x_{ed3}, x_{s3}) = [\theta, \theta] \), the effect of the time shift failure is always present at the synchronization time. There are two possible situations. The first one is when \( \forall n \in \mathbb{Z}, t_1 = D_{u_1}(n), t_2 = D_{u_2}(n) \) such that \( a + \theta + b + t_1 > c + d + t_2 \) (i.e. the time shift failure is along the path \( p_1, p_3 \) and its effect makes this path always slower than the path \( p_2, p_4 \)). As \( \Delta(x_{ed1}, x_{s1}) = [0,0] \), the time shift failure \( \theta \) is necessarily in place \( p_3 \). Indeed, \( x_1 = \bigoplus_{k=0}^{N} \gamma_{nk} s^k + a \) and the next transition is \( x_3 = \bigoplus_{k=0}^{K} \gamma_{nk} s^k D_{u_2}(n) + a + b + c + d \). If a time shift failure \( \theta \) is in place \( p_3 \) the transition is \( x_{ed1} = \bigoplus_{k=d_1}^{K} \gamma_{nk} D_{u_1}(n) + c + a + b + \theta \) and \( x_{ed2} = \bigoplus_{k=d_2}^{K} \gamma_{nk} D_{u_2}(n) + c + a + b + \theta \). Therefore \( x_{ed3} = \bigoplus_{k=d_3}^{K} \gamma_{nk} D_{u_3}(n) + a + b + \theta \). The second situation is the symmetrical one that is if \( a + b + t_1 < c + d + t_2 \), and for the same reasons, the time shift failure is in \( p_4 \). Row 6 describes the case where the effect of the time shift failure is intermittent which leads to \( \Delta(x_{ed3}, x_{s3}) = [0, \theta] \) instead of \( \Delta(x_{ed3}, x_{s3}) = [\theta, \theta] \). In this case \( p_3 \) is suspected under the weaker condition that \( \exists n, t_1 = D_{u_1}(n), t_2 = D_{u_2}(n) \) such that \( a + \theta + b + t_1 > c + d + t_2 \). Note that here the indicator takes advantage of the underlying STD estimated state as the one of [PLCPV20] would generally return \([0, \theta] \) in any of these cases.

V. CONCLUSIONS

This paper defines a method for detecting and localizing time shift failures in systems modeled as TEG using observer

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2In every Table, notation \( p_i \) means that the localization is \( p_i \) while the notation \( (p_i) \) means that the localization is \( p_i \) or a place in the upstream of \( p_i \) in the TEG.

3Note that transitions \( u_1 \) and \( u_2 \) are actually not part of the synchronization structure that we describe here.
that estimates their sensitive-to-disturbance states. An indicator is proposed using these STD estimated states whose interval values help to localize the failures in three types of elementary TEG structures.

A first direct perspective is to improve the localization for an other TEG elementary structure called loop, meaning any elementary TEG structures. Then, all the conclusions made separately have to be unified in a general conclusion about where the failure is localized on the TEG. Finally, we will look for exploiting more deeply the interval values of the indicator to characterize the nature of the time shift failure.

### Table II

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### Table III

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### References


