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► **To cite this version:**

Wan Wang, Julio Andrés Iglesias Martinez, Yabin Jin, Bahram Djafari-Rouhani, Abdelkrim Khelif. Experimental realization of a pillared metasurface for flexural wave focusing. *APL Materials*, 2021, 9, pp.051125 (7). 10.1063/5.0052278 . hal-03414231

**HAL Id: hal-03414231**

**<https://hal.science/hal-03414231>**

Submitted on 4 Nov 2021

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# Experimental realization of a pillared metasurface for flexural wave focusing

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## Abstract

A metasurface is an array of subwavelength units with modulated wave responses that show great potential for the control of refractive/reflective properties in compact functional devices. In this work, we propose an elastic metasurface consisting of a line of pillars with a gradient in their heights, erected on a homogeneous plate. The change in the resonant frequencies associated with the height gradient allows to achieve transmitted phase response covering  $2\pi$  range while the amplitude response remaining at a relatively high level. We employ the pillared units to design focusing metasurface and compare the properties of the focal spots through simulation and experiment. Subwavelength transverse and lateral full width at half maximum (FWHM) of the focusing intensity profiles are observed in both simulation and experiment with the underlying mechanism being the interference and diffraction of the scattered waves from the resonant pillars as well as the boundaries (especially for experiment). The good correspondence between the experimental and simulated relative focal length shows the robustness of the focusing pillared metasurfaces with respect to fabrication imperfections. This proposed compact, simple and robust metasurface with unaffected mechanical properties provides a new platform to elastic wave manipulation for energy harvesting, wave communication, sensing, non-destructive testing, among others.

Keywords: elastic metasurface; focusing; robustness; Lamb wave; generalized Snell's law

## 1. Introduction

A metasurface consists of an array of subwavelength thickness units exhibiting inhomogeneous or modulated phase response that can arbitrarily manipulate refracted or reflected wavefronts. This field was rapidly developed in the domains of optics [1-5], microwave [6-8], and acoustics [9-18]. Recently it has been extended to elastic waves [19-21] especially for Lamb waves in plates with diverse potential applications at different scales. The key feature to design an elastic metasurface is to realize a  $2\pi$

43 phase span response by the constituting units while keeping a relative high level of the  
44 wave amplitude. As known by the classical wave motions, the propagating speeds of  
45 the Lamb waves depend on the elastic properties of the host medium and the plate's  
46 thickness (for flexural mode)[22, 23]. A large number of works focus on tailoring the  
47 plates to design composite in-plane geometries in order to fulfill the  $2\pi$  phase shift span  
48 requirement[24-34]. Although in-plane geometries provide a wide platform to  
49 manipulate wave responses with different mechanisms, they will significantly reduce  
50 the stiffness of plates which is very important as concerns the mechanical property of  
51 the structure.

52 An alternative solution is to design added out-of-plane units on the plate, for  
53 instance, by adding a thin patch with a size of the order of the wavelength for reflected  
54 wave response[35], or a set of slender pillars with a subwavelength total length for  
55 transmitted wave response[36, 37]. Given that a pillar is able to exhibit rich resonant  
56 properties[38-43], one can take advantage of the phase shift around the resonance to  
57 build the units of the array in the metasurface. For example, a set of graded pillars can  
58 produce different phase shifts in their transmission coefficient. However, in general, the  
59 variation of the phase around a resonance span a region of  $\pi$  instead of  $2\pi$ . In ref. [38],  
60 some of the authors showed that the phase shift can cover a range of  $2\pi$  if we superpose  
61 at the same frequency the fundamental compressional mode of the pillar with one of its  
62 bending modes. Based on this result, ref. [44] proposed theoretically the design of a  
63 metasurface, constituted by a set of pillars with graded heights, to achieve  
64 subwavelength focusing and imaging of flexural Lamb waves. The purpose of the  
65 present paper is the first experimental realization of such a design.

66 In this work, we experimentally realize a pillared metasurface consisting of a  
67 transverse line of graded height resonant pillars with identical subwavelength diameter  
68 that is able to design various wavefront functions. Each pillar of the metasurface acts  
69 as a secondary excitation source with different transmitted phases through the  
70 interference between the incident wave and the re-emitted wave. It is found that the  
71 transmitted phase response covers  $2\pi$  shift while the amplitude remains at sufficiently  
72 high level. In Sec.2, we present the design process of the metasurface unit for phase  
73 and amplitude manipulation which is further adopted to design plane wave focusing  
74 effect. In Sec.3, we fabricate the pillared metasurface and experimentally demonstrate  
75 focusing effect in good comparison with the numerically predicted results. Finally, we  
76 make a summary of this work in section 4.

## 77 78 **2. Pillared metasurface design**

79  
80 We first consider the transmission through a line of identical pillars arranged on  
81 the plate in the frequency domain, as shown in Fig.1(a). Periodic conditions are applied  
82 on the two sides of the unit cell along the  $x$  direction to simulate the infinite line of  
83 uniform pillars. Perfect matching layers (PML) are also applied to the two ends of the  
84 unit cell along  $y$  direction to avoid wave reflections from the edges. An incident plane  
85 flexural wave (the dominant component is the out of plane displacement) source is  
86 excited by an out of plane face force, and the transmitted wave is detected at a point on

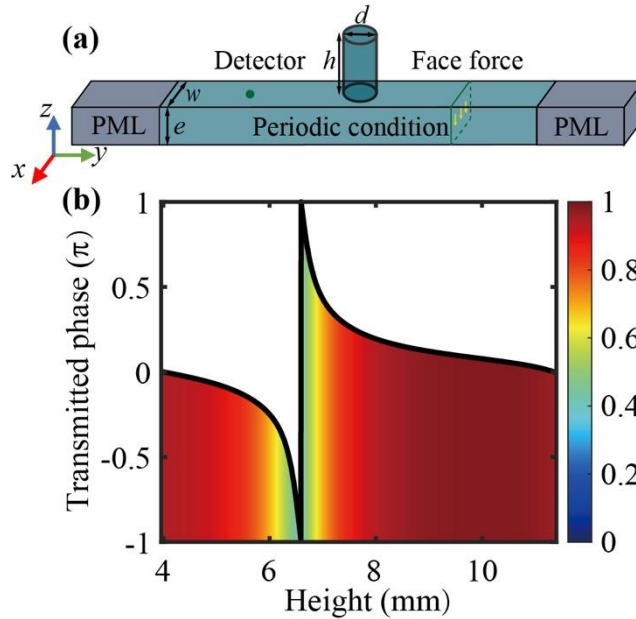
87 the plate's surface after the pillar. By analyzing the wavenumber of the transmitted  
 88 wave with Fourier transformation method, it is found that the transmitted wave is still  
 89 dominated by the flexural wave, as seen the appendix for details. The transmission  
 90 coefficient is defined as

$$91 \quad T = \left| \frac{w_0}{w_{\text{ref}}} \right|$$

92 where  $w_0$  and  $w_{\text{ref}}$  are the out-of-plane displacement of the detection point with and  
 93 without the pillar respectively. Generally, two types of resonant modes, namely bending  
 94 and compressional modes, can be excited by the incident plane wave; then the pillars  
 95 play as a secondary source and emit out-of-phase scattering wave. The transmitted wave  
 96 results from the interference between the incident and re-emitted waves[38]. For a  
 97 specific case, the bending and compressional modes can occur at the same frequency  
 98 to enhance the re-emitting effect, making the amplitude of the scattering wave as about  
 99 1.55 times that of the incident wave. After the destructive interference of the scattering  
 100 and the incident waves, the transmitted wave is dominated by the scattering wave, with  
 101 an out-of-phase transmission coefficient of amplitude 0.55.

102 We set the pillar's diameter  $d = 3.6$  mm, pillar's height  $h = 6.6$  mm, plate thickness  
 103  $e = 6.0$  mm, unit width  $w = 4.8$  mm. The pillar and plate are made of 3D-printed  
 104 materials polylactic acid (PLA) with Young's modulus  $E$  as 3.5GPa, Poisson's ratio as  
 105 0.36 and density as 1240 kg/m<sup>3</sup> (provided from the manufacturer). The above  
 106 parameters are chosen appropriately such that the superposition effect of the bending  
 107 mode and compressional mode can be achieved at 59.2 kHz corresponding to the  
 108 simulated wavelength  $\lambda = 13.3$  mm. The diameter of the pillar is only  $0.27\lambda$ , being deep  
 109 subwavelength. It should be mentioned that the size of the pillars can be scaled up and  
 110 down to drive the working frequency to lower or higher range, respectively.

111



112

113

FIG. 1. (a) Illustration of the pillared metasurface unit. The flexural wave is excited by the out  
 114 of plane face force and detected at a point on the plate's surface after the pillar. Periodic conditions  
 115 are applied to the two boundaries along  $x$  direction. The cyan area represents PLA materials, and

116 the dark gray areas are the perfect matched layers. (b) Transmitted phase (black curve) and  
 117 amplitude (color level) as a function of the pillar height  $h$  in the metasurface unit.

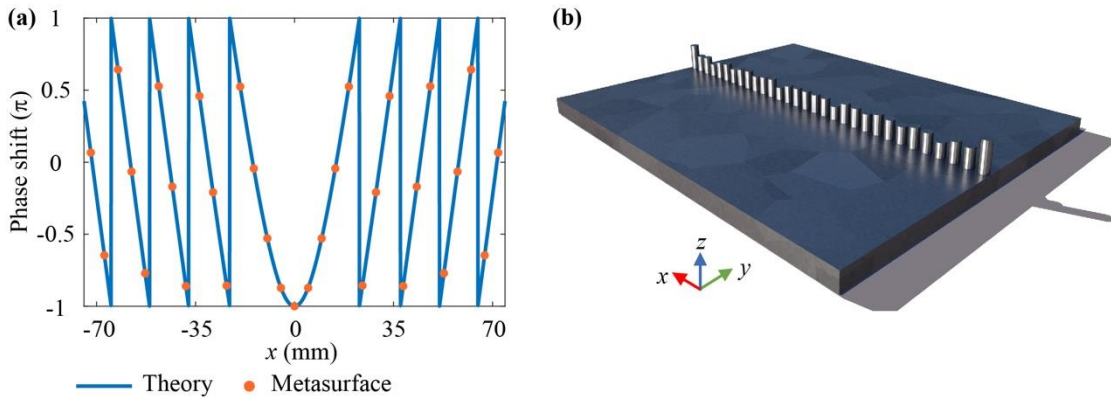
118

119 By sweeping the pillar's height at 59.2 kHz while maintaining other parameters as  
 120 mentioned above, Fig. 1(b) shows the transmitted amplitude and phase responses of the  
 121 displacement  $u_z$ . It is found that when the pillar's height is swept from 4 mm to 11.4  
 122 mm, the transmitted phase can fully cover the range from  $-\pi$  to  $\pi$ , as shown by the black  
 123 curve, and the transmitted amplitude keeps a relatively high level, as shown by the color  
 124 map. Then it is possible to manipulate the transmitted wavefront based on the  
 125 generalized Snell's law for various advanced functions, such as beam deflection,  
 126 focusing, source illusion, wave suppression, among others. Focusing is one of the  
 127 widest effects studied in wave physics, which is widely used in non-destructive testing  
 128 or signal reception as for sensing application[45]. Therefore, we select it to demonstrate  
 129 the functionality of the proposed pillared metasurface. To design a plane wave focusing  
 130 effect, it requires a gradient phase response along the transverse  $x$  axis in the  
 131 metasurface. The continuous phase response profile  $\varphi(x)$  can be given as

$$132 \quad \varphi(x) = \frac{2\pi}{\lambda} (\sqrt{F^2 + x^2} - F) + \varphi(x = 0) \quad \text{Eq. (1)}$$

133 where  $F$  and  $x$  are the focal length and  $x$ -coordinate position along the metasurface,  
 134 respectively,  $\lambda$  is the working wavelength. Since the real metasurface is made up of  
 135 individual pillars and cannot represent continuous phase shift, the continuous phase  
 136 profile needs to be discretized into the required phase points according to pillar's  
 137 positions along the metasurface. In Fig. 2(a), we calculate the continuous phase profile  
 138 (blue curve) by Eq. (1) for the plane wave focusing with a focal length  $F=\lambda$ , and  
 139 discretize it into 31 phase points (orange dots) for a pillared metasurface composed of  
 140 31 pillars. The number of pillars is limited by the maximum size capacity of 3D printer.  
 141 The phase of the central unit is set as  $-\pi$ , corresponding to the strongest resonant status  
 142 of the pillar. However, it should be noted that the choice of the central phase is not the  
 143 key factor for the focusing effect, and other phases can be set as well. Once the discrete  
 144 phases of metasurface units are obtained, the corresponding pillar' heights can be easily  
 145 retrieved from the black curve in Fig. 1(b). The final designed metasurface composed  
 146 of 31 gradient pillars is shown in Fig.2(b).

147



148

149 FIG. 2. (a) Theoretical phase profile (blue line) and discrete phases (orange dots) of the 31  
 150 pillars metasurface along  $x$  axis with the designed focal length  $F=\lambda$ . (b) Illustration of the designed  
 151 pillared metasurface.

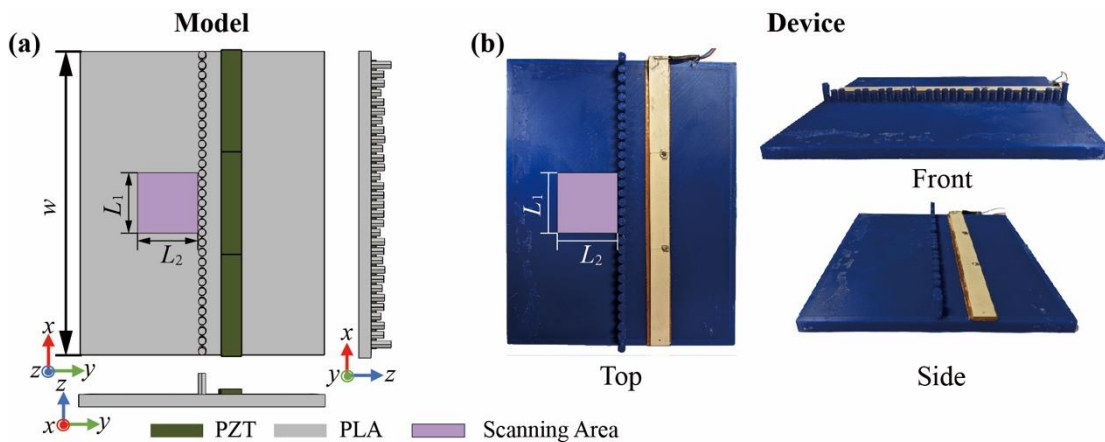
152

### 153 3. Numerical and Experimental demonstrations of focusing effect

154

155 A focusing metasurface containing 31 pillars with the focal length as  $F=\lambda$  is  
 156 consequently designed as shown in Fig. 3(a). To launch the incident wave, we followed  
 157 a process close to the experiment. Namely, three piezoelectric ceramic transducers PZT  
 158 patches with dimension of 2.58 mm thick, 50 mm length and 10 mm width, as the dark  
 159 green area shown in Fig. 3(a), are arranged on the upper surface of the plate to excite  
 160 flexural waves by applying an electric field of amplitude of 10V between the electrodes.  
 161 The PZT patch made of lead zirconate titanate have the followings material properties  
 162 with piezoelectric constants  $e_{31}=e_{32}=-5.2 \text{ C m}^{-2}$ ; dielectric permittivity  $\epsilon_{33}=663.2 \epsilon_0$ ,  
 163 with  $\epsilon_0$  being the vacuum permittivity. We fabricate the sample with PLA material for  
 164 experiment by using 3D printing technology and its picture is presented in Fig.3(b). The  
 165 three identical PZT patches are glued to the upper surface of the plate for exciting  
 166 flexural waves, as shown by the yellow stripe in Fig. 3(b); the patches are driven with  
 167 a signal at the frequency of 59.2kHz and an electric amplitude of 10V. The out of plane  
 168 displacement is measured with a Doppler laser vibrometer, MSA-500 by polytec  
 169 supplied by analog displacement decoder model DD-300 with sensitivity of  $50\text{nmV}^{-1}$ .  
 170 The interested area of the focusing field is chosen as a square purple zone,  $L_1=L_2=30$   
 171 mm, and further divided in rectangular subsections which is as big as the field of view  
 172 of the objective with size 3.5 mm by 4.5 mm. The displacement is scanned along  $x$ - $y$   
 173 plane with a step of  $335\mu\text{m}$  in each subsection. Absorbing materials are applied to the  
 174 surrounding of the sample during the experimental measurement in order to reduce the  
 175 boundary reflecting effect as much as possible.

176



177

178 FIG. 3. Illustration of the simulating geometric model (a) and the experimental device (b).  
 179 The scanning area is also shown as purple color.

180

181 We present the simulated ( $E=3.5 \text{ GPa}$ ) and experimental results of the focusing  
 182 effect in Fig.4(a) and (c). A clear focusing spot is observed with both approaches,

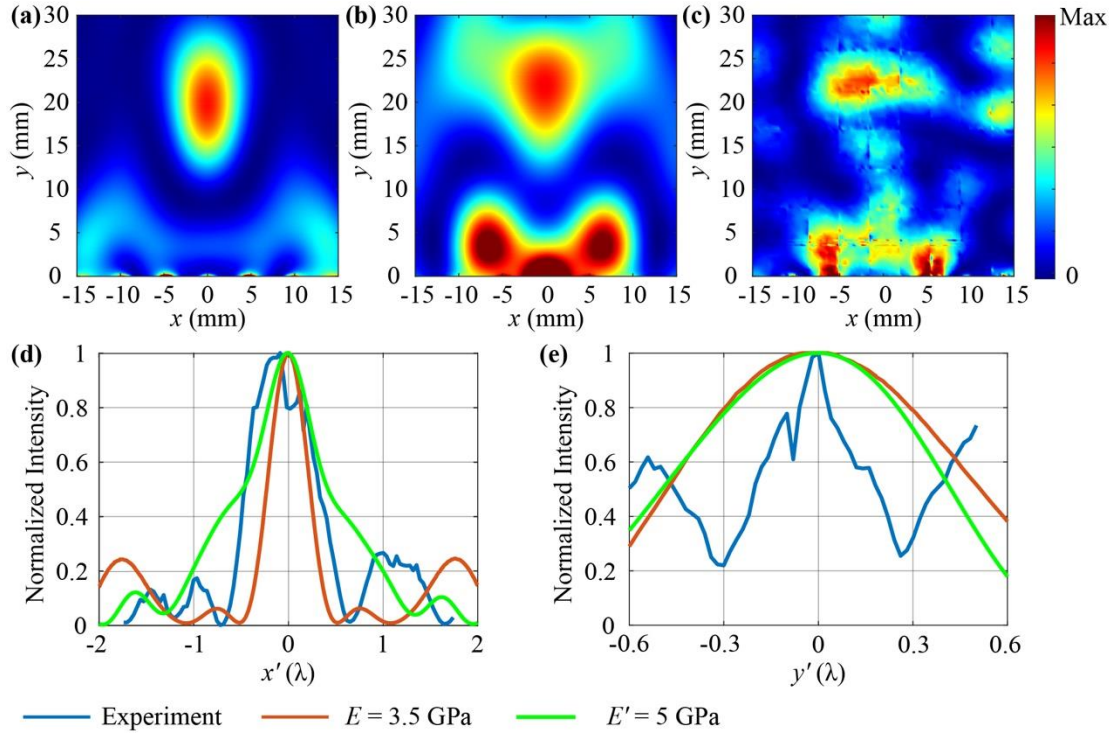
183 supporting the focusing functionality of the pillared metasurface. The simulated focal  
184 length is found to be 19.9mm corresponding to  $1.50\lambda$ , since the wavelength of the  
185 flexural Lamb wave is  $\lambda=13.3\text{mm}$  as mentioned in the previous section. In the  
186 experiment, the wavelength of the Lamb wave is found to be 15.2mm and the measured  
187 focal length is 22.6mm corresponding to  $1.48\lambda$ . Therefore, the two focal lengths are very  
188 close to each other when expressed in units of the wavelength, while they show 15%  
189 deviation in their absolute values. We infer this to a deviation of the Young's modulus  
190  $E'$  of the actual 3D printed sample as compared to the value  $E$  provided by the  
191 manufacturer. To make a more quantitative comparison with experiment, we simulated  
192 the behavior of the fabricated sample at the same frequency of 59.2kHz with a Young's  
193 modulus of  $E'=5\text{ GPa}$  that reproduces the experimental wavelength. The result is shown  
194 in Fig.4(b) where one can note a focal length of 22.0 mm (corresponding to  $1.45\lambda$ ), in  
195 good agreement with experiment. Additionally, the new simulation reproduces well the  
196 two experimental hot spots in the close vicinity of the metasurface, at the level of the  
197 two pillars on both sides of the central pillar. However, caution should be taken that  
198 with the new value  $E'$  of the Young's modulus, the pillars in the fabricated sample no  
199 longer satisfy the exact phase shift conditions chosen from Fig. 2 during the initial  
200 design process. Another difference between the experimental results and the  
201 simulations is about the asymmetry of the measured pattern. We think that this can be  
202 attributed to some fabrication and experimental imperfections that will be mentioned  
203 below.

204

205 For a more quantitative comparison, we compare the size of the simulated and  
206 experimental focal spots in terms of their corresponding wavelengths, Fig. 4(d) and (e)  
207 display the intensity profile along the  $x$  and  $y$  directions crossing the focal spot,  
208 respectively. For simulated ( $E=3.5\text{ GPa}$ ) case, the full width at half maximum (FWHMs)  
209 obtained from the intensity field along  $x$  and  $y$  axes are  $0.46\lambda$  and  $0.99\lambda$ , respectively,  
210 as shown by the brown curves. The FWHM along  $x$  axis in simulation is subwavelength,  
211 originating from the interference and diffraction of the gradient secondary sources[11,  
212 44, 46]. For simulated ( $E'=5\text{ GPa}$ ) case, the FWHMs along  $x$  and  $y$  axes become  $0.90\lambda$   
213 and  $0.91\lambda$ , respectively, as shown by the green curves. One can note that we lose the  
214 subwavelength FWHM along  $x$  which can be explained by the deviation of the actual  
215 pillars from an accurate design based on the correct value  $E'$  of the Young's modulus.  
216 For the experimental case, the FWHMs along  $x$  and  $y$  axes are  $0.78\lambda$  and  $0.39\lambda$ ,  
217 respectively, as shown by the blue curves in Fig. 4(d) and (e). The FWHM along  $x$  is  
218 also above  $\lambda/2$  and qualitatively close to the second simulation. As concerns the  
219 experimental FWHM along  $y$ , it has a complicated shape due to the asymmetry of the  
220 whole transmission pattern. This asymmetry may result from different imperfections in  
221 the sample or possibly in the experimental excitation of the incident wave, in particular  
222 the lateral boundary conditions and the induced reflection in the experimental  
223 configuration, the fabrication imperfections and the gluing, the bandwidth of the  
224 transducers, the proximity of the transducers to the sample which can generate an  
225 imperfect plane wave.

226

227 From the above discussion, one can conclude that the focusing effect is observed  
 228 in all the three above approaches and the relative focal lengths in terms of  
 229 corresponding wavelength are very close to each other. This indicates a robust focusing  
 230 property of the metasurface despite the actual differences between the experiment and  
 231 simulation. It is further expected that if the ratio between the focal length and the  
 232 metasurface length decreases, one can achieve a deep sub-diffraction focusing effect  
 233 with the transverse FWHM much smaller than half a wavelength, and the focusing  
 234 effect can be robustly conserved over a certain frequency range [44].  
 235



236 — Experiment —  $E = 3.5$  GPa —  $E' = 5$  GPa  
 237 FIG. 4. Intensity fields of the scanning area of focusing effect obtained by simulation with (a)  
 238  $E=3.5$  GPa, (b)  $E'=5$  GPa and (c) experiment. Intensity profiles along the  $x$  axis (d) or  $y$  axis (e)  
 239 crossing the focusing point. Brown, green and blue curves represent results from simulation with  
 240  $E=3.5$  GPa,  $E'=5$  GPa and experiment, respectively.  $x'$  and  $y'$  are local coordinate systems with the  
 241 focal point as the origin.  
 242

#### 243 4. Conclusion

244 In summary, we numerically and experimentally demonstrated a pillared  
 245 metasurface, consisting of a line of pillars with gradient heights, for focusing an  
 246 incident plane flexural wave into a spot. We took the advantage of the superposition of  
 247 the bending and compressional modes of the pillar on a homogeneous plate which is  
 248 able to enhance the out-of-plane scattering wave with the amplitude larger than that of  
 249 the incident wave. After the destructive interference between the scattering and incident  
 250 waves, the transmitted wave is out of phase. The phase response of the pillared units  
 251 spans a  $2\pi$  shift range and the amplitude response keeps at a relatively high level. We  
 252 designed and fabricated a focusing metasurface, and compared the measured focusing  
 253 spots quantitatively with simulations. The FWHM along  $x$  for simulated spot is



254 subwavelength due to the interference and diffraction of the re-emitted waves in the  
255 near field. The experimental and simulated relative focusing lengths have a good  
256 agreement, showing a strong robustness of the focusing metasurface. This metasurface  
257 can also be extended to surface waves, such as Rayleigh waves [47]. The simple,  
258 compact and robust design of the proposed pillared metasurfaces without tailoring the  
259 plate open a new avenue for advanced elastic wave functions in great potential  
260 applications for MEMS, civil engineering, aerospace engineering, marine engineering,  
261 and so on.

262

### 263 **Acknowledgment**

264 This work was supported by the National Natural Science Foundation of China  
265 (Grant No. 11902223), the Shanghai Pujiang Program (Grant No. 19PJ1410100), the  
266 program for professor of special appointment (Eastern Scholar) at Shanghai Institutions  
267 of Higher Learning, the Fundamental Research Funds for the Central Universities, and  
268 the High-Level Foreign expert Program. This work is also partially supported by the  
269 French EIPHI Graduate School (contract "ANR-17-EURE-0002").

270

### 271 **Data Availability**

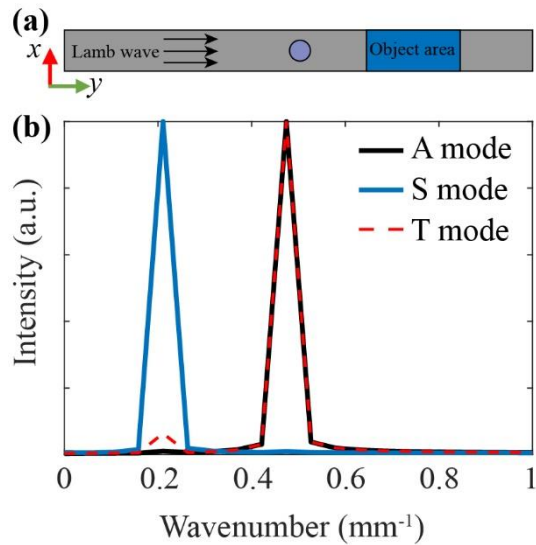
272 The data that support the findings of this study are available from the  
273 corresponding author upon reasonable request.

274

### 275 **Appendix. Mode analysis**

276

277 In Fig.5(a), the out of plane displacements in the ‘Object area’ are selected to  
278 perform the Fast Fourier Transformation (FFT) to analyze the wavenumber of  
279 transmitted waves. First, both flexural/antisymmetric (black) and symmetric (blue)  
280 Lamb modes incident waves are independently excited in the bare plate without the  
281 pillars. The positions of the black and blue intensity peaks correspond to the  
282 wavenumber values of the antisymmetric and symmetric Lamb modes, respectively.  
283 Then we excite the antisymmetric Lamb wave in the plate with the pillars, the intensity  
284 is plotted as the dotted red line. It is found that there are two peaks occurred at the  
285 wavenumbers of the antisymmetric and symmetric Lamb modes. The intensity value of  
286 the antisymmetric Lamb mode is over one magnitude bigger than that of the symmetric  
287 Lamb mode, supporting that the transmitted waves after the pillars are dominated by  
288 the flexural/ antisymmetric Lamb wave.



289

290 Fig.5. (a): Transmission model (top view). Periodic boundary conditions are applied to the two  
 291 edges along x directions and perfectly matched layers (not shown) are applied to the two edges  
 292 along y directions to avoid wave reflection from edges. (b): Wavenumber spectra after FFT for  
 293 flexural/antisymmetric (A mode) and symmetric (S mode) Lamb incident waves without the pillar  
 294 and for the transmitted wave (T mode) after the pillars with a flexural incident wave.

295

296

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