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# Elastic metasurfaces for deep and robust subwavelength focusing and imaging

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## Abstract

Metasurfaces are planar metamaterials with a flat surface and a subwavelength thickness that are able to shape arbitrary wavefronts such as focusing or imaging. There is a broad interest in the literature about subwavelength focusing/imaging based on bulk metamaterials while the utilization of metasurfaces for elastic waves has rarely been reported. Here, we present a new type of elastic metasurface consisting of a line of gradient resonant pillars for robust deep subwavelength focusing and imaging of elastic waves in a plate. Numerical approaches supported by analytic Huygens-Fresnel demonstrations show that the subwavelength full width at half maximum (FWHM) behaves linearly as a function of the ratio  $F/D$  where  $F$  is the measured focal length and  $D$  the metasurface length. We discuss the range of  $F/D$  where FWHM remains smaller than half a wavelength in the near field. The focal length  $F$  and the FWHM exhibit stable performances when submitted to disorder perturbations in the geometrical parameters and to frequency fluctuations. We show that the enhanced focusing resolution with smaller FWHM can be very beneficial for energy harvesting since the output electric power can be increased by more than one order of magnitude. The proposed elastic metasurfaces bring a new way for high resolution focusing and imaging which is useful for applications in various domains such as energy harvesting, wave sensing, communication, nondestructive evaluation.

## 31 **1. Introduction**

32 A metasurface is an array of subwavelength thickness units with nonhomogeneous wave  
33 response that defines a new principle for wave control[1-3], in particular by introducing a gradient  
34 in the phase or amplitude of the reflection and transmission coefficients of neighboring units. The  
35 wave response of the metasurface can be tailored by designing properly the geometry of the units  
36 to realize diverse abnormal applications. In recent years, this concept has been widely developed  
37 for optical[4-6] and acoustical [7-11] waves and extended to elastic waves [12-18] and mechanical  
38 systems, especially owing to the significant role of elastic and mechanical waves as information  
39 carrier for communication[19, 20]. Comparing to phononic crystals and acoustic  
40 metamaterials[21-25], acoustic metasurfaces have the advantage of a compact size that promote  
41 the design of thin and lightweight structures for different applications such as wave focusing,  
42 imaging, absorption, isolation, modal conversion, among others.

43 The focusing resolution of acoustic/elastic waves in the far-field suffers from diffraction limit  
44 due to the loss of evanescent waves which carry subwavelength information and decay  
45 exponentially from the source/object. This is referred to as Abbé or Rayleigh criterion. It is often  
46 mentioned in most references that the full width at half maximum (FWHM) of a focusing  
47 transverse intensity field is of the order of  $\lambda/2$  ( $\lambda$  is wavelength) due to the diffraction limit.  
48 Different mechanisms are proposed to break the diffraction limit so that a smaller FWHM than  $\lambda/2$   
49 can be achieved to improve the focusing resolution. The development of phononic crystals and  
50 acoustic metamaterials provides negative effective refractive index[26-31] by dispersion analysis  
51 and homogenization scheme to overcome the diffraction limit by collecting or magnifying the  
52 evanescent waves. Super-oscillation acoustic wave packets can be generated and further  
53 superimposed to a diffraction-limit-broken spot by designing acoustic meta-lenses which  
54 originates from the Schrodinger equation in quantum mechanical system[32, 33]. An alternative  
55 approach is based on the utilization of time reversal acoustics [34, 35]. In this approach, regardless  
56 of the complexity of the scattering media, a set of waves are created by a distribution of transducers  
57 that precisely retraces all of the complex paths of the scattered field and synchronously converges  
58 to the original source. Recently, a gradient index acoustic metasurface is exhibited to achieve  
59 focusing spots beyond the diffraction limit via spoof surface acoustic wave modulation[36]. Far-  
60 field imaging and edge detection of subwavelength objects have also been proposed by designing

61 a resonator array to amplify spatial frequencies and a binary phase grating to convert  
62 subwavelength components to propagating mode[37]. Focusing of spherical waves beyond the  
63 diffraction limit is demonstrated with anti-causal Green's function based on membrane-type  
64 acoustic metasurface[38]. One can notice that these realizations are for air-borne sound waves;  
65 therefore, their principles to beat the diffraction limit cannot be directly applied to mechanical  
66 waves where multiple polarizations and a coupling between the vibrational components are  
67 involved.

68 In this work, we propose a new type of pillared elastic metasurface for the purpose of  
69 subwavelength focusing and imaging whose FWHM is much smaller than half a wavelength. It  
70 consists of a line of resonant pillars with gradient phase response whose diameters are  
71 subwavelength. It is found that the transmitted wave can be regarded as a sum of the incident wave  
72 and the scattered waves from resonant pillars. Each resonant pillar plays as a secondary emitter  
73 source to generate point-like scattering waves[14]. Then, several transmitted waves with a gradient  
74 phase shift fully covering a  $2\pi$  range can be realized by designing an array of gradient resonant  
75 pillars as shown in Sec.2. In Sec.3 focusing phenomena are achieved whose FWHM of the  
76 transverse intensity profile is significantly less than half a wavelength. This is based on the  
77 physical mechanism of interference and diffraction between the gradient sub-sources in the near  
78 field region. The subwavelength focusing of the designed metasurface is numerically demonstrated  
79 by using the finite element method and the trend of FWHM as a function of the focal distance  $F$   
80 and the metasurface length  $D$  is investigated. The numerical results are theoretically supported on  
81 the basis of the Huygens-Fresnel principle applied in the frame of a Green's function method. In  
82 Sec.4 it is further demonstrated that the subwavelength focusing is robust against disorders in the  
83 geometrical parameters or frequency change of the incident wave, which would be beneficial for  
84 the practical fabrication and measurement issues. We show that the decrease in the FWHMs of  
85 focusing spots makes the pillared metasurfaces of significant interest for the purpose of energy  
86 harvesting. Indeed, they allow an enhancement by more than one order of magnitude of the output  
87 electric power that can be recovered by piezoelectric materials placed at the location of the spots.  
88 In Sec.5, we also demonstrated the subwavelength imaging effect by the proposed metasurface  
89 with FWHM less than half a wavelength. We finally summarize the work in Sec.6.

90

## 91 2. Metasurface units

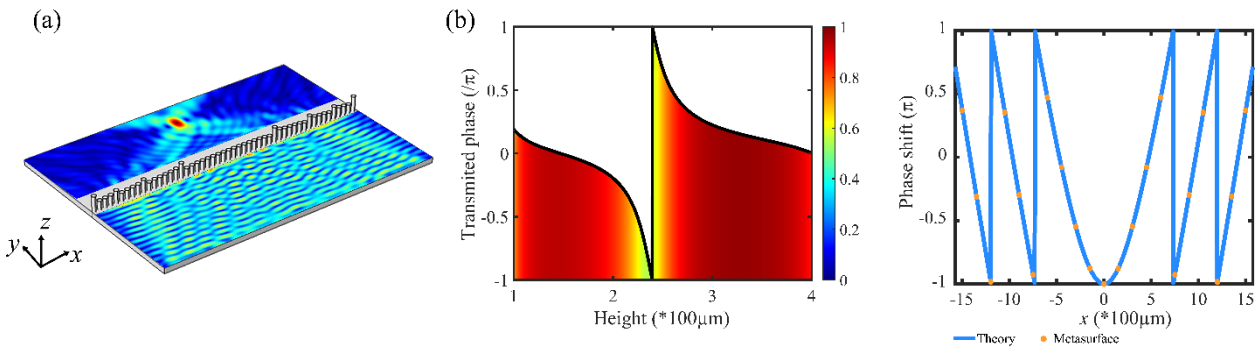
92 For a line of identical pillars on a plate, there are two main low frequency resonances, namely  
93 compressional and bending modes. The resonant frequencies are directly related to the geometrical  
94 parameters of the pillars, such as height and diameter[14]. We set the initial geometric parameters  
95 as follows: pillar's diameter  $d=120\mu\text{m}$ , pillar's height  $h=239\mu\text{m}$ , plate's thickness  $e=145\mu\text{m}$ ,  
96 period of the array  $a=150\mu\text{m}$ . Cubic silicon is chosen for the entire solid structure unless  
97 specifically mentioned. The choice of material is not a key factor for the observed trends and other  
98 common materials such as aluminum for machining process and steel or polymer for 3D printer  
99 can be used as well. Also, the results can be produced at different frequency scales by changing all  
100 the geometrical parameters by the same scale factor. The above geometrical parameters are chosen  
101 appropriately such that the second bending and first compressional modes are superposed at  
102 8.05MHz. This choice is beneficial in order to keep a high level of transmission while allowing a  
103 span of the phase shift over  $2\pi$  when changing gradually the height of the pillars.

104 The object is to design a line of gradient pillars (e.g. gradient in pillar's height) to steer  
105 incident wavefronts into desired forms. In Fig.1a, a plane wave focusing effect is illustrated with  
106 a pillared metasurface. The resonant properties of one pillar in the metasurface are obtained by  
107 considering a periodical structure of period  $a=150\mu\text{m}$  with this pillar and applying the periodic  
108 boundary condition based on Bloch theorem. The fundamental antisymmetric Lamb wave (flexural  
109 wave with main component along  $z$  axis perpendicular to the plate, called  $A_0$ ) is excited and  
110 propagated along  $y$  axis to interact with the pillars and the vibrating information about the resonant  
111 frequencies and transmission coefficients are obtained by the finite element method. To be  
112 mentioned that Fig.1a is not the whole space of simulation in our design, for instance, the  
113 surrounding perfect matched layers around the sample are not shown.

114 For the pillar's parameters mentioned above, the second bending and first compressional  
115 modes are superposed at 8.05MHz. The amplitude and phase of the out-of-plane displacement  
116 component  $u_z$  in the downstream on the plate with respect to the reference (same detected point on  
117 the plate without pillars) are respectively shown in Fig.1b by sweeping the pillar's height in the  
118 unit cell from  $100\mu\text{m}$  to  $400\mu\text{m}$  while keeping their diameter  $d$  and the period  $a$  constant for the  
119 sake of simplicity. It is found that the transmitted phase spans a range from  $-\pi$  to  $\pi$  as revealed by  
120 the black curve while the transmitted amplitude keeps a relatively high level as shown by the level

121 of color in Fig. 1b. The average value of the squares of the transmission coefficient where phase  
 122 covers  $2\pi$  is 0.79, which supports an overall sufficient energy in the transmitted field. It should  
 123 also be noted that the structure made of one line of pillars can possibly convert part of the incident  
 124  $A_0$  mode into  $S_0$  mode; however, we have checked that this conversion remains weak, so that the  
 125 transmitted wave is still dominated by  $A_0$  mode. Therefore, the phase and amplitude in Fig.1b keep  
 126 the same no matter the detected position in the downstream of the pillars' line, namely very close  
 127 to the pillars (e.g. until half a wavelength) or in the far field. This is in contrast to the case of a  
 128 single-sided pillared phononic crystal plate (a set of pillars deposited on one side of the plate in  
 129 the propagation direction), where the incident  $A_0$  mode wave is able to generate a strong  
 130 coexistence of  $A_0$  and  $S_0$  modes, as shown in Ref[39].

131



132

133 Figure 1. (a) Illustration of the gradient pillared metasurface for focusing effect; (b) the variation response of  
 134 transmitted phase (black curve) and amplitude (color level) in a periodic array as a function of the pillar's height  
 135  $h$  in the unit cell. (c) the theoretical phase profile (blue line) for  $F=\lambda$  and  $\Phi(x=0)=-\pi$  from Eq.(1) and the discrete  
 136 phases of the 21 selected pillars for the metasurface (yellow dots).

137

138 Taking advantage of a full  $2\pi$  phase span and sufficient high transmission amplitudes, we can  
 139 design a gradient phase meta-line of pillars to realize anomalous transmitted wavefronts based on  
 140 the generalized Snell's law. To be noted, the thickness of this metasurface, *i.e.* the diameter of the  
 141 pillars, is less than  $1/3$  of the incident wavelength, namely  $\lambda = 423 \mu\text{m}$  for the  $A_0$  mode at 8.05  
 142 MHz. In our metasurface approach, the bearing capacity and rigidity of the plate are conserved,  
 143 showing an excellent candidate in compact solid devices at micro or macro scales. As an example,  
 144 a gradient metasurface can be designed for plane wave focusing as shown in Fig.1a. Different from  
 145 a metasurface unit with plate's tailoring[12, 13], the resonant pillars will directly make impact on  
 146 the wave focusing while keeping the rigidity of the plate.

147

148 **3. Sub-wavelength focusing**

149 With full tunability of the transmitted phase, a metasurface consisting of different units can  
 150 be designed for desired wavefront functions based on the generalized Snell's law. The aim here is  
 151 to design a focusing effect with an incident plane wave. Therefore, the phase profile  $\Phi(x)$  along  
 152 the  $x$  direction can be easily derived as

$$153 \quad \Phi(x) = \frac{2\pi}{\lambda} (\sqrt{F^2 + x^2} - F) + \Phi(x = 0) \quad \text{Eq.(1)}$$

154 where  $\lambda = 423 \mu\text{m}$  is the wavelength of the  $A_0$  Lamb mode at 8.05MHz, and  $F$  is the focal length.

155 First, 21 pillars are considered along  $x$  for the metasurface with an interval of  $150 \mu\text{m}$ , about  
 156  $1/3$  of the wavelength, and the focal length  $F$  is fixed to  $\lambda = 423 \mu\text{m}$ . Also, the phase of the central  
 157 pillar ( $x=0$ ) is set to  $-\pi$  corresponding to the strongest resonant status. The discrete required phase  
 158 for the other pillars (yellow dots in Fig.1c) can be easily calculated by Eq. (1) and the  
 159 corresponding heights can be further obtained from the relationship (black line) in Fig.1b. The  
 160 transmitted intensity field of the focusing effect by the pillared metasurface is shown in Fig.2a.  
 161 The measured focal length is found to be  $1.06 \lambda$  and the FWHM (obtained from the transverse  
 162 intensity along the  $x$  axis crossing the spot) reaches a subwavelength value of  $0.37\lambda$  (Fig. 2c). To  
 163 validate the subwavelength focusing effect, we adopt the Huygens-Fresnel principle with a  
 164 calculation based on the Green's function method (Fig. 2b).

165 The Huygens-Fresnel principle would consist of superposing the emitted fields of the pillars  
 166 while assuming that each pillar is a point source emitting with the amplitude  $A_i$  and phase  $\varphi_i$  as  
 167 defined from Fig. 1b and c. Such a calculation can be analytically performed using a Green's  
 168 function approach, namely the displacement field response is given by

$$169 \quad w_i(r_i) = A_i W(r_i) e^{-i\varphi_i} \quad \text{Eq. (2)}$$

170 Here  $W(r)$  is the Green function associated with the equation of motion of flexural wave in  
 171 the plate and satisfies the following equation

$$172 \quad (\nabla^4 - k^4)W(r) = \delta(r) \quad \text{Eq. (3)}$$

173 where  $\nabla^4$  is the biharmonic operator,  $k$  is the wavenumber. The solution can be found in the form  
 174 [40]

175

$$W(r) = \frac{i}{8k^2} [H_0(kr) + \frac{2i}{\pi} K_0(kr)] \quad \text{Eq. (4)}$$

176

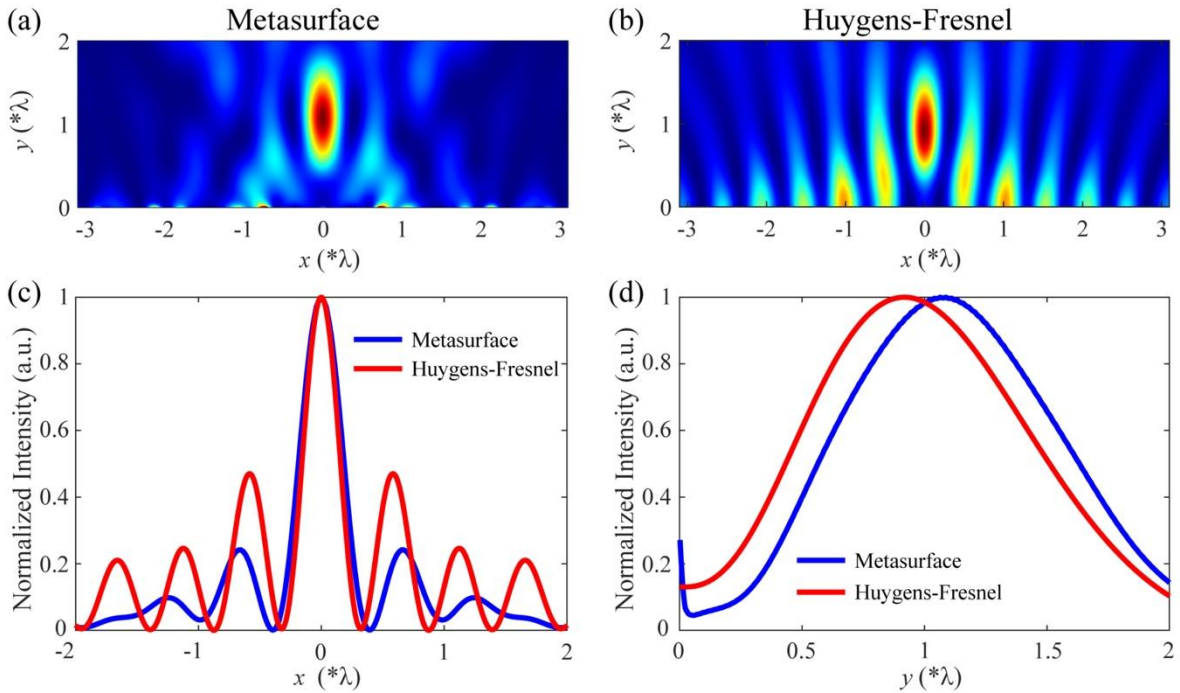
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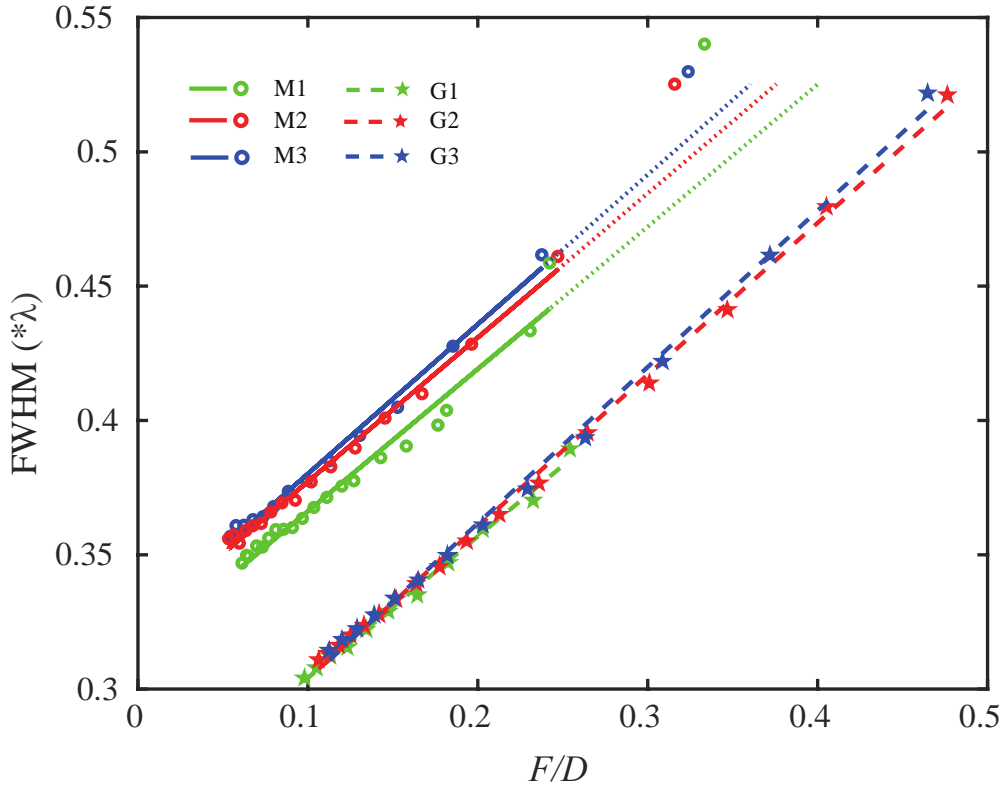
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Figure 2. Intensity field of plane wave focusing effect by (a) pillared metasurface, (b) analytically calculated Huygens-Fresnel principle with the Green's function approach. (c) the intensity along the  $x$  axis crossing the focusing spot. (d) the intensity along the  $y$  axis crossing the focusing point.

The subwavelength focusing is simultaneously dependent upon the focal length  $F$  and the metasurface length  $D$ . The length  $D$  is defined along the  $x$  axis and directly depends on the number of the pillars. We study this dependence for several design values of  $F$ , namely  $\lambda$ ,  $3\lambda$  and  $5\lambda$ . For each  $F$  value, we sweep the metasurface length  $D$  and calculate the intensity field distributions and the corresponding FWHM. These calculations are performed either numerically for the actual metasurface (called M1, M2, M3) or in the frame of the Huygens-Fresnel principle based on the Green's function approach (called G1, G2, G3). For both methods, we find in general a linear relationship between the FWHM and the ratio  $F/D$  as shown by the circles and star dots in Fig.3.



195 To be noted that in the ratio  $F/D$  we use the actual measured focal length  $F$  rather than the nominal  
 196 one. We define a fitting function  $\text{FWHM}=a*F/D+b$  for the two approaches as shown by the solid  
 197 and dashed lines with the fitting coefficient  $a, b$  shown in Table 1. For the metasurface simulation  
 198 approach, the circles above  $F/D=0.3$  (corresponding to FWHM over  $0.5\lambda$ ) are excluded from the  
 199 fitting process because for a low number of pillars (or a small  $D$ ), the focusing quality is lost and  
 200 the results deviate too much from the extrapolated straight line. For a similar reason, we disregard  
 201 in the G1 case the upper limit star where  $F/D=0.254$  corresponding to  $\text{FWHM}=0.389$ .



202  
 203 Figure 3. The relation between the  $F/D$  and FWHM by the metasurface simulation (M1, M2 and M3 stand for  
 204 designed  $F$  as  $\lambda, 3\lambda, 5\lambda$ ), and the Green's function based Huygens-Fresnel (G1, G2 and G3 stand for designed  $F$   
 205 as  $\lambda, 3\lambda, 5\lambda$ ) approaches with their corresponding fitting curves (solid lines for M and dotted lines for G) in the  
 206 form of  $\text{FWHM}=a*F/D+b$ . To be noted that in the ratio  $F/D$  we use the measured focal length  $F$ .

207

208

Table. 1 Fitting coefficient

Case	$a$	$b$
M1	0.530	0.313
M2	0.536	0.324
M3	0.557	0.325

G1	0.527	0.252
G2	0.565	0.247
G3	0.582	0.245

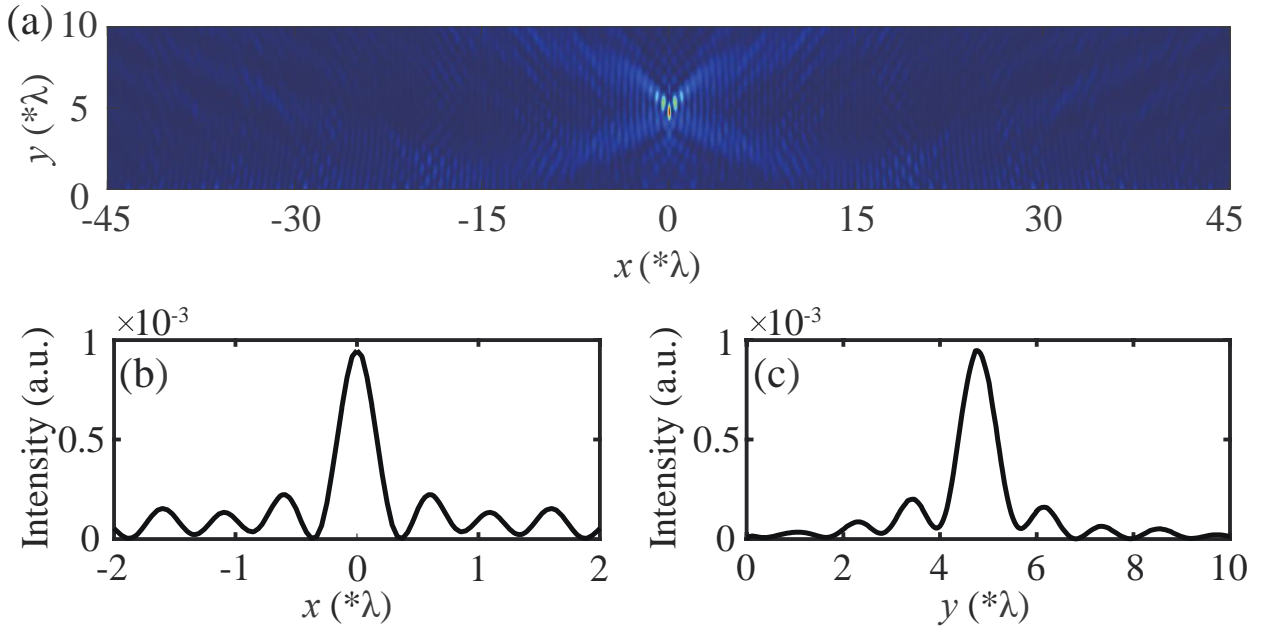
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210 For the Green's function based Huygens-Fresnel approach, each pillar is assumed as a point  
 211 source which is defined by a delta function force. The gradient sources are realized by setting the  
 212 amplitude and phase shown in Fig.1. Generally, the slopes of the fitting curves by the two  
 213 approaches are similar for different cases. However, the Huygens-Fresnel approach do not take  
 214 into consideration the coupling effect among the pillars in the metasurface, which explains the  
 215 mismatch in fitting coefficient  $b$  and the different configurations of side lobes as shown in Fig.2a  
 216 and b.

217 From Fig.3, one can observe a broad parameter's space for realizing subwavelength focusing  
 218 with FWHM less than  $0.5\lambda$  which the physical mechanism is the interference and diffraction of  
 219 the gradient sub-sources in the near field region[41, 42] with the near field length defined as  $L_N =$   
 220  $D^2/4\lambda$ . When the ratio  $F/D$  goes above 0.3 and 0.4, the FWHM becomes larger than  $0.5\lambda$  for the  
 221 simulation and analytical methods, respectively. For different designed focal lengths, the linear  
 222 relationships remain valid and keep stable with both approaches, showing a guidance for the design  
 223 of subwavelength focusing metasurfaces with proper  $F/D$ .

224 As long as the ratio  $F/D$  keeps at a low level in the near field, the subwavelength focusing  
 225 effect (FWHM less than  $0.5\lambda$ ) can be maintained. To give an illustration, we further propose a  
 226 metasurface consisting of 255 pillars for a designed focal length of  $5\lambda$  and show the focusing  
 227 information in Fig.4, where a focusing spot is clearly observed in Fig.4a. The measured FWHM  
 228 and focal length are  $0.34\lambda$  and  $4.8\lambda$  as can be seen from Fig.4b and c, respectively, corresponding  
 229 to  $F/D=0.079$ .

230



231

232 Figure 4. (a) The intensity field of plane wave focusing effect from pillared metasurface with 255 pillars for a  
 233 designed focal length of  $5\lambda$ . (b) the intensity along the x axis crossing the focusing spot showing a FWHM of  
 234  $0.34\lambda$  (c) the intensity along the y axis crossing the focusing point showing a focal length of  $F=4.8\lambda$ .

235

#### 236 4. Robustness of subwavelength focusing and application to energy harvesting

237

238 In this section we first demonstrate the robustness of sub-wavelength features against disorder  
 239 in the geometrical parameters of the metasurface or a shift in the precise frequency of the incident  
 240 wave. Since the focusing properties of the proposed pillared metasurface depend on the geometry  
 241 and not the choice of the constituent material, we employ in this section a plate made of a very  
 242 common material such as aluminum and propose a design of the metasurface in the millimeter  
 243 scale.

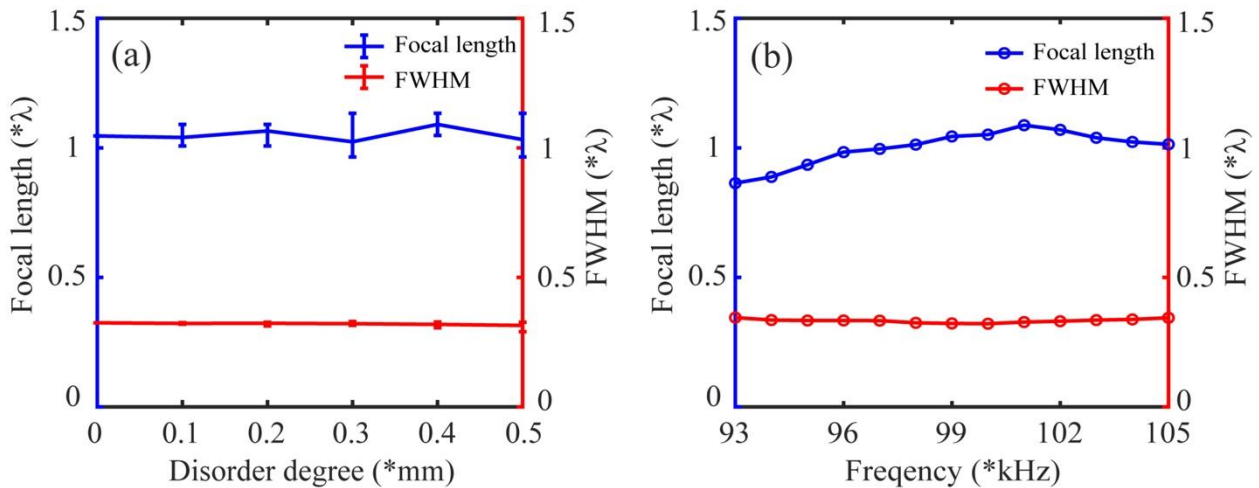
244

245 Following the same principle as in Sec.2, the geometrical parameters of the proposed  
 246 aluminum metasurface consisting of 51 pillars are chosen as follows: plate's thickness  $e=10\text{mm}$ ,  
 247 width of unit cell  $7.5\text{mm}$ , pillar's radius  $3\text{mm}$ . The pillar's height varies from  $5.7\text{mm}$  to  $18\text{mm}$  to  
 248 fully cover  $2\pi$  phase shift for a working frequency of  $99\text{kHz}$  corresponding to a wavelength of  
 249  $23.8\text{mm}$ . The ratio between the metasurface's thickness and the wavelength is only  $1/4$ . The  
 250 designed focal length is chosen as  $\lambda$  and the phase of the central pillar is chosen as  $\Phi(x=0) = -\pi$ .  
 251 We consider a disorder value in position, radius and height for all the pillars simultaneously in the  
 metasurface as it can appear in practical fabrications. The disorder degree is quantified as  $\gamma$ ,  
 meaning that the introduced disorder value is randomly chosen between  $0$  and  $\gamma\text{mm}$ . A maximum

252 disorder degree  $\gamma=0.5$  mm is considered which corresponds to a maximum error of 16.7% for  
 253 pillar's radius, much higher than most error levels in practical machining process fabrication.

254 In Fig.5a, we plot the measured focal length and FWHM as a function of the disorder  
 255 parameter  $\gamma$ . Each disorder case is sampled 5 times. For each disorder level, the vertical bars stand  
 256 for the obtained maximum and minimum values and the solid line stands for the average values  
 257 over the 5 times sampling. Without disorder ( $\gamma=0$ ), the focal length and the FWHM of the focusing  
 258 spot are  $1.04\lambda$  and  $0.32\lambda$ , respectively. It can be noticed that the curves are almost horizontal which  
 259 clearly supports the strong robustness of subwavelength features against such disorders that can  
 260 happen in fabrications and experiments. Although the metasurface is designed for single frequency  
 261 at 99kHz, we evaluate the subwavelength focusing with a frequency shift since experimentally the  
 262 exciting wave packet is broadened over a certain frequency range. In Fig.5b, we consider a  
 263 frequency bandwidth ratio (bandwidth divided by the central frequency) about 12%, with a  
 264 frequency range from 93kHz to 105kHz. The position of the focal length has a small variation  
 265 around the designed value of  $\lambda$ . On the other hand, the FWHM of the focusing spot almost keeps  
 266 flat below  $0.5\lambda$ , with a high robustness. The analysis of robustness helps to ensure the validity of  
 267 the subwavelength focusing in real sample fabrication and experimental measurements.

268



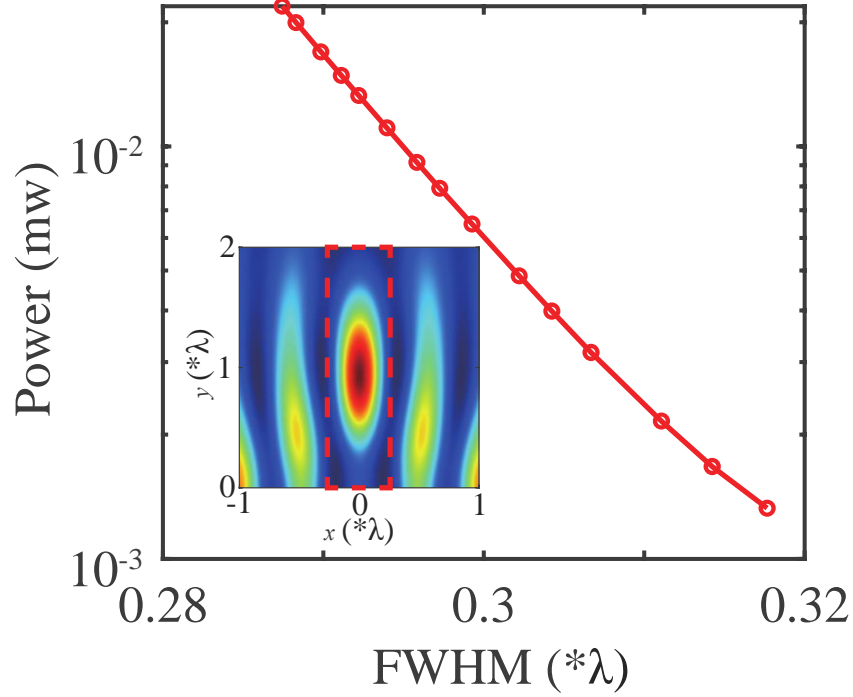
269 Figure 5. Robustness of subwavelength focusing against disorder effects. (a) we show the evolutions of the focal  
 270 length and the FWHM of the focusing spots as a function of the disorder degree. (b) the measured focal length  
 271 and the FWHM of subwavelength focusing against frequency shifts  
 272  
 273

274 Besides improving the resolution in non-destructive detection, a reduction in the FWHM  
 275 focusing will increase the confinement of elastic energy that can be used for energy harvesting.

276 We design a PZT thin layer attached to the opposite side of the plate located at the red dotted  
 277 rectangle in the inset of Fig.6. For the AC electrical circuit, the output power  $P$  can be derived  
 278 as[43]

$$279 \quad P = \frac{1}{2} I^2 R = \frac{R}{2} \left( \frac{\omega |\eta|}{1 + \omega \varepsilon_{33} R L_1 L_2 / t} \right)^2, \quad \eta = -\frac{e}{2} \int_A (e_{31} \frac{\partial^2 w}{\partial x^2} + e_{32} \frac{\partial^2 w}{\partial y^2}) dA \quad \text{Eq.(5)}$$

280 where  $I$  is the current,  $R$  the resistance chosen as  $1200\Omega$ ,  $e_{31}=e_{32}=-6.62281\text{Cm}^{-2}$  the piezoelectric  
 281 constants of PZT,  $\varepsilon_{33} = 1433.6\varepsilon_0$  the dielectric permittivity with  $\varepsilon_0$  being the vacuum  
 282 permittivity,  $L_1=13.1\text{mm}$  and  $L_2=47.55\text{mm}$  the sizes of the red dotted rectangle along  $x$  and  $y$ ,  
 283  $e=10\text{mm}$  the plate's thickness,  $t=1\text{mm}$  the thickness of PZT layer. Once the geometry and the  
 284 material properties of the piezoelectric layer are fixed, the main factor affecting the output power  
 285 is the curvature of the plate's surface along  $x$  and  $y$  in the red dotted rectangle. These curvatures  
 286 appear in the integral defining the parameter  $\eta$  in Eq. (5). We adopt the focusing field from the  
 287 Green's function approach (as in Fig.2b) and calculate the output power for different FWHM  
 288 values of G1 case in Fig.3. The results for the output power are presented as the dotted-line in  
 289 Fig.6. One can notice that the power is 16 times higher at  $\text{FWHM}=0.287\lambda$  than at  $\text{FWHM}=0.318\lambda$ .  
 290 Since the elastic strain energy is more confined for higher focusing resolution situations, the  
 291 surface's curvatures will be also enhanced, resulting in higher output electric powers. Therefore,  
 292 the subwavelength focusing performance of the elastic metasurfaces brings a significant impact  
 293 on the increase of the energy density for harvesting application.



294  
 295 Figure 6. The relationship between the output electric power by PZT layer attached to the plate covering the  
 296 focusing spot (marked as the red dotted line in the inset) as a function of the measured FWHM. The continuous  
 297 line is a guide for the eyes.

298

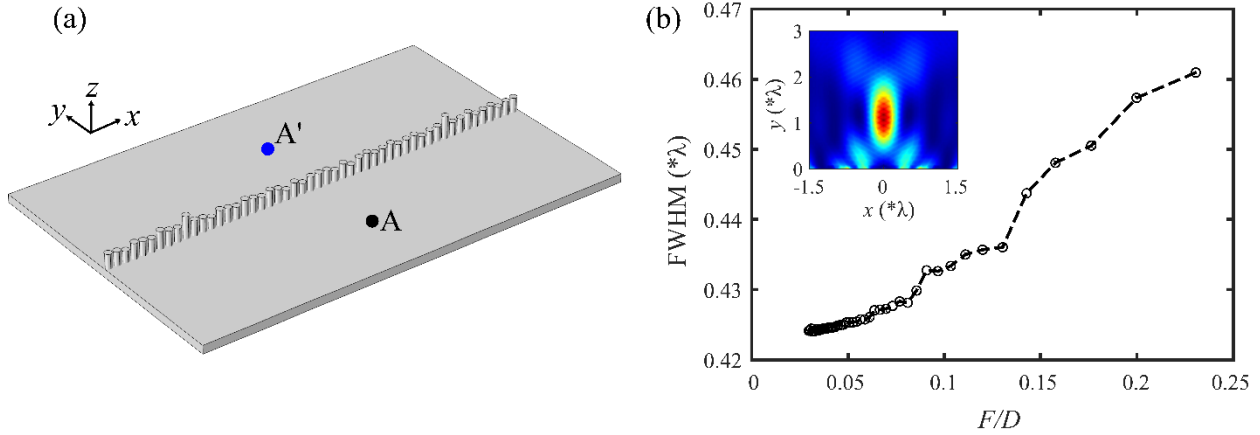
### 299 5. Sub-wavelength imaging

300 Similar to the plane wave focusing, we can also achieve sub-wavelength imaging with the  
 301 proposed pillared metasurface. In Fig.7a, we show schematically a point source A located at one  
 302 side of a metasurface with a distance  $F$ , and its image A' appearing at the opposite side with a focal  
 303 length  $F$ . In another word, the point source A and its image A' are symmetrically designed with  
 304 respect to the metasurface. The corresponding required phase profile can be derived as

$$305 \quad \Phi(x) = 2 \times \frac{2\pi}{\lambda} (\sqrt{F^2 + x^2} - F) + \Phi(x=0) \quad \text{Eq.(6)}$$

306 We also adopt  $\Phi(x=0) = -\pi$  for the central pillar corresponding to the strongest resonant status,  
 307 and the focal length  $F$  is chosen as  $F = \lambda = 423 \mu\text{m}$ . The employed material and geometric  
 308 parameters except pillar's height are the same as in Fig.3. The transmitted intensity field of the  
 309 imaging effect by the metasurface with 21 pillars is shown as the inset in Fig.7b. The measured  
 310 focal length is  $1.06 \lambda$  (corresponding to  $F/D=0.14$ ) and the FWHM reaches a subwavelength value  
 311 of  $0.44 \lambda$ . Then we fix the designed focal length at one wavelength and make a sweep in the  
 312 metasurface's length  $D$  (number of pillars). The behavior of FWHM as a function of  $F/D$  is shown

313 as the dotted curve in Fig.7b. Similar to Fig.3, the measured focal length is used in the ratio  $F/D$ .  
 314 From Fig.7b, one can observe that the FWHM of the imaging spot can also achieve values below  
 315  $0.5\lambda$  with the same physical mechanism as in Fig.3, showing a quasi linear relationship with  $F/D$ .  
 316 It is found that when the metasurface consists of 13 or more pillars, the focal length of the image  
 317 always keeps stable as  $1.06\lambda$ . In Fig.7b, the highest point with  $F/D=0.23$  and  $\text{FWHM}=0.46\lambda$   
 318 corresponds to the number of pillars equal to 13.



319  
 320  
 321 Figure 7. (a) Illustration of imaging effect with the pillared metasurface where A is the point source and A' is the  
 322 imaging spot. (b) The relation between  $F/D$  and FWHM when sweeping the number of pillars in the metasurface  
 323 while keeping the other parameters fixed. The inset shows the intensity field of imaging effect when the object  
 324 is at a distance  $\lambda$  of the metasurface and the number of pillars is 21.

325  
 326 **6. Summary**

327 In summary, we proposed a new type of pillared elastic metasurface consisting of a line of  
 328 gradient resonant pillars with subwavelength diameter on a homogeneous plate, which is able to  
 329 provide full phase range manipulation and high transmissions. Subwavelength focusing is  
 330 achieved by designing gradient pillared metasurface and the results are validated by the Green's  
 331 function-based Huygens-Fresnel principle (analytical) and the finite element method (numerical)  
 332 approaches. We also show linear relation between  $F/D$  and FWHM in the subwavelength focusing  
 333 effect. There is a wide range of parameters to keep FWHM smaller than half a wavelength as long  
 334 as  $F/D$  remains within a certain limit with the underlying physical mechanism being the  
 335 interference and diffraction of the gradient sub-sources in the near field. It is also found that the  
 336 minima of FWHM by the two approaches can be predicted from the data fitting. We further  
 337 demonstrated an excellent robustness to maintain the subwavelength behaviors against disorder in

338 the geometrical parameters or in the frequency shifts of the incident wave that may exist in real  
339 sample fabrication and experimental measurement of the focal length and the FWHM. Since the  
340 surface's curvatures will be also enhanced when FWHM is smaller, the pillared metasurfaces bring  
341 significant impact on the increase of the energy density for harvesting application, namely  
342 promoting the output electric power by more than one order of magnitude. We finally demonstrated  
343 subwavelength imaging effect with FWHM smaller than half a wavelength by the proposed  
344 metasurface. Since the rigidity of the homogenous plate is conserved, the proposed metasurfaces  
345 bring a new way for subwavelength wave manipulation that can be useful for several applications  
346 such as energy harvesting, sensing, wave communication or nondestructive evaluation.

347

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354

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