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Elastic metasurfaces for deep and robust subwavelength focusing and imaging

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Abstract

Metasurfaces are planar metamaterials with a flat surface and a subwavelength thickness that are able to shape arbitrary wavefronts such as focusing or imaging. There is a broad interest in the literature about subwavelength focusing/imaging based on bulk metamaterials while the utilization of metasurfaces for elastic waves has rarely been reported. Here, we present a new type of elastic metasurface consisting of a line of gradient resonant pillars for robust deep subwavelength focusing and imaging of elastic waves in a plate. Numerical approaches supported by analytic Huygens-Fresnel demonstrations show that the subwavelength full width at half maximum (FWHM) behaves linearly as a function of the ratio F/D where F is the measured focal length and D the metasurface length. We discuss the range of F/D where FWHM remains smaller than half a wavelength in the near field. The focal length F and the FWHM exhibit stable performances when submitted to disorder perturbations in the geometrical parameters and to frequency fluctuations. We show that the enhanced focusing resolution with smaller FWHM can be very beneficial for energy harvesting since the output electric power can be increased by more than one order of magnitude. The proposed elastic metasurfaces bring a new way for high resolution focusing and imaging which is useful for applications in various domains such as energy harvesting, wave sensing, communication, nondestructive evaluation.

31 **1. Introduction**

32 A metasurface is an array of subwavelength thickness units with nonhomogeneous wave
33 response that defines a new principle for wave control[1-3], in particular by introducing a gradient
34 in the phase or amplitude of the reflection and transmission coefficients of neighboring units. The
35 wave response of the metasurface can be tailored by designing properly the geometry of the units
36 to realize diverse abnormal applications. In recent years, this concept has been widely developed
37 for optical[4-6] and acoustical [7-11] waves and extended to elastic waves [12-18] and mechanical
38 systems, especially owing to the significant role of elastic and mechanical waves as information
39 carrier for communication[19, 20]. Comparing to phononic crystals and acoustic
40 metamaterials[21-25], acoustic metasurfaces have the advantage of a compact size that promote
41 the design of thin and lightweight structures for different applications such as wave focusing,
42 imaging, absorption, isolation, modal conversion, among others.

43 The focusing resolution of acoustic/elastic waves in the far-field suffers from diffraction limit
44 due to the loss of evanescent waves which carry subwavelength information and decay
45 exponentially from the source/object. This is referred to as Abbé or Rayleigh criterion. It is often
46 mentioned in most references that the full width at half maximum (FWHM) of a focusing
47 transverse intensity field is of the order of $\lambda/2$ (λ is wavelength) due to the diffraction limit.
48 Different mechanisms are proposed to break the diffraction limit so that a smaller FWHM than $\lambda/2$
49 can be achieved to improve the focusing resolution. The development of phononic crystals and
50 acoustic metamaterials provides negative effective refractive index[26-31] by dispersion analysis
51 and homogenization scheme to overcome the diffraction limit by collecting or magnifying the
52 evanescent waves. Super-oscillation acoustic wave packets can be generated and further
53 superimposed to a diffraction-limit-broken spot by designing acoustic meta-lenses which
54 originates from the Schrodinger equation in quantum mechanical system[32, 33]. An alternative
55 approach is based on the utilization of time reversal acoustics [34, 35]. In this approach, regardless
56 of the complexity of the scattering media, a set of waves are created by a distribution of transducers
57 that precisely retraces all of the complex paths of the scattered field and synchronously converges
58 to the original source. Recently, a gradient index acoustic metasurface is exhibited to achieve
59 focusing spots beyond the diffraction limit via spoof surface acoustic wave modulation[36]. Far-
60 field imaging and edge detection of subwavelength objects have also been proposed by designing

61 a resonator array to amplify spatial frequencies and a binary phase grating to convert
62 subwavelength components to propagating mode[37]. Focusing of spherical waves beyond the
63 diffraction limit is demonstrated with anti-causal Green's function based on membrane-type
64 acoustic metasurface[38]. One can notice that these realizations are for air-borne sound waves;
65 therefore, their principles to beat the diffraction limit cannot be directly applied to mechanical
66 waves where multiple polarizations and a coupling between the vibrational components are
67 involved.

68 In this work, we propose a new type of pillared elastic metasurface for the purpose of
69 subwavelength focusing and imaging whose FWHM is much smaller than half a wavelength. It
70 consists of a line of resonant pillars with gradient phase response whose diameters are
71 subwavelength. It is found that the transmitted wave can be regarded as a sum of the incident wave
72 and the scattered waves from resonant pillars. Each resonant pillar plays as a secondary emitter
73 source to generate point-like scattering waves[14]. Then, several transmitted waves with a gradient
74 phase shift fully covering a 2π range can be realized by designing an array of gradient resonant
75 pillars as shown in Sec.2. In Sec.3 focusing phenomena are achieved whose FWHM of the
76 transverse intensity profile is significantly less than half a wavelength. This is based on the
77 physical mechanism of interference and diffraction between the gradient sub-sources in the near
78 field region. The subwavelength focusing of the designed metasurface is numerically demonstrated
79 by using the finite element method and the trend of FWHM as a function of the focal distance F
80 and the metasurface length D is investigated. The numerical results are theoretically supported on
81 the basis of the Huygens-Fresnel principle applied in the frame of a Green's function method. In
82 Sec.4 it is further demonstrated that the subwavelength focusing is robust against disorders in the
83 geometrical parameters or frequency change of the incident wave, which would be beneficial for
84 the practical fabrication and measurement issues. We show that the decrease in the FWHMs of
85 focusing spots makes the pillared metasurfaces of significant interest for the purpose of energy
86 harvesting. Indeed, they allow an enhancement by more than one order of magnitude of the output
87 electric power that can be recovered by piezoelectric materials placed at the location of the spots.
88 In Sec.5, we also demonstrated the subwavelength imaging effect by the proposed metasurface
89 with FWHM less than half a wavelength. We finally summarize the work in Sec.6.

90

91 2. Metasurface units

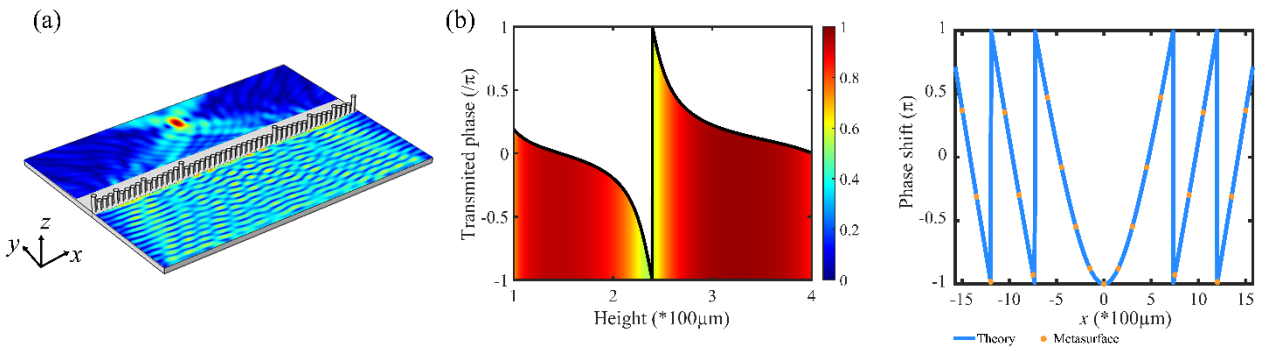
92 For a line of identical pillars on a plate, there are two main low frequency resonances, namely
93 compressional and bending modes. The resonant frequencies are directly related to the geometrical
94 parameters of the pillars, such as height and diameter[14]. We set the initial geometric parameters
95 as follows: pillar's diameter $d=120\mu\text{m}$, pillar's height $h=239\mu\text{m}$, plate's thickness $e=145\mu\text{m}$,
96 period of the array $a=150\mu\text{m}$. Cubic silicon is chosen for the entire solid structure unless
97 specifically mentioned. The choice of material is not a key factor for the observed trends and other
98 common materials such as aluminum for machining process and steel or polymer for 3D printer
99 can be used as well. Also, the results can be produced at different frequency scales by changing all
100 the geometrical parameters by the same scale factor. The above geometrical parameters are chosen
101 appropriately such that the second bending and first compressional modes are superposed at
102 8.05MHz. This choice is beneficial in order to keep a high level of transmission while allowing a
103 span of the phase shift over 2π when changing gradually the height of the pillars.

104 The object is to design a line of gradient pillars (e.g. gradient in pillar's height) to steer
105 incident wavefronts into desired forms. In Fig.1a, a plane wave focusing effect is illustrated with
106 a pillared metasurface. The resonant properties of one pillar in the metasurface are obtained by
107 considering a periodical structure of period $a=150\mu\text{m}$ with this pillar and applying the periodic
108 boundary condition based on Bloch theorem. The fundamental antisymmetric Lamb wave (flexural
109 wave with main component along z axis perpendicular to the plate, called A_0) is excited and
110 propagated along y axis to interact with the pillars and the vibrating information about the resonant
111 frequencies and transmission coefficients are obtained by the finite element method. To be
112 mentioned that Fig.1a is not the whole space of simulation in our design, for instance, the
113 surrounding perfect matched layers around the sample are not shown.

114 For the pillar's parameters mentioned above, the second bending and first compressional
115 modes are superposed at 8.05MHz. The amplitude and phase of the out-of-plane displacement
116 component u_z in the downstream on the plate with respect to the reference (same detected point on
117 the plate without pillars) are respectively shown in Fig.1b by sweeping the pillar's height in the
118 unit cell from $100\mu\text{m}$ to $400\mu\text{m}$ while keeping their diameter d and the period a constant for the
119 sake of simplicity. It is found that the transmitted phase spans a range from $-\pi$ to π as revealed by
120 the black curve while the transmitted amplitude keeps a relatively high level as shown by the level

121 of color in Fig. 1b. The average value of the squares of the transmission coefficient where phase
 122 covers 2π is 0.79, which supports an overall sufficient energy in the transmitted field. It should
 123 also be noted that the structure made of one line of pillars can possibly convert part of the incident
 124 A_0 mode into S_0 mode; however, we have checked that this conversion remains weak, so that the
 125 transmitted wave is still dominated by A_0 mode. Therefore, the phase and amplitude in Fig.1b keep
 126 the same no matter the detected position in the downstream of the pillars' line, namely very close
 127 to the pillars (e.g. until half a wavelength) or in the far field. This is in contrast to the case of a
 128 single-sided pillared phononic crystal plate (a set of pillars deposited on one side of the plate in
 129 the propagation direction), where the incident A_0 mode wave is able to generate a strong
 130 coexistence of A_0 and S_0 modes, as shown in Ref[39].

131



132

133 Figure 1. (a) Illustration of the gradient pillared metasurface for focusing effect; (b) the variation response of
 134 transmitted phase (black curve) and amplitude (color level) in a periodic array as a function of the pillar's height
 135 h in the unit cell. (c) the theoretical phase profile (blue line) for $F=\lambda$ and $\Phi(x=0)=-\pi$ from Eq.(1) and the discrete
 136 phases of the 21 selected pillars for the metasurface (yellow dots).

137

138 Taking advantage of a full 2π phase span and sufficient high transmission amplitudes, we can
 139 design a gradient phase meta-line of pillars to realize anomalous transmitted wavefronts based on
 140 the generalized Snell's law. To be noted, the thickness of this metasurface, *i.e.* the diameter of the
 141 pillars, is less than $1/3$ of the incident wavelength, namely $\lambda = 423 \mu\text{m}$ for the A_0 mode at 8.05
 142 MHz. In our metasurface approach, the bearing capacity and rigidity of the plate are conserved,
 143 showing an excellent candidate in compact solid devices at micro or macro scales. As an example,
 144 a gradient metasurface can be designed for plane wave focusing as shown in Fig.1a. Different from
 145 a metasurface unit with plate's tailoring[12, 13], the resonant pillars will directly make impact on
 146 the wave focusing while keeping the rigidity of the plate.

147

148 **3. Sub-wavelength focusing**

149 With full tunability of the transmitted phase, a metasurface consisting of different units can
 150 be designed for desired wavefront functions based on the generalized Snell's law. The aim here is
 151 to design a focusing effect with an incident plane wave. Therefore, the phase profile $\Phi(x)$ along
 152 the x direction can be easily derived as

$$153 \quad \Phi(x) = \frac{2\pi}{\lambda} (\sqrt{F^2 + x^2} - F) + \Phi(x = 0) \quad \text{Eq.(1)}$$

154 where $\lambda = 423 \mu\text{m}$ is the wavelength of the A_0 Lamb mode at 8.05MHz, and F is the focal length.

155 First, 21 pillars are considered along x for the metasurface with an interval of $150 \mu\text{m}$, about
 156 $1/3$ of the wavelength, and the focal length F is fixed to $\lambda = 423 \mu\text{m}$. Also, the phase of the central
 157 pillar ($x=0$) is set to $-\pi$ corresponding to the strongest resonant status. The discrete required phase
 158 for the other pillars (yellow dots in Fig.1c) can be easily calculated by Eq. (1) and the
 159 corresponding heights can be further obtained from the relationship (black line) in Fig.1b. The
 160 transmitted intensity field of the focusing effect by the pillared metasurface is shown in Fig.2a.
 161 The measured focal length is found to be 1.06λ and the FWHM (obtained from the transverse
 162 intensity along the x axis crossing the spot) reaches a subwavelength value of 0.37λ (Fig. 2c). To
 163 validate the subwavelength focusing effect, we adopt the Huygens-Fresnel principle with a
 164 calculation based on the Green's function method (Fig. 2b).

165 The Huygens-Fresnel principle would consist of superposing the emitted fields of the pillars
 166 while assuming that each pillar is a point source emitting with the amplitude A_i and phase φ_i as
 167 defined from Fig. 1b and c. Such a calculation can be analytically performed using a Green's
 168 function approach, namely the displacement field response is given by

$$169 \quad w_i(r_i) = A_i W(r_i) e^{-i\varphi_i} \quad \text{Eq. (2)}$$

170 Here $W(r)$ is the Green function associated with the equation of motion of flexural wave in
 171 the plate and satisfies the following equation

$$172 \quad (\nabla^4 - k^4)W(r) = \delta(r) \quad \text{Eq. (3)}$$

173 where ∇^4 is the biharmonic operator, k is the wavenumber. The solution can be found in the form
 174 [40]

175

$$W(r) = \frac{i}{8k^2} [H_0(kr) + \frac{2i}{\pi} K_0(kr)] \quad \text{Eq. (4)}$$

176

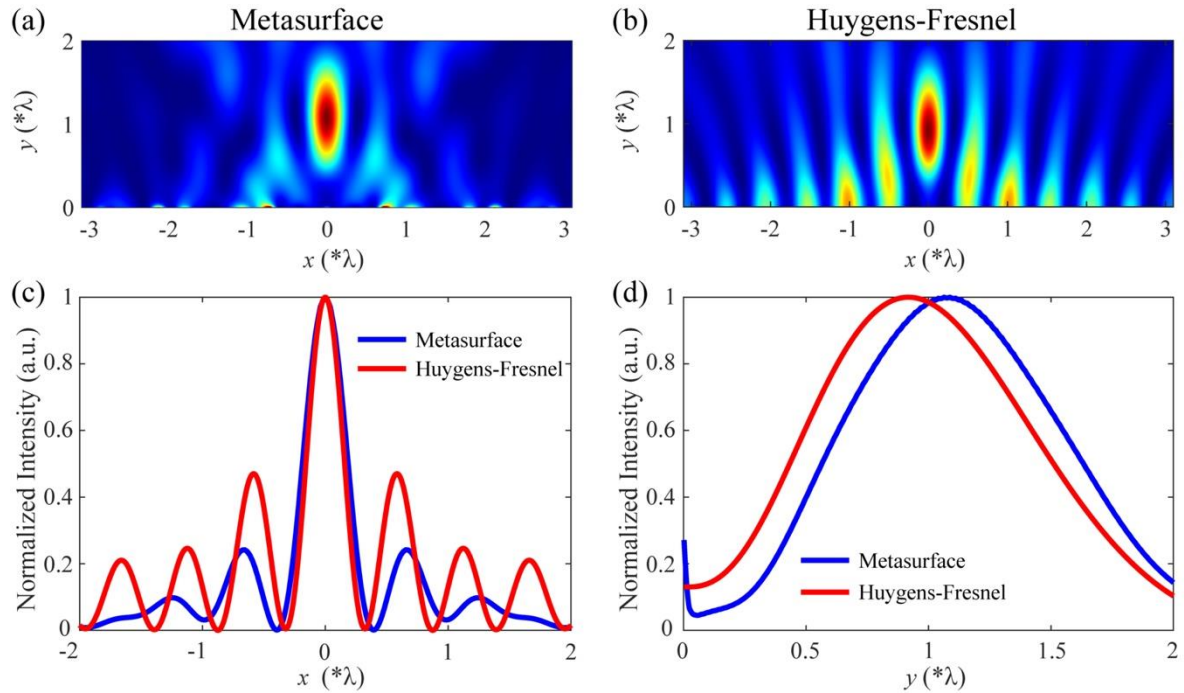
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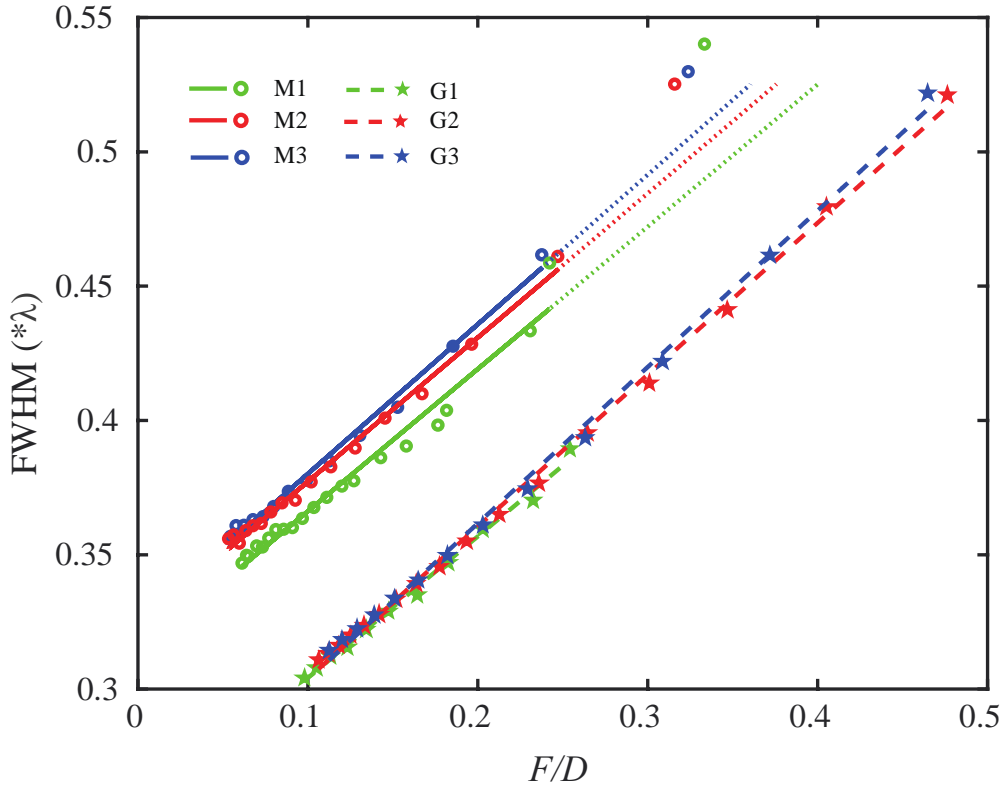
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194

Figure 2. Intensity field of plane wave focusing effect by (a) pillared metasurface, (b) analytically calculated Huygens-Fresnel principle with the Green's function approach. (c) the intensity along the x axis crossing the focusing spot. (d) the intensity along the y axis crossing the focusing point.

The subwavelength focusing is simultaneously dependent upon the focal length F and the metasurface length D . The length D is defined along the x axis and directly depends on the number of the pillars. We study this dependence for several design values of F , namely λ , 3λ and 5λ . For each F value, we sweep the metasurface length D and calculate the intensity field distributions and the corresponding FWHM. These calculations are performed either numerically for the actual metasurface (called M1, M2, M3) or in the frame of the Huygens-Fresnel principle based on the Green's function approach (called G1, G2, G3). For both methods, we find in general a linear relationship between the FWHM and the ratio F/D as shown by the circles and star dots in Fig.3.

195 To be noted that in the ratio F/D we use the actual measured focal length F rather than the nominal
 196 one. We define a fitting function $\text{FWHM}=a*F/D+b$ for the two approaches as shown by the solid
 197 and dashed lines with the fitting coefficient a , b shown in Table 1. For the metasurface simulation
 198 approach, the circles above $F/D=0.3$ (corresponding to FWHM over 0.5λ) are excluded from the
 199 fitting process because for a low number of pillars (or a small D), the focusing quality is lost and
 200 the results deviate too much from the extrapolated straight line. For a similar reason, we disregard
 201 in the G1 case the upper limit star where $F/D=0.254$ corresponding to $\text{FWHM}=0.389$.



202
 203 Figure 3. The relation between the F/D and FWHM by the metasurface simulation (M1, M2 and M3 stand for
 204 designed F as λ , 3λ , 5λ), and the Green's function based Huygens-Fresnel (G1, G2 and G3 stand for designed F
 205 as λ , 3λ , 5λ) approaches with their corresponding fitting curves (solid lines for M and dotted lines for G) in the
 206 form of $\text{FWHM}=a*F/D+b$. To be noted that in the ratio F/D we use the measured focal length F .

207

208

Table. 1 Fitting coefficient

Case	a	b
M1	0.530	0.313
M2	0.536	0.324
M3	0.557	0.325

G1	0.527	0.252
G2	0.565	0.247
G3	0.582	0.245

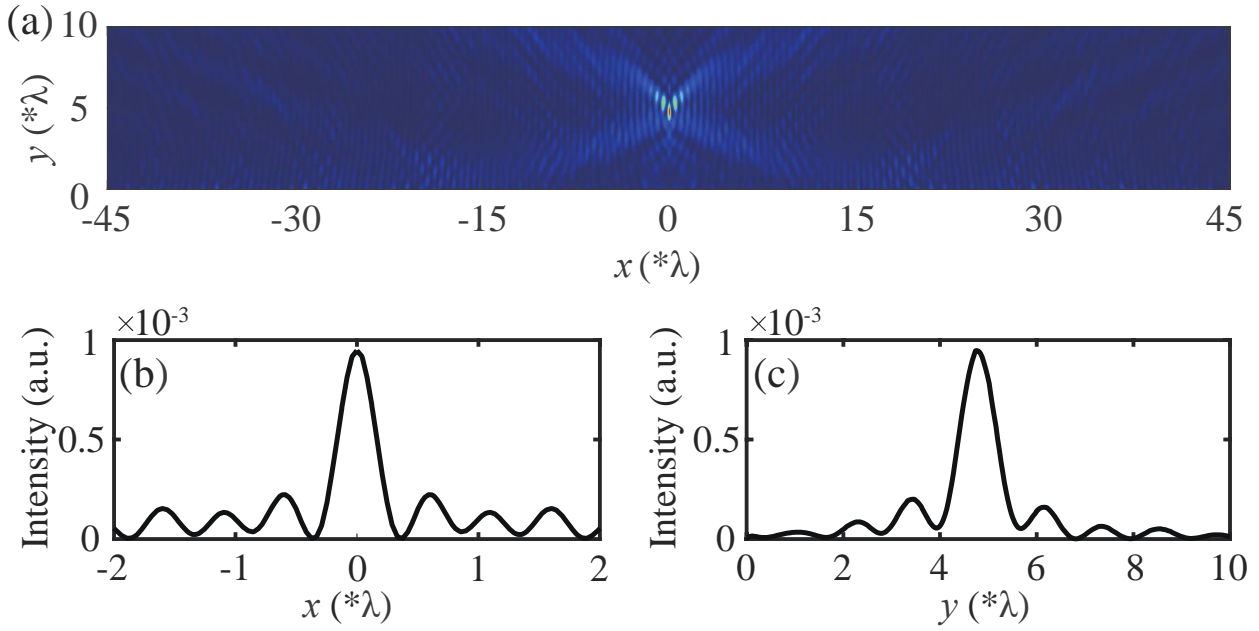
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210 For the Green's function based Huygens-Fresnel approach, each pillar is assumed as a point
 211 source which is defined by a delta function force. The gradient sources are realized by setting the
 212 amplitude and phase shown in Fig.1. Generally, the slopes of the fitting curves by the two
 213 approaches are similar for different cases. However, the Huygens-Fresnel approach do not take
 214 into consideration the coupling effect among the pillars in the metasurface, which explains the
 215 mismatch in fitting coefficient b and the different configurations of side lobes as shown in Fig.2a
 216 and b.

217 From Fig.3, one can observe a broad parameter's space for realizing subwavelength focusing
 218 with FWHM less than 0.5λ which the physical mechanism is the interference and diffraction of
 219 the gradient sub-sources in the near field region[41, 42] with the near field length defined as $L_N =$
 220 $D^2/4\lambda$. When the ratio F/D goes above 0.3 and 0.4, the FWHM becomes larger than 0.5λ for the
 221 simulation and analytical methods, respectively. For different designed focal lengths, the linear
 222 relationships remain valid and keep stable with both approaches, showing a guidance for the design
 223 of subwavelength focusing metasurfaces with proper F/D .

224 As long as the ratio F/D keeps at a low level in the near field, the subwavelength focusing
 225 effect (FWHM less than 0.5λ) can be maintained. To give an illustration, we further propose a
 226 metasurface consisting of 255 pillars for a designed focal length of 5λ and show the focusing
 227 information in Fig.4, where a focusing spot is clearly observed in Fig.4a. The measured FWHM
 228 and focal length are 0.34λ and 4.8λ as can be seen from Fig.4b and c, respectively, corresponding
 229 to $F/D=0.079$.

230



231

232 Figure 4. (a) The intensity field of plane wave focusing effect from pillared metasurface with 255 pillars for a
 233 designed focal length of 5λ . (b) the intensity along the x axis crossing the focusing spot showing a FWHM of
 234 0.34λ (c) the intensity along the y axis crossing the focusing point showing a focal length of $F=4.8\lambda$.

235

236 4. Robustness of subwavelength focusing and application to energy harvesting

237

238 In this section we first demonstrate the robustness of sub-wavelength features against disorder
 239 in the geometrical parameters of the metasurface or a shift in the precise frequency of the incident
 240 wave. Since the focusing properties of the proposed pillared metasurface depend on the geometry
 241 and not the choice of the constituent material, we employ in this section a plate made of a very
 242 common material such as aluminum and propose a design of the metasurface in the millimeter
 243 scale.

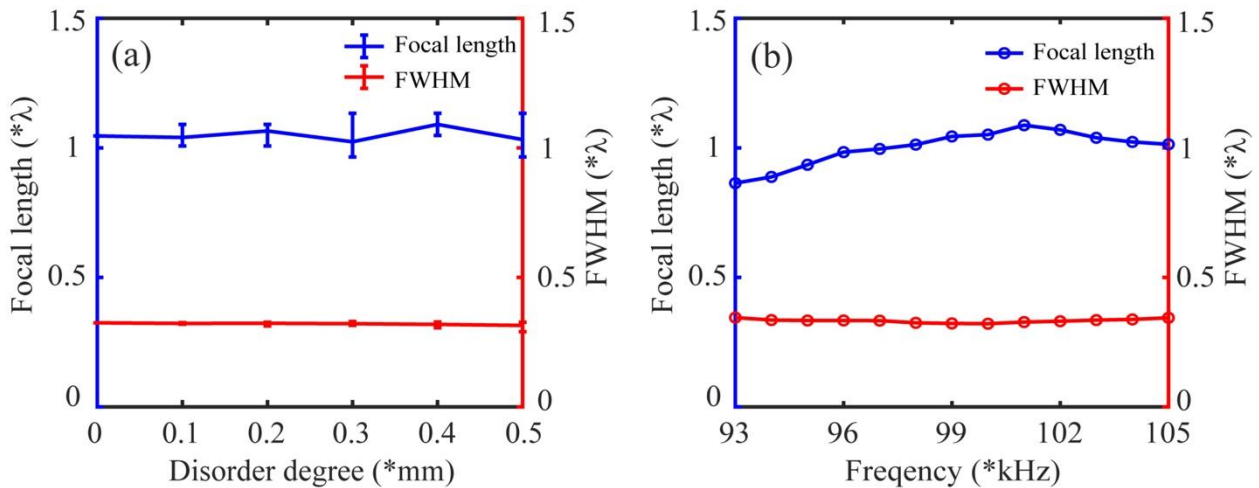
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245 Following the same principle as in Sec.2, the geometrical parameters of the proposed
 246 aluminum metasurface consisting of 51 pillars are chosen as follows: plate's thickness $e=10\text{mm}$,
 247 width of unit cell 7.5mm , pillar's radius 3mm . The pillar's height varies from 5.7mm to 18mm to
 248 fully cover 2π phase shift for a working frequency of 99kHz corresponding to a wavelength of
 249 23.8mm . The ratio between the metasurface's thickness and the wavelength is only $1/4$. The
 250 designed focal length is chosen as λ and the phase of the central pillar is chosen as $\Phi(x=0) = -\pi$.
 251 We consider a disorder value in position, radius and height for all the pillars simultaneously in the
 metasurface as it can appear in practical fabrications. The disorder degree is quantified as γ ,
 meaning that the introduced disorder value is randomly chosen between 0 and γmm . A maximum

252 disorder degree $\gamma=0.5$ mm is considered which corresponds to a maximum error of 16.7% for
 253 pillar's radius, much higher than most error levels in practical machining process fabrication.

254 In Fig.5a, we plot the measured focal length and FWHM as a function of the disorder
 255 parameter γ . Each disorder case is sampled 5 times. For each disorder level, the vertical bars stand
 256 for the obtained maximum and minimum values and the solid line stands for the average values
 257 over the 5 times sampling. Without disorder ($\gamma=0$), the focal length and the FWHM of the focusing
 258 spot are 1.04λ and 0.32λ , respectively. It can be noticed that the curves are almost horizontal which
 259 clearly supports the strong robustness of subwavelength features against such disorders that can
 260 happen in fabrications and experiments. Although the metasurface is designed for single frequency
 261 at 99kHz, we evaluate the subwavelength focusing with a frequency shift since experimentally the
 262 exciting wave packet is broadened over a certain frequency range. In Fig.5b, we consider a
 263 frequency bandwidth ratio (bandwidth divided by the central frequency) about 12%, with a
 264 frequency range from 93kHz to 105kHz. The position of the focal length has a small variation
 265 around the designed value of λ . On the other hand, the FWHM of the focusing spot almost keeps
 266 flat below 0.5λ , with a high robustness. The analysis of robustness helps to ensure the validity of
 267 the subwavelength focusing in real sample fabrication and experimental measurements.

268



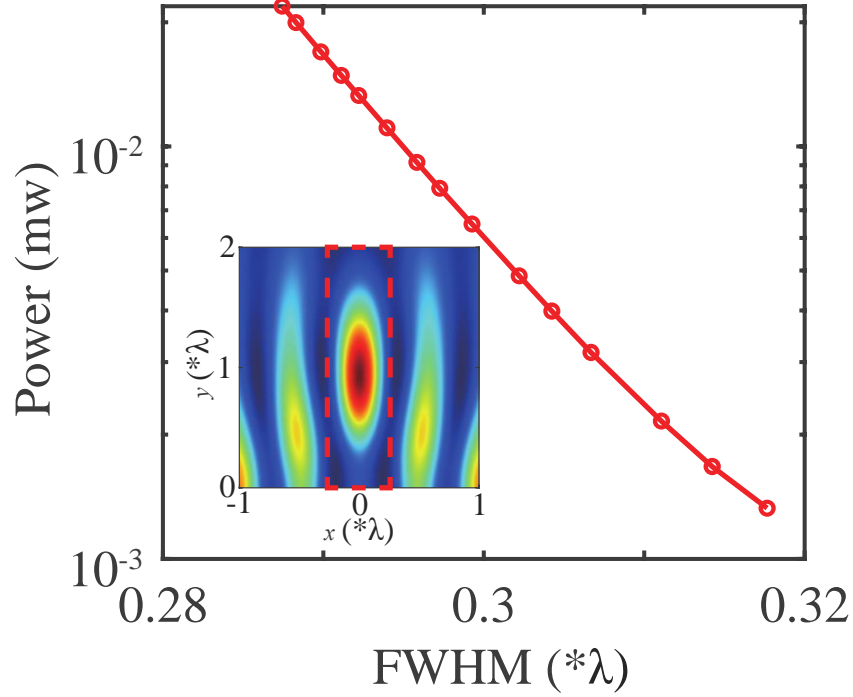
269 Figure 5. Robustness of subwavelength focusing against disorder effects. (a) we show the evolutions of the focal
 270 length and the FWHM of the focusing spots as a function of the disorder degree. (b) the measured focal length
 271 and the FWHM of subwavelength focusing against frequency shifts
 272
 273

274 Besides improving the resolution in non-destructive detection, a reduction in the FWHM
 275 focusing will increase the confinement of elastic energy that can be used for energy harvesting.

276 We design a PZT thin layer attached to the opposite side of the plate located at the red dotted
 277 rectangle in the inset of Fig.6. For the AC electrical circuit, the output power P can be derived
 278 as[43]

$$279 \quad P = \frac{1}{2} I^2 R = \frac{R}{2} \left(\frac{\omega |\eta|}{1 + \omega \varepsilon_{33} R L_1 L_2 / t} \right)^2, \quad \eta = -\frac{e}{2} \int_A (e_{31} \frac{\partial^2 w}{\partial x^2} + e_{32} \frac{\partial^2 w}{\partial y^2}) dA \quad \text{Eq.(5)}$$

280 where I is the current, R the resistance chosen as 1200Ω , $e_{31}=e_{32}=-6.62281\text{Cm}^{-2}$ the piezoelectric
 281 constants of PZT, $\varepsilon_{33} = 1433.6\varepsilon_0$ the dielectric permittivity with ε_0 being the vacuum
 282 permittivity, $L_1=13.1\text{mm}$ and $L_2=47.55\text{mm}$ the sizes of the red dotted rectangle along x and y ,
 283 $e=10\text{mm}$ the plate's thickness, $t=1\text{mm}$ the thickness of PZT layer. Once the geometry and the
 284 material properties of the piezoelectric layer are fixed, the main factor affecting the output power
 285 is the curvature of the plate's surface along x and y in the red dotted rectangle. These curvatures
 286 appear in the integral defining the parameter η in Eq. (5). We adopt the focusing field from the
 287 Green's function approach (as in Fig.2b) and calculate the output power for different FWHM
 288 values of G1 case in Fig.3. The results for the output power are presented as the dotted-line in
 289 Fig.6. One can notice that the power is 16 times higher at $\text{FWHM}=0.287\lambda$ than at $\text{FWHM}=0.318\lambda$.
 290 Since the elastic strain energy is more confined for higher focusing resolution situations, the
 291 surface's curvatures will be also enhanced, resulting in higher output electric powers. Therefore,
 292 the subwavelength focusing performance of the elastic metasurfaces brings a significant impact
 293 on the increase of the energy density for harvesting application.



294
 295 Figure 6. The relationship between the output electric power by PZT layer attached to the plate covering the
 296 focusing spot (marked as the red dotted line in the inset) as a function of the measured FWHM. The continuous
 297 line is a guide for the eyes.

298

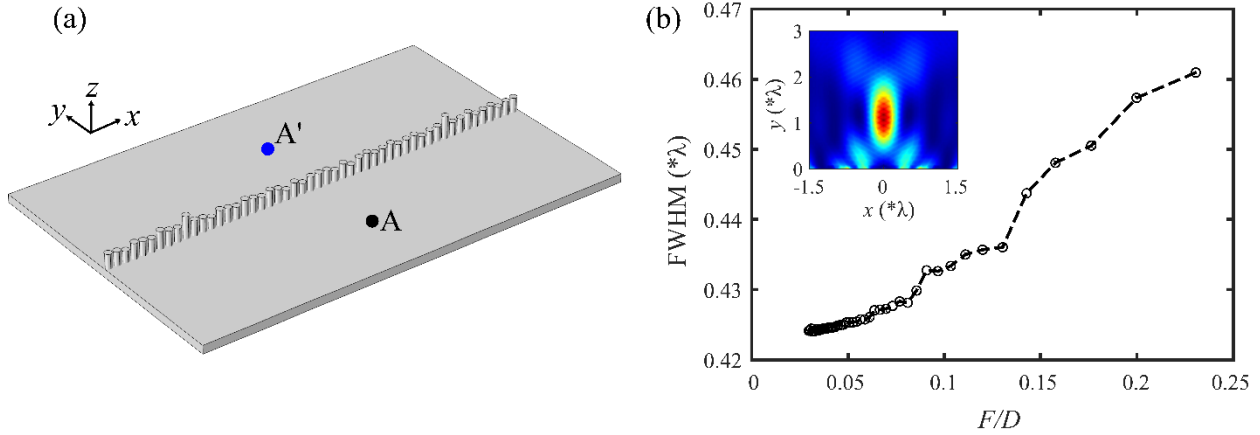
299 5. Sub-wavelength imaging

300 Similar to the plane wave focusing, we can also achieve sub-wavelength imaging with the
 301 proposed pillared metasurface. In Fig.7a, we show schematically a point source A located at one
 302 side of a metasurface with a distance F , and its image A' appearing at the opposite side with a focal
 303 length F . In another word, the point source A and its image A' are symmetrically designed with
 304 respect to the metasurface. The corresponding required phase profile can be derived as

$$305 \quad \Phi(x) = 2 \times \frac{2\pi}{\lambda} (\sqrt{F^2 + x^2} - F) + \Phi(x=0) \quad \text{Eq.(6)}$$

306 We also adopt $\Phi(x=0) = -\pi$ for the central pillar corresponding to the strongest resonant status,
 307 and the focal length F is chosen as $F = \lambda = 423 \mu\text{m}$. The employed material and geometric
 308 parameters except pillar's height are the same as in Fig.3. The transmitted intensity field of the
 309 imaging effect by the metasurface with 21 pillars is shown as the inset in Fig.7b. The measured
 310 focal length is 1.06λ (corresponding to $F/D=0.14$) and the FWHM reaches a subwavelength value
 311 of 0.44λ . Then we fix the designed focal length at one wavelength and make a sweep in the
 312 metasurface's length D (number of pillars). The behavior of FWHM as a function of F/D is shown

313 as the dotted curve in Fig.7b. Similar to Fig.3, the measured focal length is used in the ratio F/D .
 314 From Fig.7b, one can observe that the FWHM of the imaging spot can also achieve values below
 315 0.5λ with the same physical mechanism as in Fig.3, showing a quasi linear relationship with F/D .
 316 It is found that when the metasurface consists of 13 or more pillars, the focal length of the image
 317 always keeps stable as 1.06λ . In Fig.7b, the highest point with $F/D=0.23$ and $\text{FWHM}=0.46\lambda$
 318 corresponds to the number of pillars equal to 13.



319
 320
 321 Figure 7. (a) Illustration of imaging effect with the pillared metasurface where A is the point source and A' is the
 322 imaging spot. (b) The relation between F/D and FWHM when sweeping the number of pillars in the metasurface
 323 while keeping the other parameters fixed. The inset shows the intensity field of imaging effect when the object
 324 is at a distance λ of the metasurface and the number of pillars is 21.

325
 326 **6. Summary**

327 In summary, we proposed a new type of pillared elastic metasurface consisting of a line of
 328 gradient resonant pillars with subwavelength diameter on a homogeneous plate, which is able to
 329 provide full phase range manipulation and high transmissions. Subwavelength focusing is
 330 achieved by designing gradient pillared metasurface and the results are validated by the Green's
 331 function-based Huygens-Fresnel principle (analytical) and the finite element method (numerical)
 332 approaches. We also show linear relation between F/D and FWHM in the subwavelength focusing
 333 effect. There is a wide range of parameters to keep FWHM smaller than half a wavelength as long
 334 as F/D remains within a certain limit with the underlying physical mechanism being the
 335 interference and diffraction of the gradient sub-sources in the near field. It is also found that the
 336 minima of FWHM by the two approaches can be predicted from the data fitting. We further
 337 demonstrated an excellent robustness to maintain the subwavelength behaviors against disorder in

338 the geometrical parameters or in the frequency shifts of the incident wave that may exist in real
339 sample fabrication and experimental measurement of the focal length and the FWHM. Since the
340 surface's curvatures will be also enhanced when FWHM is smaller, the pillared metasurfaces bring
341 significant impact on the increase of the energy density for harvesting application, namely
342 promoting the output electric power by more than one order of magnitude. We finally demonstrated
343 subwavelength imaging effect with FWHM smaller than half a wavelength by the proposed
344 metasurface. Since the rigidity of the homogenous plate is conserved, the proposed metasurfaces
345 bring a new way for subwavelength wave manipulation that can be useful for several applications
346 such as energy harvesting, sensing, wave communication or nondestructive evaluation.

347

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