

Handling plant model uncertainties in adaptive regulation using dual Youla-Kucera parametrization. Application to Active Noise Control

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Outline

- Introduction and Motivation
- Adaptive regulation (nominal case)
- Introduction of plant model uncertainties
- Existence of solutions and stability analysis
- Experimental results (active noise control in ducts)
- Concluding remarks





Introduction

Adaptive regulation:

Attenuation of disturbances with unknown/time varying dynamic characteristics (models) while plant model is assumed to be known and almost constant



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Control of unknown/ time varying plant models

while the model (dynamic characteristic) of the disturbance is assumed to be known and almost constant

Combined adaptive control and regulation is not solved in the general case





uncertainties. Application to ANC

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Introduction

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A prototype adaptive regulation problem:

Rejection of single/multiple narrow band disturbances with unknown frequencies Basic ingredients: Internal Model Principle, Youla-Kucera (YK) parametrization, Parameter adaptation algorithms

Application areas : Active noise and vibration control





Closed loop system. Internal Model Principle



$$d(t) = \frac{N_d(q^{-1})}{D_d(q^{-1})}\delta(t)$$

(poles of D_d are on the unit circle)

Controller:

 $R(q^{-1}) = R'(q^{-1}) \cdot H_R(q^{-1});$ $S(q^{-1}) = S'(q^{-1}) \cdot H_S(q^{-1}).$

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Internal model principle: $H_S(q^{-1}) = D_d(q^{-1})$

Controller should contain the model of the disturbance

Output:
$$y(t) = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})} \cdot p(t) = S_{yp}(q^{-1}) \cdot p(t) = \frac{A(q^{-1})H_s(q^{-1})S'(q^{-1})}{P(q^{-1})} \cdot \frac{N_d(q^{-1})}{D_d(q^{-1})} \cdot \delta(t) \longrightarrow \lim_{t \to \infty} y(t) = 0$$

CL poles: $P(q^{-1}) = A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1})$

Nominal plant + Central Controller + YK parametrization + IMP

- Nominal system $G_0(q^{-1}) = \frac{B_0(q^{-1})}{A_0(q^{-1})}$, Central controller $C_0(q^{-1}) = -\frac{R_0(q^{-1})}{S_0(q^{-1})}$
- Disturbance $d(t) = \frac{N_d(q^{-1})}{D_d(q^{-1})} \delta(t)$ (poles of D_d are on the unit circle)
- Closed-loop characteristic polynomial : $P_0(q^{-1}) = A_0(q^{-1})S_0(q^{-1}) + B_0(q^{-1})R_0(q^{-1})$





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- YK parametrization:

Poles of the CL remain unchanged !





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- $R(q^{-1}) = R_0(q^{-1}) + A(q^{-1})Q(q^{-1});$ $S(q^{-1}) = S_0(q^{-1}) \cdot B(q^{-1})Q(q^{-1}) \qquad Q(q^{-1}) = q_0^Q + q_1^Q q^{-1} + \dots + q_{n_0}^Q q^{-n_Q}$ • YK parametrization: $\delta(t)$ N_d Primary Poles of the CL remain unchanged ! $\overline{D_d}$ path Secondary path Internal Model Principle with YK parametrization: ٠ B_0 1 yt) u(t) $\overline{S_0}$ $S_0 = \underbrace{D_d S'}_{S} + \mathbf{Q} B_0$ $\overline{A_0}$ B_0 A_0 Minimal order of Q: $\deg(Q) = \deg(D_d) - 1$ Image of the Q w(t)disturbance R_0 Landau & al. Adaptive regulation in the presence of plant 11 uncertainties. Application to ANC



Unknown disturbances: Direct adaptive feedback regulation

- Youla-Kucera parametrization allows to express the residual error(y(t)) as a function of the errors in the Q parameters (with respect to the optimal ones) $y(t+1) = \left[Q(q^{-1}) - \hat{Q}(t,q^{-1})\right] \frac{B_0^*(q^{-1})}{P(q^{-1})} w(t) + v(t+1)$
- *Consequence:* A direct adaptive regulation scheme can be built

Vanishing term

- No SPR condition for Asymptotic Stability. Various adaptation gain policies
- Particularly dedicated to multiple narrow band disturbances or limited band disturbances





Direct adaptive feedback regulation Vanishing term *a priori* adaptation error $e^{\circ}(t+1) = y(t+1) = [Q(q^{-1}) - \hat{Q}(t,q^{-1})] \frac{B_0^*(q^{-1})}{P(q^{-1})} w(t) + v(t+1)$ $\theta = [q_1, q_2, \dots]; \hat{\theta}(t) = [\hat{q}_1(t), \hat{q}_2(t) \dots]$ $\phi(t) = [w(t), w(t-1) \dots]$ $L(q^{-1}) = B_0^*/P$ $\psi(t) = L(q^{-1})\phi(t)$ $e^{0}(t+1) = \left[\theta - \hat{\theta}(t)\right]^{T} \psi(t)$

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Direct adaptive feedback regulation Vanishing term a priori adaptation error $e^{\circ}(t+1) = y(t+1) = \left[Q(q^{-1}) - \hat{Q}(t,q^{-1})\right] \frac{B_0^*(q^{-1})}{P(q^{-1})} w(t) + v(t+1)$ Parameter Adaptation Alagorithms (PAA) scalar $\theta = [q_1, q_2, \dots]; \hat{\theta}(t) = [\hat{q}_1(t), \hat{q}_2(t) \dots]$ $\hat{\theta}(t+1) = \hat{\theta}(t) + F\psi(t) \frac{e^0(t+1)}{1+\psi^T(t)F\psi(t)} ; \quad F = \gamma I; \gamma > 0$ adaptation $\phi(t) = [w(t), w(t-1) \dots]$ gain $L(q^{-1}) = q^{-d}B_{G}^{*}/P$ matrix $\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\psi(t) \frac{v^0(t+1)}{1+v^T(t)F(t)v^0(t)}$ F(t)>0 adaptation $\psi(t) = L(q^{-1})\phi(t)$ gain $e^{0}(t+1) = \left[\theta - \hat{\theta}(t)\right]^{T} \psi(t)$ $\mathbf{r} e(t+1) = \left[\theta - \hat{\theta}(t+1)\right]^{T} \psi(t)$ *a posteriori* adaptation error Standard form for the synthesis of stable adaptive schemes

• Performance will depend of course also upon the design of the parameter adaptation algorithms

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How to take into account large variations of the plant model in adaptive regulation?



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A disruptive approach:

Can we solve this problem by overparametrization of the YK filter used in adaptive regulation? *Expected benefit:* no need for adding testing signals,

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Ingredients for the theoretical analysis:

-Dual YK parametrization, Small gain theorem, Parameter adaptation algorithms with projection



Taking into account plant model uncertainties

The real plant model:
$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})}$$





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The real plant model:
$$G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})}$$

- System uncertainties can be represented with the Dual-YK parametrization: $G(q^{-1}) = \frac{\Gamma(q^{-1})B_0(q^{-1}) + \Delta(q^{-1})S_0(q^{-1})}{\Gamma(q^{-1})A_0(q^{-1}) \Delta(q^{-1})R_0(q^{-1})}$
- $\Gamma(q^{-1})$ monic polynomial, $\Delta(q^{-1})$ has no scalar term.
- The sensitivity function becomes:
- $y(t) = \frac{\Gamma A_0 \Delta R_0}{\Gamma + Q\Delta} \frac{S_0 QB_0}{P_0} d(t)$ • $w(t) = \frac{\Gamma A_0 - \Delta R_0}{\Gamma + \Delta Q} d(t)$

Can become unstable!

Nominal case : Γ =1, Δ =0

 $w(t)=A_0d(t)$

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Existence of a finite dimensional solution

Does it exist a filter Q of finite dimension such that:

The solution for the internal model principle (non necessarily minimal) stabilizes the closed loop for a given set of uncertainties

i.e. such that Q satisfies the two simultaneous conditions:

- 1) $S_0 = D_d S' + Q B_0$ (internal model principle condition)
- 2) $\Gamma + \Delta Q$ is an asymptotically stable polynomial



Conditions for the existence of a finite dimension solution

• For the frequencies ω_j of the disturbance corresponding to $D_d(e^{-i\omega_j}) = 0$, the following inequality is satisfied:

$$\left|\frac{S_o(e^{-i\omega_j})}{B_o(e^{-i\omega_j})}\right| < \left|\frac{A(e^{-i\omega_j})S_o(e^{-i\omega_j}) + B(e^{-i\omega_j})R_o(e^{-i\omega_j})}{B(e^{-i\omega_j})A_o(e^{-i\omega_j}) - A(e^{-i\omega_j})B_o(e^{-i\omega_j})}\right|$$



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• For all other frequencies, the following inequality is satisfied:

$$\left| Q(e^{-i\omega}) \right| < \left| \frac{A(e^{-i\omega})S_o(e^{-i\omega}) + B(e^{-i\omega})R_o(e^{-i\omega})}{B(e^{-i\omega})A_o(e^{-i\omega}) - A(e^{-i\omega})B_o(e^{-i\omega})} \right|$$

This condition gives the domain where the system can be used for disturbance attenuation



These sufficient conditions give hints for the design of the central controller



From where these conditions come?

 $\Gamma + \Delta Q$ should be an asymptotically stable polynomial









Adaptive case (unknown disturbances) in the presence of plant uncertainties

- Use of the same PAA as for the nominal case, but
- Since one uses an overparametrized solution and « richness of excitation » argument can not more be used, a **projection** has to be added to the PAA
- A sufficient stability condition appears in terms of sizes of uncertainties (Δ , Γ) and projection domain.



Parameter Adaptation Algorithm with Projection

$$\widehat{\theta}(\mathsf{t}+1) = \widehat{\theta}_{p}(\mathsf{t}) + F\psi(t) \frac{e^{0}(t+1)}{1+\psi^{T}(t)F\psi(t)}$$

$$\widehat{\theta}_{p}(t+1) = \begin{cases} \widehat{\theta}(t+1) & \text{if } \|\widehat{\theta}(t+1) - \theta_{0}\| \leq R \\ \theta_{0} + R \frac{\widehat{\theta}(t+1) - \theta_{0}}{\|\widehat{\theta}(t+1) - \theta_{0}\|} & \text{if } \|\widehat{\theta}(t+1) - \theta_{0}\| > R \end{cases}$$



The optimal vector heta is assumed to be inside domain \mathscr{D}

One can use as θ_p any value of θ inside domain $\mathcal{D}a$ nd in particular θ_0

Hypotheses

- 1) Controller C_0 stabilizes both G_0 and G
- 2) Disturbance d(t) is bounded
- 3) It exists Q of finite dimension such that $\Gamma + \Delta Q$ has roots inside the unit circle

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$$e(t+1) = \left[\theta - \hat{\theta}(t+1)\right]^T \psi(t) \quad \text{if } \psi(t) \text{ is bounded} \quad \blacksquare \quad \lim_{t \to \infty} e(t) = 0$$

But: $\psi(t) = f(w(t))$

w(t) should be bounded

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Remark : For the nominal case : $\Delta = 0$, $\Gamma = 1$

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Attenuation of single and multiple tonal noises in ducts





Attenuation of single and multiple tonal noises in ducts





Model	n _A	n_B	d
Secondary path Go	38	32	8
Secondary path G	27	20	7

Many low damped complex poles and zeros



Experimental results 1a





Experimental results 1b







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Experimental results 2a



Experimental results 2b





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Experimental results 3







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170 Hz

Parameter Drift and PAA with projection - Experimental results



Step changes in frequency every 30 s for interference phenomenon arround 160 Hz +160.5 Hz



zoom





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Concluding remarks

- Overparametrization works for handling plant model uncertainties
- Requires specific design of the central controller + PAA with projection
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Open problems:

- Design of the central controller for a set of plant models in order to maximize the operation region
- To be proven: augmenting the size of the YK filter Q reduces the $|Q|_{\infty}$





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Thank you for your attention!