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# Losses in interacting quantum gases: ultra-violet divergence and its regularization

Isabelle Bouchoule, Léa Dubois and Léo-Paul Barbier[1]

[1] *Laboratoire Charles Fabry, Institut d'Optique Graduate School, CNRS, Université Paris-Saclay, 91127 Palaiseau cedex, France*

We investigate the effect of losses on an interacting quantum gas. We show that, for gases in dimension higher than one, assuming together a vanishing correlation time of the reservoir where dissipation occurs, and contact interactions leads to a divergence of the energy increase rate. This divergence is a combined effect of the contact interactions, which impart arbitrary large momenta to the atoms, and the infinite energy width of the reservoir associated to its vanishing correlation time. We show how the divergence is regularized when taking into account the finite energy width of the reservoir, and, for large energy width, we give an expression for the energy increase rate that involves the contact parameter. We then consider the specific case of a weakly interacting Bose Einstein condensate, that we describe using the Bogoliubov theory. Assuming slow losses so that the gas is at any time described by a thermal equilibrium, we compute the time evolution of the temperature of the gas. Using a Bogoliubov analysis, we also consider the case where the regularization of the divergence is due to the finite range of the interaction between atoms.

The effect of the coupling of a many-body quantum system to an environment attracted a lot of attention in the last years, in the context of cold atoms experiments. Engineered coupling was proposed to realize particular many-body states [1, 2], including strongly correlated phases or highly entangled states [3]. It can also be used as a resource for quantum computation [4]. A particular coupling to an environment, that has received a lot of attention recently, is realized when the gas suffers from losses. Losses can produce highly correlated phases [5–8], induce Zenon effect [9–11], drive phase transition [12], lead to non-thermal states [13–16], and produce cooling [17–19]. In all the works mentioned above, the coupling to the environment is described assuming that the correlation time of the environment is much smaller than any characteristic evolution time of the system. Then the time evolution of the system obeys a universal Lindblad equation (see the review [20]) describing the coupling to an environment of vanishing correlation time. In this paper we show that this approximation is not always correct.

For a homogeneous single atom loss process, the universal Lindblad equation reads, for a gas in the continuous space,

$$\frac{d\rho}{dt} = -(i/\hbar)[H_0,\rho] + \Gamma \int d^d\mathbf{r} \left\{ -\frac{1}{2}\{\psi_{\mathbf{r}}^+\psi_{\mathbf{r}},\rho\} + \psi_{\mathbf{r}}\rho\psi_{\mathbf{r}}^+ \right\} \tag{1}$$

where $H_0$ is the Hamiltonian of the quantum gas, $\rho$ is its density matrix, $d$ is the dimension of system, $\psi_r$ annihilates an atom at position $\mathbf{r}$, and $\Gamma$ is the loss rate. For simplicity we consider here a single specy gas. Eq. (1) is universal in the sense that the loss process is characterized by a single parameter $\Gamma$, details of the reservoir being irrelevant.

The evolution under the above Lindblad equation is simple if one assumes the state of the gas is uncorrelated, for instance within a mean-field approximation : the population of each single particle state decreases exponentially [9]. However, interactions between atoms introduce correlations, which highly complicates the calculation of the effect of losses. In cold atoms experiments, the range of the interaction potential between atoms is typically much smaller than all length scales in the problem. Then the effect of interactions is well modeled by a contact interaction term. This description of interactions is also a universal model: details of the interaction potential is irrelevant and interactions are described by a single parameter, the scattering length. In this paper, we show that the combination of the two above universal models leads to unphysical predictions in dimensions higher than one: for a gas with contact interaction evolving under Eq. (1), the increase rate of the energy diverges.

The divergence of the energy increase rate originates from the following process. The contact interaction in the gas is responsible for singularities of the many body wavefunction when two atoms meet [21], leading, in dimension higher than one, to a diverging kinetic energy, this divergence being counterbalanced by the interaction energy such that the total energy is finite. The Lindblad dynamics of Eq. (1) assumes that loss events are instantaneous with respect with the gas dynamics: within the quantum trajectory description equivalent to the Lindblad dynamics [20] a loss event corresponds to the instantaneous action of the jump operator $\psi_{\mathbf{r}}$. Thus, just after a loss event occurred, the many-body wavefunction of the remaining atoms is equal to its value just before the loss event. This wave function presents a singularity when the position of an atom approaches the position of the lost atom. The divergence of the kinetic energy associated with this singularity, is no longer counterbalanced by interaction energy: it amounts to an infinite value of the energy in the system. Note that the infinitely large increase of the energy is made possible by the infinite energy available in the reservoir involved in the loss process: the vanishing correlation time is associated to an infinite energy width.

Several mechanisms could lead to a regularization of the above divergence. First, the finite range of the interaction between the atoms will introduce a cutoff that

prevents the divergence of the kinetic energy. Second, the reservoir has in practice a finite energy width which limits the maximum energy a loss event can deposit in the system. In this paper, we consider both regularizations, with an emphasis on the effect of the finite reservoir energy-width.

We first propose a model for the loss mechanism, with a finite energy width $E_{\rm res}$. Using an analysis of the 2-atoms case, we then derive the expected value of the energy density increase rate for a gas with contact interactions, valid for large $E_{\rm res}$. We find a general expression, that involves the contact parameter. Although our derivation concentrates on the Bosonic case for simplicity of notations, our results are general. To compute the evolution of the system beyond this limit of large $E_{\rm res}$, one needs a many-body model of the system that includes correlations between atoms introduced by interactions. We will concentrate on the case of a weakly interacting Bose-Einstein condensate and we use the Bogoliubov description. Within this framework, we compute the evolution of the energy. Assuming a loss rate much smaller than the relaxation rate of the gas, the system can be described locally at any time by a thermal equilibrium state. We compute the expected evolution of the temperature under the effect of losses. Within the Bogoliubov treatment, we also consider the case where the regularization comes from the finite range of the interactions.

*Model for the loss process.* We consider a gas made of particles of mass $m$ in dimension $d = 1$ (1D), $d = 2$ (2D) or $d = 3$ (3D), and we use periodic boundary conditions in a box of size $L^d$. A homogeneous one-body loss process occurs if, at each point, the atoms are coupled to a continuum. In a gas confined in 1D or 2D, the frozen dimension(s) could serve as the continuum, if atoms are coupled to an untrapped state. In 3D, the loss mechanism could be the de-excitation of the atoms, if the latter are in a metastable state, in which case the momentum of the emitted photon provides the continuum for the loss mechanism. Here instead we will consider a simpler, yet equivalent model [22], where the loss mechanism is induced by a noisy coupling to an untrapped internal state, the different Fourier components playing the role of the continuum. This would correspond to the effect of a noisy magnetic field for magnetically trapped atoms [19]. More precisely, we consider a coupling to the reservoir which writes

$$V = \int d^d\mathbf{r}\Omega(t)\psi_{\mathbf{r}} b^+_{\mathbf{r}} + h.c. = \sum_{\mathbf{p}} \Omega(t)\Psi_{\mathbf{p}} B^+_{\mathbf{p}} + h.c., \quad (2)$$

where $\psi_{\mathbf{r}}$, resp. $b_{\mathbf{r}}$, annihilates an atom of the system, resp. of the reservoir, at position $\mathbf{r}$, $\Psi_{\mathbf{p}}$, resp. $B_{\mathbf{p}}$, annihilates an atom of the system, resp. of the reservoir, of momentum $\mathbf{p}$, $h.c.$ is the abbreviation of "hermitian conjugate" and $\Omega(t)$ is a noisy function. $\mathbf{p}$ takes discrete values whose coordinates are multiple of $2\pi\hbar/L$, and we note $p = |\mathbf{p}|$. We define the energy-dependent rate $\Gamma(E)$ from the spectral density of $\Omega(t)$ according to

$$\Gamma(E) = \frac{1}{\hbar^2}\int d\tau e^{-iE\tau/\hbar}\langle\Omega^*(\tau)\Omega(0)\rangle \quad (3)$$

and we note $\Gamma_0 = \Gamma(0)$. We assume a Gaussian correlation function such that $\Gamma(E) = \Gamma_0 e^{-E^2/(2E^2_{\rm res})}$, where $E_{\rm res}$ is the energy width of the loss process, corresponding to a correlation time $\hbar/E_{\rm res}$. The energy of the state of momentum $\mathbf{p}$ in the reservoir is $p^2/(2m)$ where $m$ is the mass of the atoms, up to a constant term that could be compensated by a shift in $E$ of $\Gamma(E)$ and that we take equal to zero. If there would be a single atom in the system, its loss rate, obtained within a Born-Markov approximation [22], would be $\Gamma_0$. The Lindblad equation (1) is obtained by making $E_{\rm res} \to \infty$ at a fixed value of $\Gamma_0$: $E_{\rm res}$ then no longer play a role and the parameter $\Gamma_0$ entirely characterizes the loss process. However, as shown below, in presence of contact interactions between atoms, such an approximation leads to a divergence of the energy increase rate in dimension $d > 1$. In this paper, we consider a finite value for $E_{\rm res}$.

*Two atoms case.* Let us first investigate the behavior expected for a system comprising initially 2 atoms. In addition to the kinetic energy term, the Hamiltonian contains a contact interaction term. We go in the center-of-mass frame so that the total momentum is vanishing and we consider a state of energy $E_0$. The two-atoms wave function writes $\varphi(\mathbf{r}_1, \mathbf{r}_2) = \varphi(\mathbf{r}_1 - \mathbf{r}_2)$, with $\int\int d^d\mathbf{r}_1 d^d\mathbf{r}_2 |\varphi(\mathbf{r}_1 - \mathbf{r}_2)|^2 = 1$. For simplicity of notation, we will consider identical Bosonic atoms and use second quantization representation, such that this state reads $|\varphi\rangle = (1/2)\sum_{\mathbf{p}} \varphi(\mathbf{p})\Psi^+_{\mathbf{p}}\Psi^+_{-\mathbf{p}}|0\rangle$, where, for $\mathbf{p} \neq \mathbf{0}$, $\varphi(\mathbf{p}) = \sqrt{2}\int d^d\mathbf{r} e^{i\mathbf{p}.\mathbf{r}/\hbar}\varphi(\mathbf{r})$. The contact interaction imposes the short distance behavior $\varphi(r) \simeq u_0(|r| - a_{1D})$ in 1D, $\varphi(\mathbf{r}) \simeq u_0 \ln(|\mathbf{r}|)/a_{2D})$ in 2D and $\varphi(\mathbf{r}) \simeq u_0(1/|\mathbf{r}| - 1/a_{3D})$ in 3D, where the parameter $u_0$ depends on $d$, $E_0$ and $L$. In momentum space, this asymptotic form leads to the large $p$ behavior [21, 23]

$$|\varphi(\mathbf{p})|^2 \underset{|\mathbf{p}|\to\infty}{\simeq} \alpha_d \frac{\hbar^4|u_0|^2}{p^4}, \quad (4)$$

where $\alpha_d = 8$ in 1D, $\alpha_d = 8\pi^2$ in 2D and $32\pi^2$ in 3D.

The 2-atoms state $|\varphi\rangle$ is coupled by $V$ to the states $|\mathbf{p}\rangle = \Psi^+_{\mathbf{p}} B^+_{-\mathbf{p}}|0\rangle$ whose energy, equal to the sum of the kinetic energies of the lost atom and the remaining atom, is $p^2/m$. For weak enough $\Gamma_0$, one can use Born-Markov approximation to compute the loss rate towards the state $|\mathbf{p}\rangle$ [22]. Using $\langle\mathbf{p}|V|\varphi\rangle = \Omega(t)\varphi(\mathbf{p})$, one finds a loss rate

$$\gamma(p) = |\varphi(\mathbf{p})|^2\Gamma(p^2/m - E_0). \quad (5)$$

We can then compute the initial rate of change of the energy for the trapped atoms: $dE/dt = -\Gamma E_0 + \sum_{\mathbf{p}} p^2/(2m)|\varphi(\mathbf{p})|^2\Gamma(p^2/m - E_0)$. Here $\Gamma = \sum_{\mathbf{p}}\gamma(\mathbf{p})$ is the total loss rate. We will assume that $E_{\rm res}$ is large enough so that there exists a momentum $p_0$ such that $E_0 \ll p_0^2/m \ll E_{\rm res}$. The first inequality ensures that

the finite energy width of the reservoir does not affect the loss events towards states of momentum smaller than $p_0$. The second inequality ensures that, for final states of momenta larger than $p_0$, $|\varphi(\mathbf{p})|^2$ takes its large $p$ asymptotic behavior given Eq. (4). Then, the contribution to $dE/dt$ of decay processes towards momentum states of the remaining atom of momentum larger than $p_0$ is

$$dE/dt)_{|\mathbf{p}|>p_0} = \Gamma_0 \frac{\alpha_d \hbar^4 L^d |u_0|^2}{(2\pi\hbar)^d m} \mathcal{B}, \tag{6}$$

where

$$\mathcal{B} = \begin{cases} \int_{p_0}^{\infty} dp \Gamma(p^2/m)/(p^2\Gamma_0) \simeq 1/p_0 & \text{in 1D} \\ \pi \int_{p_0}^{\infty} \frac{dp}{p} \Gamma(p^2/m)/\Gamma_0 \simeq \frac{\pi}{4} \ln(\sqrt{mE_{\text{res}}}/p_0) & \text{in 2D} \\ 2\pi \int_{p_0}^{\infty} dp \Gamma(p^2/m)/\Gamma_0 \simeq \nu\sqrt{mE_{\text{res}}} & \text{in 3D.} \end{cases} \tag{7}$$

where $\nu = 6.769\ldots$. In 1D, the result does no longer depend on $E_{\text{res}}$: the energy change rate has a well defined finite value when $E_{\text{res}} \to \infty$. In the following we consider only gases in dimension $d > 1$. Then $\mathcal{B}$ presents a UV divergence when $E_{\text{res}} \to \infty$, which leads to the diverging energy change rate announced in the introduction. The finite value of $E_{\text{res}}$ regularizes this divergence. For large enough $E_{\text{res}}$ however, $\mathcal{B}$ is large enough such that Eq. 6 gives the main contribution to $dE/dt$ and in the next paragraph we assume $dE/dt$ is simply given by Eq. 6. Note finally that this two-atoms result could have been derived for two different atoms, such as two different fermions, providing losses affect both atomic specy in the same way.

*Many-body case: role of the contact.* The results above can be generalized to many-body systems containing $N$ atoms since, for large enough $E_{\text{res}}$, the physics will be dominated by the 2-body physics presented above. More precisely, one expects the above results to hold provided one does a sum over the pairs of atoms. The relevant quantity will be the contact $C$, which quantifies the number of pairs in the gas[21, 23, 24]. The contact is defined by the amplitude of the $1/p^4$ tails of the momentum distribution. More precisely, $C = \lim_{p\to\infty} W(p)p^4$, where the momentum distribution is normalized to $\int d^d\mathbf{p} W(\mathbf{p}) = N$. In the two atoms case discussed above, $W(p) = |\varphi(\mathbf{p})|^2 L^d/(2\pi\hbar)^d$ such that $C = \hbar^{4-d}\alpha_d |u_0|^2 L^d/(2\pi)^d$. Thus, Eq. (6) generalizes to a many-body system as

$$dE/dt = \Gamma_0 \frac{C}{m} \mathcal{B} \tag{8}$$

We emphasize the broad applicability of this expression: it is valid both in 2D and 3D, and for Fermions or Bosons. Its validity domain is however restricted to very large $E_{\text{res}}$. To go beyond this approximation, and to estimate its applicability range, one should know the details of the many-body physics. In the following we do the calculation in the case of a weakly interacting Bose gas described by the Bogoliubov theory.

*Exact treatment for a gas described by Bogoliubov* In this section we suppose the gas is a Bose condensed gas of density $n$. Beyond-mean field physics is captured, to first approximation, by the Bogoliubov theory. In this theory, the Hamiltonian reduces to

$$H_{BG} = e_0 L^d + \sum_{\mathbf{p}\neq\mathbf{0}} \epsilon_p a^+_{\mathbf{p}} a_{\mathbf{p}}, \tag{9}$$

where $a^+_{\mathbf{p}}$ creates a Bogoliuov excitation of momentum $\mathbf{p}$ whose energy is $\epsilon_p = \sqrt{p^2/(2m)(p^2/(2m) + 2gn)}$, and $e_0$ is the ground state energy density. Bogoliubov operators are bosonic operators which fulfill $[a_{\mathbf{p}} a^+_{\mathbf{p}}] = 1$, and they are related to the atomic operators by the Bogoliubov transform

$$\begin{cases} \Psi_{\mathbf{p}} = u_p a_{\mathbf{p}} + v_p a^+_{-\mathbf{p}} \\ \Psi^+_{-\mathbf{p}} = v_p a_{\mathbf{p}} + u_p a^+_{-\mathbf{p}} \end{cases} \tag{10}$$

where $u_p^2 - v_p^2 = 1$ and $v_p^2 = (f_p + f_p^{-1} - 2)/4$, and $f_p = p^2/(2m\epsilon_p)$. We set $u_0 = 1$ and $v_0 = 0$ such that the above equation also holds for $\mathbf{p} = \mathbf{0}$. Note that we use the symmetry breaking Bogoliubov approach that does not converse atom number[25].

Using the Bogoliubov transformation, the coupling to the reservoir, given Eq. (2), reads

$$V = \sum_{\mathbf{p}} a_{\mathbf{p}} (u_p \Omega(t) B^+_{\mathbf{p}} + v_p \Omega^*(t) B_{-\mathbf{p}}) + h.c. \tag{11}$$

We compute the master equation describing the time evolution of the density matrix of the system, $\hat{\rho}$: using second order perturbation theory, omitting fast oscillating terms, whose effect averages out, and making the Born-Markov approximation, we obtain [22]

$$\begin{aligned} \frac{d\hat{\rho}}{dt} = & -(i/\hbar)[H_0, \hat{\rho}] \\ & - \textstyle\sum_{\mathbf{p}} \Big\{ \Gamma(\frac{p^2}{2m} - gn - \epsilon_p) u_p^2 \left( \frac{1}{2}\{a^+_{\mathbf{p}} a_{\mathbf{p}}, \rho\} - a_{\mathbf{p}} \rho a^+_{\mathbf{p}} \right) \\ & + \Gamma(\frac{p^2}{2m} - gn + \epsilon_p) v_p^2 \left( \frac{1}{2}\{a_{\mathbf{p}} a^+_{\mathbf{p}}, \rho\} - a^+_{\mathbf{p}} \rho a_{\mathbf{p}} \right) \Big\} \end{aligned} \tag{12}$$

where the function $\Gamma(E)$ is given in Eq. (3). For non interacting atoms, $v_p = 0$, $u_p = 1$, $gn = 0$ and $\epsilon_p = p^2/(2m)$, such that the above equation reduces to Eq. (1), as expected. Correlations between atoms introduced by the interactions are responsible for the anomalous terms in $v_p^2$.

The first effect of losses is to decrease the density $n$. The difference between $dn/dt$ and $-\Gamma_0 n$ is of the order of the density of atoms in the modes of wave vector $p > \sqrt{mE_{\text{res}}}$. We assume that $E_{\text{res}}$ is large enough so that we can make the approximation $dn/dt \simeq -\Gamma_0 n$. Let us now investigate the evolution of the energy $E = \langle H_0 \rangle$. We use the Bogoliuobv approximation $H_0 \simeq H_{BG}$, where $H_{BG}$ is given in Eq. (9), such that

$$\frac{dE}{dt} = -\Gamma_0 n A + \sum_{\mathbf{p}} \epsilon_{\mathbf{p}} \left( \frac{d\langle a^+_{\mathbf{p}} a_{\mathbf{p}} \rangle}{dt} \right)_{BG} \tag{13}$$

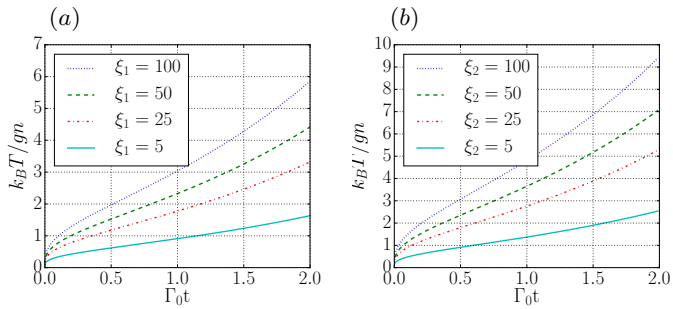


FIG. 1. Evolution of the temperature of a 3D weakly interacting Bose-Einstein condensate under the effect of losses. Temperature is rescaled to the time-dependent chemical potential $\mu \simeq gn \simeq gn_0 e^{-\Gamma_0 t}$, where $n$ is the atomic density, equal to $n_0$ at $t = 0$, $g$ the interaction strength and $\Gamma_0$ is the single atom loss rate. The initial value is $k_BT(0)/(gn_0) = 0.1$. In (a) interactions are contact interactions but the reservoir has a finite energy width $E_{\rm res}$, parameterized by the dimensionless parameter $\xi_1 = \sqrt{E_{\rm res}/(gn_0)}$. In (b), we assume a reservoir of infinite energy width, or equivalently of vanishing correlation time, but interactions have a finite range $\sigma$ (see text) and $\xi_2 = \hbar/(\sigma\sqrt{mgn_0})$.

where $A = L^d de_0/dn + \sum_{\mathbf{p}} \langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle d\epsilon_{\mathbf{p}}/dn$ and $(d\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle/dt)_{BG}$ is the evolution of $\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle$ within the Bogoliubov approximation. Inverting the Bogoliubov transform Eq. (10), we find that $a^+_{\mathbf{p}}$ and $a_{\mathbf{p}}$ depend explicitly on time, via the dependence of $u_p$ and $v_p$ on $n$. However, we assume that losses are slow enough so that one has adiabatic following: $\langle d(a^+_{\mathbf{p}} a_{\mathbf{p}})/dt\rangle \simeq 0$ [22]. Then $(d\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle/dt)_{BG}$ reduces to $(d\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle/dt)_{BG} = Tr(a^+_{\mathbf{p}} a_{\mathbf{p}} d\hat{\rho}/dt)$ and injecting Eq. (12) we obtain

$$\left(\frac{d\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle}{dt}\right)_{BG} = -\Gamma(\tfrac{p^2}{2m} - gn - \epsilon_p) u^2_{\mathbf{p}} \langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle + \Gamma(\tfrac{p^2}{2m} - gn + \epsilon_p) v^2_p \left(1 + \langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle\right). \tag{14}$$

In the case of a reservoir of infinite energy width, for which $\Gamma(E) = \Gamma_0$ for any $E$, the above equation reduces to $(d\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle/dt)_{BG} = \Gamma_0(-\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle + v^2_p)$. We recover here the results derived for 1D Bose gases in the quasicondensate regime [26], although those results were derived very differently. In particular, since $v^2_p \simeq (mgn)^2/\mathrm{p}^4$ at large $p$, we find that $\langle a^+_p a_p\rangle$ develops $1/p^4$ tails. In dimension 1, such tails are responsible for a failure of Tan's relation [16]. In dimension 2 and 3, such tails lead to the unphysical result that $dE/dt$ diverges. Proper physical results are obtained in higher dimensions only taking into account the finite energy width of the reservoir. For very large $E_{\rm res}$, $dE/dt$ is dominated by the second term of the r.h.s of Eq. (13), itself dominated by the large $p$ terms for which $v^2_p \simeq (mgn)^2/p^4$, $\epsilon_p \simeq p^2/(2m)$, and $\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle \simeq 0$. Evaluating the sum, and using the fact that the contact within Bogoliubov theory is $C = L^d(mgn)^2/(2\pi\hbar)^d$, we recover Eq. (8).

The system is ergodic in dimension $d > 1$: beyond Bogoliubov terms in $H_0$ include couplings between Bogoliubov modes which, in absence of losses and as long as local observables are concerned, ensure relaxation towards a thermal state. Here we assume that $\Gamma_0$ is much smaller than the relaxation rate so that, $\hat{\rho}$ relaxes at any time to the density matrix of a thermal state. The latter is characterized by the atomic density $n$ and the energy density, or equivalently by $n$ and the temperature $T$. The energy of the gas fulfills $E = E_{\rm th}(n, T)$, where $E_{\rm th}(n, T)$ is evaluated injecting the occupation factors $\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle = (e^{\epsilon_p/(k_BT)} - 1)^{-1}$ into Eq. (9). The time evolution of the gas is entirely characterized by the functions $n(t) = n_0 e^{-\Gamma_0 t}$ and $T(t)$. In order to compute $T(t)$, we evaluate $dT/dt$ in the following way. Since $E$ is conserved by the thermalization process, the calculation of $dE/dt$ with Eq. (13) and (14) is valid, providing one injects $\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle = (e^{\epsilon_p/(k_BT)} - 1)^{-1}$ in the r.h.s of Eq. (14). Once $dE/dt$ has been computed one can compute $dT/dt$ using $dE/dt = -\Gamma_0 n(\partial E_{\rm th}/\partial n)_T + dT/dt(\partial E_{\rm th}/\partial T)_n$. Calculations are detailed in the appendix.

In Fig. 1 we present time evolution of the temperature of the system, for different values of $E_{\rm res}$. We find that the ratio $k_BT/(gn)$ is a growing function of time. This contrasts with the prediction obtained for phonons, which are the Bogoliubov modes of momentum $p \ll \sqrt{mgn}$ : in absence of rethermalization between Bogoliubov modes, and for $E_{\rm res} \gg gn$, one expects that, for phonons, $k_BT/(gn)$ takes the asymptotic value $k_BT/(gn) = 1$ [13, 27]. The growing of $k_BT/gn$ is due to the contribution of high-$p$ Bogoliubov modes. The growing rate increases with $E_{\rm res}$, as expected: we expect that $dT/dt$ diverges as $E_{\rm res}$ goes to infinity.

*Regularization by a finite interaction range.* The Bogoliubov analysis can serve also to describe the regularization of the UV divergence due to finite interaction range. We consider here a two-body interaction potential $V(r) = ge^{-r^2/(2\sigma^2)}/((2\pi)^{3/2}\sigma^3)$, where $r$ is the distance between the two atoms and $\sigma$ is the interaction range. The Bogoliubov transform given in Eq. (10) is still valid, using the Bogoliubov spectrum [28] $\epsilon_{\mathbf{p}} = \sqrt{p^2/(2m)(p^2/(2m) + 2gne^{-p^2\sigma^2/(2\hbar^2)})}$. One can then compute the effect of losses as above. In the limit of infinite $E_{\rm res}$, the divergence of $dE/dt$ is regularized by the finite interaction range $\sigma$. For very small $\sigma$, $dE/dt$ is dominated by the large $p$ term of the sum in Eq. (13), for which $v^2_p \simeq (mgn)^2(e^{-p^2\sigma^2/\hbar^2})/p^4$. Evaluating the sum, we recover Eq. (8) with $\mathcal{B} = \pi^{3/2}\hbar/\sigma$. As above, from the calculation of $dE/dt$ due to losses, we compute the time evolution of the temperature in the system. Fig. 1 shows the time evolution of the temperature, for different values of $\sigma$. The evolution of $k_BT/(gn)$ is qualitatively similar to what is observed for contact interactions but finite energy width of the reservoir, $\hbar^2/(m\sigma^2)$ playing the role of $E_{\rm res}$.

*Conclusion.* Remarkably, although losses are ubiquitous in experiments, the description and the understanding of their effect is still at its infancy. Before this work,

effect of losses has been studied using the universal Lindblad equation Eq. (1). However, studies where made either in 1D, in which case the divergence of the energy increase rate does not exists, or for a gas confined in the lowest band of a lattice, in which case the lattice period provides a cut-off that prevents the divergence, or using a mean-field approximation that neglect correlations between atoms. This paper provides the first prediction for the effect of losses on an interacting quantum gas in higher dimension and in the continuum. Predictions of this paper could be tested experimentally using an engineered noisy coupling to an untrapped state [19] whose energy width $E_{\rm res}$ can be varied. Alternatively, the measurement of the temperature evolution could be used to infer the energy width of the reservoir. This work raises many questions. How can we extend the results obtained with Bogoliubov to quasi-condensate describing 2D gases at thermodynamic limit ? How can the results presented in this paper be extended to 2-body or 3-body losses ? How can we extend the calculations done in this paper to other models of quantum gases, such as two-components fermionic gases ? Our work put in question the fundamental relation giving the time evolution of the density under the effect of losses. In this paper, we assumed a reservoir energy width large enough so that $dn/dt$ remains close to $-\Gamma_0 n$. However, one expects a small correction involving 2-body processes, whose calculation deserves investigation.

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[22] See Supplemental Material for ($i$) the calculation of the loss rate for a single atom, ($ii$) the derivation of formula (5), ($iii$) the derivation of the equivalence between a noisy time-dependent coupling and a couling to a continuum of states, ($iv$) the derivation of the master equation Eq. (12), ($v$) the justification of the fact that $\langle a^+_{\mathbf{p}} a_{\mathbf{p}}\rangle$ is not affected by the slow time-variation of $u_p$ and $v_p$, and ($vi$) the calculation of the time-evolution of the temperature.

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[25] See appendix C of the Supplemental material for a discussion on this point.

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