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► **To cite this version:**

Foivos Karakostas, Nicholas Schmerr, Ross Maguire, Quancheng Huang, Doyeon Kim, et al.. Scattering Attenuation of the Martian Interior through Coda Wave Analysis. Bulletin of the Seismological Society of America, 2021, 10.1785/0120210253 . hal-03406123

HAL Id: hal-03406123

<https://hal.science/hal-03406123>

Submitted on 27 Oct 2021

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Scattering Attenuation of the Martian Interior through Coda Wave Analysis

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2021

CONFLICT OF INTEREST

The authors acknowledge there are no conflicts of interest recorded.

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Abstract

We investigate the scattering attenuation characteristics of the Martian crust and uppermost mantle to understand the structure of the Martian interior. We examine the energy decay of the spectral envelopes for 21 high-quality Martian seismic events from Sol 128 to Sol 500 of InSight operations. We use the model of Dainty et al. (1974b) to approximate the behavior of energy envelopes resulting from scattered wave propagation through a single diffusive layer over an elastic half-space. Using a grid search, we mapped the layer parameters that fit the observed InSight data envelopes. The single diffusive layer model provided better fits to the observed energy envelopes for High Frequency (HF) and Very High Frequency (VF) than for the Low Frequency (LF) and Broadband (BB) events. This result is consistent with the suggested source depths (Giardini et al., 2020) for these families of events and their expected interaction with a shallow scattering layer. The shapes of the observed data envelopes do not show a consistent pattern with event distance, suggesting that the diffusivity and scattering layer thickness is non-uniform in the vicinity of InSight at Mars. Given the consistency in the envelope shapes between HF and VF events across epicentral distances and the tradeoffs between the parameters that control scattering, the dimensions of the scattering layer remain unconstrained but require that scattering strength decreases with depth and that the rate of decay in scattering strength is fastest near the surface. This is generally consistent with the processes that would form scattering structures in planetary lithospheres.

39 INTRODUCTION

40 Scattering of seismic waves from random heterogeneities is a well-studied phenomenon (Aki
41 (1980); Ishimaru (1978); Wu (1982) and others) that depends strongly on the relative length scale
42 of the heterogeneity and wavelength of seismic waves. For elastic waves, random heterogeneities
43 are defined by contrasts between materials of differing seismic wave speeds, that are related to
44 changes in shear rigidity, bulk modulus, and density of materials. Such changes are common in
45 geologically complex materials, and expected where there are non-uniform variations in materi-
46 als with depth and location. A common simplifying assumption is to use linear scaling between
47 wavespeed and density when approximating these variations (Sato, 1990). A further simplifying
48 assumption in (semi)analytic approaches is that scatterers are isotropic, although full waveform
49 methods enable the treatment of anisotropic scatterers (Cormier, 1999). The strength of the pertur-
50 bation (i.e., random heterogeneity) is thus defined as the relative change in the seismic wave speed
51 that occurs across a discontinuous boundary in the material, usually represented with a percentile
52 of the relevant parameter. The other aspect of heterogeneity is size; scales of random perturbations
53 within a medium are typically characterized using an autocorrelation function, where the corre-
54 lation distance is an approximation for the size of the heterogeneities within the medium (see a
55 review of the topic by Shearer (2007)). In materials where the sizes of heterogeneities are large
56 compared to the seismic wavelength, weak forward scattering dominates. If the heterogeneities
57 are considerably larger than the seismic wavelength, then scattering effects become negligible.
58 Likewise, if heterogeneities are considerably smaller than the seismic wavelength, then scatter-
59 ing effects disappear and the medium behaves as a homogeneous solid. Scattering effects are
60 strongest when the sizes of the scatterers and seismic wavelength are similar and there is a large

61 seismic velocity contrast between neighboring heterogeneities (Aki and Richards, 2002). Thus,
62 by examining the contribution of scattering effects across a range of wavelengths (or equivalently
63 frequencies), it becomes possible to constrain the strength and size distribution of scatterers within
64 the medium through which the waves propagate. In this study we look into a range of events with
65 frequencies ranging from below 1 Hz to above 9 Hz. In the discussion section, we analyze the
66 frequency-dependence of the inferred scattering properties.

67 The analysis of the coda of seismic body waves (e.g., P- and S- waves) at different frequencies
68 provides valuable insight into the material properties of the medium through which they travel.
69 This is because the rate of decay of coda energy and the signature of energy loss are related to both
70 the intrinsic attenuation structure ($1/Q_i$) and seismic scattering ($1/Q_s$) in the subsurface. Wesley
71 (1965) and Aki and Chouet (1975) used single scattering and diffusion theory to describe how Q
72 can be inferred from S-wave coda analysis. The multi-scattering case and a diffusion equation was
73 used by Margerin et al. (1998) in order to develop the a radiative transfer equation to study the coda
74 waves in an inhomogeneous layered medium. Similar approaches that used the diffusion equation
75 in order to take into account the leakage from a diffusive layer to an underlying elastic half-space
76 were developed by Margerin et al. (1999) and Wegler (2003).

77 In most of the Earth's interior, seismic waves experience relatively weak scattering, allowing
78 for the direct observation of individual seismic body waves (like P- and S-waves) and surface
79 waves. Where scattering is present, it manifests as later-arriving codas that directly follow the
80 main arrivals and decay with time. Scattering is typically associated with small-scale composi-
81 tional heterogeneities in the crust (Revenaugh, 1999), lithosphere (Kennett and Furumura, 2016),
82 mantle (Mancinelli et al., 2016), core-mantle boundary region (Kim et al., 2020; Ma and Thomas,
83 2020), and even inner core (Leyton and Koper, 2007) of Earth. From a wide range of studies,

84 small-scale heterogeneities appear to be omnipresent throughout the Earth, with the exception
85 of the well-mixed outer core, and are typically assigned to compositional variations, as thermal
86 anomalies would not be long-lived in the Earth due to thermal diffusion (Shearer, 2007). Although
87 some environments like fault zones and volcanic edifices (Prudencio et al., 2013) show multiple
88 scattering, the majority of the Earth is typically characterized by relatively weak scattering that is
89 concentrated in the near surface, and allows for observations of direct seismic arrivals with weak
90 codas.

91 In contrast, seismic waves propagating within the Moon are dominated by scattering effects
92 (see Nunn et al. (2020) for a review). A significant fraction of the seismic energy produced by
93 moonquakes undergoing intense scattering in the near surface to the point that body waves cannot
94 be readily identified on seismograms, and surface waves are non-existent. The lunar crust has
95 undergone billions of years of impact gardening (Cintala, 1992) that has produced an upper surface
96 layer of regolith, with a seismic P-wave velocity of 100 – 300 m/s (Kovach and Watkins, 1976),
97 and underlain by a megaregolith consisting of fractured and cracked materials that may extend
98 to 30 km depth (Lognonné et al., 2003). This impact-modified layer and a lack of intragranular
99 fluids create a strongly-scattering environment for seismic waves. Dainty et al. (1974b) inferred the
100 properties of a scattering layer by analytically relating them to the energy envelopes of natural and
101 artificial impact events. They identified the density of the scatterers in the lunar shallow structure,
102 as well as an attenuation factor of $Q = 5000$ for events with dominant frequencies 0.5 to 1 Hz.
103 Since these foundational studies, the thickness of the lunar scattering layer still remains under
104 debate, with recent estimates extending the scattering layer to 100 km or more, with a $Q_s \leq 10$
105 (Blanchette-Guertin et al., 2012, 2015; Garcia et al., 2019; Gillet et al., 2017). As strong scattering
106 in the lunar crust and megaregolith likely results from nearly ubiquitous fractures and cracks that

107 extend into lunar rocks, as well as from the thick blanket of ejecta deposits and impact melt that
108 persists in the shallower crust, the properties and thickness of the layer are expected to be highly
109 variable across the surface of the Moon (Nakamura et al., 1975). Although the upper layer of
110 scattering produces seismic codas that last well over an hour and obfuscate the detection of body
111 waves, signal processing and polarization filtering have enabled successful detections of a lunar
112 core (Garcia et al., 2011; Weber et al., 2011) as well as other internal interfaces (see Lognonné and
113 Johnson (2015) for a review).

114 The landing of InSight in late 2018 (Banerdt et al., 2020) now presents the opportunity to
115 study the nature of seismic waves propagating within Mars. While Mars has extensive surface
116 impact cratering, it also possesses an atmosphere and evidence of resurfacing through erosion and
117 deposition of sediments by liquid water and lava flows (Carr and Bell, 2014). Therefore, it would
118 be expected that the planet might lie somewhere between the weak scattering regime present on
119 Earth and the highly diffusive wave propagation found on the Moon.

120 InSight is the first ever seismometer to operate directly on the surface of Mars, with the Viking-
121 1 and 2 landers having also brought seismometers that were both lander mounted (Anderson et al.,
122 1976). Unfortunately Viking-1's seismometer failed to uncage, and only a single seismic event was
123 identified on the Viking-2 seismometer (Lazarewicz et al., 1981). InSight has successfully detected
124 over 500 marsquakes during its two first years of operation on the surface (InSight Marsquake
125 Service, 2021), enabling studies of scattering and seismic attenuation in the Martian interior. Based
126 on the methodology of Margerin et al. (1998), a radiative transfer model was used by Lognonné
127 et al. (2020) to constrain, for the first time, the attenuation and scattering structure of the Martian
128 crust. Lognonné et al. (2020) reported different diffusivities in the Martian upper crust depending
129 on the frequency content of the examined events. Receiver function analyses of InSight data has

130 revealed crustal layering, including a shallow, 10-km thick, low velocity layer below the InSight
131 lander (Knapmeyer-Endrun et al., 2021; Lognonné et al., 2020).

132 Additional marsquakes, available in the 5th version of the Mars Seismic Catalog (InSight
133 Marsquake Service, 2021), offer the opportunity to examine the scattering properties of Mars more
134 thoroughly and constrain the characteristics of the scattering attenuation in the upper crust. There-
135 fore, in this study, we use this extended marsquake dataset and systematically explore a series
136 of parameters that control the seismic scattering on Mars. Additionally, we further examine and
137 analyze the frequency-dependence of diffusivity reported by Lognonné et al. (2020). Our objec-
138 tive is to provide further insights and explore the limits of scattering analysis given the available
139 marsquake dataset and the context of a single seismometer on a planet.

140 Martian seismic events are classified by their spectral properties Giardini et al. (2020) into
141 different event types that occur across a range of dominant frequencies and distances from the
142 InSight lander. According to Clinton et al. (2021), when the dominant frequency range of the
143 event is below the 2.4 Hz resonance (Ceylan et al., 2021; Kim et al., 2021a), it is considered
144 a Low Frequency event, whereas the Broadband events include seismic energy excitation below
145 and above 2.4 Hz. High Frequency and Very High Frequency events are dominantly above the
146 threshold defined by the 2.4 Hz resonance.

147 These are the basic features of the main categories of the events that are examined in this study:

- 148 • Low Frequency (**LF**) events show a very rapid coda decay (less than 1 minute). The seismic
149 catalog (InSight Marsquake Service, 2021) locates these events to epicentral distances greater
150 than 30° .
- 151 • Broadband (**BB**) events are located at relatively large distances (more than 30° and their

152 coda decay is around 1.5 times longer than that of Low Frequency events. Previous studies
153 (Giardini et al., 2020) located these events in the Cerberus Fossae region.

- 154 • High Frequency (**HF**) events are located in the $20^\circ - 30^\circ$ distance range. However, the
155 distance estimates depend substantially on the choice of velocity model. HF decay times are
156 longer than those of families where lower frequencies (below 1 Hz) are dominant.
- 157 • Very High Frequency (**VF**) events are further classified in two families according to their
158 epicentral distance: one, very near family closer than 20° ; and a second, more distant family,
159 further than 30° . The coda decay duration of VF events is relatively long, comparable to that
160 of HF events, and does not appear to correlate with the epicentral distance.

161 Teleseismic events with identifiable P and S waves are characterized by dominant frequencies
162 $f < 1$ Hz, and the body waves show some degree of coda following the initial arrivals. The lack of
163 strong scattering and detectable surface waves in these events is interpreted to imply that they must
164 have a deeper hypocenter and therefore the recorded waves were generated in a medium where
165 scattering is weak. Local events are associated with higher frequency content, typically having
166 their energy as $f > 2.4$ Hz, and are characterized by strong codas following the P and S waves.
167 There are some events across all the different types that are located at an epicentral distance of
168 30° . Among them, the lower frequency events (LF and BB) exhibit a different spectral character
169 than the higher frequency ones (HF and VF) which is interpreted to result from shallow sources
170 and wave interaction with scatterers in the low velocity layer found in the uppermost 10 km of the
171 crust.

172 van Driel et al. (2021) conducted an analysis of the High Frequency seismic events detected by
173 InSight. They modeled the spectral envelopes of High Frequency events with the Spectral Element

174 Method (SEM) which could replicate the behavior of the HF event energy envelopes by placing
175 a layer of strong scattering in the uppermost portion of their crustal model. They concluded that
176 to explain the observations of high frequency seismic energy and its propagation over significant
177 time and distance, the shallow Martian crust had to possess high Q and some degree of scattering.
178 However, they did not attempt to directly constrain the scattering structure with their models.

179 In this study, we investigate the observation that Martian events dominated by lower frequency
180 energy appear to have shorter coda decays than those with higher frequency content. It is also
181 observed that the envelopes of the BB events for frequencies lower than 1 Hz appear to have longer
182 coda decays than the respective of LF events. We hypothesize that the frequency signature of these
183 events and the coda decay are associated with their ray paths through the Martian interior. Giardini
184 et al. (2020) suggested the LF events are associated with rays that crossed the mantle depths and
185 a mantle low velocity zone (described by Khan et al. (2021)), whereas the HF (and VF) event
186 rays are primarily trapped in the diffusive and lower velocity part of the crust (see an analysis
187 by Knapmeyer-Endrun et al. (2021)). This hypothesis could explain the longer coda decays for
188 the HF and VF events. We systematically model the codas observed on the vertical component
189 for an expanded dataset of both lower (LF, BB) and higher frequency (HF, VF) types of events,
190 in order to quantify the scattering properties of the Martian interior. Our approach investigates
191 the characteristics of the S-wave coda decay for these marsquake event types, and examines the
192 tradeoffs between the layer diffusivity, thickness, velocity ratio, and background intrinsic Q of
193 the Martian crust. We investigate how these constraints vary across event types to expand upon
194 results identified in three previous studies of the Martian seismic attenuation (Lognonné et al.,
195 2020; Menina et al., 2021; van Driel et al., 2021), and determine the shallow diffusive structure
196 that is consistent across event types.

197 **DATA**

198 **The Martian Seismic Events**

199 We analyze waveforms of 21 marsquakes recorded on the vertical component of the Very Broad
200 Band (VBB) seismometer (InSight Mars SEIS Data Service., 2019; InSight SEIS Science Team,
201 2019; Lognonné et al., 2019). We select seismic events from the Martian Seismic Catalog (InSight
202 Marsquake Service, 2021), which classifies seismic events based on their quality and frequency
203 content. We use 19 quality B events, which are defined by the identification of either multiple
204 clear phases but no polarization (identifiable distance but no location) or polarization but no clear
205 phase picks (identifiable azimuth but no distance), and 2 quality A events, which are defined by
206 both clear phase picks and polarization and therefore their distance and azimuth are identifiable
207 (Clinton et al., 2021).

208 We select InSight raw data (InSight Mars SEIS Data Service., 2019; InSight SEIS Science
209 Team, 2019) for a time window that starts 30 minutes before and ends 90 minutes after the indicated
210 time of each seismic event in the Events Catalog InSight Marsquake Service (2021). In each
211 seismogram we remove the instrument response through deconvolution, and then rotate the data
212 from the modified Galperin arrangement (Lognonné et al., 2019) to vertical (Z) and horizontal
213 (North, East) components. We then taper the data using a window size of 5% of the length of the
214 seismogram with a Tukey window.

215 We select the events on the basis of the clarity of the S-wave coda decay, particularly the ab-
216 sence of any glitches (see Scholz et al. (2020)) that would affect the examined signal and therefore
217 contaminate our analysis. Because the seismic data recorded by SEIS include a number of peculiar

218 signals arising from coupling between different InSight sensors and spacecraft components (Kim
219 et al., 2021a), we manually examine each event waveform to ensure that the effect of those signal
220 irregularity was minimal for our analysis.

221 The 5th version of the Seismic Catalog (InSight Marsquake Service, 2021) contains 1 LF and
222 1 BB, Quality A events, and 6 LF, 2 BB, 32 HF and 10 VF, Quality B events. We exclude from our
223 analysis waveforms with glitches present or events with no discernible S-wave arrival, yielding
224 a final dataset comprised of 21 marsquakes that occurred between the Sols 128 and 500 of the
225 InSight operations on Mars. Our dataset includes events in all four marsquake families; 5 Low
226 Frequency (LF), 3 Broadband (BB), 8 High Frequency (HF), and 5 Very High Frequency (VF)
227 events, as shown in Table 1.

228 **Spectral envelopes selection**

229 We calculate spectral envelopes by first computing spectrograms of the vertical velocity timeseries
230 for each seismic event using a window length of 50 s with 90% overlap. The time window for each
231 spectrogram starts 30 minutes before and ends 90 minutes after the official time of the event listed
232 in the Seismic Catalog. The event appears as a distinct region of higher amplitudes. We use a
233 visual interactive tool in order to manually choose the desired lower and upper frequency of the
234 energy envelope and the start and end of the event timeseries. The selected frequency range for
235 each event can be found in Table 1 and the time window of the selected waveforms is shown in
236 Figure 1. The spectral amplitude is then summed over a desired frequency range at each point in
237 time to create the smoothed event envelope, with a sampling rate of 5 Hz.

238 The 1 Hz tick noise, which is discussed in the study of Ceylan et al. (2021), affected the
239 selection of the examined envelopes. The amplitude of this periodic signal is high enough to affect

240 the quality of the data (see an analysis by Kim et al. (2021a)) and therefore required us to select
241 only the portion of BB-type events with energy below this threshold. In the frequency range below
242 1 Hz we observe that the coda decay times of the BB events do not differ significantly from the LF
243 events.

244 Examples of energy envelopes from each frequency dependent event type are shown in Figure
245 1. In the timeseries of the event, we select only the part of the S-wave arrival and coda decay
246 for our analysis. We manually pick the S-wave arrival and we use the envelope until the end of
247 the selected time window in our analysis, in order to include the whole S-wave coda decay. We
248 note that the selected time window almost certainly includes Sg and other crustal seismic phases;
249 however, we demonstrate later in the Methodology section that inclusion of these phases does not
250 affect the results of our investigation.

251 Figure 2 presents the complete dataset that is used in this study, and identifies where several
252 glitches that are excluded from our analysis exist. When InSight seismograms are plotted as time
253 series, it is difficult to immediately identify glitches. However, when we analyze the signal with our
254 visualization tool that shows the spectrogram of each event, these glitches are readily identified by
255 a characteristic band of energy with a broadband signature that extends from the highest to lowest
256 frequencies, in the range of $f < 0.1$ Hz. The low frequency energy is diagnostic of a glitch, as
257 none of the cataloged marsquakes possess substantial energy at these low frequencies.

258 **METHODOLOGY**

259 We use the InSight SEIS-VBB seismogram dataset above to investigate the characteristics of scat-
260 tering attenuation in the Martian crust and upper-most mantle. To study scattering, we define a

261 structural model consisting of a diffusive layer overlying an elastic halfspace. We assume the pres-
262 ence of a shallow diffusive structure similar to Lognonné et al. (2020), and focus upon the strong
263 scattering found in the near surface of planetary bodies. The underlying elastic layer in our inves-
264 tigation is assumed to be a half-space, which means that we do not infer the depth of the Moho or
265 other underlying seismic interfaces (Kim et al., 2021b; Knapmeyer-Endrun et al., 2021). Further-
266 more, we show that the characteristics of the elastic half-space do not affect our results: the only
267 parameter of the elastic half-space that enters our analysis is its average seismic velocity, which is
268 only used to define velocity ratio between it and the overlying, scattering layer.

269 The diffusive layer is assumed to be an isotropic homogeneous layer, with a unique average
270 diffusivity and average seismic velocity. The current data from Mars are not extensive enough to
271 enable the mapping of lateral and azimuthal variations in these parameters. As is explained in the
272 following section, the effective thickness of the diffusive layer seen by the seismic waves depends
273 upon the epicentral distance and the velocity ratio with the underlying elastic layer.

274 **The average seismic ray approach**

275 Figure 3 shows the geometry of the two layers used in our model. The diffusive layer is assumed
276 to contain a homogeneous distribution of scatterers. Seismic waves are generated at the source and
277 travel in all directions. The wave propagation direction from source to receiver is thus represented
278 by a seismic ray. The seismic rays, which are shown in white, change directions when they interact
279 with a scatterer, as the waves are either reflected, refracted, or absorbed. The reflection of the
280 waves by the scatterers is discussed here, whereas the effect of absorption and therefore the loss of
281 energy in the diffusive layer is summarized through a single quality factor, Q .

282 Margerin et al. (1998) examined in detail the transition from the elastic to diffusive regime
283 by modeling coda waves using a solution for the radiative transfer equation (Papanicolaou and
284 Burridge, 1975). In this study, we use the approach of an average seismic ray, from the source to
285 the station, in order to represent the average path of the seismic rays in the diffusive and elastic
286 layer. The direction of this average ray is not affected by the presence of the scatterers and it
287 follows the path of a seismic wave propagating in two elastic layers with different velocities. In
288 other words, the diffusivity in the top layer is translated into a lower apparent seismic velocity. The
289 range of the seismic ray is defined by the velocity ratio between the two layers, which is the ratio
290 of the apparent velocity in the top diffusive layer and the real seismic velocity in the underlying
291 elastic layer. The seismic velocities are related to the speed of S-waves in each medium as our
292 investigation is focused on S-wave related attenuation.

293 **The geometry of the seismic ray**

294 The geometry of the average seismic ray is shown in Figure 4. The diffusive layer with an apparent
295 S-wave velocity V_d overlies the elastic half-space, which has an S-wave velocity V_e . In reality, V_d
296 is not constant in the crust and it changes with depth. The reader should note that the thickness
297 of the diffusive layer, h , is exaggerated in the Figure. In addition, it should be noted that the only
298 discontinuity shown in this schematic representation is the one between the aforementioned two
299 layers, whereas other discontinuities of the planetary internal structure (e.g. Moho, Core-Mantle
300 Boundary) are omitted. Therefore, the elastic layer is a space extended towards the center of the
301 planet, but its properties are only relevant in the depth range traveled by the examined seismic ray.

302 In the same Figure 4, the planet's radius is noted by R , and the epicentral distance in units of
303 length by Δ and radians by ϵ . The range of the seismic ray corresponds to the projection on the

304 surface of the planet of the ray path in the diffusive layer, and it is noted with r in length units.
 305 On the left side, where the ray is refracted into the elastic layer, this range is divided in two equal
 306 parts, one on the source side and one on the station. On the right side, where the ray represents
 307 the reflection, the range, r , coincides with the epicentral distance, Δ . The range is expressed in
 308 radians by θ_r , which is divided in two equal angles on the left (refraction) part and coincides with
 309 the epicentral distance on the right (reflection). In the refracted ray case (on the left) the incident
 310 angle is noted with i and the angle of refraction with ψ .

311 We use this simplified geometry to compute the range of the seismic ray in the diffusive layer,
 312 depending on the thickness of the layer and the velocity ratio V_d/V_e . The reflection case on the right
 313 is part of the refraction case on the left, which the refraction involving additional wave propagation
 314 in the elastic layer. The relationship among the layer thickness to the range and velocity ratio is
 315 given in Equation 6.

316 For the right part of Figure 4 we have:

$$\sin\left(\frac{\theta_r}{2}\right) = \frac{AC}{R} \quad (1)$$

$$\sin\left(\frac{\theta_r}{2}\right) = \frac{GD}{h} \quad (2)$$

$$\cos\frac{\theta_r}{2} = \frac{GF}{h} \quad (3)$$

317 This is used to compute the angle of the incident ray:

$$\tan(i) = \frac{AC}{EC} = \frac{R \sin\left(\frac{\theta_r}{2}\right)}{h - R\left(1 - \cos\left(\frac{\theta_r}{2}\right)\right)} \quad (4)$$

318 Meanwhile, we have:

$$\psi = \frac{\pi}{2} - \frac{\varepsilon - \theta_r}{2} \Rightarrow \sin(\psi) = \cos\left(\frac{\varepsilon - \theta_r}{2}\right) \quad (5)$$

319 Therefore, using Snell's law, the velocity ratio can be expressed as:

$$\begin{aligned} \frac{V_e}{V_d} &= \cos\left(\frac{\varepsilon - \theta_r}{2}\right) \frac{1}{\sin\left(\arctan\left(\frac{R \sin\left(\frac{r}{2R}\right)}{h - R(1 - \cos\left(\frac{r}{2R}\right))}\right)\right)} \Rightarrow \\ \arcsin\left(\frac{V_d}{V_e} \cos\left(\frac{\varepsilon - \theta_r}{2}\right)\right) &= \arctan\left(\frac{R \sin\left(\frac{r}{2R}\right)}{h - R(1 - \cos\left(\frac{r}{2R}\right))}\right) \Rightarrow \\ \frac{R \sin\left(\frac{r}{2R}\right)}{h - R(1 - \cos\left(\frac{r}{2R}\right))} &= \tan\left(\arcsin\left(\frac{V_d}{V_e} \cos\left(\frac{\varepsilon}{2} - \frac{r}{2R}\right)\right)\right) \end{aligned}$$

320 which gives:

$$h = \frac{R \sin\left(\frac{r}{2R}\right)}{\tan\left(\arcsin\left(\frac{V_d}{V_e} \cos\left(\frac{\varepsilon}{2} - \frac{r}{2R}\right)\right)\right)} + R \left(1 - \cos\left(\frac{r}{2R}\right)\right) \quad (6)$$

321 A consequence of this equation is that for a given epicentral distance, there exists a unique pair
 322 of range and diffusive layer thickness for a given velocity ratio (i.e., a ray parameter that satisfies
 323 both). In Figure 5 we show the values obtained for the range of the seismic ray in a unit layer
 324 thickness ($h = 1$) and epicentral distance $\varepsilon = 20^\circ$, 50° and 100° as a function of the velocity ratio,
 325 V_d/V_e .

326 Computation of energy envelopes

327 A common approach in seismic coda analysis is to model the energy envelope of the seismic
 328 wavefield, which discards information about polarity and phase in favor of the shape of the coda
 329 decay. To fit the shape of the S-coda energy envelope, we use the Dainty et al. (1974b) equation to
 330 compute a theoretical energy envelope of the S-coda waves, considering an impulse at the source:

$$i(t) = \frac{4}{\pi \xi_H t h} \exp\left(-\frac{r^2}{\xi_H t} - \frac{wt}{Q}\right) \sum_{n=1}^{\infty} \frac{\alpha_n}{2\alpha_n + \sin 2\alpha_n} \exp\left(-\frac{t \xi_V \alpha_n^2}{4h^2}\right) \quad (7)$$

331 where ξ_H and ξ_V are the horizontal and vertical diffusivity components, t the time, h the diffu-
 332 sive layer thickness, r the seismic ray range in cylindrical coordinates, w the frequency calculated
 333 as $w = \frac{(f_{min} + f_{max})}{2}$, where f_{min} and f_{max} are respectively the minimum and maximum frequency
 334 of the selected envelope from the data, Q the attenuation factor, and α the positive roots of the
 335 equation:

$$\alpha \tan \alpha = \frac{4hv}{\xi_V} \quad (8)$$

336 where v is the seismic velocity in the underlying elastic halfspace. This means that v corre-
 337 sponds to the V_e term of Equation 6, so that holding fixed the velocity in the diffusive layer, v will
 338 increase with the diffusive layer thickness, h . Therefore, the boundary condition that is described
 339 by Equation 8 depends on the diffusive layer thickness.

340 This equation was developed for lunar impacts and the shown form is valid only for shallow
 341 seismic events, with depth $z = 0$ in cylindrical coordinates. As we discuss later, this assumption
 342 makes the methodology less appropriate when applied analyzing coda of deep marsquakes.

343 **Diffusivity computation and its dependence on the geometry**

344 We assume a diffusivity in the diffusive layer $D = \xi_H = \xi_V$ although we note that in practice the
 345 ratio of the horizontal and vertical diffusivity is typically greater than one owing to the additional
 346 seismic energy contributed by surface waves to the horizontal component of motion. Dainty et al.
 347 (1974a) defined the relationship of the diffusivity, D , with the free mean path, l , and the velocity

348 of wave propagation, v , as following:

$$D = \frac{vl}{3} \quad (9)$$

349 The transport mean path, l is given by:

$$l = (\sigma n)^{-1} \quad (10)$$

350 where σ is the cross-sectional area and n the number of particles per unit volume. If we con-
 351 sider two layers with the same seismic wave velocity, $v_1 = v_2$, we find that the diffusivities D_1
 352 and D_2 are proportional to l . In order to compute its value we can consider a cuboid of dimen-
 353 sions h (the diffusive layer thickness) and r (the range of the seismic ray). The number of events
 354 corresponding to the cross section of this cuboid is equal in all directions, as we assumed earlier
 355 that $\xi_H = \xi_V$. Given that n is the number of particles per unit volume, corresponding to a cross
 356 sectional area σ , the number of scattering events, N , in a cross-sectional area $S = hr$ is:

$$N = Sn = hrn \quad (11)$$

357 Starting with an impulsive signal, in order to obtain the same envelope for $v_1 = v_2$ but different
 358 size of the diffusive layer, we need to have $N_1 = N_2$. Using the Equation 11, we have:

$$N_1 = N_2 \Rightarrow h_1 r_1 n_1 = h_2 r_2 n_2 \Rightarrow \frac{n_1}{n_2} = \frac{h_2 r_2}{h_1 r_1} \quad (12)$$

359 Then we solve this on the basis of the definition of diffusivity (Equation 9) to obtain:

$$\frac{D_1}{D_2} = \frac{vl_1}{vl_2} = \frac{l_1}{l_2} = \frac{\sigma n_2}{\sigma n_1} = \frac{n_2}{n_1} \quad (13)$$

360 Using Equation 12 we have the relationship:

$$\frac{D_1}{D_2} = \frac{h_1 r_1}{h_2 r_2} \Rightarrow D_1 = \left(\frac{h_1 r_1}{h_2 r_2} \right) D_2 \quad (14)$$

361 This relationship between the thickness of the diffusive layer and therefore the range (which is
362 itself uniquely related to the velocity ratio) of the seismic ray, leads to a tradeoff with the diffusivity
363 that is discussed in the results. Thus, in order to explore the effect of each parameter, we perform
364 a grid search over the model space. Similarly, Lognonné et al. (2020) used 3 different crustal
365 thicknesses ($h = 20, 40$ and 60 km) to investigate the respective diffusivity for the Martian crust.

366 In Figure 6 we show the effect of this tradeoff on the computed energy envelopes. On both
367 left and right panels we use the same velocity ratio between the diffusive and elastic layer, the
368 same Q attenuation factor and the same frequency, ω , as these variables are contained in Equation
369 7. We compute energy envelopes for 3 different layer thicknesses, $h = 10, 20$ and 30 km and
370 solve Equation 6 for the self-consistent value of r . In the left panel, we fix the diffusivity to
371 $d = 0.1 \text{ km}^2/\text{s}$, and can observe that the coda decay duration is longer for a thicker diffusive layer.
372 In the right panel, we adjust the diffusivity for each layer, using Equation 14 and the thickness
373 of 20 km as a reference. We observe that for all three layer thicknesses, the adjusted diffusivities
374 produce identical envelope shapes. Due to this complete tradeoff between diffusivity and thickness,
375 we only need to compute the results for a given layer thickness, obtain the other parameters of
376 Equation 7 for that layer thickness, and then adjust their values accordingly for the thickness of the
377 diffusive layer.

378 **Reverberation dependence**

379 A possible complication to our average ray path assumption is the effect of rays that do not fol-
380 low the direct path of the refracted ray, but rather reverberate within the scattering layer(s). As
381 described earlier in this section, with the approximation of the average seismic ray, due to the
382 reflection on the interface with the elastic layer, the reverberations in the diffusive layer can be
383 considered part of the diffusion of the seismic waves. Therefore, a hypothetical n number of rever-
384 berations of range r can be modeled as a unique seismic ray of range $n \cdot r$ in one layer of a given
385 thickness. The diffusivity in this case should be adjusted with the use of Equation 14 for the new
386 range of the seismic ray.

387 In order to observe the effect of this adjustment, we perform a test that is shown in Figure 7.
388 On the left panel, we show in blue the envelope for a single ascending ray in a 20 km layer, with
389 a $Q = 1000$ and $d = 1 \text{ km}^2/\text{s}$. Using the same layer thickness and Q , we compute the energy
390 envelope for a seismic ray that is generated on top of the diffusive layer and is reflected on the
391 interface with the elastic layer. If we define the range of the single ascending ray as r , in the case
392 of a single reflection, we are computing two distinctive envelopes. The first is the result of an
393 impulse, using the Equation 7, a range $r/2$ and diffusivity $2 \text{ km}^2/\text{s}$ (with the use of Equation 14).
394 The second is the result of the same computation, however instead of inputting an impulse, we use
395 the result of the first envelope computation. The result of this single reflected ray is shown in red.
396 We apply the same methodology accordingly to the cases of a double and triple reflection in the
397 diffusive layer, showing the respective results in yellow and purple. On the right panel, the same
398 computations are performed for an initial diffusivity $d = 5 \text{ km}^2/\text{s}$.

399 We observe that the computed envelopes do not differ significantly by adding more reverbera-

400 tions to the ray path. More precisely, the computed deviation of every case of reverberations versus
401 the case of the ascending ray is shown in Table 2. This deviation does not appear to depend on the
402 number of the reverberations or the diffusivity, whereas in all cases it is much less than 0.1 which
403 is approximately the best misfit that we find in the results presented in the next section.

404 It is important to note that there is no ballistic wave in the diffusion model, hence no reflected
405 waves in the traditional sense. As illustrated by Margerin et al. (1998), the range of validity of the
406 diffusion models extends to a point where the mean free path, l , is longer than the thickness of the
407 scattering/diffusive layer; modeling beyond this range of validity is outside the scope of this study.

408 **RESULTS**

409 Here we examine the fit between the observed spectral energy envelopes from the Martian data
410 and the theoretical scattering model. We perform a grid search by using a range of values for the
411 parameters of Equation 7, which define the characteristic diffusivity and scattering attenuation in
412 the shallow Martian lithosphere. Models that minimize misfit with the data are considered possible
413 structures for the interior of Mars.

414 Previous studies (Lognonné et al., 2020) have noted that it is impossible to constrain the diffu-
415 sive layer thickness based on the S-coda wave analysis alone, because layer thickness has a tradeoff
416 with the diffusivity. As discussed in the Methodology section, we do not need to vary the layer
417 thickness in the computations because the results for any desired layer thickness can be calculated
418 given the range of the seismic ray and the diffusivity. In our grid search, we use a fixed layer thick-
419 ness, $h = 20$ km, and adjust the other parameters (thickness, range) accordingly. Figure 8 shows
420 the necessary adjustment of the diffusivity, depending on the layer thickness.

421 The range of the seismic ray depends on the epicentral distance and the velocity ratio between
 422 the diffusive and elastic layer, and is obtained by solving Equation 6 for r . The epicentral distance
 423 for all the studied events is given in the Seismic Catalog (InSight Marsquake Service, 2021) and
 424 reproduced in Table 1, and we perform our investigation for 5 different velocity ratios, $V_d/V_e =$
 425 0.15, 0.18, 0.20, 0.25, 0.30. One can note that the value for this ratio is much smaller than
 426 the velocity ratio expected between typical seismic discontinuities; for example, the crust and the
 427 mantle on Earth has a velocity ratio of around 0.6, and other crustal layers may be even higher.
 428 However, because we base our modeling approach on the average seismic ray through the non-
 429 scattering and scattering medium, we expect the apparent velocity in the diffusive layer to be
 430 lower due to the scattering. Finally, we search in the range of $Q_s = 100$ to 2000 for the quality
 431 factor of in the scattering layer.

432 For each case of the aforementioned parameters, we select a time window for the computation
 433 of the misfit between the data (spectral envelopes) and the prediction (computed energy envelopes).
 434 To calculate misfit, we first align the observed and modeled envelopes on their peak, and trim the
 435 model data series in the appropriate time window that corresponds to the S-wave arrival and coda
 436 of the data, as explained in the Data Processing section. We finally compute the Root Mean Square
 437 Error (RMSE) between the model and data:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (d_i - s_i)^2}{N}} \quad (15)$$

438 where d_i is the observed spectral envelope and s_i the synthetic one.

439 In Figure 9 we show the data and computed models for an event of each frequency type. The
 440 black curve shows the spectral envelope computed from the vertical component of the velocity

441 seismogram. The red area indicates the margins of normalized amplitudes for a range of models
442 that provide an RMSE lower than the indicated threshold on the top of each example. It is noted
443 that the RMSEs are much higher for the LF and BB events, whereas the HF and VF events show
444 a better match to the model prediction. This is consistent with the suggested location and focal
445 depth of the events, as was analyzed by Giardini et al. (2020). The LF and BB events are located
446 at teleseismic distances and are inferred to be deep, possibly sub-crustal marsquakes. Their ray
447 paths would therefore travel longer in the elastic region of the Martian interior than the respective
448 HF and VF events. The latter, which are typically located at shorter distances and assumed to be
449 events near the surface, would have waves that are propagated through the highly diffusive layer
450 (or region) near the Martian surface.

451 The grid search results for every event are examined in a summary plot, and an example is
452 shown in Figure 10 for the event S0231b (HF). The results are shown for a layer thickness of
453 $h = 20$ km. The diffusivities shown in this summary should be adjusted for each desired layer
454 thickness, as described earlier in this section and shown in Figure 8. The white curves show the
455 lowest misfit for every pair of Q and diffusivity values and correspond to the curves that are shown
456 on the right bottom side, where the RMSE is plotted as a function of Q for each case of velocity
457 ratio. The gray dashed line in this subplot corresponds to the best misfit among all the velocity
458 ratios tested.

459 We observe that for higher velocity ratio, there is a narrow region of low misfit, which indicates
460 a preference for a specific small range of Q . For smaller values of the velocity ratio, this range of
461 preferred Q increases and the associated curve reaches a flat region for the lower RMSEs. The
462 shape of the gray dashed line shows that we cannot choose a specific attenuation factor based on
463 this analysis. Importantly, its shape is not identical for every event family (LF, BB, HF and VF)

464 which allows us to deduce information about the properties of Mars by a comparative analysis.

465 The results of that analysis are shown in the bottom right part of Figure 10 for each event, and
466 are presented collectively, for the events of each type in Figure 11. The best and worst misfit (lower
467 and higher RMSEs) as a function of Q are shown in red, whereas the mean value of all the curves
468 is shown in blue.

469 Due to the small number of lower-frequency type events (LF and BB), the results for these
470 event types can be considered as more uncertain. For the LF events, the lowest misfit corresponds
471 to the curve for event S0409d and the highest misfit for event S0189a. As seen in Table 1, we
472 do not observe any correlation of the envelope fit and the frequency content of the data envelopes
473 or the epicentral distance of the events. For the BB events, there are only 2 computed curves,
474 with the minimum misfit corresponding to event S0235b and the maximum to event S0185a. This
475 could be evidence that the misfit, in the flat region of higher Q , correlates with the epicentral
476 distance of the events, but the same correlation is not found across all the event types, and remains
477 unconstrained. For the LF and BB events, we note the inability of our modelling approach to
478 provide good envelope fits independently of the parameter ranges used in the grid search.

479 On the other hand, more information about the structural properties of Mars is provided through
480 the analysis of the HF and VF events. As shown in Figure 10 for the HF event S0231b, at lower
481 values of Q , the spectral envelopes can be fit with a smaller diffusivity, while at higher Q values,
482 the diffusivity must also increase; best fits (lowest RMSE) are found for intermediate Q values.
483 This local minimum in the RMSE as a function of Q is even more apparent for VF events, where
484 optimal fits are provided by Q values in the 400 – 640 range. For VF events, we also find that
485 as the frequency content of the events increases, a greater quality factor provides better fits to the
486 data. However, this is based on the analysis of only 5 events and more event data are necessary

487 in order to draw any firm conclusions from the frequency and scattering quality factor correlation.
488 The same correlation is not observed in analysis of the HF events.

489 Due to the tradeoff between the layer thickness and the diffusivity, which is discussed in the
490 Methodology section and summarized by Equation 14, it is not possible to constrain a specific
491 diffusivity in the elastic layer. This tradeoff is further illustrated in Figure 12 using the results of
492 the analysis of VF event S0128a. On the left panel, the minimum RMSE is plotted as a function
493 of Q . The different color curves correspond to different velocity ratios, which result in a different
494 range for the seismic ray and therefore distinct ray lengths in the diffusive layer, as indicated in the
495 legend. We observe no preference for any specific diffusive layer thickness as they can all satisfy
496 the envelope equally well, for a different choice of diffusivity. These corresponding diffusivities
497 are shown in the center panel, with increasing values for increasing Q . This feature is a direct
498 consequence of Equation 7. To fit the data envelope, there is a large range of acceptable diffusivity
499 values that trade off directly with the chosen layer thickness, as shown on the right panel of Figure
500 12.

501 The complete trade off between layer thickness and diffusivity is a major obstacle for the inter-
502 pretation of our results in terms of scattering layer thickness and strength. The obtained diffusivity
503 results that correspond to a specific diffusive layer thickness can be adjusted at will, by following
504 the relationship of Equation 14, as it is demonstrated by the test shown in Figure 6. However there
505 is another element of the analysis that can be used to constrain the structure of the diffusive part of
506 the Martian lithosphere. As shown in Figure 12, the diffusivity varies with the change of the ve-
507 locity ratio between the studied diffusive layer and the underlying elastic half-space. When V_d/V_e
508 increases, the diffusivity will increase as well. In order to investigate if this is another artifact due
509 to the tradeoff between the diffusivity and the dimensions of the seismic ray path in the diffusive

510 layer (defined by h and r) we use the results of the VF events, as shown in Figure 12 for event
 511 S0128,a and test if the computed diffusivities can be obtained by only using the Equation 14 for
 512 a constant layer thickness and the respective range of the seismic ray, corresponding to different
 513 velocity ratios. The computed diffusivities depend on the change of the range, r , of seismic ray,
 514 however not linearly but quadratically, which means that for any given diffusivity D_1 , for a veloc-
 515 ity ratio $(V_d/V_e)_1$ with corresponding range r_1 and a layer thickness h , there is a diffusivity D_2 for
 516 $(V_d/V_e)_2$ that gives a seismic ray range r_2 for the same layer thickness and their relationship is:

$$\frac{D_1}{D_2} = \frac{r_1^2}{r_2^2} \quad (16)$$

517 This relationship can be obtained through the joint solution of Equations 6 and 14. Therefore,
 518 for the interpretation of the data, we need to analyze the range of best fitting diffusivities and Q
 519 pairs for one given velocity ratio and layer thickness and therefore range of the seismic ray.

520 **DISCUSSION**

521 In our analysis we investigate the scattering properties of the Martian interior, based on the compu-
 522 tation of energy envelopes used previously for lunar impacts by Dainty et al. (1974b). By system-
 523 atically investigating the effects of all the model parameters (7), we establish the existence of key
 524 tradeoffs between these parameters. More precisely, we show that there are trade offs between the
 525 dependence of the diffusivity, the diffusive layer thickness, the velocity ratio between the diffusive
 526 layer and the underlying elastic half space, and the range of the seismic ray in the diffusive layer.
 527 This means that by knowing any one of these parameters independently, we can use our model-
 528 ing to place definite bounds on parameters controlling the scattering in the Martian interior. On

529 the other hand, if we do not have independent constraints for these parameters, we end up with a
530 multidimensional space of possibilities for models that fit the data.

531 Lognonné et al. (2020) showed results for specific models of scattering, with separate analyses
532 assuming a diffusive layer thickness of $h = 20, 40$ and 60 km. More precisely, they performed
533 a preliminary analysis of VF event S0128a, LF event S0173a and BB event S0235b. The key
534 purpose of their work was to demonstrate the compatibility of observed envelope shapes with a
535 multiple-scattering origin. In the case of the VF event, they used forward modeling based on radia-
536 tive transfer equations in a few sets of statistically uniform random models with ad-hoc statistical
537 properties. These authors inferred that a diffusivity of the order of $D = 90 \text{ km}^2/\text{s}$ at a frequency
538 of 7.5 Hz was compatible with the observations. However since the modeling did not include
539 any depth dependence, our work shows that this value may be an overestimate. In the case of the
540 LF/BB events, Lognonné et al. (2020) considered a simplified model where a single plane wave
541 impinges vertically on a scattering crust from below. While their approach bears some similarity
542 with the present one, there are some important differences with our work: the bottom of the scat-
543 tering layer in Lognonné et al. (2020) coincides exactly with the Moho, whereas any scattering
544 effect on the downgoing part of the ray is neglected. In that study it is estimated a broad range of
545 values for the diffusivity at 0.5 Hz , from $200 \text{ km}^2/\text{s}$ to $2000 \text{ km}^2/\text{s}$. In the present work, we con-
546 siderably expand the initial dataset, thereby covering a broader range of frequencies and epicentral
547 distances. We also extend the range of values of the parameters that control scattering. Our study
548 investigates in details the possible trade-offs between assumptions for scattering strength, crustal
549 thickness, velocity contrast at the Moho and absorption. Thereby, we offer a more comprehensive
550 view of the current uncertainties on the scattering properties in the Martian lithosphere.

551 Through our analysis of envelope shapes, we find that many models could fit the data, and that

552 drawing conclusions based on a single unique model that can fit the data would be misleading, as
553 it would be reflect assumption(s) made to remove tradeoffs among several parameters that control
554 scattering. For example, previous numerical modeling of scattering proposed candidate structures
555 that fit the InSight data (van Driel et al., 2021), for a specific layer thickness of $h = 10$ km, velocity
556 ratios $v_d/v_e > 0.5$ and scattering ranges from 10% to 100%. These ranges of parameters are all
557 found to yield acceptable fits to the coda envelopes analyzed in our study. More precisely, if we
558 apply Equations 6 and 16 to the results that we present in Figure 12 for HF event S0128a, we find
559 that for this range of parameters, the diffusivity that provides the best fit is $D = 0.7$ km²/s, which
560 is in agreement with the findings of the van Driel et al. (2021) paper.

561 In Figure 13, we present a comparison between envelopes computed through 2D numerical
562 wave propagation simulations of van Driel et al. (2021) and those computed in our study using the
563 analytical methodology based on the theory by Dainty et al. (1974b). For the estimated range of
564 diffusivity used in that study ($D = 0.5 - 0.7$ km²/s), our predicted coda decays provide an imper-
565 fect but good fit to their synthetic coda envelope, as seen in the left panel of Figure 13. In the right
566 panel, we increase the range of diffusivity values to $D = 0.3 - 0.7$ km²/s, and plot the coda decays
567 predicted by the approach developed in our paper. We find that the lower diffusivity values provide
568 improved fits to the S-coda envelopes computed through numerical wave propagation simulations,
569 mainly for epicentral distances greater than 15° (receiver index greater than 7 in Figure 13). At
570 smaller epicentral distances, higher diffusivity values fit the modeled data better, which may be
571 the reason that we find a good agreement for the results for event S0128a, a Very High Frequency
572 event at an estimated epicentral distance of 7.79° (in the middle range between receiver index 3 and
573 7). This comparison with coda decays obtained through numerical wave propagation simulations
574 of van Driel et al. (2021) demonstrates that the relationships developed in this study yield correct

575 estimates for the parameters that control scattering.

576 In their study of the lunar interior, Dainty et al. (1974b), who originally used the modeling
577 equation that was chosen for our analysis, suggest a unique model that can fit the ensemble of the
578 data. However, in their model they suggest an apparent thickness for the diffusive layer which
579 varies with the frequency of the examined data. In order to do this, they use a factor for the
580 amplitude of the body waves generated by the Lunar Module and the Saturn-IV B impacts, that
581 was computed by the study of Toksöz et al. (1972). We showed that modeling based on Equation
582 7 yielded better fits to the observed S-coda envelopes of higher frequency Martian events (HF and
583 VF). This finding is consistent with previous studies that suggested that these are shallow events
584 (Giardini et al., 2020; van Driel et al., 2021). We also showed that the approach based on the mean
585 ray path can describe with fairly small errors eventual reverberations of the diffusive waves in a
586 shallow diffusive layer. As shown in Table 2 and Figure 7, these errors are much smaller than the
587 RMSE between the computed energy envelopes and the spectral envelopes of the data.

588 In this study we examine only the first arriving S-wave signals in our data but excluded the P-
589 wave coda which also contains complementary information on scattering. The P-waves propagate
590 with a different velocity and frequency content, and future analysis of their coda properties could
591 provide an independent constraint on scattering in Mars. Furthermore, we normalize the maximum
592 amplitudes of each event envelope to unity, removing information on the absolute energy loss.
593 While the magnitude of the energy loss in the elastic layer does not strongly affect the envelope
594 shape, an analysis of absolute amplitude at varying event distances would provide constraints on
595 the intrinsic Q of the mantle, a task we do not explore further here. Finally, we assumed the elastic
596 region underlying the scattering layer to be a homogeneous half-space, which does not affect the
597 envelope shapes and therefore the coda decay. This means that we do not compute the actual

598 seismic rays but only an average interpretation of their paths. Future work could combine data
599 analysis to constrain the model space of scattering properties, with more sophisticated full wave
600 propagation modeling to further refine the fits to marsquake waveforms.

601 We can interpret the success of modeling VF and HF events and the relative inability to model
602 LF and BB events in terms of the likely source characteristics for these event types Giardini et al.
603 (2020). Compared to the HF or VF envelopes, we observe a very rapid coda decay for the LF events
604 and slightly longer, but still rapid decay for the BB events. Our approach for the computation of
605 the envelopes in a diffusive layer cannot be effective when we try to model the body waves that
606 propagate in the elastic part, which would be expected for body waves from deep marsquakes.
607 For deep events, multiple scattering that can be modeled as a diffusion process happens only in
608 the vicinity of the station, when the impulse of seismic energy broadened only by the effects of
609 attenuation arrives from below. Thus, we interpret the inability of our model to fit the coda of LF
610 and BB events to be further evidence that these events are deep marsquakes.

611 This might be due to the fact that the waves travel shorter distances in the diffusive layer
612 when they occur deeper in the Martian interior, which has also a correlation with their frequency
613 content. An argument that supports this hypothesis is the better fit of the equation developed for
614 lunar impacts (Dainty et al., 1974b) (i.e. shallow and surficial) to higher frequency events. It is
615 in coherence with the suggestion that lower frequency events occur in greater depths. However,
616 the modeling of these events is not able to show a critical result to constrain the thickness of the
617 diffusive layer. In addition, regarding the poor fit of LF and BB events, we know now that they
618 are composed of multiple energy injections that arrive in the coda. These injections are not taken
619 into account in the modeling and this contributes to the difficulty fitting the data. It is additionally
620 worthy to note that Lognonné et al. (2020) performed an analysis of the LF and BB events, showing

621 that respective spectral envelopes can be modeled through the analysis of the ratio between the
622 ballistic and S-coda waves, as well as the coda decay, but they do not follow the diffusion model.

623 The epicentral distances of the HF events have a small range in variation, between 21.4° and
624 28.4° (see Table 1) which limits our ability to study how scattering changes over distance. Trans-
625 lated into *km*, these values correspond to a distance between 1262.6 km and 1675.6 km. This
626 means that the recorded waves should either cross the crust-mantle boundary and travel in an
627 elastic regime in the lithosphere, or become trapped and reverberate in the crust, as suggested by
628 Giardini et al. (2020) and van Driel et al. (2021). In the case of many reverberations, the apparent
629 speed in the diffusive layer should be very high according to our analysis. This high seismic ve-
630 locity corresponds to a very low Q in our results and very low diffusivity, which corresponds to a
631 region where our results are saturated. However, for the VF event S0128a, located at a relatively
632 small epicentral distance (7.79°), we find that a low $Q = 200 - 300$ fits the data better than a higher
633 Q . It is therefore unclear if the crustal waveguide, or properties of the scattering in the crust are
634 producing these difference between more distant or closer marsquakes. As more HF events are
635 recorded at different distances, this behavior can be investigated in more detail.

636 The analysis of the VF events shows that they are better modeled with Equation 7, as would
637 be expected if their sources were indeed shallow or surficial (Giardini et al., 2020). Epicentral
638 distances of VF events vary widely from 6.44° to 36.8° . Despite this distribution in terms of
639 distance, we do not observe a correlation between their distance and their S-coda decay time or for
640 inferred values of diffusivity, as would be expected for a near surface layer of some given thickness
641 and scattering properties. One possibility is that the VF events have a distribution of azimuths
642 with respect to the InSight lander position and thereby sample very different scattering structures.
643 Unfortunately, for most of the VF events, it is not possible to robustly determine backazimuth.

644 Nevertheless, future determinations of VF backazimuths would make it possible to infer lateral
645 variations in the Martian crust, and argue against a single, uniform diffusive layer.

646 To interpret the range of structures that fit the obtained results, we can assume for example,
647 a relatively thin diffusive layer confined to the shallow-most crust. Underneath, the elastic half-
648 space contains part of the diffusive region of the Martian crust and upper mantle's structure. The
649 diffusivity in the thinner top layer should be smaller than the one obtained for a thicker one. This
650 tradeoff suggests that the number of scattering events depends not only on the number density of
651 scatterers in a given cross section (Equation 14) but also depends on the square of the range of the
652 seismic ray (Equation 16).

653 Given this interpretation, a last question is whether the examined diffusive layer structure, with
654 its lateral variations, corresponds to a part of the Martian crust or extends deeper in the crust-
655 mantle or even in the upper mantle. If we assume that future analyses or events yield reliable
656 backazimuth estimates, the answer to this question depends on the level of the general knowledge
657 for Mars interior and more precisely the structure of the lithosphere, as this approach will be
658 able only to constrain the thickness of diffusive layer but not the thickness of the crust, which
659 is a matter of debate in the literature. Wiczorek et al. (2021) performed a review of the studies
660 that defined the average crustal thickness of Mars. The suggested values vary from $H_C < 29$ km
661 for a model of isostatically compensated crust in Hellas Planitia (Wiczorek and Zuber, 2004), to
662 $H_C < 115$ km for a model with viscous relaxation of dichotomy boundary and Hellas basin (Nimmo
663 and Stevenson, 2001). Moreover, they suggested a crustal thickness in the vicinity of InSight either
664 20 km or 37 km through the assumption of a 2-layered or 3-layered crust model respectively.
665 More recently, Knapmeyer-Endrun et al. (2021) computed autocorrelations and receiver functions
666 using InSight data and suggested an average crust thickness varying between 24 – 70 km, with the

667 presence of either one or two seismic interfaces in the Martian crust. The relevance of these crustal
668 thickness constraints for the interpretation of results presented in our study hinges on whether the
669 seismic waves of the HF and VF events are crossing the crust-mantle interface or if they travel only
670 within the diffusive layer considered as part of the crust defined by intra-crustal interfaces.

671 Information about the energy loss towards the inner depths of the planet, which can be provided
672 through analogies of the expected amplitudes as it was done in previous works (for example Dainty
673 et al. (1974b) used a known analogy for the amplitudes of the Lunar Module and Saturn IV artificial
674 impacts, provided by Toksöz et al. (1972) in order to suggest a unique structure model) can be a
675 valuable element to constrain this feature. Furthermore, it will be useful if future experiments are
676 performed in a region close to the InSight seismic experiment (more precisely in a range of around
677 30°, as this is an average distance of the High Frequency events) and this geographical setting will
678 allow a joint analysis and further interpretation of the currently available InSight seismic data. The
679 existence of such a network will improve the ability of phase peaking and location identification
680 of the events and therefore it will give an extra constrain for an analysis similar to this study's,
681 which is the structure of each event's waves propagation, with more data coming from events that
682 are now characterized of lower quality in the Seismic Catalog (Clinton et al., 2021).

683 Mars appears to be intermediate between the Earth and Moon in terms of seismic scattering and
684 attenuation, since Martian seismograms exhibit a shorter coda durations than lunar seismograms
685 (van Driel et al., 2021). However, the HF and VF Martian events have long codas and exhibit
686 some resemblance to Moonquakes and strongly scattered Earth seismograms, while the LF and BB
687 events resemble regional tectonic events on Earth. Like the Moon, the origin of the scattering on
688 Mars likely lies in the crust or uppermost lithosphere. On the Moon, the scattering is produced by
689 impact processes that have produced a shallow layer of regolith and deeper megaregolith of highly

690 fractured bedrock. The scattering properties of lunar events were measured by Gillet et al. (2017),
691 who derived a model of the scattering and attenuation properties of the Moon using diffusion
692 theory. They found very low wave diffusivity ($D \approx 2 \text{ km}^2/\text{s}$) in the uppermost 10 km of the
693 Moon. They noted that these values correspond to some volcanic areas on Earth, which are the
694 most heterogeneous regions on our planet. Below the surface layer, the diffusivity rises slowly
695 up to a depth of 80 km, where it increases abruptly by about one order of magnitude. Gillet
696 et al. (2017) suggested that the megaregolith corresponds to the region of low diffusivity, and that
697 it is 100 km thick (much larger than previous estimates). When looking at the seismic layers at
698 Mars, there is a low velocity surface layer (Knapmeyer-Endrun et al., 2021; Lognonné et al., 2020),
699 which probably represents the ejecta rubble and severely cracked rock produced by lunar meteorite
700 bombardment (Goins, 1978). Our Martian data require low diffusivity (D is generally lower than
701 $1.5 \text{ km}^2/\text{s}$), which suggests that the diffusivity in the top 10 km beneath Elysium Planitia on Mars
702 are similar to the low diffusivity found on the Moon. However, there is weak geological evidence
703 on Mars for a thick megaregolith layer, which is further substantiated by the existence of the Low
704 Frequency and Broadband Marsquake events (Giardini et al., 2020) that appear to occur below
705 the scattering layer. Therefore, Mars appears to be more complicated than a simple intermediate
706 between Earth and the Moon. Instead, it shares some of the properties of these two bodies, the
707 only other seismically-investigated bodies in the Solar System.

708 CONCLUSION

709 We investigated the seismic attenuation in the Martian crust and upper mantle by examining the S-
710 wave codas of a series of InSight detected marsquakes. For our investigation, we used the spectral

711 envelopes 21 marsquakes in 4 different event families – classified by their frequency content – with
712 source parameters from the Seismic Catalog (InSight Marsquake Service, 2021). We assumed a
713 diffusive layer over an elastic half-space model, and computed the mean raypath of the seismic
714 waves from the shallow source to the station for a given epicentral distance and for free variables
715 of scattering, Q , diffusivity, and velocity ratio between the diffusive and the elastic layer.

716 In our study, we observed that the Low Frequency (LF) and Broadband (BB) events, with
717 frequency content below the threshold of the 1 Hz tick noise, could not be fit by our model. The
718 spectral envelopes of the S wave codas of these events showed a very rapid decay which suggests
719 that they do not have an extensive propagation path in the diffusive layer. This observation is in
720 agreement with the suggestion of previous studies (Giardini et al., 2020) that these events are deep
721 marsquakes and travel through the upper mantle of Mars.

722 Based on the results of the High Frequency (HF) and Very High Frequency (VF) events, we
723 observed a range of possible paths and diffusivities that can satisfy the data, and we investigated
724 the tradeoffs between the parameters in the modeling equation (Dainty et al., 1974b) that controls
725 the shape of the energy envelope for the events. The analysis of these tradeoffs shows that the
726 S-wave coda do not uniquely constrain the depth of the diffusive region in the Martian crust and
727 the upper mantle. Our analysis of HF and VF events, is consistent with previous studies (Giardini
728 et al., 2020; van Driel et al., 2021) which argued that that these events are shallow-sourced.

729 However, the observation that the lower frequency event families cannot satisfy the model,
730 showing a very rapid S-coda decay, suggests the possibility that the Martian lithosphere may differ
731 compared to the lunar or terrestrial one. The diffusive region on Mars is comparable to the lunar
732 regolith, however the regolith on Mars is not extended to great depths, as demonstrated by deep
733 marsquakes that appear to propagate in the elastic region of the Martian lithosphere. We find that

734 the equation used by Dainty et al. (1974b) to fit moonquakes is not able to fit the Martian data at all
735 the frequency ranges. Therefore, we deduce that the scattering structure of the Martian lithosphere
736 is not similar to the Moon, and that Mars is unlikely to have a deep megaregolith.

737 The results of this study illustrate the challenges of working with single station seismic data
738 where independently determined event location information, including distance, azimuth, and
739 depth are crucial for understanding the lateral variation in seismic properties of a planet.

740 **DATA AND RESOURCES**

741 Seismic data used for this study were collected as part of the Seismic Experiment of Internal Struc-
742 ture (SEIS) (Lognonné et al., 2019) of the NASA InSight Mission to Mars (Banerdt et al., 2013).
743 They can be obtained from the IRIS Data Management Center (<https://www.iris.edu/hq/sis/insight>),
744 the NASA PDS Geoscience Node (InSight SEIS Science Team, 2019) and the IPGP SEIS Data
745 portal (InSight Mars SEIS Data Service., 2019).

746 **ACKNOWLEDGEMENTS**

747 We acknowledge NASA, CNES, their partner agencies and Institutions (UKSA, SSO, DLR, JPL,
748 IPGP-CNRS, ETHZ, IC, MPS-MPG) and the flight operations team at JPL, SISMOC, MSDS,
749 IRIS-DMC and PDS for providing SEED SEIS data. F.K., N.S., Q.H. and D.K. were supported
750 by NASA grant #80NSSC18K1628. V.L. acknowledges support by the Packard Foundation. C.N.
751 was supported by strategic funds from the Jet Propulsion Laboratory, California Institute of Tech-
752 nology, under a contract with the National Aeronautics and Space Administration. We thank two
753 anonymous reviewers for their constructive comments. This is InSight contribution 182.

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Table 1: The complete list of the seismic events that are used in this study. The minimum and maximum frequency of the spectral envelopes is obtained with the use of a visual tool that allows the pick of the event signal from their spectrograms and the epicentral distance is estimated by InSight Marsquake Service (2021)

	Min f (Hz)	Max f (Hz)	Distance ($^{\circ}$)	Quality Type
Low Frequency Events				
S0173a	0.16	0.86	29.3	A
S0189a	0.41	0.81	32.7	B
S0290b	0.37	0.80	29.5	B
S0407a	0.23	0.86	28.6	B
S0409d	0.18	0.82	30.4	B
Broadband Events				
S0185a	0.26	0.84	58.4	B
S0235b	0.15	0.81	27.8	A
S0484b	0.35	0.84	30.9	B
High Frequency Events				
S0185b	1.67	3.90	27.3	B
S0228c	1.32	4.08	21.4	B
S0231b	1.64	3.53	23.4	B
S0260a	1.11	5.69	25.2	B
S0340a	1.66	3.24	27.1	B
S0352a	1.55	4.52	28.4	B
S0432a	1.77	3.38	24.7	B
S0490a	1.55	6.58	24.7	B
Very High Frequency Events				
S0128a	1.42	4.51	7.79	B
S0263a	1.79	6.97	6.44	B
S0334a	1.30	9.03	19.8	B
S0421a	1.04	7.38	36.8	B
S0500a	1.84	8.05	12.0	B

Table 2: The energy envelopes for seismic rays with 1, 2 and 3 reverberations are compared to the simple ascending ray for 2 different models of the diffusive layer, with layer thickness $h = 20$ km and diffusivity $D = 1 \text{ km}^2/\text{s}$ and $D = 5 \text{ km}^2/\text{s}$. The Table shows the deviation of the computed envelopes for the reflected rays versus the case of an ascending ray, from the bottom to the top of the diffusive layer.

Deviation of reflected ray's envelope vs Ascending ray envelope			
	1 reflection	2 reflections	3 reflections
$D = 1 \text{ km}^2/\text{s}$	0.0096	0.0236	0.0112
$D = 5 \text{ km}^2/\text{s}$	0.0044	0.0254	0.0120

1009 **LIST OF FIGURE CAPTIONS**

1010 **Figure 1**

1011 The spectral envelopes of example events for each of four different event types defined according
1012 to their frequency content. The envelopes are obtained through visual selection on the event spec-
1013 trograms. The event envelope is shown in black, whereas the S wave arrival and coda decay that
1014 is used in our analysis are shown in red. The gray thin line corresponds to a part of the data that
1015 is not used in our analysis, corresponding either to the noise level or glitches and features that are
1016 not part of the event signal.

1017 **Figure 2**

1018 The complete dataset of seismic events used in this study for vertical velocity seismograms (gray)
1019 and envelopes (thick line). The seismograms and the respective envelopes are organized by the
1020 epicentral distance of the events. The black color represents the time window of the selected
1021 signal, whereas the part of the S-coda decay that was used in our analysis is shown in red.

1022 **Figure 3**

1023 Schematic showing the approximate average seismic raypath from the source to the station. The
1024 seismic waves that are produced at the source are scattered in the diffusive layer due to the presence
1025 of scatterers. These "real" ray paths are represented by the white rays, which are scattered in the
1026 diffusive layer. The approximation of the ensemble of these rays is represented by the blue curve,
1027 which is not scattered, but corresponds to a lower-than-true, apparent velocity in the diffusive layer.

1028 **Figure 4**

1029 The mean ray path (red) of the seismic waves from the source (A) to the station (D) through an
1030 elastic layer that overlies a diffusive layer. The refracted waves case is shown on the left and the
1031 reflected waves on the right. The thickness of the diffusive layer is noted with h , the range of the
1032 seismic ray in the diffusive layer with r in length units and θ_r in radians, the S-wave velocity in the
1033 diffusive layer with V_d and in the elastic layer with V_e . The epicentral distance is noted with Δ in
1034 length units and ε in radians. R is the planet's radius.

1035 **Figure 5**

1036 The range of the seismic ray in the diffusive layer, for a layer thickness $h = 1$ and epicentral
1037 distances of 20° , 50° and 100° for a range of velocity ratio between the diffusive and elastic layer
1038 from 0.1 to 1.

1039 **Figure 6**

1040 The energy envelopes obtained for an impulse source at the surface. In both panels, the blue,
1041 red and yellow curves correspond to layer thicknesses of 10, 20 and 30 km respectively. On the
1042 left panel, the diffusivity is $D = 0.1 \text{ km}^2/\text{s}$ for all the examples. We observe longer decay rates
1043 for bigger diffusive layer thickness. On the right panel, the diffusivity is adjusted using Equation
1044 14. With this adjustment, which follows the tradeoff between layer thickness and diffusivity, the
1045 computed envelopes remain unchanged.

1046 **Figure 7**

1047 The effects of adding multiple reverberations into the envelope calculation. The blue line represents
1048 a seismic ray traveling from the discontinuity between the diffusive and the elastic layer, upwards
1049 towards the station. The red line represents a seismic ray of a surface event with one reverberation
1050 and the yellow and purple line represent 2 and 3 reverberations respectively. In both panels, the
1051 diffusive layer thickness is set to $h = 20$ km and attenuation factor $Q = 1000$. The diffusivity on
1052 the left panel is $D = 1$ km²/s and in the right panel $D = 5$ km²/s.

1053 **Figure 8**

1054 The tradeoff between diffusivity and layer thickness. Using a diffusivity $D = 1$ km²/s for a thick-
1055 ness of $h = 20$ km, we use Equation 14 in order to show how the diffusivity should be adjusted,
1056 depending on the layer thickness, in order to obtain the same results in our modeling.

1057 **Figure 9**

1058 The fit of the scattering diffusive model to spectral envelopes of the data. The spectral envelopes
1059 of 4 seismic events, one for each family, are shown in black. The synthetic envelopes that provide
1060 an RMSE lower than a specific threshold (0.4 for the LF, 0.16 for the BB, 0.14 for the HF and 0.1
1061 for the VF events) are shown in red. We observe that the modeling approach works better for the
1062 HF and VF events, which are considered to be located at closer epicentral distances and be sourced
1063 near the surface.

1064 **Figure 10**

1065 Results for models fits of HF event S0231b. The 5 colormaps show the Root Mean Square between
1066 the data spectral envelope and the computed modeled envelope for the velocity ratio shown on the
1067 top of the subplot, the Q on the y-axis and the diffusivity given in km^2/s for a layer thickness of
1068 $h = 20$ km. The white curves note the best misfit for every pair of Q and diffusivity. The best misfit
1069 is also shown on the right bottom side, in respect to Q for each case of velocity ratio. The gray
1070 dashed line in this subplot corresponds to the best misfit among all the velocity ratio associated
1071 curves.

1072 **Figure 11**

1073 The curves of the best misfit as a function of Q for each event family. The dashed lines correspond
1074 to the results of every event, whereas the minimum and maximum values of the former are shown
1075 in red and their average value is shown in blue. The specified frequency range corresponds to the
1076 lower and upper frequency that was used for the filtering of the ensemble of the data of each event
1077 family.

1078 **Figure 12**

1079 The analysis of the results for VF event S0128a. a) The best fit between the data and the computed
1080 envelopes for each Q is shown. Different colors correspond to the velocity ratio between the
1081 diffusive and elastic layers, which controls the length of the ray path in the diffusive layer, as
1082 indicated in the legend. The green cross shows the minimum RMSE for a $Q = 400$. b) The
1083 corresponding diffusivities are shown and we observe an increasing diffusivity that satisfies the

1084 data envelope as the ray path travels a longer distance in the diffusive layer. c) The results for
1085 $Q = 400$ are given for a range of layer thickness $h = 1 - 60$ km with each color corresponding to
1086 a different V_d/V_e velocity ratio.

1087 **Figure 13**

1088 The envelopes obtained with the analytical method in this study (red, orange) are compared to
1089 envelopes computed through 2D numerical wave propagation modeling (black) (van Driel et al.,
1090 2021). In the left panel, we use the same parameters and a range for the diffusivity, $D = 0.5 -$
1091 $0.7 \text{ km}^2/\text{s}$ (i.e. the estimate of van Driel et al. (2021)) to compute the envelopes. The S-coda
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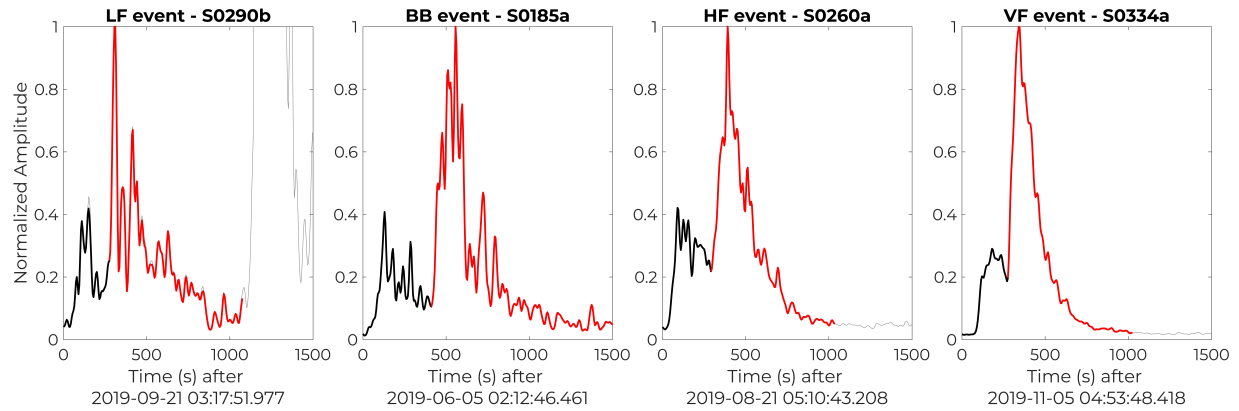


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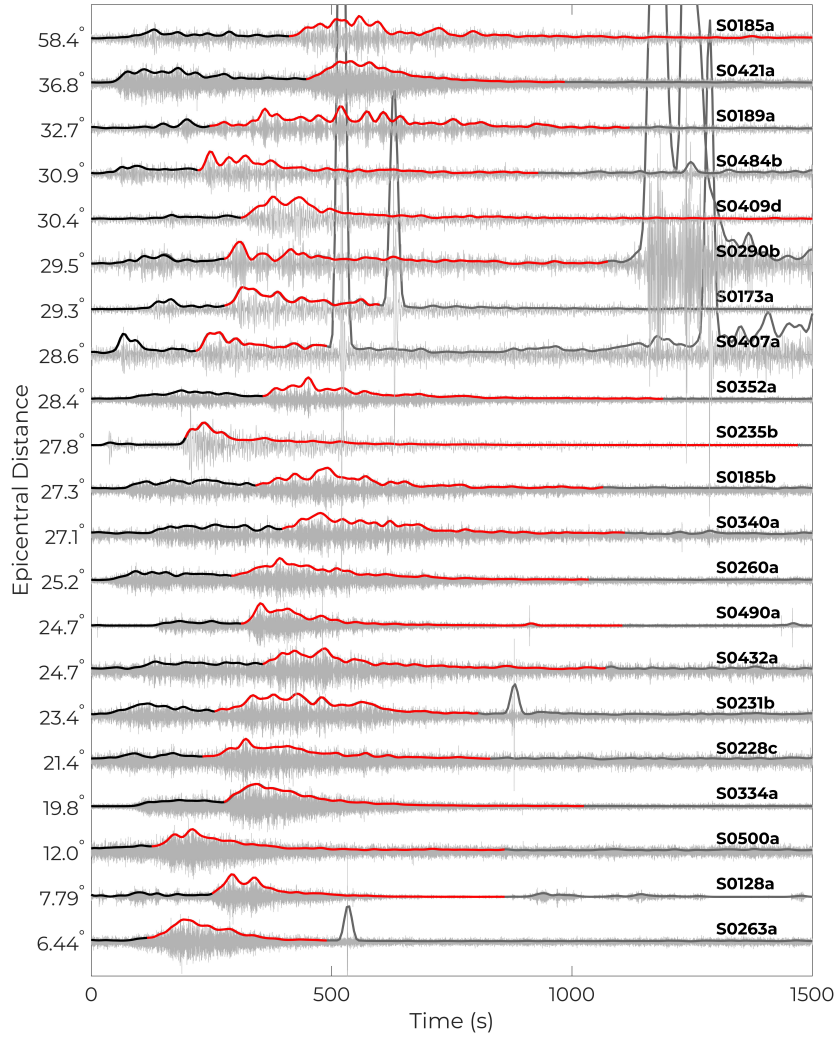


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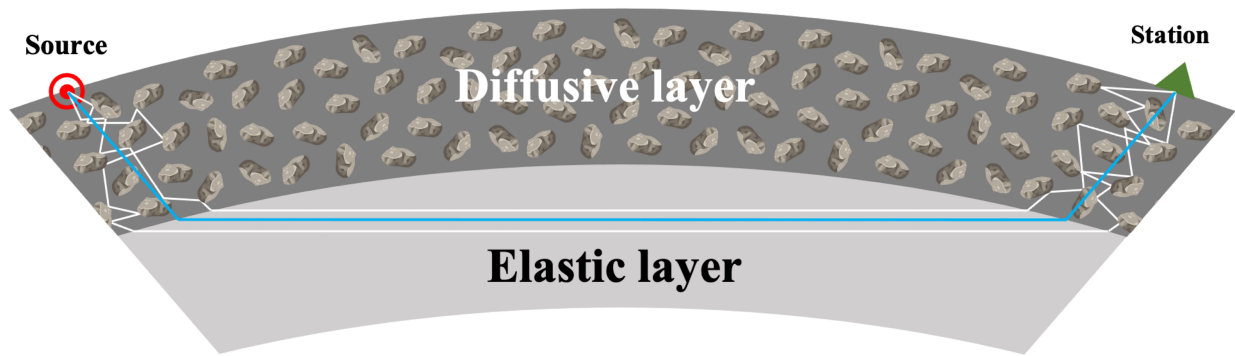


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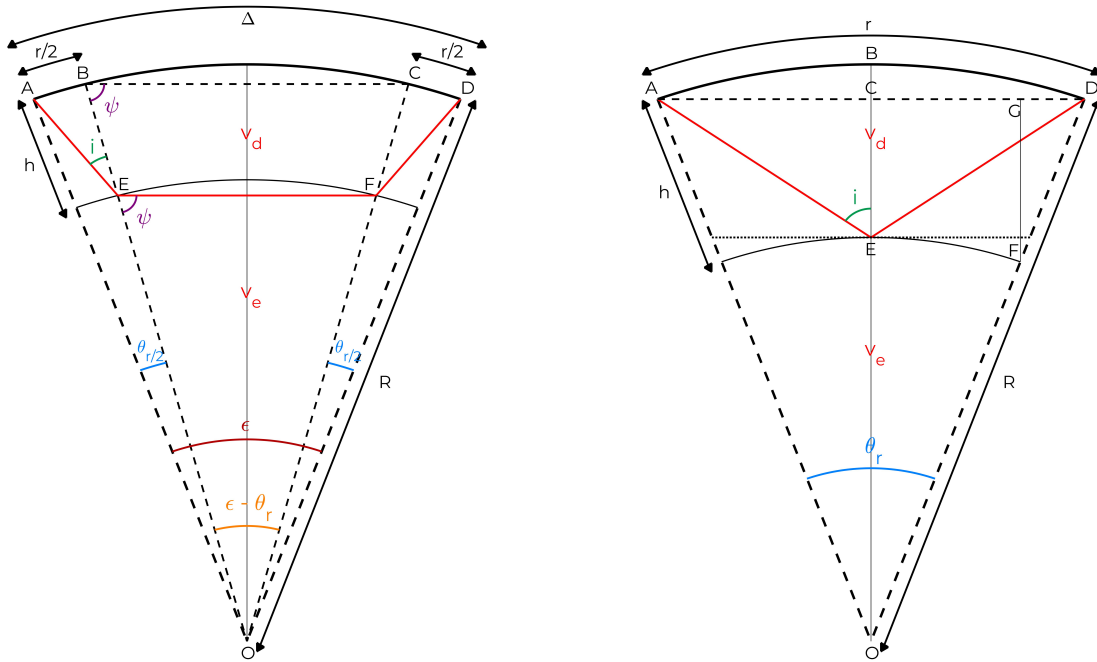


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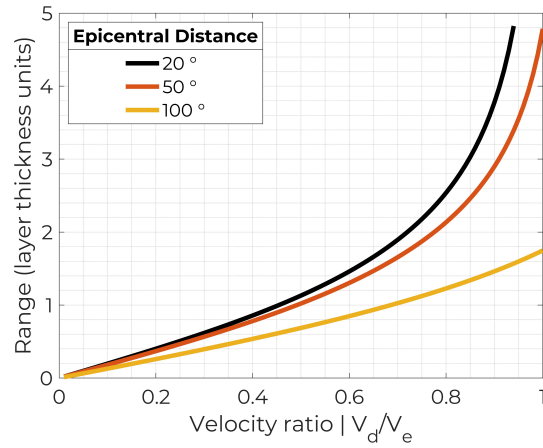


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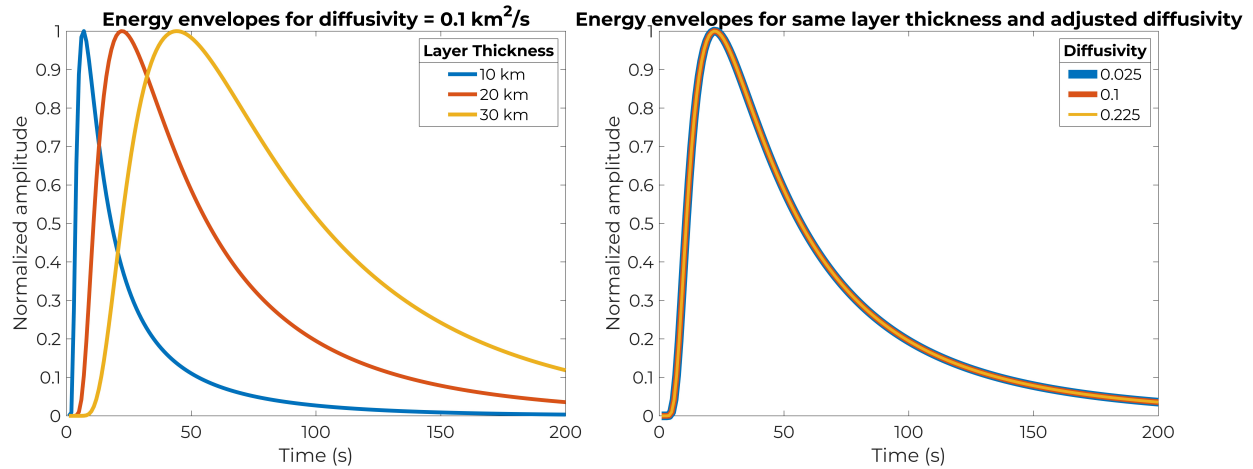


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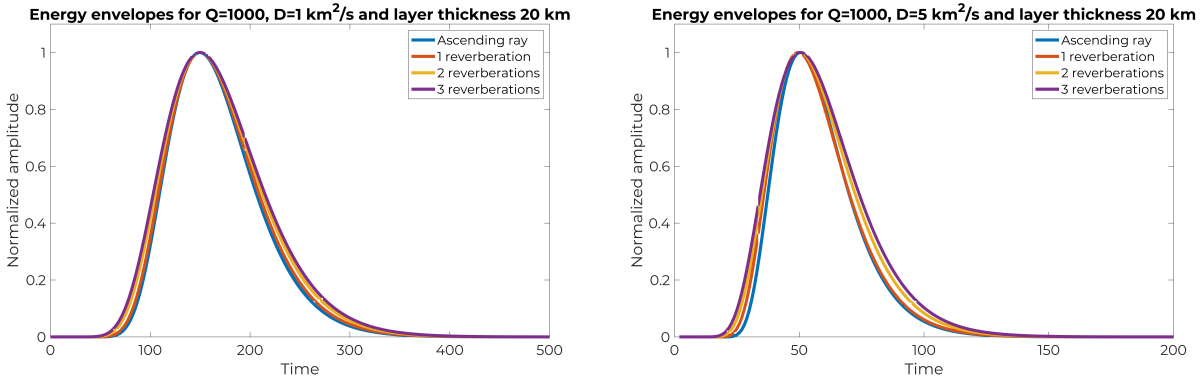


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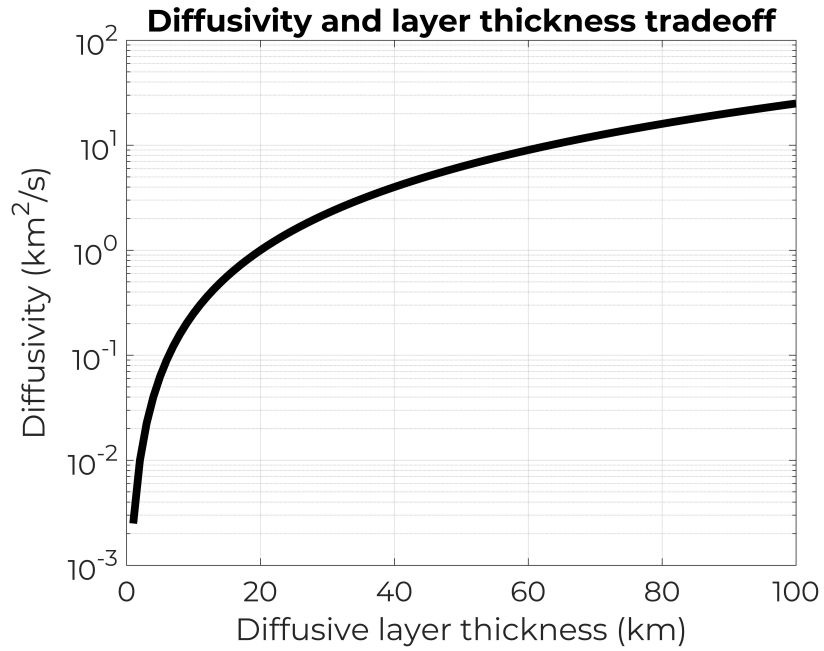


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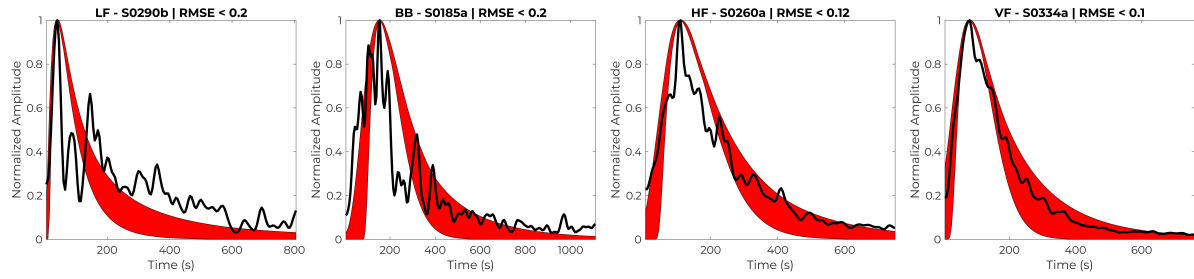


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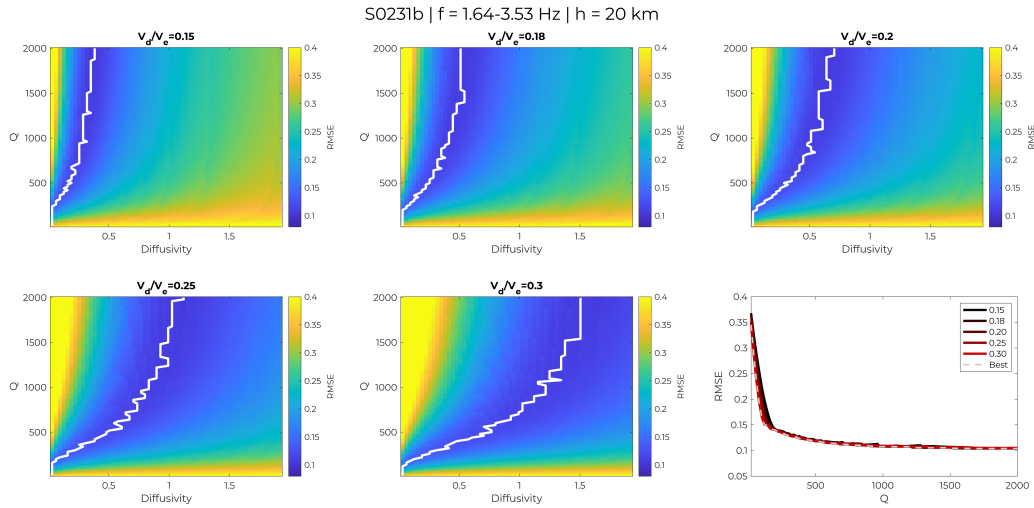


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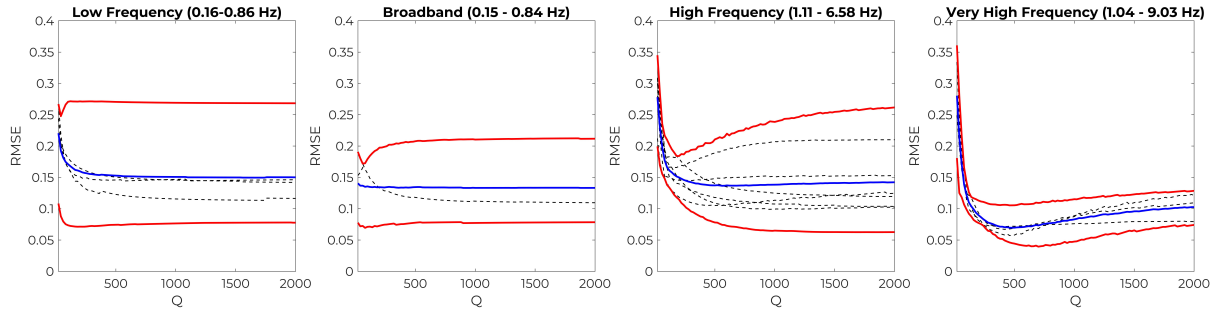


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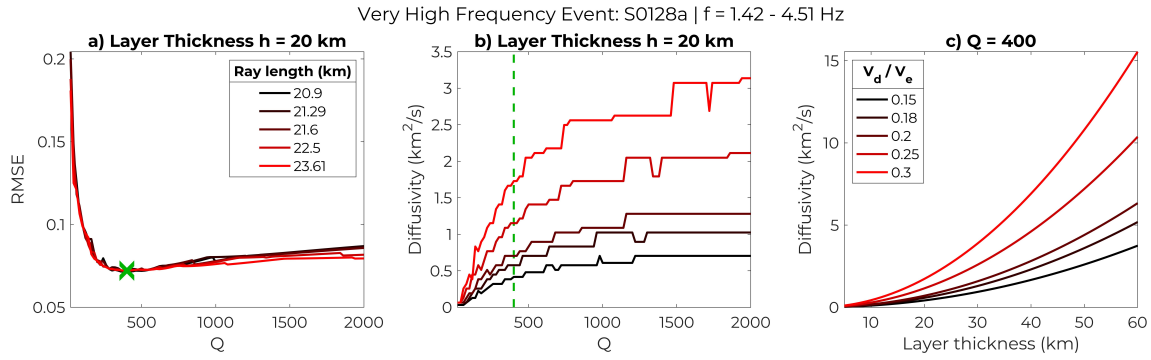


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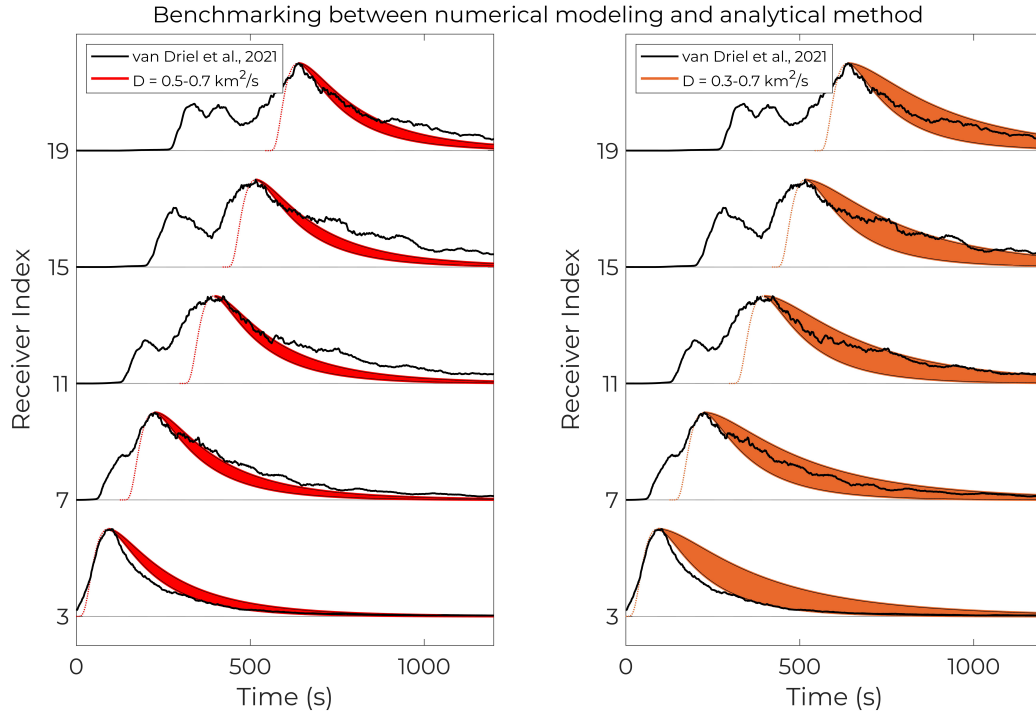


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