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Numerical development of heat transfer correlation in asymmetrically heated turbulent channel flow

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Abstract

This study deals with forced convective heat transfer. It is divided into two parts. Firstly, a review of heat transfer involving forced convection in tubes and channels is proposed. In the case of channel geometry, studies dealing with symmetrically and asymmetrically heated flows as well as one side heated flows are reported. Then, a new correlation for symmetrically and asymmetrically heated channels associated with a turbulent flow is developed using Large Eddy Simulations. The Proposed Correlation, also referred to as PC, involves a specific term to handle asymmetric conditions. The domain covered by simulations is the following: heat fluxes vary between 4 and 580 kW/m², Prandtl number ranges from 0.76 to 3.18 and bulk Reynolds number from 10 600 to 177 000. Three correlations of the literature are compared to the one proposed in this paper. The conventional expressions tend to overestimate the Nusselt number at the hot side of asymmetrically heated channel independently of the temperature ratio. They, also, underestimate the heat transfer at the cold side, especially when the bulk temperature is close to the cold wall temperature. The proposed correlation provides great results in both heating conditions. The global accuracy associated with each correlation is quantified.

Keywords: Heat Transfer, Channel flow, Asymmetric heating, Forced convection, Large Eddy Simulation

1. Introduction

Forced convective heat transfers through rectangular ducts are commonly studied in engineering field to design systems such as heat exchangers [1, 2, 3, 4, 5, 6], nuclear reactors [7, 8], nuclear waste repositories [9], electronic devices [10] and solar receivers [11, 12, 13, 14, 15] among others. Heat transfer correlations are widely used to estimate the heat flux between fluid and surface. These expressions mostly depend on the geometry, the working fluid and the operating conditions. Laminar flow characteristics can be calculated with analytic solutions considering few assumptions. However, for turbulent flows, experimental data or numerical simulations are required to estimate heat transfer.

Large Eddy Simulation (LES) is a numerical approach in which Navier-Stokes equations are solved in every cell of a discretized threedimensional domain. It allows to precisely estimate time dependent flow characteristics with a reasonable calculation cost, compared to Direct Numerical Simulations (DNS), by resolving larger scales of motion and modeling small scale effects on the larger scales [16, 17, 18, 19, 20]. LES is, then, very suitable to produce heat transfer results in various conditions.

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Nomenclature

- heat capacity at constant pressure J/kg/K
- $C_p \\ D_h$ hydraulic diameter m
- Darcy friction coefficient
- Graetz number $Re_b Pr_b D_h/L_x$
- dimensionless thermal length $L_x/(Re_bPr_bD_h)$
- $\begin{array}{c}
 f \\
 Gz \\
 L^* \\
 L_i \\
 N_i
 \end{array}$ length of the channel in the direction i m
- number of cells in the direction i Nusselt number based on hydraulic diameter
- $\frac{Nu}{Pr}$ Prandtl number
- heat flux kW/m^2
- ${q \over R^2}$ coefficient of determination
- ReReynolds number based on hydraulic diameter
- Re_{τ} friction Reynolds number
- RiRichardson number based on hydraulic diameter Ttemperature K
- T_m mean wall temperature K
- axial position in the channel m x_p

Greek symbols

- δ half channel height m
- Δ_i^+ dimensionless cell size in the direction i $\operatorname{Re}_{\tau}\Delta_{i}/\delta$
- ϵ relative error %
- thermal conductivity W/K/m λ
- dvnamic viscosity kg/m/s μ
- cinematic viscosity \tilde{m}^2/s ν
- density kg/m^3 ρ

Subscripts

- b bulk cold wall
- chhot wall
- mmean
- wwall
- xstreamwise direction
- wall normal direction y
- spanwise direction z

Results produced need to be averaged in space and time to provide an estimation of the heat fluxes. The calculation time and the procedure required to convert raw data into usable results are still obstacles for most of engineering companies. For that reasons, LES is commonly used by researchers in order to provide more adapted tools to engineering domain such as heat transfer correlations.

Asymmetrically heated channels are of particular interest in many of the engineering fields discussed above; especially in the concentrated solar power plant for which only the irradiated wall of the receiver is subjected to a very strong solar flux which leads to asymmetric heating of the fluid [21, 22, 23]. To the authors' knowledge, there are very few studies dealing with turbulent flows in asymmetrically heated channels, despite its frequent use, and no correlation has been developed in these difficult to study but widely encountered conditions. The present investigation has been undertaken to fulfill this gap in the literature.

A review of existing studies dealing with forced convection heat transfer for tube and channel geometries is presented in section 1. In section 2, the method and the characteristics of performed numerical simulations are explained. Then, in section 3, a correlation for asymmetrically heated channels under turbulent flow conditions is proposed and compared with selected correlations from the literature. In section 4, conclusions are drawn.

2. Forced convection heat transfer in the literature

Studies relative to heat transfer in tubes are presented in subsection 2.1. Subsection 2.2 deals with heat transfer in channels.

2.1. Heat transfer in tubes

In 1933, Colburn proposed a first general method for the correlation of forced convection heat transfer and pressure drop [24]. He links the Nusselt number to the Reynolds and Prandtl numbers. Film properties are involved and the correlation is applicable for a Reynolds number upper than 10000 and a Prandtl number ranging from 0.6 to 160. However, Colburn expresses doubts on the validity of his correlation for $Pr_b >$ 100. Since this pioneer paper on heat and mass transfer, a plethora of articles has been published to estimate heat transfer in various conditions. Sieder and Tate [25] base their work on Colburn's study. They propose to simplify the correlation of the latter using mainstream properties instead of film values and use a ratio of dynamic viscosity to permit estimating heating as well as cooling conditions. The applicable domain is similar to the one proposed by Colburn. Whitaker [26] investigates forced convection heat transfer correlation for flows in pipes as well as flows past flat plate, single-cylinder, single-sphere and flow in packed beds and tubes bundles. He proposes a heat transfer correlation for each condition. The

tested Prandtl numbers depend on the geometry but it covers at least the range $0.71 < Pr_b < 380$ (excepted for the flow in packed beds where only a Prandtl number equal to 0.7 is assessed). In 1976, Gnielinski [27] modified the correlation of Petukhov [28] and proposed a new equation for heat transfer in turbulent pipe flows. Dittus and Boetler [29] investigate heat transfer in automobile radiators of the tubular type. They propose to adapt the exponent of the Prandtl number to the heat transfer direction. Barnes and Jackson [30] study air, carbon dioxide and helium flows in circular tubes with large surface/gas temperature ratio. Experiments are performed under approximately uniform wall heat flux, low subsonic turbulent flow and fully developed profiles of velocity and temperature. Their work aims at measuring the effect of temperature ratio on Nusselt number for various fluids. Tests are performed in the range $4000 < Re_b < 60000$. An increase of the surface/gas temperature ratio leads to a reduction of the Nusselt number for a given Reynolds number. They propose a new correlation for each fluid. The expression developed for air was numerically tested by Dailey *et al.* [31] with a constant wall heating rate. It gives satisfying results for low and high fluid heating in turbulent channel flows. Gnielinski [32] moves forward heat transfer in tube flows and pointed out that there is no efficient correlation in this region due to the gap in the Nusselt number at $Re_b = 2300$. To fulfill this shortage in the literature, he realizes a linear interpolation of the Nusselt number between $Re_b = 2300$ and $Re_b = 4000$ using existing correlations. Experimental investigation of circular pipe flow is performed by Bertsche etal. [33] in the range 500 $< Re_b < 23000$ and $7 < Pr_b < 41$. The correlations of Gnielinski [34, 32] and Petukhov [28] are assessed. A good agreement is found. Recently, Taler [35] provides simple correlations to estimate heat transfer in turbulent tube flow for three various Prandtl number ranges: $0.1 < Pr_b < 1$, $1 < Pr_b < 3$ and $3 < Pr_b < 1000$. The exponent of the Prandtl number and the multiplicative constant decrease when the Prandtl number grows. The exponent of the Reynolds number evolves in the opposite

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way. The presented correlations are summarized in Table 1. Note that tube sections are the most well documented in terms of correlations. These correlations are sometimes used for rectangular geometries.

A graphical view of the applicable domain, in terms of Reynolds and Prandtl numbers, of each tube correlations is presented in Fig. 1. The existing correlations cover large ranges of Prandtl and Reynolds numbers.



Figure 1: Applicable range of heat transfer correlations for tubes. The two letters at the top right corner of the domains refer to the first author of the correlations reported in Table 1. Red contours signify that the correlation is applicable in the entrance region.

2.2. Heat transfer in rectangular ducts

In this subsection, symmetrically heated channel studies are reported in 2.2.1. Asymmetrically and one sided heated channel works are presented in 2.2.2.

2.2.1. Symmetric heating

Dennis *et al.* [36] were among the first authors studying forced heat convection in rectangular ducts. They consider laminar flows and supplement the researches of Clark and Kays [37] with heat transfer analyses in the thermal entry region. In 1970, Battista and Perkins [38] investigated heat transfer and skin friction coefficients

Expression	Domain
$Nu = \left(1 - \frac{Re_b - 2300}{4000 - 2300}\right) Nu_{lam, 2300} + \frac{Re_b - 2300}{4000 - 2300} Nu_{turb, 4000}$	$2300 < Re_b < 4000$
$Nu = 0.015 Re_b^{0.83} Pr_b^{0.42} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	$\begin{array}{l} 0.48 < Pr_b < 592 \\ 2300 < Re_b < 100000 \\ 0.44 < (\mu_b/\mu_w) < 2.5 \end{array}$
$Nu = \frac{(f/2)(Re_b - 1000)Pr_b}{1 + 12.7\sqrt{f/2}(Pr_b^{2/3} - 1)} \left[1 + \left(\frac{D_h}{L_x}\right)^{2/3} \right] K$	$\begin{array}{l} 3000 < Re_b < 1000000 \\ 0.5 < Pr_b < 200 \end{array}$
$Nu = 0.02155 Re_b^{0.8018} Pr_b^{0.7095}$	$\begin{array}{l} 3000 < Re_b < 1000000 \\ 0.1 < Pr_b < 1 \end{array}$
$Nu = 0.01253 Re_b^{0.8413} Pr_b^{0.6179}$	$\begin{array}{c} 3000 < Re_b < 1000000 \\ 1 < Pr_b < 3 \end{array}$
$Nu = 0.00881 Re_b^{0.8991} Pr_b^{0.3911}$	$\begin{array}{l} 3000 < Re_b < 1000000 \\ 3 < Pr_b < 1000 \end{array}$
$Nu = 0.023 Re_b^{0.8} Pr_b^{0.4} \left(\frac{T_w}{T_b}\right)^{n_{Ba}}$	$4000 < Re_b < 60000$ $Pr_b = 0.7$
$Nu = 0.023 Re_f^{0.8} Pr_f^{1/3}$	$Re_b > 10000 \\ 0.6 < Pr_b < 160$
$Nu = 0.023 Re_b^{0.8} Pr_b^{n_{DB}}$	$Re_b > 10000 \\ 0.7 < Pr_b < 120$
$Nu = 0.027 Re_b^{0.8} Pr_b^{0.33} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$	$\begin{array}{l} 10000 < Re_b < 110000 \\ 0.7 < Pr_b < 16700 \end{array}$
	$\begin{split} & \text{Expression} \\ & Nu = \left(1 - \frac{Re_b - 2300}{4000 - 2300}\right) Nu_{lam,2300} + \frac{Re_b - 2300}{4000 - 2300} Nu_{turb,4000} \\ & Nu = 0.015 Re_b^{0.83} Pr_b^{0.42} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \\ & Nu = \frac{(f/2)(Re_b - 1000)Pr_b}{1 + 12.7\sqrt{f/2}(Pr_b^{2/3} - 1)} \left[1 + \left(\frac{D_h}{L_x}\right)^{2/3}\right] K \\ & Nu = 0.02155 Re_b^{0.8018} Pr_b^{0.7095} \\ & Nu = 0.02155 Re_b^{0.8413} Pr_b^{0.6179} \\ & Nu = 0.01253 Re_b^{0.8413} Pr_b^{0.6179} \\ & Nu = 0.023 Re_b^{0.8} Pr_b^{0.4} \left(\frac{T_w}{T_b}\right)^{n_{Ba}} \\ & Nu = 0.023 Re_b^{0.8} Pr_f^{1/3} \\ & Nu = 0.023 Re_b^{0.8} Pr_b^{1/3} \\ & Nu = 0.023 Re_b^{0.8} Pr_b^{n_{DB}} \\ & Nu = 0.027 Re_b^{0.83} Pr_b^{0.33} \left(\frac{\mu_b}{\mu_w}\right)^{0.14} \end{split}$

Table 1: Heat transfer correlations for tube flows. ${}^{1}K = (Pr_b/Pr_w)^{0.11}$ for liquid and $K = (T_w/T_b)^{n_{Gn}}$ for gas. In the case of air flow, $n_{Gn} = 0.45$. 2 In the case of air flow $n_{Ba} = -0.4$. $n_{DB} = 0.4$ for fluid heating and $n_{DB} = 0.3$ for cooling. 3 The fluid properties are evaluated at the film temperature.

in air flow through vertical square ducts with significant heating. According to [39], the correlation can be extended to bulk Reynolds numbers of at least 4000. The same expression is valid for equilateral triangular ducts [40]. Heat transfer appears to be 10 % below the results obtained by Perkins and Worsoe-Schmidt [41] with circular tubes. Sudo et al. [7] assess Dittus-Boelter correlation with experimental data in turbulent channel flows symmetrically heated and heated from one side. It appears that the estimation of heat transfer proposed by Dittus-Boettler [29] gives prediction within an error of ± 20 % for symmetrically heated channels. For the one side heated conditions, at high Reynolds number, some predicted Nusselt numbers scatter outside the ± 20 % error lines. Experimental studies in a narrow rectangular channel have been carried out by Ma et [8] with deionized water. They point out al. that the non-heated friction factor and the average Nusselt number are well predicted with traditional relations. Ma et al. [8] notice that, for channel flows, the transition regime appears at

a slightly larger Reynolds number than for tube flows. They propose another correlation valid for $4000 < Re_b < 13000$ and $3.9 < Pr_b < 4$. Liang et al. [42] perform experiments in vertical and horizontal narrow rectangular channels with deionized water for bulk Reynolds numbers between about 300 and 6200. The Nusselt number seems independent of the channel orientation, meaning that the force of gravity has little effect in these conditions. The heat transfer coefficient increases quickly with the mass flow and slowly with the heat flux. Wang *et al.* [43] study the laminar to turbulent transition region in rectangular channel. Their experiments are in the range $1000 < Re_b < 20000$ and $2.1 < Pr_b < 3.5$. The lower and upper critical Reynolds numbers of the laminar to turbulent transition increase with the decrease in Prandtl number. Another experimental investigation of forced convection heat transfer in narrow rectangular channels has been performed by Jo et al. [44] with demineralized water. Upward and downward flows are compared. The authors highlight that the flow orientation

has no significant effect on heat transfer. Correlations for laminar and turbulent flows in the entrance region and in the fully developed region are proposed. The error relative to the turbulent flow in the fully developed region is ± 18.5 %. Ghione *et al.* [45] carried out rectangular channel flow experiments with water under strong heat fluxes and high mass flows. Two channel heights are evaluated. They find that the correlation of Dittus-Boetler underestimates the heat transfer for high Reynolds numbers, which is in agreement with [46]. The authors also notice that reducing the channel height leads to heat transfer enhancement. Moreover, they propose correlations for fully developed boiling flow. Mortean and Mantelli [3] adapt the correlation of Stephan and Preußer [47] developed for tube flows to channel flows: the first term (4.364 in [47]) is replaced by 3.63 which is the Nusselt number adopted by the literature for fully developed flow in square channels. Mortean *et al.* [4] provide an expression for the transition region $1800 < Re_b < 4000$ which combines laminar and turbulent Nusselt number correlations in the manner of Gnielinski [32]. The mean difference between this equation and the experimental data is 13.9 %. With the same data, the heat transfer expression proposed by Gnielinski for tubes has a mean difference of 19.8 %. In a recent experimental study, Sun et al. [48] perform non uniform and uniform heating of a rectangular channel flow. The results indicate that the average Nusselt number is affected by the form of the transverse heating. They test 15 heat transfer equations and develop a new one for laminar to beginning of turbulence region of water flows. The correlation is applicable for $3000 < Re_b < 10000$ and $3.5 < D_h < 4.5$. The aspect ratio width to depth of the channel is between 20 and 30.

Heat transfers are also studied thanks to numerical simulations. In 1994, Wang *et al.* [49] investigated heat transfer in a 180 degrees turning duct with several turn configurations. Their numerical study uses an extended version of the $k-\epsilon$ turbulence model and agrees favorably with experimental data for Nusselt estimation. Qin and Pletcher [50] performed LES of turbulent mixed convection heat transfer in a variable-property

thermally developing rotating square. A fairly satisfactory agreement between prediction and experimental results is obtained. Dailey *et al.* [31] carried out LES of a channel flow under constant heat flux boundary conditions. They study strong heating and cooling leading to high wall-to-bulk temperature ratios $(T_w/T_b = 1.5 \text{ in the heated})$ case and $T_w/T_b = 0.56$ in the cooling case). Fluid properties are temperature dependent. High heating and cooling as well as low heating conditions are investigated and compared to empirical correlations. The high cooling and low heating cases are in good agreement with most of the heat transfer correlations. However, for high heating conditions, the Nusselt number obtained by correlations is lower than the one predicted by LES. The effects of channel aspect ratio on thermally developing flow and heat transfer are studied by Lee and Garimella [10]. Boundary conditions are circumferentially constant wall temperature and axially constant wall heat flux. The authors propose generalized Nusselt correlations for microchannel heat sinks and compare them with other conventional correlations and experimental data. The results indicate that the more the channel is flattened, the bigger the Nusselt number is. Sato et al. [51] investigate the effect of the Prandtl number on heat transfer enhancement in a dimpled-channel flow. Their work covers the range $1000 < Re_b < 10600$. Simulations are conducted for Prandtl numbers of 0.2, 0.7 and 3. They define a thermal performance parameter (also seen in [11]), which is the ratio between the Nusselt number to the pressure drop compared to the same ratio for a flat channel flow at the same bulk Reynolds number, and plot it against various mass flows. The results suggest that the best heat transfer enhancement thanks to the dimples is reached for high Prandtl number at a bulk Reynolds number of 2000 which is in the end of the laminar region. Thermal entrance length in microchannel flows is analyzed, for different aspect ratios of channels, by Su et al. [52] thanks to simulations. The Reynolds number is lower than 2000. The results of this study show that the longest thermal entrance length is reached for a channel three times wider than deep.

Furthermore, the Nusselt number is found to increase with the aspect ratio which is in agreement with [10, 45, 46].

A summary of the most used correlations, depending on the entrance length, for rectangular channels is proposed in Table 2. Correlations for fully developed conditions are reported in Table 3.

A graphical view of the applicable domain, in terms of Reynolds and Prandtl numbers, of each channel correlations is presented in Table 2. The Prandtl number range between 1.2 and 5.9 is very well covered by the existing correlations for laminar to highly turbulent region. However, there is a lack of correlations for Prandtl number inferior to one, especially at high Reynolds number.



Figure 2: Applicable range of heat transfer correlations for rectangular channels. The two letters at the top right corner of the domains refer to the first author of the correlations reported in tables 2 and 3. Red contours signify that the correlation is applicable in the entrance region.

2.2.2. Asymmetric and one sided heated channel studies involving forced convection

Asymmetrically heated channel flow is commonly studied with natural convection [53, 54, 55]. These conditions are mainly investigated for building applications [56, 57, 58]. However, few publications handle asymmetric heating associated with forced convection and even less deal-

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ing with turbulent flows. Osborne and Incropera [59] investigate laminar flow in the thermal entrance region through a horizontal asymmetrically heated plate channel. In this experimental study mixing natural and forced convection, various small heat fluxes are fixed (inferior to 6 kW/m^2). Vertical temperature distributions are plotted, flow visualization is achieved and heat transfer analysis is carried out. In their study, the top plate conditions are poorly influenced by buoyancy. When the flow is symmetrically heated, the heat transfer is significantly enhanced by buoyancy at the bottom wall. Mixed convection of air in a horizontal channel heated from one side has been investigated by Gau et al. [60]. The authors show that the increase of the buoyancy parameter leads to an elevation of the Nusselt number normalized by $Re_b^{0.4}$. Heat transfer in the studied configuration is much greater than in the case involving vertical channel flow. Hwang *et al.* [61] study forced convection in an asymmetrically heated packed channel filled with glass spheres. Following their experimental runs, they provide a correlation to estimate heat transfer thanks to the Reynolds number and diameter of the solid particles. The authors state that, at similar Reynolds number, forced convective heat transfer of Freon-113 in a packed channel is about three times higher than in an empty channel. Conventional Nusselt number at the cold wall of an asymmetrically heated channel becomes infinite and then negative when the bulk temperature tends toward and overtakes the cold wall temperature. To avoid this singularity, Nield [62] defines a mean Nusselt number as:

$$Nu_m = \frac{D_h q_m}{\lambda_w (T_m - T_b)} \tag{1}$$

with q_m the mean heat flux and T_m the mean wall temperature. An analytic study leads to Nusselt expressions for slug flow, Poiseuille flow and Couette flow. Nield [62] studies laminar flows and shows that, as long as the velocity profile is symmetric with respect to the midline of the channel, the Nusselt number is independent of the asymmetry. He demonstrates the validity of his method for the isoflux and isotemperature cases.

Author (date)	Expression	Domain
	4.00	$400 < Re_b < 2400$
Mortean $et al.$ (2019)	$Nu = 3.63 + 0.086 \frac{(1/L^*)^{1.33}}{1+0.1Pr} (Re, D, /L)^{0.83}$	0.7 < Pr < 7
	$1+0.177_b(Re_bD_h/L_x)^{+1}$	L* > 0.03
Jo et al. (2013)	$N_{at} = 0.011 P_{c} 0.1418 D_{m} 0.082 C_{a} 0.296$	$Re_b < 3000$
	$Nu = 0.911 Re_b$ FT_b Gz	$2.64 < Pr_b < 6.46$
Wang <i>et al.</i> (2012)	$N = 0.04 (D^{2/3} - 100) D^{15} [1 + (D_{b})^{2/3}] (\mu_{b})^{0.11}$	$2500 < Re_b < 7500$
	$N u = 0.04 \left(Re^{2/3} - 100 \right) Pr_b^{2/3} \left[1 + \left(\frac{\pi u}{L_x} \right)^{2/3} \right] \left(\frac{\pi u}{\mu_w} \right)$	2.2 < Pr < 3.5
Battista and Perkins ¹ (1970)		$4000 < Re_b < 49000$
	$N_{\mu} = 0.001 D [0.8 D [0.4 (T_{\mu})^{-0.7} [1 + (x_{\mu})^{-0.7} (T_{\mu})^{0.7}]$	$Pr_b = 0.7$
	$N u = 0.021 Re_b^{ord} P T_b^{ord} \left(\frac{\overline{w}}{T_b} \right) \qquad \left[1 + \left(\frac{\overline{D}}{D_h} \right) - \left(\frac{\overline{w}}{T_b} \right) \right]$	Tw/Tb < 2.13
		$22 < x_p/D_h < 155$
Jo et al. (2014)	$N_{at} = 0.0007 P_{c} 0.7705 D_{m} 0.143 C \sim 0.217$	$5000 < Re_b < 54000$
	$Nu = 0.0097 Re_b^2 \cdots Pr_b^2 Gz^{-12}$	$2.64 < Pr_b < 6.46$

Table 2: Heat transfer correlations for channel flow in the entrance region. 1 The temperatures are expressed in degrees Fahrenheit and the correlation is for square ducts.

Moreover, Nield [62] explains how to get wall information (temperatures or heat flux depending on boundary conditions) from his average Nusselt number. Nevertheless, it seems difficult to operate with turbulent conditions. Mitrovic et al. [63] study the asymmetric Graetz problem, which is defined as steady-state forced convection heat transfer, with plane laminar flow confined by two parallel plates that are kept at constant but different temperatures. Their numerical simulation of the problem agrees excellently with the analytic solution. It shows that the singularity of the Nusselt number occurs at different axial positions and depends on the thermal asymmetry and fluid inlet conditions. The authors pointed out that there is no correlation in the literature that accounts for possible jumps of the Nusselt number relative to the cold wall. Another asymmetric study of channel flow has been achieved by Repata and Satvamurti [64]. Their numerical work assumes laminar and incompressible, steady flow of a Newtonian fluid with constant thermophysical properties. They show that the heat flux varies monotonically with the axial distance even though the conventional Nusselt number is discontinuous. The modified Nusselt number proposed by Nield [62] is continuous. The authors affirm that this modified Nusselt number is applicable for the case of equal wall temperatures and coincides with the classical definition. In

the laminar case, the fully developed condition is reached when the temperature varies linearly from the temperature at one wall to the temperature at the other wall. Kuznetov and Nield [65], investigate forced convection in a channel partly occupied by a bidisperse porous medium under asymmetric heating. Thanks to an analytic study, they show that a singularity of the modified Nusselt number proposed by Nield in [62] appears under certain heating conditions. When plotting against an asymmetrical heating parameter, this singularity relies on the bidisperse porous medium thickness. Asymmetric heating of air in a square microchannel has been investigated by van Male [66]. Nusselt correlations are developed et al. for laminar and plug flows. For laminar flow, the Reynolds number is between 200 and 500. An analytic description of heat and mass transfer is provided for plug flow. In [46], Ma et al. perform experimental tests on heat transfer enhancement with longitudinal vortices in a narrow rectangular channel heated from one side. The used fluid is water. The results indicate that in the laminar region, longitudinal vortices enhance the heat transfer of about 100 % for an increase of flow resistance of only 11 %. In the case of turbulent flow, the flow resistance modification is higher than the heat transfer (an increase of 100 % for a 87 % enhancement of heat transfer).

Torii and Yang [67] carry out numerical simula-

Author (date)	Expression	Domain
Liang et al. (2012)	$N_{11} = 2.077 D_{10} 0.109 D_{11} 1/3$	$Re_b < 2300$
	$Nu = 3.975 Re_b^{abov} Pr_b^{abov}$	Deionized water
I_{2} at al. (2012)	$N_{22} = 2.0120 C \cdot 0.3756$	$Re_b < 3000$
Jo <i>et al.</i> (2013)	$Nu = 2.0129Gz^{-1.012}$	$2.64 < Pr_b < 6.46$
Montoon at $al (2012)$	$\Gamma(x, 6, x, 6, x, 6, x)^{1/6}$	$1800 < Re_b < 4000$
Mortean $et al. (2012)$	$N u_{transi} = \left[\left(N u_{lam}^{\circ} + N u_{turb}^{\circ} \right) \right]^{2}$	Water
Liang et al. (2012)	$N_{au} = 0.00666 P_{c} 0.933 P_{m} 0.4$	$2300 < Re_b < 7000$
	$Nu = 0.00000 Re_b \Gamma T_b$	Deionized water
Sun et al. (2020)	$N_{\rm res} = 0.212 (D_{\rm r} 0.6 - 0.24) D_{\rm r} 0.4 (1 - D_{\rm r})^{2/3}$	$3000 < Re_b < 10000$
	$Nu = 0.212 \left(Re_b^{0.0} - 83.4 \right) Pr_b^{0.4} \left(1 - \frac{D_h}{L_x} \right)$	Deionized water
	$N_{22} = 0.00254 D_{22} D_{22}^{-0.4}$	$4000 < Re_b < 13000$
Ma $et at. (2011)$	$N u = 0.00554 R e_b P T_b$	$3.9 < Pr_b < 4.1$
	0.7	$4000 < Re_b < 49000$
Battista and Perkins ^{$*$} (1970)	$Nu = 0.021 Re_{1}^{0.8} Pr_{1}^{0.4} \left(\frac{T_{w}}{T_{w}}\right)^{-0.7}$	$Pr_b = 0.7$
	$b \qquad b \qquad \left(\begin{array}{c} I_{b} \end{array} \right)$	Tw/Tb < 2.13
$I_{-} = + -1 (2014)$	$N_{\rm ev} = 0.00 r_0 D_{\rm e} 0.9383 D_{\rm e} 0.4$	$5000 < Re_b < 54000$
Jo et al. (2014)	$Nu = 0.0058Re_b^{chose}PT_b^{ch}$	$2.64 < Pr_b < 6.46$
	0.14	$10000 < Re_b < 269000$
Ghione et al. (2016)	$Nu = 0.0044 Re_{b}^{0.960} Pr_{b}^{0.568} \left(\frac{\mu_{b}}{\mu_{b}}\right)^{0.14}$	$1.2 < Pr_b < 5.94$
	$v = v = (\mu_w)$	$2\delta = 2.161mm$
Ghione <i>et al.</i> (2016)	0.14	$10000 < Re_b < 177000$
	$Nu = 0.00184 Re_{b}^{1.056} Pr_{b}^{0.618} \left(\frac{\mu_{b}}{\mu_{b}}\right)^{0.14}$	$1.18 < Pr_b < 5.70$
	μ_w	$2\delta = 1.509mm$

Table 3: Heat transfer correlations for channel flow in the fully developed region. 1 The temperature are expressed in degrees Fahrenheit and the correlation is for square ducts.

tions of thermal fluid-flow transport phenomena in a channel under asymmetric heat flux. They use the $k - \epsilon$ turbulence model and two equations heat transfer model. The authors compare theoretical solutions for turbulent and laminar heat transfer proposed by Kays and Crawford [37] to assess the laminarization process. Their results show that if a channel is heated from exclusively one wall the fluid flow cannot be laminarized contrary to the case of tube flow. Few correlations for tube flows laminarization are compared to flow laminarization in channel heated from both sides.

No correlation of asymmetrically heated turbulent channels flows is proposed in the literature, however, physical effects of the coupling between temperature and velocity have been numerically investigated in [68, 69, 70, 71] for similar working conditions. These studies show that the coupling between temperature and dynamic should be considered to properly describe the flow in case of asymmetric heating conditions. It is illustrated by the poor efficiency of the conventional correlations when handling asymmetric heating (see subsection 4.3). The specificity of heat transfer in asymmetrically heated channel flows requires a new correlation to be properly estimated. It is the purpose of this paper.

3. Numerical simulations

3.1. Physical and numerical settings

3.1.1. Physical setting

Large Eddy Simulations (LES) are performed to investigate convective heat transfer of fully developed turbulent channel flow. Navier-Stokes equations are solved under the low Mach number approximation proposed by Paolucci [72]. This assumption permits considering the coupling effect between temperature and dynamic, as advised by the literature [68, 69, 70, 71] while neglecting acoustic effects, which are very small in the current conditions. The flow is driven by forced convection. Indeed, the Richardson number is about 10^{-3} for the simulations at lowest

Reynolds numbers, meaning that the buoyancy effects are negligible compared to the kinetic effects. Newtonian fluid and Fourier's law are assessed. Equations are expressed using a Favre filtered variable built with density weighted Favre filter $(\widetilde{\cdot})$ defined for any field ψ as $\psi = \rho \psi / \overline{\rho}$. The AMD model [73] is applied to estimate the momentum convection subgrid term $\left(\widetilde{U_jU_i} - \widetilde{U_j}\widetilde{U_i}\right)$, where U_i and U_j are respectively the i-th and j-th component of velocity, and density-velocity correlation subgrid term $\left(\widetilde{U_j/\rho} - \widetilde{U_j}/\overline{\rho}\right)$. This model has shown a good agreement with Direct Numerical Simulations (DNS) in asymmetrically heated channel associated with turbulent flows [74, 75]. The fluid is air. Sutherland's law [76] is applied to calculate the viscosity. In each simulation, the Prandtl number is constant. The heat capacity at constant pressure is also fixed.

3.1.2. Geometry and mesh

The channel is periodic in the streamwise (x)and spanwise (z) directions and enclosed by two plane walls in the wall-normal direction (y). The temperatures of the plates are fixed. Note that Avellaneda *et al.* [70] studied the influence of thermal boundary conditions (fixed wall temperatures versus imposed heat flux) for channel flow under asymmetric high temperature gradient and show that there are no significant differences on the main turbulence statistics. Figure 3 presents the geometry in case of asymmetrical heating. The coolest wall is referred to as "cold wall" and hottest is referred to as "hot wall". The domain size is $L_x \times L_y \times L_z = 4\pi\delta \times 2\delta \times 4/3\pi\delta$ with $\delta = 0.003$ m. LES are carried out using a finite difference method in a staggered grid system. Mesh is uniform in the homogeneous directions and follows a hyperbolic tangent law in the wallnormal coordinate direction. For practical reasons, the same mesh is used for all calculations and is composed of 2.3 millions of cells distributed as follow : $N_x \times N_y \times N_z = 160 \times 152 \times 96$. The dimensionless cell size varies with the conditions of simulation. Meshes characteristics are presented in Table 4. The coarsest LES has a dimensionless size of the first cell of 1.39 in the wall nor-



Figure 3: Geometry of the channel flow.

mal direction which is in the viscous sublayer. A streamwise volume force is added in the momentum equation to balance wall friction. Similarly, a source term is included in the energy equation to permit testing various profiles of temperature inducing different heat fluxes.

Table 4: Meshes characteristics at cold wall (highest friction velocity). Std means standard deviation.

Meshes characteristics	Δ_x^+	$\Delta_y^+(0)$	$\Delta_y^+(\delta)$	Δ_z^+
Coarsest mesh	103.5	1.39	42.5	57.5
Finest mesh	10.7	0.14	4.4	5.9
Average mesh	48.9	0.64	20.1	27.1
Std	25.4	0.37	10.4	14.1

3.2. Computation of Nusselt numbers using Large Eddy Simulation

Comparisons with Direct Numerical Simulations (DNS), performed for different heating conditions and various mass flows, indicate that LES systematically underestimate heat flux and Nusselt number. However, this error is predictable because it is proportional to the dimensionless cell sizes and can be corrected by introducing a multiplication factor. For that reason, in this paper, all results of LES are corrected to reproduce the heat fluxes that would have been obtained with DNS. For each wall, the Nusselt number is calculated using the temporally and spatially averaged heat flux of the wall, q_w . Its expression is given by equation 2.

$$Nu_w = \frac{q_w D_h}{\lambda_w (T_w - T_b)} \tag{2}$$

Where D_h is the hydraulic diameter, λ_w the thermal conductivity, T_w and T_b respectively the wall and bulk temperatures.

The calculation of the Nusselt number with this methodology is validated against experiments achieved by Jo *et al.* [44]. Nine comparisons are performed for three Reynolds numbers and two Prandtl numbers (see Table 5). The results show satisfying agreement. Indeed, the relative error, $r_i = (Nu^{LES} - Nu^{Jo}) / Nu^{Jo}$, obtained is inferior to 8.3 % for each tested configuration, which is smaller than the error range of experimental measures. The validation with experimental data of Jo *et al.* are performed until a bulk Reynolds number of 35620 corresponding to a fully turbulent flow.

Table 5: Comparison of the Nusselt number obtained by the experiments of Jo *et al.* and the performed simulations. In the first column, X-X-X refers to Reynolds number, Prandtl number and temperature difference: "L" means low, "M" medium, and "H" high. r_i is the relative error and is calculated as follow: $r_i = (Nu^{LES} - Nu^{Jo}) / Nu^{Jo}$

Characteristics	Re_b	Pr_b	$T_w - T_b$	r_i
L-L-M	11816	2.64	24.6	2.4~%
L-L-H	12029	2.64	48.4	1.9~%
M-H-L	23376	3.18	9.4	-5.9~%
M-L-L	23377	2.64	9.5	-1.9 %
M-H-M	23529	3.18	18.1	-1.9 %
M-L-H	23685	2.64	27.0	-1.7 %
H-L-L	35082	2.64	10.0	-8.3 %
H-L-M	35276	2.64	17.4	-5.5~%
H-L-H	35620	2.64	30.2	-3.6 %

As for as asymmetric heating conditions, LES have been compared to DNS results for three bulk Reynolds numbers (23000, 60000 and 82000) at a Prandtl number of 0.8. Two velocity profiles have been investigated. Results show a good agreement with 3% of error on velocity and temperature profiles.

4. Correlation for asymmetrically heated channel associated with turbulent flows

4.1. Development of the proposed correlation

To the authors' knowledge, no expression is available in the literature to estimate heat transfer in an asymmetrically heated channel associated with turbulent flows. The Proposed Correlation, referred to as "PC" in figures and tables, is developed thanks to 70 LES divided into 30 simulations with symmetric heating and 40 simulations with asymmetric heating. The Battista and Perkins [38] form of the correlation was assumed and a new term, involving the wall temperature, T_w , bulk temperature, T_b and mean wall temperature, $T_m = (T_c + T_h)/2$, is added to account for asymmetric conditions:

$$Nu = aRe_b^{0.8}Pr_b^c \left(\frac{T_w}{T_b}\right)^d \left(\frac{T_w}{|T_w - T_b|}\right)^{e\left(1 - \frac{T_w}{T_m}\right)\frac{T_b}{T_w}}$$
(3)

The exponent of the Reynolds number is fixed to 0.8 because this value is widely used in similar conditions [30, 29, 38]. A term accounting for asymmetric heating conditions is added in the equation to reproduce the heat transfers at hot and cold walls. Notice that this term involves variables in the exponent. The singularity of the Nusselt number, encountered when the bulk temperature tends towards the cold wall temperature and explained in [62], is reproduced thanks to the difference of temperature at the denominator of the asymmetric group. The coefficients of the correlation (a, c, d and e) are optimized using a least-squares fit. The best-fitting correlation for the simulation results is:

$$Nu = 0.024 Re_b^{0.8} Pr_b^{0.4} \left(\frac{T_w}{T_b}\right)^{-0.9} \\ \left(\frac{T_w}{|T_w - T_b|}\right)^{1.4 \left(1 - \frac{T_w}{T_m}\right) \frac{T_b}{T_w}}$$
(4)

In case of symmetric heating, the asymmetric term is equal to one and the expression can be simplified in:

$$Nu = 0.024 Re_b^{0.8} Pr_b^{0.4} \left(\frac{T_w}{T_b}\right)^{-0.9}$$
(5)

In accordance with the literature, the proposed correlation has a bigger multiplicative constant than the one proposed by Battista and Perkins [38] for square ducts. Indeed, channel flow experiments performed by Jo *et al.* [44] indicate that the correlation of Battista and Perkins [38] underestimates the heat transfer in this geometry. Furthermore, several studies show that the more the shape of the channel is flattened, the bigger the Nusselt number is [10, 52, 45]. The proposed correlation is investigated in the commonly encountered working conditions of heat exchangers and solar receivers.

- For symmetric conditions
 - $12000 < Re_b < 177000$

$$-0.47 < T_b/T_w < 0.99$$

- For asymmetric conditions
 - $10600 < Re_b < 145000$
 - $-1.1 < T_h/T_c < 2.0$
 - $-0.63 < T_b/T_c < 0.95$
 - $-0.44 < T_b/T_h < 0.85$

The wall temperatures range from 293 K to 1300 K. The fluid temperature varies from 342 K and 1237 K, Prandtl number is between 0.76 and 3.18, and heat fluxes are between 4 kW/m² and 578 kW/m².

4.2. Results of the proposed correlation

The proposed correlation is firstly assessed using the indicators described below. The relative error is defined in Eq. 6.

$$\epsilon_{Nu,i} = \frac{Nu_i^{correlation} - Nu_i^{LES}}{Nu_i^{LES}} \tag{6}$$

The determination coefficient is evaluated to assess the global revelancy of the correlation (Eq. 7).

$$R^{2} = 1 - \frac{\sum_{i=1}^{N} \left(Nu_{i}^{LES} - Nu_{i}^{correlation} \right)^{2}}{\sum_{i=1}^{N} \left(Nu_{i}^{LES} - \overline{Nu}^{LES} \right)^{2}} \quad (7)$$

where $\left(\overline{Nu^{LES}}\right)$ is the average of the Nusselt numbers obtained with LES. The mean error and

the standard deviation are calculated as follow (Eq. 8 and Eq.9).

$$Mean = \frac{1}{N} \sum_{i=1}^{N} \epsilon_{Nu,i} \tag{8}$$

where N is the number of results.

$$Std = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |\epsilon_{Nu,i} - Mean|^2} \qquad (9)$$

All the Nusselt numbers estimated with the proposed correlation have a relative error inferior to ± 9.1 % and more than three-quarter of them are in the error bound ± 5 %. The determination coefficient is 0.993. The mean error is 0.29 % and standard deviation is 4.1 %.

The Nusselt number obtained by the proposed correlation is plotted as a function of the Nusselt number calculated by numerical simulations in Fig. 4. The Nusselt numbers range from 20 to 705. High Nusselt number values are reached by the cold Nusselt number under asymmetric conditions when the bulk temperature is close to the cold wall temperature. As discussed above, all points are below the ± 10 % error lines. For symmetric calculations, in most cases, the correlation slightly overestimates the Nusselt number. Indeed, the mean error is 1.6 %. For asymmetric conditions, points relative to hot and cold walls are equally spread out on either side of the 0 %error line.

Fig. 5 shows the error on the Nusselt number against various parameters of the simulations. The graphs are described from left to right. The effect of the bulk Reynolds number is observed on the first graph. For symmetric conditions, the biggest errors are obtained for small Reynolds numbers. In the case of asymmetric heating, the Nusselt number is underestimated at the cold wall for Reynolds numbers under 88000 and overpredicted for higher values. As for the hot wall, the accuracy of the correlation is enhanced for low Reynolds numbers. Indeed, for Reynolds numbers inferior to 80000, all points are below the ± 5 % error bound. The standard deviation increases with the Reynolds number. The second graph presents



Figure 4: Nusselt numbers obtained by the proposed correlation (PC) against Nusselt numbers calculated by Large Eddy Simulations.

the evolution of the error on the Nusselt number depending on the friction Reynolds number. For symmetric heating and cold wall, the tendencies are the same as in the first graph. For the hot side, no significant tendency is observed. In the third graph, the results are plotted against the ratio of bulk and cold wall temperatures. As explained in subsection 4.1, in the particular case where the bulk temperature is superior to the cold wall temperature (meaning that the fluid is cooled at the cold wall), this correlation is only able to predict the hot Nusselt number because of the singularity, described in [62], which occur at the cold wall. For that reason, some points corresponding to the hot side have a ratio of bulk and cold wall temperatures superior to 1. The fourth graph shows the error on the Nusselt number depending on the ratio of hot and cold wall temperatures. The evolution of the relative error seems to have no link with the temperature ratio.

A synthetic view of the result is proposed in Fig. 6. The errors on the Nusselt number are plotted as a function of the bulk Reynolds number and the ratio of bulk and cold wall temperatures.

Points are colored according to the error on the Nusselt number. The same behaviors as those mentioned in the analysis of Fig. 5 are observed.

4.3. Comparison with selected correlations from the literature

In this section, the correlations of Jo *et al.* [44], Battista and Perkins [38], Gnielinski [27] and the proposed expression are compared. None of the expression existing in the literature is applicable in these conditions. However, the ones selected are the most promising due to their validity domain and/or their formulation. Pro and cons, with regard to the present configuration, of these correlations are reported in Table 6.

The performances of the four correlations are compared, in Table 7, for simulations with symmetric heating only, with asymmetric heating only and for the entire data set. Coefficient of determination (R^2) , Mean error, standard deviation (Std) as well as minimum and maximum values (Min and Max) in terms of error are specified. Min and Max values of respectively -2.3 % and



Figure 5: From left to right, errors on the Nusselt number depending on the bulk Reynolds number, on the friction Reynolds number, on the ratio of bulk and cold wall temperatures and on the ratio of hot and cold wall temperatures.

Table 6: Pro and cons of the selected correlations regarding the present study configuration.

Author	pro	cons
Jo et al. (2014)	turbulent channel flow	water
Gnielinski (1976)	turbulent air flow, wall temperature involved	circular tubes
Battista and Perkins (1970)	turbulent air flow, wall temperature involved	square ducts

145.1 % indicate that the estimation of the correlation fluctuates between -2.3 % and 145.1 % error bounds. The results show that all the assessed correlations of the literature are more accurate when the heating conditions are symmetric which was expected since they are developed for this type of heating. The results of the correlation proposed by Jo *et al.* [44] do not show a good agreement with the simulations: overestimation of the Nusselt number in symmetric heating conditions and for the hot wall of asymmetric heating. The correlation of Battista and Perkins and the one of Gnielinski are quite accurate for symmetric heating (respectively $R^2 = 0.970$ and $R^2 = 0.963$). Gnielinski expression has a lower mean error than the one of Battista and Perkins but a bigger standard deviation. Considering the

asymmetric case, the two correlations have the same drawbacks: underestimation of the Nusselt number at the cold wall and overestimation at the hot wall. The expression proposed in this paper is the best of those studied here and agrees very well with the simulations. Indeed, it is accurate in both heating conditions in terms of R^2 , mean error and standard deviation. Regarding the entire data set, the mean error is 0.3 %, the standard deviation is 4.1 % and the correlation provides prediction within ± 9.1 % error.

Fig. 7 exposes the behavior of the four correlations. Nusselt numbers obtained with the correlation are plotted against Nusselt number obtained by simulations. The three correlations of the literature tend to overestimate the hot Nusselt number in asymmetric conditions and they un-



Figure 6: Errors on the Nusselt number as a function of the bulk Reynolds number and the ratio of bulk and wall temperature.

derestimate the cold Nusselt number. The error committed at the hot side and in the case of symmetric heating seems to evolve in the same way: for the correlation proposed by Jo *et al.* [44], the error increases with the Nusselt number whereas for the two others, the deviation is independent of the Nusselt number. The underestimation observed for the three correlations of the literature is worsened with the increase of the Nusselt number. For the proposed correlation, Nusselt numbers relative to cold and hot walls are equally spread out over the 0 % error line which traduces the relevance of the term accounting for asymmetric heating.

The errors committed by correlations are presented against the bulk Reynolds number in Fig. 8. In the symmetric conditions and hot side of the channel, the deviation of the Jo *et al.* [44] correlation increases with the Nusselt number. These results can be explained by the fact that this correlation has been developed for bulk Reynolds numbers below 50000. For the Nusselt number predicted by Jo *et al.* at the cold wall and the one estimated by the other correlations, there is no significant influence of the bulk Reynolds number on the error committed.

Fig. 9 shows the evolution of the gap in the Nusselt number against the ratio of bulk and cold wall temperatures for the selected correlations. In the symmetric case, the correlation suggested by Jo et al. [44] gradually overestimates the Nusselt number as the ratio of temperatures becomes small. The same behavior, at a lesser extent, is observed for the Gnielinski correlation. The expression proposed by Battista and Perkins is very mildly affected by the evolution of the temperature ratio. At the cold wall, the error increases with the ratio of temperatures for the three expressions of the literature. This is explained by the fact that when the bulk temperature tends toward the cold wall temperature, the Nusselt number skyrockets as shown in [62, 64]. This behavior of the Nusselt number at the cold wall is not considered by the existing correlations since they are developed for symmetric heating conditions. As a result, the heat transfer is significantly underestimated. The

Type of simulation	Correlations	R^2	Mean [%]	Std [%]	Min [%]	Max [%]
Symmetric	Jo	-1.670	36.14	34.9	-2.3	145.1
	Perkins	0.970	-7.41	2.9	-14.1	-0.3
	Gnielinski	0.963	3.67	5.7	-6.0	22.9
	PC	0.992	1.61	3.6	-5.8	8.6
Asymmetric cold side	Jo	0.658	-6.21	28.7	-48.7	68.2
	Perkins	0.239	-37.11	13.8	-64.3	9.0
	Gnielinski	0.298	-31.19	16.7	-63.4	2.4
	PC	0.990	-0.81	5.5	-9.1	8.3
	Jo	-7.226	108.36	29.8	62.4	193.5
Asymmetric hot side	Perkins	0.641	19.72	14.3	-6.2	51.3
Asymmetric not side	Gnielinski	0.180	36.27	12.3	10.2	63.9
	PC	0.991	-0.44	3.36	-7.7	8.6
Entire data set	Jo	-0.103	57.65	56.9	-48.7	193.5
	Perkins	0.519	-2.66	25.5	-64.3	51.3
	Gnielinski	0.518	9.70	29.6	-63.4	63.9
	PC	0.993	0.29	4.1	-9.1	8.6

Table 7: Performances of the four correlations for symmetric heating, for cold and hot side of asymmetric heating conditions, and for all simulations.

asymmetrical term involved in the proposed correlation permits reproducing this particular evolution of the Nusselt number. At the hot wall, the error committed by the correlation of Jo *et al.* [44] seems to be independent of the temperature ratio. However, for the expression of Battista and Perkins [38] and Gnielinski [27], the error is constant from $T_b/T_c = 0.63$ to $T_b/T_c = 1$ then it increases with the ratio of temperature. This tendency is not observed for the proposed correlation and its errors remain equally spread out over the 0 % error line and between the ±10 % limit.

5. Conclusions

In this paper, a review of heat transfer studies for tube and channel flows is proposed. Symmetrically and asymmetrically heated flows as well as one sided heated channel flows are presented. For the two geometries, the main correlations are summarized in tables. A graphical view of the applicable domain of correlations is proposed. It follows from this literature review that there is a lack of correlations for asymmetrically heated channels under turbulent flow conditions.

A new correlation for turbulent flows in symmetrically and asymmetrically heated channels is investigated using a Large Eddy Simulation database composed of more than 90 Nusselt number values. The simulations have been performed in a bi-periodic channel representing a channel with a small height compared to width. The domain covered by simulations is large (heat fluxes varying between 4 and 580 kW/m², bulk temperature ranging from 340 K to 1240 K and bulk Reynolds number from 10 600 to 177 000).

The Nusselt number obtained by the proposed expression is compared to three correlations of the literature which have been selected for their relative closeness with the studied configuration. The coefficient of determination, mean error and standard deviation between the estimation provided by correlations and the results of Large Eddy Simulations are calculated. Moreover, the error committed by correlations is plotted against various flow characteristics in order to analyze their behavior.

The evaluation of the turbulent heat transfer coefficient shows that the correlations of the literature tend to overestimate the Nusselt number at the hot side of asymmetrically heated channel independently of the temperature ratio. They, also, underestimate the heat transfer at the cold side, especially when the bulk temperature is close to cold wall temperature. On the contrary, the proposed correlation is very accurate for both heating



Figure 7: Nusselt number obtained by correlations against Nusselt number calculated by simulations.

conditions and provides reliable predictions. Indeed, the coefficient of determination is 0.993, the mean error is 0.29 % and all the results obtained are below the 9.1 % error limit.

The term accounting for asymmetrical heating involves variables in exponent. To the authors' knowledge it is the first time that such a formulation is used to estimate heat transfer and its relevancy has been proved in this study. With a possible adjustment, this term may be added to other correlations to extend their domain of validity to asymmetric heating.

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Figure 8: Comparison of errors on the Nusselt number obtained by the four correlations depending on the bulk Reynolds number.

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Figure 9: Comparison of errors on the Nusselt number obtained by the four correlations depending on the ratio of bulk and cold wall temperatures.

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