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# Optimal paths on the ocean

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## 1 Introduction

### 1.1 Context

The ocean is a vast area that plays a major role in our lives and in the global economy (90% of goods are transported by sea). From merchant ships (more than 56, 000) and fishing vessels (4.6 million) to recreational and races sailboats (more than 30 million), many different users are sailing across the world. The ocean might also be dangerous due to severe weather conditions and the potentially isolated location of ships. Fishing is indeed the most dangerous work in France [18].

Optimizing ship navigation is therefore a crucial issue, which has been studied for decades [43]. Since each kind of ship has specific objectives and constraints, the general weather routing problem is complex to solve. For example, sailing ships often wish to minimize voyage duration while merchant ships wish to minimize voyage consumption with a maximum travel time constraint.

Ship weather routing is an important topic today for various reasons. First, races sailboats have new flying behavior thanks to the foil technology, which results in new optimization constraints. Moreover, due to climate change, extreme weather events are stronger, which highlights the need for safer routing. And finally, ship weather routing is being pushed forward by new IMO2020 international regulations [13] to reduce the shipping industry carbon footprint. This regulation encourages shipowners to optimize the operation of their ships, and also to consider wind-assisted propulsion. These new kinds of hybrid propelled ships have complex physic and their performances can be enhanced by weather routing.

The interest of investigating this topic is that today, there is no off-the-shelf integrated solution for performing route optimization on either sailing, motor, and hybrid propelled ships. Developing a flexible enough solution is a challenging issue, as these ships may have a different set of control variables, and also have a different sensitivity to the environment. For example, sailing ships are extremely sensitive to wind forecasts and, thus, may have optimal routes further away from the great circle route than a cargo ship. Also, as mentioned before, usual objectives and constraints differ depending on the ship's propulsion.

## 1.2 Workflow

The general process of ship route optimization can be summarized as in Fig. 1 and is based on the following elements:

- **Weather forecast:** simulating the environment is a complex process that is externalized from the routing solution. Depending on the weather quantity (such as swell or wind waves for waves, and steady or gusts for wind) and on the granularity requested, various weather forecasts can be used. Weather data is given as input on a tri-dimensional grid. We can distinguish two families of weather data: deterministic and ensemble forecasts. While the first only simulates the evolution of one weather scenario, the second generates various forecasts from perturbed initial scenarios, which allows the estimation of weather uncertainties.
- **Ship model:** evaluating ship interaction with the environment is a complex issue. It is therefore also externalized from the routing solution. From ship hull design to propulsion characteristics, various methods (such as computational fluid dynamics or velocity prediction programs [29, 30]) are used to estimate ship speed depending on weather conditions and ship controls. Wave impact on ship motion and the generated added resistance are still a current topic of research. Many environmental parameters influence the ship model. As an example for cruises vessels, half of the power is consumed by the propulsion, and half by board needs (hostel, freshwater production, air conditioning, etc). While wind, waves, and current impact the propulsion, temperature (air and water) and humidity impact the board needs. To have an accurate estimation of ship consumption, all these parameters need to be taken into account.
- **Constraints:** various constraints, which can be time-dependent or not, have to be satisfied in optimization for navigation purposes (restricted areas, channels, etc), operational needs (total voyage duration, engine power changes, etc), and safety (maximum ship motions and weather conditions).
- **Objectives:** depending on ship operation, different criteria have to be optimized, simultaneously or not, such as voyage duration, consumption, ship fatigue, comfort, etc.

Then, based on these elements, an appropriate *route optimization algorithm* computes some optimal or near-optimal path(s).

## 1.3 Problem formulation

Considering a vessel moving on the ocean with a set of control variables modeled as scalars (for example heading angle  $h$  and engine power  $p$ ), we can introduce the following vectors: the ship position vector  $\underline{x}(t) = [\varphi(t), \lambda(t)]^T$  where  $\varphi, \lambda$  are respectively ship latitude and longitude at time  $t$ , the control vector  $\underline{u}(\underline{x}, t) = [h(\underline{x}, t), p(\underline{x}, t)]^T$ , and the environmental conditions vector  $\underline{e}(\underline{x}, t)$  composed of wind, waves, and (ocean) current at ship position  $\underline{x}$  and time  $t$ . Other environmental variables can be considered if needed. These vectors may have admissible values because of operational constraints (navigation area, propulsion setup, maximum weather conditions, etc...). The ship velocity  $\underline{v}$  can be determined using the ship position  $\underline{x}$ , the control vector  $\underline{u}$ , and the environmental condition vector  $\underline{e}$  as mentioned in section 1.2.

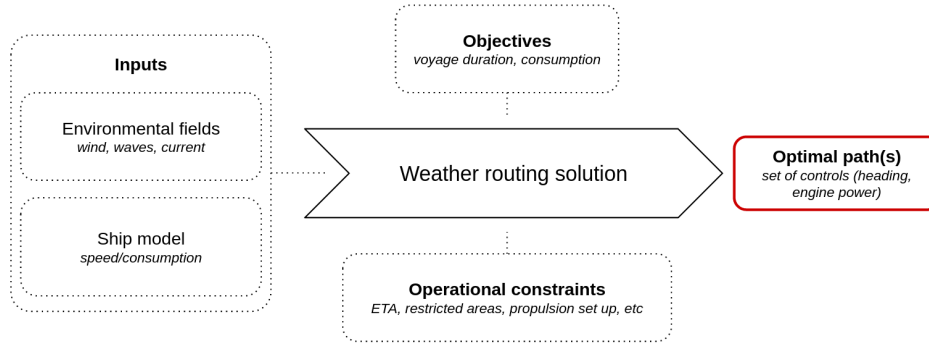


Figure 1: Workflow of the weather routing process

Considering a route from a starting position and time  $(\underline{x}_s, t_s)$  to an arrival position and time  $(\underline{x}_f, t_f)$ , with  $\underline{C}(\underline{u}, \underline{x})$  being a  $k$ -dimensional vector-valued cost function, the route cost vector can be expressed as:

$$\underline{J} = \int_{t_s}^{t_f} \underline{C}(\underline{u}, \underline{x}) dt \quad (1)$$

The control problem is to find the optimal controls  $\underline{u}^*(t)$  that minimize  $\underline{J}$ . The route  $\underline{x}^*(t)$  determined by  $\underline{u}^*(t)$  is called the ‘optimal route’. When  $k > 1$  (multi-objective optimization), we may have conflicting objectives to optimize, so determining optimality may be impossible: instead, we aim at finding a set of *Pareto-optimal* solutions.

## 2 Formulations

Even if the general problem is continuous due to the nature of the ocean, methodologies for finding paths are generally discrete. More precisely, the control variables considered in the aforementioned problem often have predefined discrete values. We can distinguish three kinds of methodologies. First, **fixed-grid** approaches that are based on a predefined discretization of the search space, such as dynamic programming or graph-based shortest paths algorithms. Second, **dynamic-grid** approaches that discover on-the-fly the search space during the algorithm, such as the isochrone and isopone methods. Finally, **evolutionary** approaches that start from an initial population of solutions and then, from mutations and crossover operations, explore other candidates. Nonetheless, some continuous approaches like calculus of variation [4] have been proposed for single-objective optimization, but are not used operationally. The choice between these methods depends on the objective(s) to optimize, the constraints to satisfy, and on the number of control variables.

### 2.1 Time-optimal weather routing problem

The most commonly used approach for this variant of the weather routing problem is the isochrone method. This algorithm successively computes the outer boundary  $B_t$  - called isochrones - of the attainable region within a time  $t$  from a departure position and time. Time is considered as a discrete variable and the time step  $\delta t$  is a parameter of the method. The algorithm stops when the arrival position is inside an isochrone, and the shortest path is computed through

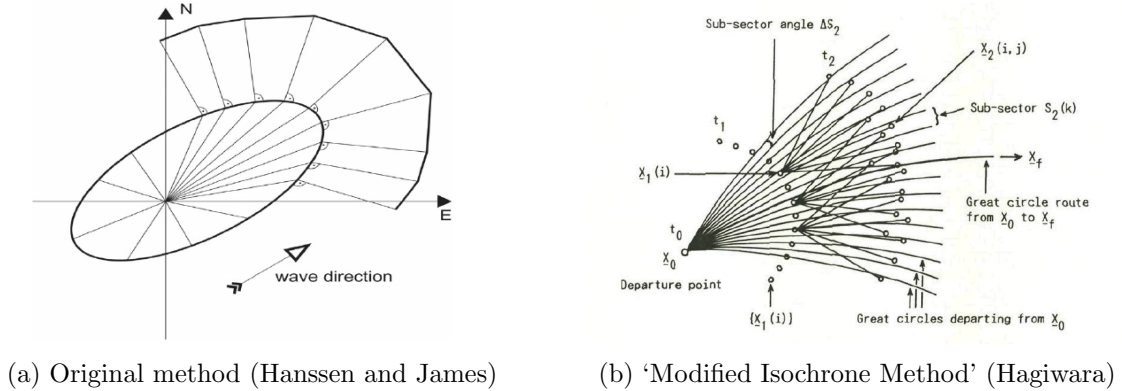


Figure 2: Scheme of some isochrones construction methods

a backward calculation. During the past decades, different variants of this method have been proposed, which differ in the way isochrones are modeled, computed, and simplified (see Fig. 2). The first version, proposed by Hanssen and James [14, 11], was appropriate for manual computation: to compute  $B_{t+\delta t}$ , each point of an isochrone  $B_t$  was progressed in the local normal direction of the isochrone. Then, Wisniewski chose to progress in the direction that maximizes the ship velocity in the local normal to the isochrone [36]. Due to weather forecasts and non-convexity in the ship model, self-loops often occur in the isochrone, making the computation more difficult. Finally, Hagiwara proposed the 'Modified Isochrone Method' [9], which is based on partitioning the navigational area into 'sectors'. Each element of an isochrone  $B_t$  progresses into many directions based on angular resolution. Then, a selection is done between all these elements to only keep the furthest one from the departure point in each sector, and, then, construct the next isochrone  $B_{t+\delta t}$ . Thanks to this selection process, self-loops are directly avoided. Szlapczynska and Smierzchalski proposed a version of isochrone which generates routes free from land crossing [36].

Some other approaches based on fixed-grids have been proposed also in the past decades, but are not widely used in the commercial solutions for this specific problem. Classical shortest paths algorithms such as  $A^*$  and Dijkstra have been applied to ship weather routing with some variations, especially to include time-dependency of costs. Kanouleas *et al.* adapted the  $A^*$  algorithm to time-dependent networks [15], and Shin *et al.* applied it to ship weather routing [33]. Mannarini *et al.* proposed a time-optimal algorithm based on Dijkstra for merchant ships [24]. Sen and Padhy also used an adaptation of the Dijkstra algorithm and focus on the wave impact on ship motions [31]. Dynamic programming approaches have also been developed: Wang and Chretienne proposed an algorithm for sailing ships route optimization [44], and Philpott and Mason a stochastic one for sailing ships under weather uncertainties [28]. They showed its impact on optimal routes for some weather scenarios and integrated the uncertainty first modeling wind speed and direction with Markov chains and then using ensemble forecasts. Allsopp proposed also a stochastic dynamic programming approach for sailing ships route optimization [2].

## 2.2 Fuel-optimal weather routing problem

Another well-known variant of the general weather routing problem, especially for merchant ships, is the optimization of voyage consumption. Operationally, this problem is often formulated

with a constraint in maximum travel time, which makes the problem more complex to solve.

The reference methods for fuel-optimal weather routing are dynamic programming algorithms. Based on Bellman's principle of optimality, this approach is particularly adapted to solve multi-stage decision problems, as ship weather routing. These algorithms consider stages and states variables whose discretization generates a predefined grid, which represents the search space. More precisely, state variables are often time or voyage progress (projection of ship position on the great circle route from departure to arrival), but more generally can be any parameter that increases with the problem-solving process. For each stage, we may have several possible values of the state variable, which can be any parameter that defines the ship state (position, motion, etc). As an example, with voyage progress as a stage variable, location and time can be taken as state variables, as shown in Fig. 3. The choice for these variables is important and is the main difference between the different dynamic programming algorithms. Chen discusses the stage variable choice between time or voyage progress and concludes that the latter is less expensive computationally [7]. Based on this discretization of the search space, all dynamic programming approaches recursively compute the fuel-optimal path to each state of successive stages, until reaching the destination. These algorithms are exhaustive, which may lead to large computing time and consumed memory.

Two main kinds of dynamic programming methods have been proposed in the literature, depending on the number of control variables considered. First, the two-dimensional dynamic programming method (2DDP), proposed by De Wit [49] and Calvert *et al.* [5], consider only one control variable (the ship heading). Secondly, the three-dimensional dynamic programming method (3DDP) is an extension of the 2DDP with two control variables (the ship heading and engine power, or vessel speed). Different versions have been proposed by Aligne *et al.* [1] and Shao and Zhou [32], the former with time as stage variable and the latter with voyage progress. Some improvements have been proposed recently. Zaccone *et al.* added some speed-up techniques to the 3DDP method [50]. More precisely, based on a lower bound of the remaining travel time from a possible state to the arrival, partial solutions that cannot respect the maximum voyage duration can be excluded. Besides, to find a good trade-off between computing time and solution accuracy, Avgouleas proposed an iterative dynamic programming approach that refines states discretization during the algorithm process [3]. Lin *et al.* proposed an adaptation of the 3DDP method, called 3DMI (for 3D Modified Isochrone method), with a heading discretization instead of predefined positions [21].

Other deterministic approaches have been proposed for this problem. Wang *et al.* proposed a discrete-time adaptation of the Dijkstra algorithm on a time-expanded graph (named 3DDA for 3D Dijkstra Algorithm), to compute the fuel-optimal path for any time instance at the arrival point [47]. Using this kind of graph model, which can be considered static, all edge costs can be pre-computed, which otherwise may be expensive to do on the fly during the algorithm (weather interpolation, ship model evaluation, etc). Park and Kim proposed a two-phase approach based first on  $A^*$  algorithm for route optimization with constant speed, and then on geometric programming for speed optimization [27]. Besides, various techniques only consider the heading as a control variable (optimizing only the ship route) and iterates on the engine power value (constant) to meet the correct arrival time. For example, Hagiwara used the previously mentioned isochrone method [9], or Takashima *et al.* adapted the Dijkstra algorithm for fuel-optimization [38]. However, as these methods consider ship propulsion constant during the voyage, results may be sub-optimal in terms of consumption reduction. Klompstra *et al.* proposed a variation of the isochrone method to optimize fuel consumption [16]. This method

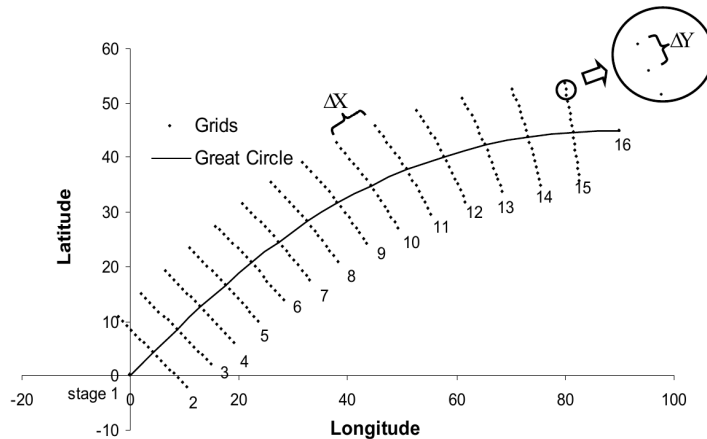


Figure 3: Dynamic Programming stage/state discretization scheme (Shao *et al.*)

successively computes isopones (outer boundary of the attainable region with a certain amount of fuel) instead of isochrones. However, as explained in [32], this method has not been adopted by mariners who prefer dynamic programming or graph-based approaches.

Finally, some stochastic algorithms are also used in ship weather routing but most of them deal with the multi-objective case (see Section 2.3). These methods generate an initial set of solutions that are improved with genetic algorithms or other techniques. Wang *et al.* combine deterministic methods such as 3DDA or 3DDP with genetic algorithms [47]. Lee *et al.* simultaneously optimize route and speed with the NSGA-II (Nondominated Sorting Genetic Algorithm) and showed the efficiency of their method compared to other state-of-the-art algorithms [19]. Cheng and Tsou proposed an approach based on an ant colony algorithm [8].

### 2.3 Multiple objectives weather routing problem

Many criteria have to be taken into account in the selection of an ‘optimal’ route by ship owners and mariners (risk at sea, weather forecast reliability, comfort, etc). Multi-objective optimization is therefore important to provide them a panel of routes that have been optimized according to some of these criteria. The methods that we present here are those which naturally generate a set of optimal routes as a result. For the particular bi-objective case with voyage duration and consumption, many approaches proposed in the literature are versatile enough to deal with more than two objectives. The methods developed in Section 2.2 can also be used to generate Pareto optimal paths by considering various arrival times.

Graph-based algorithms are widely used for this variant of the weather routing problem. However, these methods often consider only one control variable, meaning that ship heading and engine power are not optimized simultaneously. Veneti *et al.* proposed two algorithms for the general bi-objective problem that differ in the way time is modeled [39]. First, a discrete-time label-setting algorithm, that iterates over time instants, and a more flexible label-setting algorithm based on Martin’s work [26], that handles discrete or continuous-time models. These algorithms have been compared to other state-of-the-art algorithms (such as Hamacher *et al.*’s work [10]) and applied to ship routing with consumption and risk as costs. They also proposed a grid pre-processing to reduce the computational complexity [41]. More recently, Chauveau

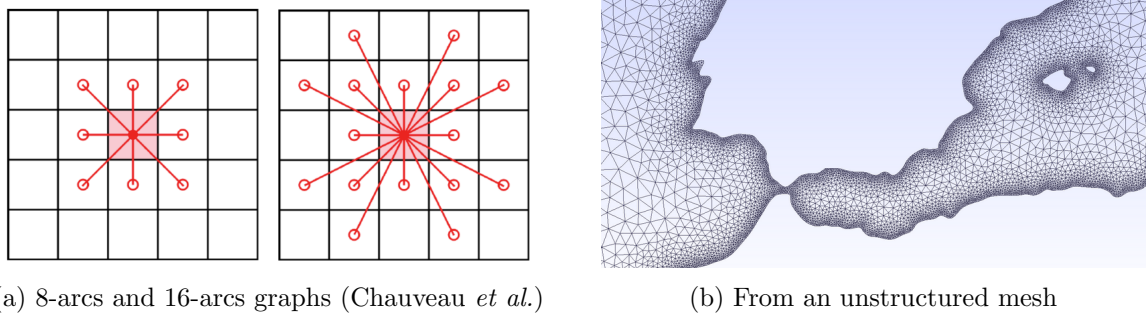


Figure 4: Different kinds of ocean graphs

*et al.* adapted the label setting algorithm NAMOA\* (New Algorithm for Multi-Objective A\*) from Mandow and Perez de la Cruz [23] to deal with time dependency and applied it to ship weather routing (called NAMOA\*-T) [6]. This approach is particularly interesting with the use of a heuristic to filter as soon as possible uninteresting solutions. More precisely, during the procedure -based on a pre-computation of a lower bound of the remaining cost from any node to the destination (called heuristic)- partial paths can be filtered (and not explored) if their heuristic cost (i.e., *partial path cost + heuristic cost to the arrival*) is dominated in the arrival node Pareto front. Besides, as Bellman's principle of optimality does not hold in the forward direction as shown in [10], the pruning procedure, which only keeps Pareto optimal paths for intermediate nodes in NAMOA\*, may lead to sub-optimal solutions in the time-dependent case and is therefore withdrawn in NAMOA\*-T. Nonetheless, Chauveau *et al.* showed that according to a time discretization and with step-wise cost functions, the algorithm remains optimal if the pruning procedure is done for the same time instant. They compared their method to the discrete-time algorithm of Veneti *et al.* and showed interesting performances in terms of computing time. Skoglund extended the Dijkstra algorithm to deal with multiple objectives and Pareto efficiency [34]. However, in the algorithm, only Pareto-optimal sub-paths to intermediate nodes are kept what may lead to sub-optimal results according to Chauveau *et al.*. Their approach is versatile enough to consider deterministic and ensemble forecasts and therefore can handle uncertainties in weather models. Furthermore, while most of the multi-objective works have been applied to merchant ships, Zyczkowski and Szlapczynski worked on the route optimization of sailing ships in terms of voyage duration, safety, and comfort [51].

Apart from these deterministic methods, various evolutionary approaches have also been proposed in the literature. As described in Section 2.2, these methods improve an initial set of solutions through evolutionary algorithms. Marie and Courteille optimize voyage duration and consumption with an algorithm based on MOGA (Multi-Objective Genetic Algorithm) [25]. Hinnenthal and Clauss used ensemble forecast to work on the robustness of routes [12]. They proposed therefore a multi-objective algorithm for voyage consumption, duration, and route robustness. To improve the computational efficiency, routes and speed profiles are modeled with B-splines. Various works propose methods to optimize simultaneously voyage duration, consumption, and risk associated with rough weather conditions: Vettor and Guedes Soares proposed a method based on the SPEA (Strength Pareto Evolutionary Algorithm) which considers two control variables and uses ensemble forecasts [42]; Szlapczynska and Szlapczynski proposed MEWRA (Multi-Objective Evolutionary Weather Routing Algorithm) which is also based on SPEA [35, 37]; Veneti *et al.* proposed a genetic algorithm with one control variable based on NSGA-II [40]. These approaches differ on the algorithms used but also on the initial



population generation: while Vettor and Guedes Soares use Dijkstra computed paths for each criterion, Szlapczynska and Szlapczynski use direct routes and time-optimal paths computed with both isochrone and  $A^*$  algorithms, and Veneti *et al.* integrate geometric distance from obstacles for safe initial routes.

Other relevant criteria have been taken into account in the optimization process by some authors. For example, Maki *et al.* tackled the risk of parametric rolling (a well-known problem for the safety of merchant ships) proposing a multi-objective optimization with consumption through an evolutionary approach, called real-code genetic algorithm [22]. Wang *et al.* considered ship fatigue damage and proposed an application of their 3DDA algorithm combined with a genetic algorithm for multi-objective optimization with voyage consumption [45].

### 3 Discussion and conclusion

#### 3.1 Guarantees

A question rise while discussing optimization techniques: do we have guarantees of optimality? This is a difficult issue in the context of ship routing, due to the continuous nature of the ocean.

Guarantees of optimality can therefore be given only for fixed-grid methods such as dynamic programming or graph-based approaches. Indeed, under several conditions, we can have the insurance to reach a global optimum, according to the predefined discretization of the search space.

More precisely, Bellman's principle of optimality has to be taken into account in the design of the algorithm to have guarantees. When it holds, only optimal sub-paths can be saved at intermediate nodes while in the opposite we cannot make this selection. For single objective optimization for example, with a cost function that does not fulfill the FIFO property (such as fuel consumption), using the Dijkstra algorithm (that only keeps optimal sub-paths) may lead to sub-optimal results [34]. For multi-objective optimization, Kostreva and Wiecek proved that, in the case of continuous-time and monotonically increasing cost functions, Bellman's principle of optimality holds in the forward direction [17]. Hamacher *et al.* showed that, without waiting policy in a graph and without specific conditions on cost functions, Bellman's principle of optimality only holds in the backward direction [10].

Otherwise, for dynamic grid methods such as the isochrone method, guarantee of optimality has never been proven; and for evolutionary algorithms, being based on an initial population of solutions that is optimized, there is no insurance to reach the global optimum [7].

#### 3.2 Critical issues

This work concerns methods to tackle the ship weather routing problem. It does not have the ambition to be exhaustive, but the most widely used techniques are presented. Depending on the kind of ship and its operation, each method has its advantages and drawbacks. For example, dynamic-grid methods are more adapted to sailing ships, whose optimal routes are strongly impacted by the weather conditions, while fixed-grid methods are more adapted to merchant ships, which need to optimize multiple objectives.

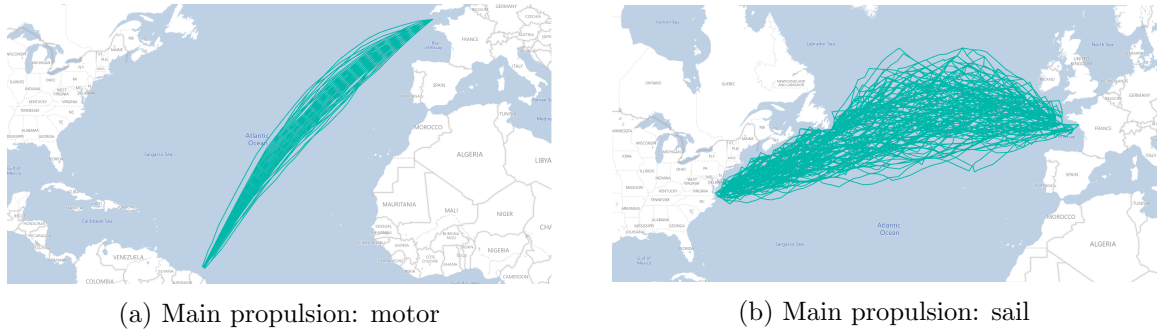


Figure 5: Optimal routes depending on wind-assisted ship main propulsion

We highlight here some interesting issues. First, the **fixed-grid generation**. As the ocean is a continuous space there is no straightforward way to mesh it, and, obviously, the grid impacts a lot the accuracy of the solutions. Various kinds of grids are used in weather routing as shown in Fig. 4, whose parameters are chosen based on the acceptable computing time, the operational constraints, and the weather forecast grid. For sailing ships, which may need small changes in direction to follow the weather evolution and sometimes big changes due to maneuvers, it is hard to find an adapted fixed grid. Some work is under development from the authors on a hybrid grid coupling a fixed one and a dynamic integration of nodes in the graph during the algorithm. Their graph-based approach deals with single and multi-objective optimization and is adapted to sailing and motor ships thanks to this hybrid grid. Besides, as mentioned in Section 1.1, the number of merchant ships with wind-assisted propulsion is growing. Depending on the wind assistance, these ships may have a behavior similar to a motor or to a sailing ship (or often in-between). To provide effective weather routing for these kinds of ships, innovative solutions - versatile enough to deal with any ship propulsion and shipping operation constraints (such as fixed voyage duration) - are needed. An example of optimal routes for two kinds of wind-assisted ships, with different operational profiles, computed through the authors' algorithm previously mentioned, is shown in Fig. 5. This highlights the impact of ship conception and operational constraints on the shape of optimal routes.

Another important parameter that has been highlighted in this work is the **number of control variables** considered in the optimization (often ship heading and engine power or vessel speed in practice). As we have shown, many solutions only optimize one control variable or propose a two-phase algorithm that first optimizes one control variable and then the other fixing the first phase results. However, as presented in many references [1, 19, 20, 32, 46], simultaneously optimizing ship heading and engine power during a voyage leads to better results, especially for fuel consumption.

Working with time-dependent fields, another issue is **time modeling**. As presented in this work, some approaches consider time as a continuous variable, while others prefer time discretization, which is often defined from the weather model temporal resolution. Wang *et al.* discussed different time models in networks [48], Veneti *et al.* proposed algorithms for both models [39], but this should be interesting to properly evaluate the impact of time models both on solutions and computing time. Another area of investigation for multi-objective optimization could be to work on clustering techniques of the Pareto front.

Finally, most of the presented works are based on deterministic weather forecasts that by nature may lead to **uncertainties** in the solutions. However, high-quality ensemble forecasts

are available and should be more integrated into the route optimization process.

### 3.3 Conclusion

As a conclusion, an introduction to the issues of the weather routing problem has been presented here with the main state-of-the-art methods. Weather routing remains a challenge and is widely studied. It can be a major player in the de-carbonation of the marine industry, as a stand-alone optimization but also thanks to its impact on wind-assisted solutions performances.

## References

- [1] F. Aligne, M. Papageorgiou, and E. Walter. Incorporating Power Variations into Weather Routeing and Why It May Lead to Better Results. *IFAC Proceedings Volumes*, 31(30):269–274, October 1998.
- [2] T. Allsopp. Stochastic weather routing for sailing vessels. Master's thesis, University of Auckland, September 1998.
- [3] K. Avgouleas. Optimal ship routing. Master's thesis, Massachusetts Institute of Technology, 2008.
- [4] S. J. Bijlsma. *On minimal-time ship routing*. PhD thesis, Delft University of Technology, Delft, 1975.
- [5] S. Calvert, E. Deakins, and R. Motte. A Dynamic System for Fuel Optimization Trans-Ocean. *The Journal of Navigation*, 44(2):233–265, May 1991.
- [6] E. Chauveau, P. Jegou, and N. Prcovic. Weather Routing Optimization: A New Shortest Path Algorithm. In *2017 IEEE 29th International Conference on Tools with Artificial Intelligence (ICTAI)*, pages 687–694, Boston, MA, November 2017. IEEE.
- [7] H. Chen. *A dynamic program for minimum cost ship routing under uncertainty*. PhD thesis, Massachusetts Institute of Technology, 1978.
- [8] H. C. Cheng and M. C. Tsou. An Ant Colony Algorithm for efficient ship routing. *Polish Maritime Research*, 20(3):28–38, October 2013.
- [9] H. Hagiwara. *Weather routing of (sail-assisted) motor vessels*. PhD thesis, Delft University of Technology, Delft, 1989.
- [10] H. W. Hamacher, S. Ruzika, and S. A. Tjandra. Algorithms for time-dependent bicriteria shortest path problems. *Discrete Optimization*, 3(3):238–254, September 2006.
- [11] G. L. Hanssen and R. W. James. Optimum Ship Routing. *Journal of Navigation*, 13(3):253–272, July 1960.
- [12] J. Hinnenthal and G. Clauss. Robust Pareto-optimum routing of ships utilising deterministic and ensemble weather forecasts. *Ships and Offshore Structures*, 5(2):105–114, August 2010.

- [13] International Maritime Organization (IMO). Sulphur 2020: Cutting Sulphur Oxide Emissions, 2020.
- [14] R. W. James. Application of wave forecasts to marine navigation. Technical report, U.S. Naval Oceanographic Office, Washington, D.C., July 1957.
- [15] E. Kanoulas, Y. Du, T. Xia, and D. Zhang. Finding Fastest Paths on A Road Network with Speed Patterns. In *Proceedings - International Conference on Data Engineering*, May 2006.
- [16] M. B. Klompstra, G. J. Olsder, and P. K. G. M. van Brunschot. The isopone method in optimal control. *Dynamics and Control*, 2(3):281–301, July 1992.
- [17] M. M. Kostreva and M. M. Wiecek. Time Dependency in Multiple Objective Dynamic Programming. *Journal of Mathematical Analysis and Applications*, 173:289–307, May 1993.
- [18] G. Lazuech. Rapport salarial, penibilite et dangerosite du travail dans le secteur de la peche artisanale francaise. *Les mondes du travail*, pages 75–87, 2016.
- [19] S. M. Lee, M. I. Roh, K. S. Kim, H. Jung, and J. J. Park. Method for a simultaneous determination of the path and the speed for ship route planning problems. *Ocean Engineering*, 157:301–312, June 2018.
- [20] X. Li, B. Sun, C. Guo, W. Du, and Y. Li. Speed optimization of a container ship on a given route considering voluntary speed loss and emissions. *Applied Ocean Research*, 94:101995, January 2020.
- [21] Y. H. Lin, M. C. Fang, and R. W. Yeung. The optimization of ship weather-routing algorithm based on the composite influence of multi-dynamic elements. *Applied Ocean Research*, 43:184–194, October 2013.
- [22] A. Maki, Y. Akimoto, Y. Nagata, S. Kobayashi, E. Kobayashi, S. Shiotani, T. Ohsawa, and N. Umeda. A new weather-routing system that accounts for ship stability based on a real-coded genetic algorithm. *Journal of Marine Science and Technology*, 16:311–322, September 2011.
- [23] L. Mandow and J. L. Perez-de-la Cruz. A New Approach to Multiobjective A\* Search. In *Proceedings - IJCAI*, 2005.
- [24] G. Mannarini, G. Coppini, P. Oddo, and N. Pinardi. A Prototype of Ship Routing Decision Support System for an Operational Oceanographic Service. *TransNav : International Journal on Marine Navigation and Safety of Sea Transportation*, Vol. 7(no. 1), 2013.
- [25] S. Marie and E. Courteille. Multi-Objective Optimization of Motor Vessel Route. page 10.
- [26] E. Q. V. Martins. On a multicriteria shortest path problem. *European Journal of Operational Research*, 16(2):236–245, May 1984.
- [27] J. Park and N. Kim. Two-phase approach to optimal weather routing using geometric programming. *Journal of Marine Science and Technology*, 20(4):679–688, December 2015.
- [28] A. Philpott and A. Mason. Optimising Yacht Routes under Uncertainty. page 11.

- [29] Yann Roux, Serge Huberson, Frdric Hauville, Jean-Philippe Boin, and Michel Guilbaud. Yacht performance prediction: towards a numerical VPP. page 8.
- [30] Yann Roux, Alban Leroyer, Michel Visonneau, Jrmie Raymond, and Frdric Hauville. Strongly coupled VPP and CFD RANSE code for sailing yacht performance prediction. page 13.
- [31] D. Sen and C. P. Padhy. An approach for development of a ship routing algorithm for application in the North Indian Ocean region. *Applied Ocean Research*, 50:173–191, March 2015.
- [32] W. Shao and P. Zhou. Development of a 3D Dynamic Programming Method for Weather Routing. In *Methods and Algorithms in Navigation*, pages 181–187. CRC Press, June 2011.
- [33] Y. W. Shin, M. Abebe, Y. Noh, S. Lee, I. Lee, D. Kim, J. Bae, and K. C. Kim. Near-Optimal Weather Routing by Using Improved A\* Algorithm. *Applied Sciences*, 10(17):6010, January 2020.
- [34] L. Skoglund. *A new method for robust route optimization in ensembleweather forecasts*. 2012.
- [35] J. Szlapczynska. Multi-objective Weather Routing with Customised Criteria and Constraints. *The Journal of Navigation*, 68(2):338–354, March 2015.
- [36] J. Szlapczynska and R. Smierzchalski. Adopted isochrone method improving ship safety in weather routing with evolutionnary approach. *International Journal of Reliability, Quality and Safety Engineering*, 14(06):635–645, December 2007.
- [37] J. Szlapczynska and R. Szlapczynski. Preference-based evolutionary multi-objective optimization in ship weather routing. *Applied Soft Computing*, November 2019.
- [38] K. Takashima, B. Mezaoui, and R. Shoji. 4 On the fuel saving operation for coastal merchant ships using weather routing. In *Marine Navigation and Safety of Sea Transportation*. CRC Press, 2009.
- [39] A. Veneti, C. Konstantopoulos, and G. Pantziou. Continuous and Discrete Time Label Setting Algorithms for the Time Dependent Bi-Criteria Shortest Path Problem. In Brian Borchers, J. Paul Brooks, and Laura McLay, editors, *INFORMS Computing Society Conference*, pages 62–73, Richmond, Virginia, January 2015.
- [40] A. Veneti, C. Konstantopoulos, and G. Pantziou. An evolutionary approach to multi-objective ship weather routing. In *2015 6th International Conference on Information, Intelligence, Systems and Applications (IISA)*, pages 1–6, July 2015.
- [41] A. Veneti, A. Makrygiorgos, C. Konstantopoulos, G. Pantziou, and I. A. Vetsikas. Minimizing the fuel consumption and the risk in maritime transportation: A bi-objective weather routing approach. *Computers & Operations Research*, 88:220–236, December 2017.
- [42] R. Vettor and C. Guedes Soares. Development of a ship weather routing system. *Ocean Engineering*, 123:1–14, September 2016.
- [43] L. Walther, A. Rizvanolli, M. Wendebourg, and C. Jahn. Modeling and Optimization Algorithms in Ship Weather Routing. *International Journal of e-Navigation and Maritime Economy*, 4:31–45, June 2016.

- [44] C. T. Wang and P. Chretienne. Routage des voiliers et programmation dynamique. *RAIRO - Operations Research*, 27(1):61–76, 1993.
- [45] H. Wang, X. Lang, and W. Mao. Voyage optimization combining genetic algorithm and dynamic programming for fuel/emissions reduction. *Transportation Research Part D: Transport and Environment*, January 2021.
- [46] H. Wang, W. Mao, and L. Eriksson. Benchmark Study of Five Optimization Algorithms for Weather Routing. In *Volume 7B: Ocean Engineering*, page V07BT06A023, Trondheim, Norway, June 2017. American Society of Mechanical Engineers.
- [47] H. Wang, W. Mao, and L. Eriksson. A Three-Dimensional Dijkstra's algorithm for multi-objective ship voyage optimization. *Ocean Engineering*, August 2019.
- [48] Yishu Wang, Ye Yuan, Yuliang Ma, and Guoren Wang. Time-Dependent Graphs: Definitions, Applications, and Algorithms. *Data Science and Engineering*, 4(4):352–366, December 2019.
- [49] C. de Wit. Proposal for Low Cost Ocean Weather Routeing. *The Journal of Navigation*, 43(3):428–439, September 1990.
- [50] R. Zaccone, E. Ottaviani, M. Figari, and M. Altosole. Ship voyage optimization for safe and energy-efficient navigation: A dynamic programming approach. *Ocean Engineering*, 153:215–224, April 2018.
- [51] M. Zyczkowski and R. Szlapczynski. Multi-Objective Weather Routing of Sailing Vessels. *Polish Maritime Research*, 24, December 2017.