



**HAL**  
open science

# A Regularized Electric Flux Volume Integral Equation for Brain Imaging

Clément Henry, Adrien Merlini, Lyes Rahmouni, Francesco Andriulli

► **To cite this version:**

Clément Henry, Adrien Merlini, Lyes Rahmouni, Francesco Andriulli. A Regularized Electric Flux Volume Integral Equation for Brain Imaging. 2020 IEEE International Symposium on Antennas and Propagation and North American Radio Science Meeting, Jul 2020, Montreal, Canada. pp.1025-1026, 10.1109/IEEECONF35879.2020.9330486 . hal-03401818

**HAL Id: hal-03401818**

**<https://hal.science/hal-03401818v1>**

Submitted on 9 Mar 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# A Regularized Electric Flux Volume Integral Equation for Brain Imaging

Clément Henry<sup>\*</sup>, Adrien Merlini<sup>†</sup>, Lyes Rahmouni<sup>\*</sup>, and Francesco P. Andriulli<sup>\*</sup>

<sup>\*</sup> Department of Electronics and Telecommunications, Politecnico di Torino, Turin, Italy

<sup>†</sup> Microwave Department, IMT Atlantique, Brest, France

**Abstract**—Volume integral equations for modeling biological tissues in the frequency domain typically suffer from ill-conditioning for high dielectric contrasts and low frequencies. These conditioning breakdowns severely compromise the accuracy and applicability of these models and render them impractical despite their numerous advantages. In this work, we present an electric flux volume integral equation (D-VIE) free from these shortcomings when computed on biologically compatible simply connected objects. This new formulation leverages on careful spectral analysis to obtain volume quasi-Helmholtz projectors capable of curing both sources of ill-conditioning. In particular, the normalization of the projectors by the material permittivity allows for an inhomogeneous re-scaling of the equation which stabilises the high contrast breakdown together with the low-frequency breakdown. Numerical results show the applicability of this new formulation in realistic brain imaging.

## I. INTRODUCTION

Realistic modeling of the electrical activity inside the human head is of crucial importance in various biomedical applications such as pre-surgical epilepsy source localization and brain computer interfaces (BCIs). Most of these scenarios leverage brain imaging strategies. Among these imaging modalities electroencephalography (EEG) has a significant advantage thanks to its superior time resolution. EEG-based neuroimaging can be split into two parts: (i) a forward problem that maps the field created by a brain current source to the potential along the scalp and (ii) an inverse problem that relies on the forward model to map a measured potential at the electrodes to the volume brain currents. The forward problem must be as accurate as possible and is derived from Poisson's equation under the quasi-static approximation. While the assumption that the brain activity can be modeled as quasi-static has been widely verified, it is not appropriate in some scenarios and can lead to significant errors. For instance, some capacitive and inductive effects, such as the ones encountered in brain stimulation, can affect the measured potential especially when the sources are stimulation pulses. Full-wave solvers are thus required to fully grasp the behaviour of the head. Unfortunately, due to the frequency dependence of the head permittivity, traditional VIE formulations suffer from ill-scalings and converge slowly, or do not converge at all, in these applications. Some remedies have been proposed in [1], [2] to address the ill-scaling in the electric current VIE (J-

VIE) formulation. Regarding the electric flux formulation, one solution to the high-contrast (HC) breakdown was proposed in [3], where a surface integral equation is added to the traditional D-VIE to artificially tune the background permittivity and thereby lower the material contrast. While they cure the high-contrast problem of the VIE, these techniques do not explicitly take its static limit.

In this work, we present a regularization scheme for the D-VIE that does not require an additional surface integral equation. It is based on the quasi-Helmholtz projectors. In particular, a normalized version of these projectors is introduced to properly re-scale the permittivity of inhomogeneous scatterers. Numerical results illustrate the stability of this new formulation and its accuracy in the EEG source localization problem.

## II. A NEW REGULARIZED D-VIE

Consider a scattering problem in which a time-harmonic wave  $\mathbf{E}^i$  is scattered in a inhomogeneous simply connected lossy dielectric object occupying the volume  $\Omega \subset \mathbb{R}^3$  embedded in free space. The scatterer is characterized by its permittivity  $\epsilon(\mathbf{r}) = \epsilon_r'(\mathbf{r}) - i\sigma(\mathbf{r})/\omega\epsilon_0$ , where  $\mathbf{r} \in \Omega$ ,  $\epsilon_r'(\mathbf{r})$  is the relative permittivity,  $\sigma(\mathbf{r})$  the conductivity, and  $\omega$  the angular frequency of  $\mathbf{E}^i$ . The incident field induces a volume current density  $\mathbf{J}(\mathbf{r}) = i\omega\kappa(\mathbf{r})\mathbf{D}(\mathbf{r})$ , where  $\mathbf{D}(\mathbf{r})$  is the electric flux and  $\kappa(\mathbf{r}) = (\epsilon(\mathbf{r}) - \epsilon_0)/\epsilon(\mathbf{r})$  is the dielectric contrast. The D-VIE relates  $\mathbf{E}^i$  and  $\mathbf{D}$  as  $\mathbf{D}(\mathbf{r})/\epsilon(\mathbf{r}) - (\mathcal{T}_A\mathbf{D})(\mathbf{r}) - (\mathcal{T}_\Phi\mathbf{D})(\mathbf{r}) = \mathbf{E}^i(\mathbf{r})$  where  $\mathcal{T}_A\mathbf{D} = k_0^2/\epsilon_0 \int_\Omega G_0(\mathbf{r}, \mathbf{r}')\kappa(\mathbf{r}')\mathbf{D}(\mathbf{r}') d\mathbf{v}'$  is the vector potential,  $\mathcal{T}_\Phi\mathbf{D} = 1/\epsilon_0 \nabla \int_\Omega G_0(\mathbf{r}, \mathbf{r}')\nabla' \cdot (\kappa(\mathbf{r}')\mathbf{D}(\mathbf{r}')) d\mathbf{v}'$  is the scalar potential,  $G_0$  is the 3D Green's function, and  $k_0$  is the wavenumber of  $\mathbf{E}^i$  in free space. The D-VIE is numerically solved using the method of moments on a tetrahedral mesh [4]. After discretizing the unknown  $\mathbf{D}$  with Schaubert-Wilton-Glisson (SWG) basis functions  $\{\mathbf{f}_m\}$  and testing the equation with  $\{\kappa\mathbf{f}_n\}$ , we obtain the system of equation  $\mathbf{Z}\boldsymbol{\alpha} = (\mathbf{Z}^G + \mathbf{Z}^A + \mathbf{Z}^\Phi)\boldsymbol{\alpha} = \mathbf{v}$  where  $[\mathbf{Z}^G]_{mn} = \langle \kappa\mathbf{f}_n, \mathbf{f}_m/\epsilon \rangle_\Omega$ ,  $[\mathbf{Z}^A]_{mn} = \langle \kappa\mathbf{f}_n, \mathcal{T}_A\mathbf{f}_m \rangle_\Omega$ ,  $[\mathbf{Z}^\Phi]_{mn} = \langle \kappa\mathbf{f}_n, \mathcal{T}_\Phi\mathbf{f}_m \rangle_\Omega$ , and  $[\mathbf{v}]_n = \langle \kappa\mathbf{f}_n, \mathbf{E}^i \rangle_\Omega$ , with  $\langle \mathbf{a}, \mathbf{b} \rangle_\Omega = \int_\Omega \mathbf{a} \cdot \mathbf{b} d\mathbf{v}$  being the  $L^2$ -inner product.

After some developments, which we omit for the sake of brevity, it can be shown that the solenoidal part of the D-VIE scales as  $\mathcal{O}(1/\epsilon_r)$  while its non-solenoidal part scales as  $\mathcal{O}(1)$ . Therefore, we need to rescale the D-VIE adequately to cure the instability associated to this different scaling. To

This work has been funded in part by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (ERC project 321, grant No.724846).

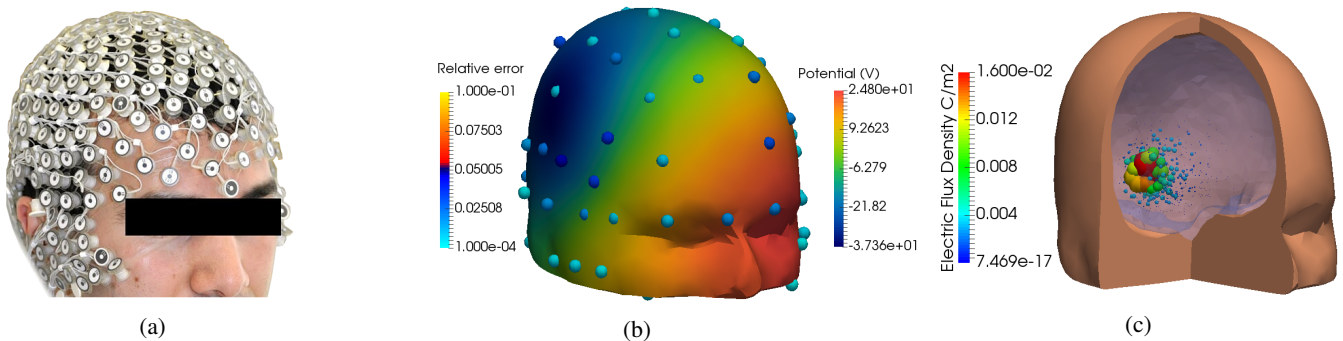


Fig. 1: (a) High-density EEG net. (b) Scalp distribution of the potential (right scale) radiated by a model of an epileptogenic source in the brain and, at the electrodes, relative error (left scale) obtained comparing the new formulation with a reference solution (FEM solver). (c) Reconstruction of the epileptogenic area via source localization.

this end, we introduce normalized quasi-Helmholtz projectors  $\tilde{\mathbf{P}}^\Sigma = \Sigma(\Sigma^T \mathbf{D} \Sigma) + \Sigma^T \mathbf{D}$  and  $\tilde{\mathbf{P}}^{\Lambda \mathbf{H}} = \mathbf{I} - \tilde{\mathbf{P}}^\Sigma$ , where  $\Lambda$  ( $\Sigma$ ) denote the loop (star) to SWG transformation matrix,  $^+$  is the Moore-Penrose pseudo inverse, and  $\mathbf{D}$  is a diagonal matrix built as  $\mathbf{D} = \text{diag}(\min(|\epsilon_{r,1}^+|, |\epsilon_{r,1}^-|) \dots \min(|\epsilon_{r,N_f}^+|, |\epsilon_{r,N_f}^-|))$  where  $\epsilon_{r,n}^{+/-}$  is the complex relative permittivity in the tetrahedron  $T_n^{+/-}$  supporting the SWG basis function  $f_n$ . Using these new projectors, we can rescale  $\tilde{\mathbf{P}}^{\Lambda \mathbf{H}}$  by  $\mathbf{D}$  to remove the permittivity ill-scaling of the D-VIE and make it stable until very low frequencies. After defining  $\mathbf{P}^\Sigma = \Sigma(\Sigma^T \Sigma) + \Sigma^T$ , we obtain the following regularized D-VIE

$$\left( \mathbf{D} \tilde{\mathbf{P}}^{\Lambda \mathbf{H}} + i \mathbf{P}^\Sigma \right) (\mathbf{Z}^G + \mathbf{Z}^A + \mathbf{Z}^\Phi) \alpha = \left( \mathbf{D} \tilde{\mathbf{P}}^{\Lambda \mathbf{H}} + i \mathbf{P}^\Sigma \right) \mathbf{v}, \quad (1)$$

which is the new equation we propose in this work.

### III. NUMERICAL RESULTS

We first evaluated the stability of the new D-VIE both at low frequencies and for high-contrast using a 3-layer sphere. Figure 2a displays the condition number for different values of the permittivity ratio between the layers and fig. 2b the conditioning of the formulation for different frequencies when the sphere is conductive. While the condition number of the traditional D-VIE depends on the permittivity and frequency, the proposed formulation remains stable with high-contrast dielectric and conductive objects until arbitrarily low frequency.

To demonstrate the accuracy of this preconditioned D-VIE in the EEG forward problem setting we computed the potential radiated by a current source in brain at  $f = 1$  Hz on the scalp (fig. 1b) and compared it to a reference solution. The preconditioned D-VIE shows good agreement with the reference solution and can be used as a forward solver in the EEG source localization problem as shown in fig. 1c.

### REFERENCES

[1] J. Markkanen, "Discrete helmholtz decomposition for electric current volume integral equation formulation," *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 12, pp. 6282–6289, Dec 2014.  
[2] C. Forestiere, G. Miano, G. Rubinacci, A. Tamburrino, L. Udpa, and S. Ventre, "A frequency stable volume integral equation method for anisotropic scatterers," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 3, pp. 1224–1235, March 2017.

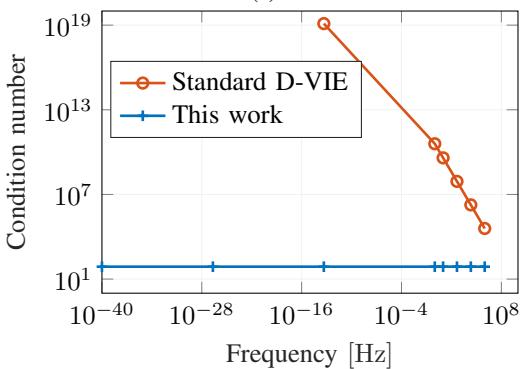
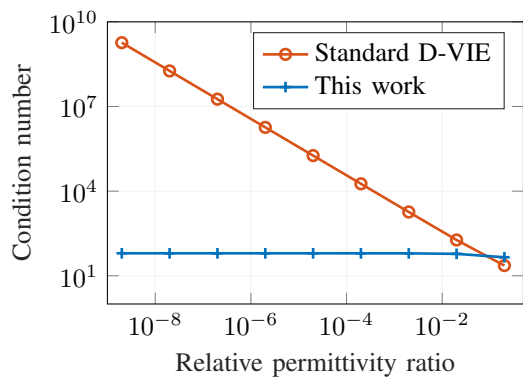


Fig. 2: Conditioning of the standard and new D-VIE as a function of (a) the relative permittivity ratio and (b) the frequency.

[3] L. J. Gomez, A. C. Yücel, and E. Michielssen, "The ICVSIE: A general purpose integral equation method for bio-electromagnetic analysis," *IEEE Transactions on Biomedical Engineering*, vol. 65, no. 3, pp. 565–574, March 2018.  
[4] D. Schaubert, D. Wilton, and A. Glisson, "A tetrahedral modeling method for electromagnetic scattering by arbitrarily shaped inhomogeneous dielectric bodies," *IEEE Transactions on Antennas and Propagation*, vol. 32, no. 1, pp. 77–85, January 1984.