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Twofold Fuzzy Sets in Single and Multiple Fault Diagnosis, Using Information about Normal Values

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Abstract

This paper proposes a general approach to diagnosis based on fuzzy pattern matching, making use of consistency and inclusion-based indices in the setting of possibility theory. The approach was first developed for binary attributes and single faults. It was then generalized to any kind of attributes (including multidimensional ones). The paper presents a refined representation (where a distinction is made between effects that are possible for sure and effects that are just not impossible, and where information about (ab)normal values is used). Moreover, an extension to multiple-fault diagnosis and to “cascading faults” is outlined.

I INTRODUCTION

There has been a continuous interest in fuzzy set-based approaches to diagnosis for about 25 years, starting with the early work of Sanchez [10], and continuing with the development of applications in medicine (see [2] for a review) and in industrial processes (e.g., see [11]). This paper presents some extensions of a diagnosis approach based on possibility theory and on fuzzy pattern matching. It somewhat departs from previous proposals by emphasizing the use of possibility distributions for representing *uncertain* pieces of knowledge, rather than using fuzzy relations for modelling the *intensity* of symptoms. This approach, which was first proposed for binary attributes [3], has been extended more recently to numerical attributes in [8]. The interest of this approach is discussed and exemplified in [5] for single-fault diagnosis.

Several further extensions of this approach are presented below. Section II provides the background and briefly discusses the handling of multidimensional attributes. Section III deals with some refinements in knowledge representation. It first introduces the use of twofold fuzzy sets (instead of simple fuzzy sets) for a better representation aiming at distinguishing between effects which are possible for sure, and effects which are just not forbidden. Besides, it allows for the representation of normal behaviour information, which can be useful for the diagnosis. Section IV takes advantage of the latter improvement in order to cope with multiple-fault diagnosis. Finally, Section V briefly discusses the detection of “cascading faults”.

II POSSIBILISTIC DIAGNOSIS

The framework developed for single-fault diagnosis is first presented and then extended to multidimensional attributes.

A Background

The approach is based on fuzzy relational knowledge: The relation between the value of each considered (binary, discrete

or continuous) attribute a and each possible fault f is described by a possibility distribution $\pi_a^f : U_a \rightarrow [0, 1]$, where U_a is the domain of attribute a . This relation can be causal, thus describing the effect of f on a by telling which values of a are (more or less) possible when f (alone) occurs. But it is not necessarily so, as it may also represent a necessary condition on attribute a for f to appear (e.g., testis and womb cancer can respectively appear only on men and women). In fact, for concrete applications, it is not feasible to ask for all π_a^f relations, especially when faults and attributes are numerous. So only known relations are described through π_a^f distributions. When π_a^f is not available, it either means that the value of a is not affected by the presence of f , or that this relation is not yet known. Then, π_a^f can be taken as equal to 1 everywhere: no information enables us to discard a value for attribute a when fault f is present.

The observations may also be pervaded with imprecision and uncertainty: a possibility distribution $\mu_a^O : U_a \rightarrow [0, 1]$ represents the (more or less) possible values of attribute a according to the observation O (e.g., reflecting sensor reliability).

Two fuzzy pattern matching indices [4] between effects (π_a^f) and observations (μ_a^O) are computed for the diagnosis:

- $cons^\pi$ evaluates how *consistent* the hypothesis of the presence of a fault (f) is with the observations:

$$cons^\pi(f) = \min_{a \in \mathcal{A}} cons^{\pi_a^f}, \quad (1)$$

where \mathcal{A} is the set of all the attributes and $cons^{\pi_a^f}$ evaluates the consistency of f with the observation of a :

$$cons^{\pi_a^f} = \sup_{u \in U_a} \min(\mu_a^O(u), \pi_a^f(u)). \quad (2)$$

$cons^{\pi_a^f}$ (resp. $cons^\pi(f)$) is 0 when the presence of f is incompatible with the observation on a (resp. an observation);

- rel^π evaluates how *relevant* to the observations a fault is:

$$rel^\pi(f) = \min_{a \in \mathcal{A}} rel^{\pi_a^f}, \quad (3)$$

where $rel^{\pi_a^f}$ evaluates to what extent the relation between the observation of a and the presence of f holds:

$$rel^{\pi_a^f} = \inf_{u \in U_a} (\mu_a^O(u) \rightarrow_D \pi_a^f(u)), \quad (4)$$

where \rightarrow_D denotes Dienes' fuzzy implication: $(x \rightarrow_D y) = \min(1 - x, y)$. $rel^{\pi_a^f}$ (resp. $rel^\pi(f)$) is 1 when the observation of

a (resp. all the observations) is (resp. are) compatible for sure with the presence of f .

Let \mathcal{K}_f denote the set of all attributes a for which there is a *Known* relation with the presence of f . Then, if $a \notin \mathcal{K}_f$, π_a^f is 1 on U_a . So, from a computational point of view, $\min_{a \in \mathcal{A}}$ can be replaced by \min in (1) and (3).

B Multidimensional attributes

The approach straightforwardly extends to multidimensional attributes, at the formal level, using joint possibility distributions such as π_{a_1, a_2}^f from $U_{a_1} \times U_{a_2}$ to $[0, 1]$ and conjunctions of observations, i.e. $\mu_{a_1, a_2}^O(u_1, u_2) = \min(\mu_{a_1}^O(u_1), \mu_{a_2}^O(u_2))$. However, it is useful in practice to identify the cases where the computation can still be done at the level of one-dimension attributes. Indeed, it is often not necessary to use explicit multidimensional distributions: Two major kinds of multidimensional attributes can be distinguished and represented in terms of one-dimension possibility distributions.

The first kind concerns *formula-linked multidimensional attributes*: For instance, assume that when fault f is present, attributes a_1 and a_2 should no longer be approximately equal. The possible values (u_1 and u_2) of a_1 and a_2 for this symptom of fault f may be represented as $\pi_{a_1, a_2}^f(u_1, u_2) = \pi_{a'}^f(u_1 - u_2)$, where a' is the new attribute $a' = a_1 - a_2$ and the closer to 0 u' is, the closer $\pi_{a'}^f(u')$ is to 0.

The second kind concerns *projection-decomposable multidimensional distributions*: Then, a symptom is described as several one-dimension manifestations linked together with logical connectors. For instance, when fault f occurs, both attribute a_1 and a_2 are high or attribute a_2 is close to 0. Then, $\pi_{a_1, a_2}^f = \max(\min(\pi_{a_1}^f, \pi_{a_2}^f), \pi_{a_2}^f)$, where $\pi_{a_1}^f$ expresses that a_1 is high, $\pi_{a_2}^f$ expresses that a_2 is high and $\pi_{a_2}^f$ expresses that a_2 is close to 0.

Of course, the two kinds of attributes can be mixed.

III REFINEMENTS OF THE REPRESENTATION

This section first introduces the use of twofold fuzzy sets for the knowledge representation. This enables us to make the distinction between what is just not impossible (because it is not ruled out by the experts' beliefs) and what is known as feasible (because it has already been observed). Moreover, under the single-fault hypothesis, the fact that fault f has no effect on attribute a means that a still takes normal values and this information can be useful to discard the presence of f when a takes abnormal values. So Section B introduces the representation of (ab)normal values of attributes and its impact on the diagnosis. Note also that a good diagnosis should explain all the abnormal behaviours observed on the attributes. For simplicity, the results below are presented for one-dimension attributes only and could be easily extended to multidimensional attributes.

A Twofold fuzzy sets

In the initial representation framework, π_a^f expresses the known restrictions on the values of attribute a linked to the presence of fault f . This idea of possibility may correspond to values which are just not known as impossible, since they are not ruled out by f . In particular, when nothing is known, π_a^f is 1

everywhere. However, it may be useful to identify among the values restricted by π_a^f those which are really known as feasible, because they have been observed in usual cases.

Thus a second possibility distribution ($\delta_a^f : U_a \rightarrow [0, 1]$) can be used for describing how typical the feasible values are known to be. For instance, we may know that when someone has influenza, some range of body temperature is guaranteed possible (according, for example, to former typical observations on people suffering from the same disease), while some other values around may not be a priori forbidden, although we are not sure that they can really take place. It will be assumed that a feasible value v for attribute a when fault f is present (i.e. $\delta_a^f(v) > 0$) should be such that $\pi_a^f(v) = 1$, for consistency reasons. So, the support of δ_a^f is included in the core of π_a^f . It entails $\delta_a^f \leq \pi_a^f$. When nothing is known, δ_a^f is 0 everywhere. Indeed, δ_a^f expresses to what extent a value of attribute a is known as feasible when f occurs.

Two other indices, $cons^\delta$ and rel^δ , may be defined (following $cons^\pi$ (1) and rel^π (3), resp.) with $cons^{\delta_a^f}$ and $rel^{\delta_a^f}$ (resp.) by replacing π_a^f by δ_a^f (in $cons^{\pi_a^f}$ (2) and $rel^{\pi_a^f}$ (4), resp.). $cons^\delta$ (resp. rel^δ) expresses to what extent it is possible (resp. certain) to have observed only typical effects or conditions of a fault.

This enables us to make the distinction between a fault f that has an unknown effect on an attribute a ($\pi_a^f = 1$ and $\delta_a^f = 0$) and a fault f' that has no effect on attribute a ($\pi_a^{f'} = 1$ and $\delta_a^{f'} = 1$), which was not possible in the initial framework.

Note that the pair (π_a^f, δ_a^f) could be summarized as a unique possibility distribution whose core would be the core of δ_a^f and whose support would be the support of π_a^f . This is a way of putting together the ideas of feasibility and non-impossibility. Yet, the pair (π_a^f, δ_a^f) provides a more refined representation.

As pointed out in [4], $cons^\pi \geq rel^\pi$ holds, so $cons^\delta \geq rel^\delta$ still holds. As $\pi_a^f \geq \delta_a^f$, it follows that $cons^\pi \geq cons^\delta$ and $rel^\pi \geq rel^\delta$. So $cons^\pi \geq \max(rel^\pi, cons^\delta) \geq \min(rel^\pi, cons^\delta) \geq rel^\delta$. There is no order relation between rel^π and $cons^\delta$.

For a cautious diagnosis, the consistency index should remain the same ($cons^\pi$, using π_a^f), discarding fault f only when (at least) one observation is for sure incompatible with the presence of f . As for the relevance index, it should select the faults whose most typical, guaranteed possible linked attributes values are all observed for sure, i.e. rel^δ (using δ_a^f).

N.B.: If $cons^\pi$ selects too many faults and rel^δ none, it is also possible to use the intermediate indices, with $cons^\delta$ and rel^π . See [7] for other refinements using *discri-min* and *leximin*, which amount to vector comparisons of $cons^{\pi_a^f}$ (or $rel^{\pi_a^f}$).

B Information on (ab)normal values

In fact, it is not fully satisfying to represent the information that the values of attribute a and the presence of fault f' are not linked with distributions $\pi_a^{f'} = 1$ and $\delta_a^{f'} = 1$. Indeed, when f' alone is present, a should take normal values in this case.

So, some information on (ab)normal behaviour of attributes is needed. Let $\rho_a : U_a \rightarrow [0, 1]$ be the possibility distribution expressing which values of attribute a are known as (more or less) abnormal. For short, it is assumed here that the normal behaviour of attribute a is defined as $\eta_a = 1 - \rho_a$.

Now, the former knowledge representation may benefit from this new information. When π_a^f and δ_a^f are not given (i.e. $a \notin \mathcal{X}_f$), it is because the expert states one of the following cases:

f (alone) is not linked to values of a , so a should keep normal values when f occurs. We get the twofold fuzzy set:

$$\pi_a^f = 1 - \rho_a; \delta_a^f(u) = 1 \text{ if } \rho_a(u) = 0; \delta_a^f \text{ is } 0 \text{ else.} \quad (5)$$

Let \mathcal{N}_f denote the set of attributes which are Not linked to f .

f could be linked to values of a but we do not know to which ones, which means that no value of a is known as impossible (resp. feasible) when f alone occurs: i.e. π_a^f (resp. δ_a^f) is 1 (resp. 0) everywhere. Let \mathcal{U}_f denote the set of attributes which have an Unknown link to f .

\mathcal{X}_f still corresponds to any other case for the attributes (Known links). So, $\forall f \in \mathcal{F}$, $(\mathcal{X}_f, \mathcal{U}_f, \mathcal{N}_f)$ is a 3-partition of \mathcal{A} . \mathcal{U}_f should be as small as possible (as every link between the presence of faults and the values of the attributes should be known, even only roughly) and \mathcal{N}_f can be very large. Then, this information on π_a^f and δ_a^f is used with the former ($cons^\pi$ and rel^δ) indices. So rel^δ is 0 as soon as $\mathcal{U}_f \neq \emptyset$ and $cons^\pi$ in equation (1) is computed with $\min_{a \in \mathcal{X}_f \cup \mathcal{N}_f}$ instead of $\min_{a \in \mathcal{A}}$.

The following toy example shows the benefits of this new information for the diagnosis. Let $\mathcal{A} = \{a_1, a_2\}$ and f_1 and f_2 be two faults s.t. $\mathcal{X}_{f_1} = \{a_1\}$, $\mathcal{X}_{f_2} = \{a_1, a_2\}$, with $(\pi_{a_1}^{f_2}, \delta_{a_1}^{f_2}) = (\pi_{a_1}^{f_1}, \delta_{a_1}^{f_1})$ (so $cons^{\pi_{a_1}^{f_1}} = cons^{\pi_{a_1}^{f_2}}$, and $\pi_{a_2}^{f_2} \leq \rho_{a_2}$ (i.e. a_2 takes abnormal values, if f_2 is present), while $\pi_{a_2}^{f_1} = 1 - \rho_{a_2}$. If $cons^{\pi_{a_1}^{f_1}} = cons^{\pi_{a_1}^{f_2}} = 1$ and $\mu_{a_2}^O \leq 1 - \rho_{a_2}$ (i.e. a_2 has normal values), then the diagnosis will select f_1 (with rel^δ) and discard f_2 (with $cons^\pi$), as expected.

Now, suppose that $cons^{\pi_{a_1}^{f_1}} = cons^{\pi_{a_1}^{f_2}} = 1$ and $\mu_{a_2}^O \leq \delta_{a_2}^{f_2}$ (i.e. a_2 takes abnormal values linked with the presence of f_2). The diagnosis should lead to f_2 . Yet, with $\pi_{a_2}^{f_2} = 1$ and $\delta_{a_2}^{f_2} = 1$, as in the approach of Sections A or II, f_2 and f_1 are both proposed. Using the information on (ab)normal values ($\pi_{a_2}^{f_1} = \delta_{a_2}^{f_1} = 1 - \rho_{a_2}$) leads to selecting f_2 with rel^δ ($cons^\pi(f_1) = 1$ and $rel^\delta(f_2) \geq 0.5$ as $\mu_{a_2}^O \leq \delta_{a_2}^{f_2} \leq \pi_{a_2}^{f_2}$) and discarding f_1 with $cons^\pi$ ($cons^\pi(f_1) \leq 0.5$ as $\mu_{a_2}^O \leq \pi_{a_2}^{f_2} \leq \rho_{a_2}$ and $\pi_{a_2}^{f_1} = 1 - \rho_{a_2}$).

Moreover, information on (ab)normal values may have an important role in multiple-fault diagnosis, as can be seen below.

IV MULTIPLE-FAULT DIAGNOSIS

This section discusses the practical handling of multiple faults, which is often left aside, since it has not the computational simplicity of the “single fault assumption”. By multiple faults is meant a combination of faults that are present simultaneously. For instance: A person may have both measles and white tonsillitis. Theoretically speaking, the approach to single-fault diagnosis can detect and identify multiple faults as well, by describing, for each possible combination of faults, the associated links to attributes values. Yet, if any set of multiple faults are possible, then one would have to define the associated links for 2^n faults, if n is the number of single faults. Such a knowledge base would be difficult to obtain in practice and would be

quite redundant as in many cases the effects linked to multiple faults are just the “sum” of the effects linked to the different single faults. For instance, measles gives fever and red spots on the body skin, and white tonsillitis gives fever (also) and white spots in the throat. Then, when someone suffers from both measles and white tonsillitis, fever, red spots on the body skin and white spots in the throat are expected.

In the following we propose a general approach to the handling of multiple faults. The idea is to represent the effects of single faults only and then to be able to compute the effects of any multiple faults (may be in a rather imprecise way).

The simplest hypothesis, called “*superposition hypothesis*”, is that effects of multiple faults are just the sum of the effects of the single faults involved in the multiple faults, i.e. the effects of all the involved single faults are present. In this case, when multiple faults are present, each of them can be selected using the indices $cons^\pi$ and rel^δ of the single fault case. The idea is to look for sets of faults with minimal cardinality (in practice one fault sets, first, then two faults sets and so on) whose joint effects are consistent and relevant w.r.t. the observations. This procedure is inspired from the parsimonious covering procedure, which was first suggested by Peng and Reggia in [9]. From a crisp point of view, the “*superposition hypothesis*” means that the effects of multiple faults on an attribute are the intersection of the effects of the corresponding single faults on the same attribute (the effects being represented by sets of possible values).

Note that when $f \in F$ and $a \in \mathcal{N}_f$, then f is not linked to a and so it has no incidence on the computation of the link between F and a . If the above situation holds $\forall f \in F$ for attribute a , then F is not linked to a ($a \in \mathcal{N}_F$): π_a^F and δ_a^F follow (5). Otherwise:

$$\begin{aligned} \forall F \subseteq \mathcal{F}, \forall a \in \mathcal{A}, \forall u \in U_a, \\ \pi_a^F(u) &= \min_{f \in \{g \in F \mid a \in \mathcal{X}_g\}} \pi_a^f(u) \\ \text{and } \delta_a^F(u) &= \min_{f \in \{g \in F \mid a \in \mathcal{X}_g \cup \mathcal{U}_g\}} \delta_a^f(u). \end{aligned} \quad (6)$$

Note that the minimum in (6) should always be based on $f \in \{g \in F \mid a \in \mathcal{X}_g \cup \mathcal{U}_g\}$. But from a computational point of view, the f such that $a \in \mathcal{U}_f$ can be left aside ($\pi_a^f = 1$). On the contrary, if $\{g \in F \mid a \in \mathcal{U}_g\} \neq \emptyset$, then $\delta_a^F = 0$.

Note that pair (π_a^F, δ_a^F) is still a twofold fuzzy set. The fuzzy intersection is justified for both π_a^F and δ_a^F as π_a^F represents the fuzzy set of values which are made impossible by none of the $f \in F$ and δ_a^F the fuzzy set of values which are known feasible for all the $f \in F$.

Yet, the “*superposition hypothesis*” is not always acceptable: Some effects cannot superpose as they are contradictory (they lead to $\pi_a^F = 0$, i.e. attribute a may not have a value when F is present!). For instance, let disease α give fever, disease β give hypothermia. What can be computed for the effect on the body temperature when those two diseases are simultaneously present? Will the effect of one fault be stronger than the effect of the other? or will both effects compensate into a normal body temperature? or will the simultaneous presence of the two diseases lead to something different, as a very strong fever? Here, the “*superposition hypothesis*” cannot hold.

In fact, all these types of effect combination can be modeled. For instance, “one effect is stronger” (but we do not know

which one: It may depend on the person or on the states of the diseases) is captured by the union, in the crisp case, and leads to: $\forall F \subseteq \mathcal{F}, \forall a \in \mathcal{A}, \forall u \in U_a, \pi_a^F(u) = \max_{f \in \{g \in F | a \in \mathcal{X}_g\}} \pi_A^f(u)$ and $\delta_a^F(u) = \max_{f \in \{g \in F | a \in \mathcal{X}_g \cup u_g\}} \delta_A^f(u)$. Note that (π_A^F, δ_A^F) is still a twofold fuzzy set. All types of effect combination can be computed. The important point is that the calculation must yield a twofold fuzzy set. Another combination of interest could be the worsening of the effects. For instance, if one disease gives fever and another too, the presence of both diseases might give a *strong* fever.

It might seem that we are back to the initial problem: For every multiple faults, and each attribute, the type of combination must be defined. This is true when there are no means to “guess” the type of combination. Yet, in most of the cases (in the domain where this multi-fault diagnosis is applied), the “superposition hypothesis” holds, unless it comes to a contradiction. In the cases where a contradiction is reached, all types of combination can be found. But, as a general approach is needed, we might just say that when “superposition hypothesis” does not hold for F on a , F has an effect on a but it cannot be computed precisely. So, $\pi_a^F = 1$ (a could possibly take any value when F is present) and $\delta_a^F = 0$ (no value is more characteristic than another). Of course, when this effect combination computation is not satisfying for multiple fault \mathcal{F} on attribute a , it is still possible to define π_a^F and δ_a^F in the knowledge base.

V CASCADING FAULTS

Another interesting phenomenon in diagnosis problems is the possible cascade from one fault to another: Fault f_1 may have some effects such that after a while fault f_2 will occur (and so on ...). For instance, if you do not stop early enough your influenza, you may get bronchitis. Then, you might have both influenza and bronchitis at the same time.

Using multiple-fault diagnosis, these cascading faults can be identified as several successive single or multiple faults. Here: First influenza takes place as a single disease, then both influenza and bronchitis. Yet, the cascade itself is not recognized through the diagnosis although it could be a good help in the diagnosis process.

Indeed, a cascade is rather a single fault inducing other faults than a multiple fault involving several independent faults. In order to take this into account in the parsimonious covering procedure, a cascade (no matter how many faults it induces or whether it is cycling or not) is counted as only one fault.

For this, let C be a binary relation in $\mathcal{F} \times \mathcal{F}$, with $C(f_1, f_2) = 1$ if and only if fault f_1 may induce fault f_2 . Then, as soon as $\exists f, \exists t_0 \text{cons}^\pi(f, t_0) > 0$ and $\text{cons}^\pi(f, t') = 0$ for $t' < t_0$, it means that the presence of f , or of a cascade starting with f (the possible cascades are given by C) is consistent with the observations. Here, $\text{cons}^\pi(f, t)$ is the result of $\text{cons}^\pi(f)$ at time t . The corresponding extension is also valid for rel^δ .

In practice the diagnosis system first only searches for single faults. After one is found (f_1), it searches for this single fault or cascades starting with this single fault. If necessary, it searches for a multiple fault of cardinality 2, made of f_1 (or one of its cascade) and another single fault (f_2). Then it searches for a multiple fault of cardinality 2 made of f_1 (or one of its cascade) and f_2 (or one of its cascade), and so on. The underlying hy-

pothesis is that a fault stays until it has been corrected, once it has occurred.

Another improvement is to have $C(f_1, f_2)$ expressing the time for f_2 to appear after f_1 has taken place (may be in a fuzzy way). This additional information would be a useful ingredient to incorporate in cons^π and rel^δ in order to identify cascades.

If the hypothesis – that a fault lasts until it is corrected – does not hold, cascades have to be described as chronicles, giving also (possibly) fuzzy time periods during which each fault lasts before it disappears (see [6]).

VI CONCLUSION

This paper has shown how the basic fuzzy pattern matching approach to diagnosis problem could be refined in order to allow for a more refined representation framework (distinguishing between effects which are surely possible and effects which are not impossible), to take advantage of normal behavior information, and to cope with multiple faults.

The approach has been implemented on a car engine dyno test bench problem, where observations arrive in real time. Preliminary results are promising [1].

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