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## Boulay *et al.* Reply

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In the preceding Comment [1] on our article [2], Garrett questions the IBFM-1 interpretation of the low-lying level structure of  $^{99}\text{Zr}$  by pointing out possible drawbacks of the wave functions (w.f.) of the  $1/2_1^+$  (ground) and  $3/2_1^+$  states, and by stating that a similar reproduction of the experimental data could be achieved with a much different set of single-particle energies (SPE) [3, 4].

The  $1/2_1^+$ ,  $3/2_1^+$  states of  $^{99}\text{Zr}$  have similar  $B$  values and magnetic moments with those of  $^{97}\text{Zr}$  and  $^{97}\text{Sr}$  [2, 5, 6]. Based on this similarity and on the observation of the  $1/2_1^+$  state in the  $^{96}\text{Zr}$  ( $d, p$ )  $^{97}\text{Zr}$  study [7], it is claimed [1] that this state should be predominantly  $\nu s_{1/2}$ , as provided by the IBFM-1 calculations of [3, 4]. However, the ( $d, p$ ) reaction in this argument concerns a different isotope, and the extremely quick variation of the properties in this region ( $^{99}\text{Zr}$  is close to the critical point of the shape phase transition at  $N \approx 59$  [2, 5])

may invalidate this conclusion. Indeed, a recent ( $d, p$ ) reaction investigation of the  $^{95,96,97}\text{Sr}$  isotopes showed a weak population of the  $1/2_1^+$  and  $3/2_1^+$  states in  $^{97}\text{Sr}$  [ $C^2S$  values of 0.07(5) and 0.25(5), respectively], compared to  $^{95}\text{Sr}$  [ $C^2S$  values of 0.41(9) and 0.53(8)] [8]. This suggests differences between the w.f. of these states in  $^{95}\text{Sr}$  and  $^{97}\text{Sr}$ . As the Sr and Zr isotones in this region are very similar, one may expect that these states of  $^{99}\text{Zr}$  are more weakly populated in the ( $d, p$ ) reaction than their analogs in  $^{97}\text{Zr}$ . There are also recent IBFM-2 calculations [5, 6] where the  $1/2^+$  g.s. of  $^{99,97}\text{Zr}$  and  $^{97}\text{Sr}$  has  $\nu s_{1/2}$  components of 21%, 16%, and 12%, respectively, compared to 1.5% in [2, 9]. The  $3/2_1^+$  state in  $^{99}\text{Zr}$  is  $\nu d_{5/2}$  dominated and has a  $\nu d_{3/2}$  component of 11% in [2, 9], while in [5] it has 32%  $\nu d_{3/2}$ . Without experimental ( $d, p$ ) data for  $^{99}\text{Zr}$  one cannot decide now which s.p. composition is more realistic.

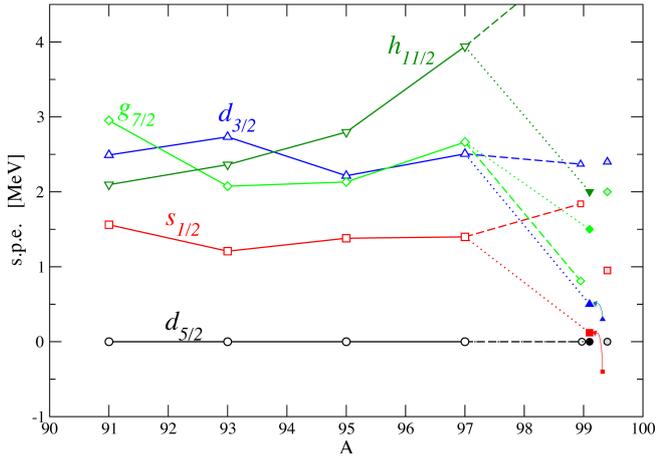


FIG. 1: Neutron SPE for  $^{91-99}\text{Zr}$  isotopes: experiment up to  $A = 97$  [10], and IBFM for  $^{99}\text{Zr}$ : filled symbols, present calculations, arrows showing the modification of values from [2, 9]; open symbols (left) [5]; gray-filled symbols (right) [3, 4]. Note that the SPE of  $\nu h_{11/2}$  in our calculations was chosen to reproduce the relative position of the positive- and negative-parity states calculated with the same Hamiltonian.

The second point in [1] concerns the SPE values used in [2, 9]. Those values report an extreme solution with an exact reproduction of the  $7/2_1^+$  state  $g$  factor [2]. For the present discussion we reviewed the old calculations and chose a different SPE set where the latter condition was relaxed. The initial calculations started from SPE close to the experimental systematics of Zr isotopes [10] and those of [3, 4]. For each SPE set we sought parameters of the boson-fermion interaction that describe the available experimental data. Many trials indicated that the  $s_{1/2}$  and  $d_{3/2}$  SPE have to be lowered. Figure 1 shows the SPE values of the present calculations and those of

[4, 5]. The present SPE differ from those in [2, 9] by an increase of 0.5 MeV for  $s_{1/2}$  and of 0.2 MeV for  $d_{3/2}$ , but they still provide a description of the experimental data similar to that in [2, 9], the predicted magnetic moment of the  $7/2_1^+$  state being now lower (+1.48 nm). Increasing more the energy of any of the two orbitals accentuates the discrepancies for some of the magnetic moments and transition probabilities. These SPE values, although not uniquely determined because of inherent ambiguities, still show deviations from the pattern of the experimental values. Some SPE used in the IBFM-2 calculations [5] also do not smoothly continue the experimental trend of the masses 91 to 97, and differ from those of [3, 4] (Fig. 1). Thus, the parameterization of the IBFM Hamiltonian (core, SPE, and boson-fermion interaction) forms a whole for each case, that must be validated by comparison with experimental data. In our work this was achieved by empirically studying the influence of the various parameters. In the IBFM-2 calculations [5, 6], the IBM-2 core Hamiltonian and the SPE were determined by microscopic calculations, leaving adjustable only the boson-fermion strengths. In the older calculations [3, 4] the choice of the SPE and boson-fermion interaction parameters was made to reproduce the properties of the lowest three states of  $^{97}\text{Sr}$  and  $^{99}\text{Zr}$ , and was guided by zeroth order shell-model expectations for the quasiparticle structure of their w.f. (and notably, a  $\nu s_{1/2}$  dominated g.s.). Overall, the relatively large number of model parameters was tuned to a rather small set of available experimental data. The calculations from [2, 9] and [5, 6] were tuned to a much larger set of experimental data [2, 9], validating, within certain limits, the w.f. of the IBFM Hamiltonian. New experimental data on Zr isotopes around mass 99, especially on one-neutron transfer, are essential to better understand this special nuclear region.

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