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Application of a mixed finite element to the analysis of laminated composite

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Résumé:

Un élément fini mixte d'interface basé sur le principe variationnel mixte de Reissner est présenté pour analyser les structures stratifiées. C'est un élément fini mixte bidimensionnel à 7 nœuds avec 5 nœuds déplacement et 2 nœuds contrainte. Cet élément assure la continuité des vecteurs déplacement et contrainte à travers l'interface entre matériaux. Les résultats obtenus, avec l'élément d'interface présenté, montrent une bonne concordance avec les solutions analytiques pour des poutres stratifiées.

Abstract:

A mixed finite element based on Reissner's mixed variational principle has been presented to analyze laminated composite. The present element is a 7-node two dimensional mixed finite element with 5 displacement nodes and 2 stress nodes. The mixed interface finite element ensures the continuity of stress and displacement vectors at the interface between two materials. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions for laminated beam.

Keywords: Mixed finite element, Interface, Reissner's mixed variational principle, laminated composite.

1 Introduction

Composite materials became an important structural component in the industrial field, in particular in civil engineering, aeronautics and automobiles. The rapid increase of the industrial use of structures composed of laminated materials has necessitated the development of new analytical and numerical tools that are suitable for the analysis and study of the mechanical behaviour of such structures. Elasticity solutions for composite laminates [1-2] indicate that interlaminar continuity of transverse normal and shear stresses as well as the layer-wise continuous displacement field through the thickness are essential requirements in analysis [3].

In this paper, a mixed interface finite element presented by Bouziane and al. [4] has been used to analyze laminated composite. The element consists on a 7-node two dimensional mixed finite element developed by using Reissner's mixed variational principle. The interface element, of 5 displacement nodes with two degrees of freedom (two displacement components u_1 , u_2) per node and 2 stress node with two degrees of freedom (two transverse stress components σ_{22} , σ_{12}) per node, ensures the continuity of transverse stress and displacement fields. A brief discussion of the formulation of the mixed interface element has been presented in the next section. Finally, the validity of the present mixed-interface element is demonstrated through some numerical for a sandwich.

2 Mixed-interface finite element

A laminated composite beam consisting of N-layers of lamina has been considered for finite element analysis. The beam has discretized using a 7-node mixed –interface finite element (RMQ-7 element: **R**eissner **M**odified **Q**uadrilateral) as shown in figure 1.

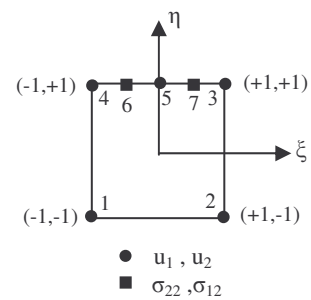
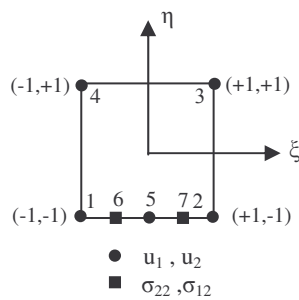
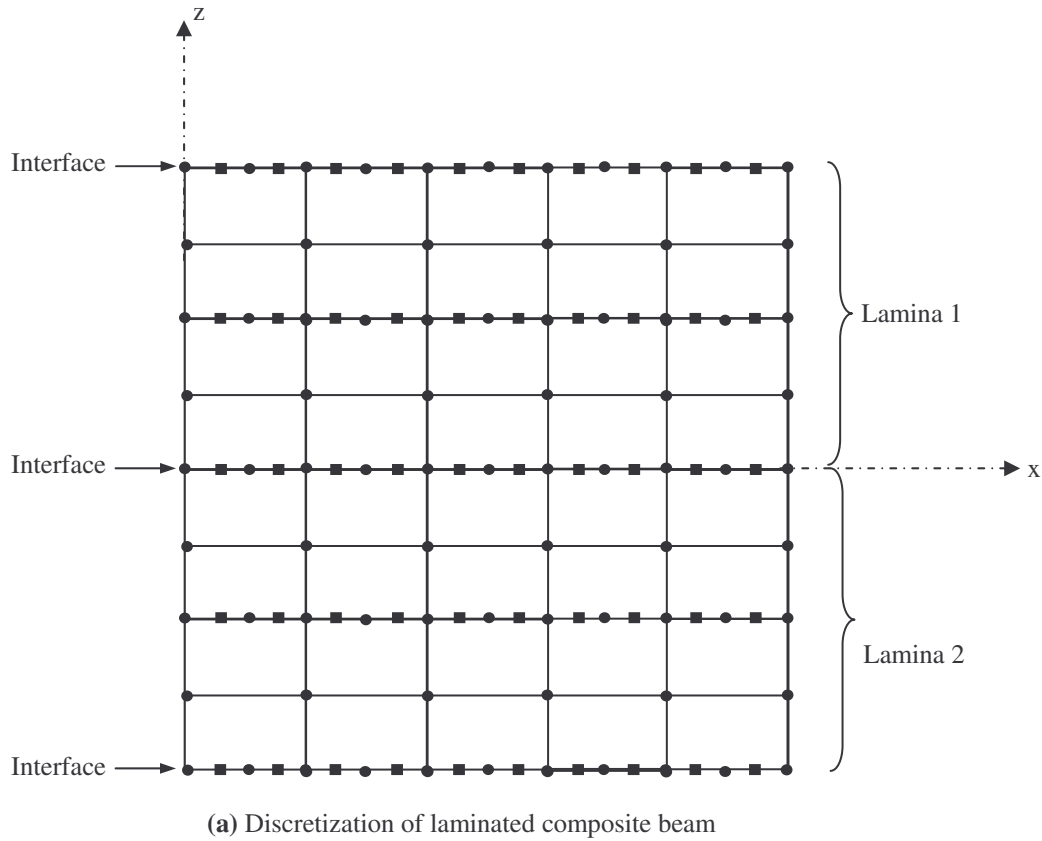


FIG. 1 - Discretization of laminated beam and RMQ-7 element

The RMQ-7 interface element is a quadrilateral mixed element with 7 nodes and 14 degrees of freedom [4]. This interface element is obtained by successively exploiting the technique of relocalisation [5] and the static condensation procedure [6]. Three of its sides are compatible with linear traditional elements and present a displacement node at each corner (figure 1(b)). The fourth side, in addition to its two displacement nodes of corner (node 1 and node 2), offers three additional nodes: a median node (node 5) and two intermediate nodes in the medium on each half-side (nodes 6 and 7), introducing the components of the stress vector along the interface. The Continuity of the

displacement and stress vectors can be taken into account on the level on this particular side, which must be placed along the interface.

The mixed-interface finite element shown in figure 1(b) has been developed by considering displacement fields $u_1(x,z)$ and $u_2(x,z)$ having linear variation along three sides and quadratic variation along the particular side.

In the configuration of figure 1(b), the shape functions, used to approximate σ_{22} and σ_{12} are given as follows:

$$\begin{aligned} M_{i2}^6 &= \frac{1}{6}(1-2\xi)(1-2\eta) & , & & M_{i2}^7 &= \frac{1}{6}(1+2\xi)(1-2\eta) \\ M_{i2}^8 &= \frac{1}{3}(1+2\xi)(1+\eta) & , & & M_{i2}^9 &= \frac{1}{3}(1-2\xi)(1+\eta) \quad , i=1,2 \end{aligned} \quad (1)$$

For the inversed RMQ-7 element (figure 1(c)), the shape functions used to evaluate σ_{22} and σ_{12} are given by

$$\begin{aligned} M_{i2}^6 &= \frac{1}{6}(1-2\xi)(1+2\eta) & , & & M_{i2}^7 &= \frac{1}{6}(1+2\xi)(1+2\eta) \\ M_{i2}^8 &= \frac{1}{3}(1+2\xi)(1-\eta) & , & & M_{i2}^9 &= \frac{1}{3}(1-2\xi)(1-\eta) \quad , i=1,2 \end{aligned} \quad (2)$$

The element matrix $[K_e]$ is given by

$$[K_e] = \begin{bmatrix} [K_{\sigma\sigma}] & [K_{\sigma u}] \\ [K_{\sigma u}]^t & [0] \end{bmatrix} \quad (3)$$

Here

$$[K_{\sigma\sigma}] = -e \int_{A^e} [M]^t [S] [M] dA^e \quad (4)$$

and

$$[K_{\sigma u}] = e \int_{A^e} [M]^t [B] dA^e \quad (5)$$

where e is the thickness, $[S]$ is the compliance matrix, $[M]$ is the matrix of interpolation functions for stresses, $[B]$ is the strain-displacement transformation matrix and A^e is the element area.

3 Numerical results and discussion

The computer program developed by incorporating the present mixed-interface finite element (RMQ-7) has been employed for the analysis of symmetric sandwich beam. This beam is subjected to uniform load and presents three layers including two identical orthotropic face sheet and an isotropic core as shown in figure 2.

The material properties of the face sheet and core material are:

- Face sheet material (orthotropic material):
 $E_x=52800$ MPa, $E_z=39713$ MPa, $G_{xz}=13200$ MPa, $\nu_{xz}=0,41$
- Core material (isotropic material):
 $E=50$ MPa, $G=21$ MPa, $\nu=0,2$

In order to evaluate the validity and the credibility of the present mixed-interface element, a study of the convergence on a sandwich beam is carried out. In this analysis several meshes are used.

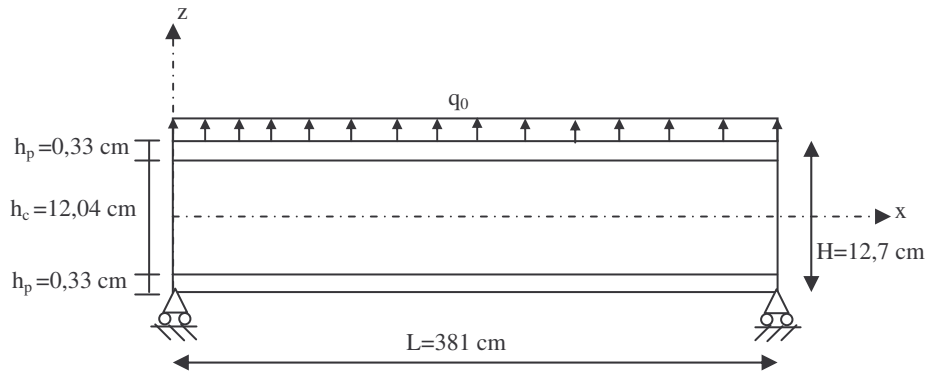


FIG. 2 - Sandwich beam analyzed

Results obtained through the present mixed element for various numbers of degrees of freedom are tabulated in tables 1, 2 and 3 where they have been compared with the elastic solutions given by Pagano [1]. Table 1 shows the deflection values obtained at $x=L/2$ according to the number of degrees of freedom. Table 2 gives the horizontal displacement at $x=L/4$. Variation of transverse shear at $x = L/8$ has been presented in table 3.

Element type	Number of elements	Number of degree of freedom	Deflection u_2 (cm)
Present mixed element	4	32	1.72
	16	98	2.65
	64	402	2.70
Pagano [1]		-	2.73

Table 1. Deflection in a sandwich beam at $x=L/2$ solved by various meshes

Element type	Number of degree of freedom	Horizontal displacement u_1 (cm)	
		$z = -h_c/2$	$z = h_c/2$
Present mixed element	32	0.0326	-0.0325
	98	0.0682	-0.0682
	402	0.0690	-0.0690
Pagano [1]	-	0.0705	-0.0705

Table 2. Horizontal displacement in a sandwich beam at $x=L/4$ solved by various meshes

Element type	Number of degree of freedom	Transverse shear σ_{12} (MPa)	
		$z = -h_c/2$	$z = h_c/2$
Present mixed element	32	0.1123	0.1123
	98	0.1137	0.1146
	402	0.1131	0.1159
Pagano [1]	-	0.115	0.115

Table 3. Transverse shear in a sandwich beam at $x=L/8$ solved by various meshes

It can be seen that the results from the present mixed element are in very good agreement with the elasticity solution [1].

Figures 3, 4 and 5 show the variation of the deflection, horizontal displacement and the transverse shear respectively with the number of degrees of freedom. It appears that the mixed interface element converges very quickly for a number relatively low of degrees of freedom.

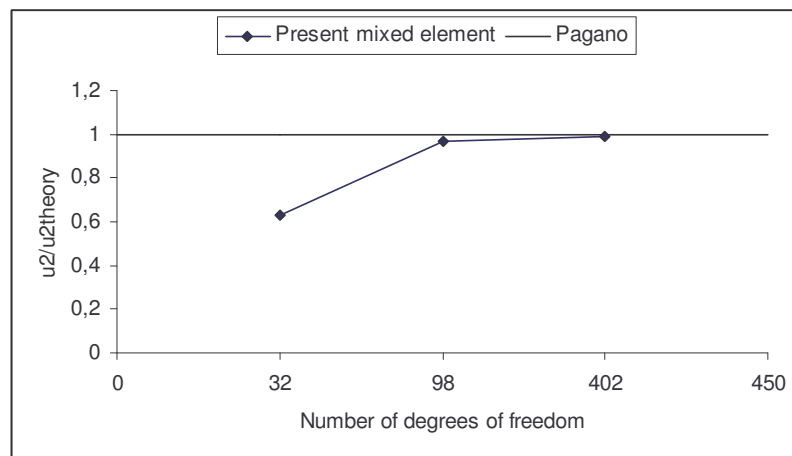


FIG. 3 – Convergence of deflection at $x=L/2$

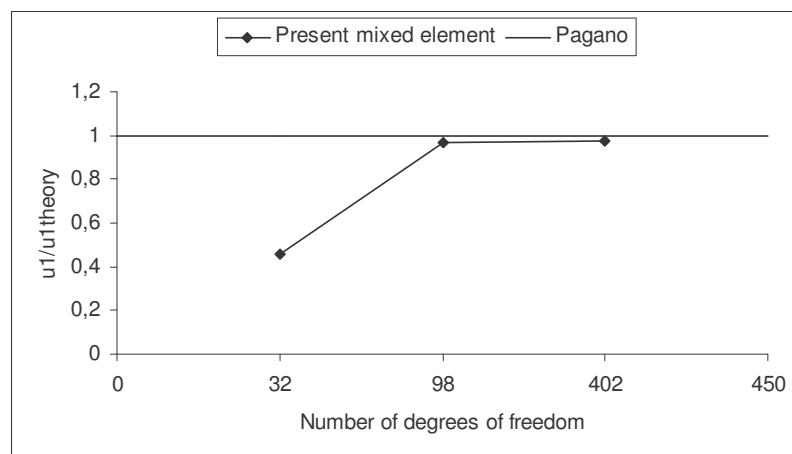


FIG. 4 – Convergence of horizontal displacement at $x=L/4$ ($z=hc/2$)

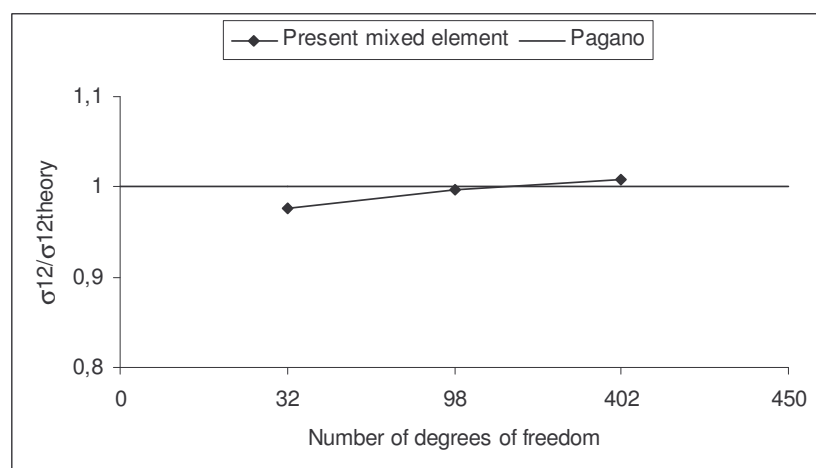


FIG. 5 – Convergence of transverse shear at $x=L/8$ ($z=hc/2$)

4 Conclusion

A novel mixed-interface finite element has been used to analyze a sandwich beam. In the formulation of this element, we used Reissner's mixed variational principle to build the parent element. The mixed interface finite element is obtained by successively exploiting the technique of relocalisation and the static condensation procedure. This element ensures the continuity of transverse stress and displacement fields at the interface. The accuracy of the element has been evaluated by comparing the numerical solution with an available analytical solution. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions. Comparison of the results with the exact elastic solutions validates the present interface element and indicates its application to laminated composite problems.

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