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To cite this version:
Achim Wirth. Dynamics of Oceanic Gravity Currents. CFM 2009 - 19ème Congrès Français de Mécanique, Aug 2009, Marseille, France. hal-03391189

HAL Id: hal-03391189
https://hal.science/hal-03391189
Submitted on 21 Oct 2021

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Non-traditional Ocean Deep Convection

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Résumé :
La convection dans un océan homogène est considérée en intégrant numériquement les équations de Boussinesq dans un repère en rotation, dans lequel la composante horizontale du vecteur de rotation n’est pas négligée. L’étude détermine les effets de l’angle entre l’axe de rotation et la gravité sur le processus convectif. Nous faisons variés deux paramètres clés : (i) la magnitude du flux de chaleur et (ii) l’angle entre l’axe de rotation et la gravité. Nous montrons que : Le théorème de Taylor-Proudman-Poincaré laisse une signature dans les structures convectives, (ii) une circulation moyenne horizontale s’établit, (iii) les moments de deuxième ordre sont augmentés.

Abstract :
Convection in a homogeneous ocean is investigated by numerically integrating the three dimensional Boussinesq equations in a tilted-rotating frame (f-F-plane) subject to a negative buoyancy flux (cooling) at the surface. The study focuses on determining the influence of the angle (tilt) between the axis of rotation and gravity on the convection process. To this end we vary two essential parameters : (i) the magnitude of the surface heat flux, and (ii) the angle (tilt) between the axis of rotation and gravity. The range of the parameters investigated is a subset of typical open ocean deep convection events. We demonstrate that when gravity and rotation vector are tilted with respect to each other : (i) the Taylor-Proudman-Poincaré theorem leaves an imprint in the convective structures, (ii) a horizontal mean circulation is established, (iii) the second order moments involving horizontal velocity components are considerably increased.

Mots clefs : Geophysical fluid dynamics, convection, traditional approximation

1 Introduction
A striking feature of open-ocean deep convection is that, although it governs a substantial part of the poleward heat transport of the atmosphere-ocean system, through its influence on the thermohaline circulation, it is nevertheless extremely localized in space and time. In fact, convection chimneys typically measure about 100 km in the horizontal directions and account only for a tiny fraction of the world ocean and a convection event typically lasts for only about one week. Convection chimneys are an ensemble of many convection plumes which typically measure less than one kilometer. These plumes, the “atoms of convection”, perform the actual (negative) buoyancy transport and mixing. A substantial part of the large-scale-long-time ocean and climate dynamics is thus slaved to what happens at these almost space-time singularities.
Convection in the atmosphere takes only a few hours while it takes a few days in the ocean. We here demonstrate that the corresponding difference in Rossby number is essential for the convection process and its parameterization. Oceanic convection is in a dynamically interesting regime as vertical velocities are large enough so that non-hydrostatic terms can not be ignored but are small enough so that rotation can not be ignored, either. Non-linearity is strong enough so that the dynamics is in a three-dimensional turbulent regime, as opposed to a quasi-two-dimensional heton dynamics.
I show that not only the magnitude of the rotation vector (as expressed in the Rossby number) is of importance but also its direction. More precisely, in the majority of numerical calculations considering ocean dynamics, the traditional approximation is employed which completely neglects the horizontal component of the rotation vector and thus its “tilt.” Rotation is thus supposed to be collinear with gravity, which is strictly only the case at the poles. The traditional approximation may be justified in instances where vertical velocities are small compared to their horizontal counterparts, that is, when non-hydrostatic terms can be neglected. However, when non-hydrostatic terms are essential for the dynamics, as in the case of convection, the horizontal component of the rotation vector (the tilt) has to be included (see Gerkema et al. 2008, for a thorough discussion of the traditional approximation).

2 Basic facts about Open-ocean deep convection
The entire deep ocean convection process is usually divided into three phases : (i) preconditioning, (ii) deep convection (vertical exchange of water masses) and (iii) lateral exchange between the (homogenized) convec-
tion site and the ambient fluid (see e.g. Marshall & Schott 1999 for a review). The first and last process happen on scales of a few tens of kilometers to the order of a few hundreds of kilometers, and are usually well represented in hydrostatic models of ocean convection with sufficient resolution. It is the second process, deep convection, that involves plumes of 100 meters to one kilometer in diameter, and which is intrinsically non hydrostatic. This process has thus to be parametrized in hydrostatic large-scale ocean models. In the present work we investigate the dynamics of the second process.

3 The Model

We found in numerical experiments that a domain spanning 8km in each horizontal direction allows for a sufficient space for a few plumes, spanning a few hundreds of meters in the horizontal direction, to descend independently. The atmospheric forcing varies on horizontal scales of 100 to 1000km so that it is adapted to use a homogeneous cooling on the surface. An integration representing a few days of dynamics is necessary to account for the descent of several plumes and to obtain a statistically stationary dynamics. The domain size of 8km in both horizontal directions is, however, far too small to obtain statistical significant results for even the lowest order quantities. We thus continued the experiment after a statistically stationary state was obtained for a couple of days and used the (supposed) ergodicity of the dynamics by averaging over space and time to obtain converged statistical estimates.

To study the convection process a non-hydrostatic ocean model is essential. The mathematical model of the here presented ocean convection experiments are the Boussinesq equations (1) of an incompressible flow (2) in a rotating frame, supplemented by boundary conditions. The flow field is given by $u$ and the scalar (temperature) field by $T$. The buoyant scalar is transported by the flow (3) and diffuses at the rate $κ$.

$$\begin{align*}
\partial_t u + u \cdot \nabla u + 2\Omega \times u + \nabla P &= \alpha g T e_\perp + \nu \nabla^2 u \\
\nabla \cdot u &= 0 \\
\partial_t T + u \cdot \nabla T &= \kappa \nabla^2 T + G.
\end{align*}$$

As we are considering only a small slice of the ocean, spanning 8 km in the horizontal directions we can safely neglect the sphericity of the earth by using a Cartesian geometry and we can also neglect the variation of the direction of the rotation vector with the latitudinal direction ($\theta$). This geometry is called the $f-F$ plane, where $f$ and $F$ are twice the normal and horizontal components of $\Omega$, respectively. That is:

$$f = \begin{pmatrix} 0 \\ F \\ f \end{pmatrix} = 2\Omega \begin{pmatrix} 0 \\ \cos \theta \\ \sin \theta \end{pmatrix}$$

The $f-F$ plane allows for the implementation of periodic boundary conditions in both horizontal directions. The mathematical model is solved numerically using a pseudo-spectral scheme entirely based on Fourier expansion. The boundary conditions are implemented using a method inspired by the immersed boundary technique. For a detailed discussion of the model and the new boundary technique we refer the reader to Wirth 2004. Our model will henceforth be called HAROMOD (HARmonic Ocean MOdel).

4 Results

A detailed discussion on the dynamics of a single convective plume is published in Wirth & Barnier (2005), where a special emphasis was put on the influence of a non-vanishing angle between the directions of gravity and the rotation-vector (tilted convection). We here emphasise on the dynamics of the convection process in an ocean that is cooled homogeneously at its surface. The resulting dynamics is that of an ensemble of convective plumes.

Due to the non-linear character of the convection process, the dynamics of turbulent plume ensembles can not be derived from the behavior of a single plume. We again have to resort to numerical simulations to determine the basic features of turbulent convection.

After the onset of cooling a negative temperature anomaly develops at the surface. In less than 12 hours this unstable situation leads to convective motion and a convective front descents into the unstratified ocean until it reaches the ocean floor after about 18 to 30 hours depending on the strength of the forcing and rotation.

The subject of this paper is the subsequent statistically stationary stage of convective dynamics in the entire water column. During this stage descending plumes surrounded by rising water mix the water column as can be seen in Fig 1. A shallow boundary layer develops at the ocean surface. As in the case of a single plume, the influence of the (no-slip) bottom slows down the front propagation 500 meters (the typical horizontal plume scale) above the bottom. This emphasizes once more the importance of the Ekman layer dynamics at the ocean floor.
4.1 Temperature

The most important variable to consider is of course the temperature or buoyancy density (both are linearly related). My numerical results show clearly that the temperature gradient is not constant. The vertical temperature gradient can be well approximated by a linear behavior away from the top and bottom boundary. With a dependence on the surface buoyancy flux \( \mathcal{B}_0 \) that is consistent with dimensional analysis. The second derivative of the mean temperature with respect to the vertical is thus constant, \( \langle \partial_{zz} T \rangle_{h,t} = \tau \times \frac{\mathcal{B}_0^2}{3H^{-7/3}/(\alpha g)} \), where we estimated the dimensionless constant \( \tau = 10 \).

In all results on temperature and its vertical gradients no significant difference between the non-tilted and tilted case could be detected, except for differences in the bottom Ekman layer. Of great importance is also the observation that in the lower 500 meters the temperature gradient is positive which means, that there is a counter gradient flux of heat. This behavior is often named “non-local transport” and is caused by the buoyancy transport of the convective plumes that extent from the surface to the bottom of the domain. Fitting an affine law to the gradient of the depth averaged temperature allows to write the buoyancy-flux as:

\[
\kappa_Z (\alpha g \partial_z T - \gamma) = -\frac{\mathcal{B}_0 z}{H},
\]

where \( \gamma \) denotes the non-local part of the flux. Equation (5) is taken from the KPP parametrization (see Large & McWilliams 1994), and the term on the right hand side is a consequence of the stationarity of the dynamics. Dimensional analysis suggests \( \kappa_Z = \kappa_0 (B_0 H^4)^{1/3} \), where the dimensionless constant \( \kappa_0 = \tau^{-1} = .1 \) best fits our data.

4.2 Velocity

The equation for the horizontally averaged velocity is given by:

\[
\partial_t \langle u \rangle_h = -\partial_z \langle uw \rangle_h + f \langle v \rangle_h + \nu \partial_z^2 \langle u \rangle_h
\]

\[
\partial_t \langle v \rangle_h = -\partial_z \langle vw \rangle_h - f \langle u \rangle_h + \nu \partial_z^2 \langle v \rangle_h
\]

These equations also reflect the fact that a vanishing total momentum (depth averaged \( \langle u \rangle_h \) and \( \langle v \rangle_h \)) is conserved at all time as the vertical integral of \( \partial_z \langle uw \rangle_h \) and \( \partial_z \langle vw \rangle_h \) vanish due to the vanishing vertical velocity at the top and bottom boundary. If we suppose that the velocity variables are statistically stationary,
the time derivative vanishes when time averages, denoted by $\langle \cdot \rangle_{t}$, are taken. If we further neglect the direct effects of viscosity in equations (6) and (7) we obtain:

$$f\langle v \rangle_{h,t} = \partial_z \langle uw \rangle_{h,t}$$
$$f\langle u \rangle_{h,t} = -\partial_z \langle vw \rangle_{h,t}$$ (8)

As upward-moving fluid parcels are deflected north and westward and downward-moving fluid parcels are deflected south and eastward by rotation (conservation of angular momentum with a tilted rotation vector), this leads to a negative correlation of $\langle uw \rangle_{h,t}$ and a positive correlation of $\langle vw \rangle_{h,t}$. The vertical velocity component ($w$) vanishes at the lower and upper boundary while the horizontal velocity components ($u,v$) vanish only on the lower (no-slip) boundary and have a local extrema at the upper boundary (vanishing first derivative). This leads to a positive velocity to a north-eastward mean velocity in the upper kilometer of the ocean and an opposed velocity in the deeper part. In Fig. 2 it can be appreciated that Eqs. (8) and (9) are almost perfectly verified for an averaging over horizontal slices of the entire domain at 30 consecutive instances of time separated by 3 hours. Small differences are visible near the upper and lower boundary. The neglected friction plays a non-negligible role for the differences at the lower boundary.

![Fig. 2](image_url)

**FIG. 2** – Left and right side of eqs. 8 (left figure) and 9 (right figure), for a cooling of 250W/m$^2$ (light blue), 500W/m$^2$ (red) and 1000W/m$^2$ (deep blue), the more wavy curves are the right side of the eqs.

### 5 Discussion

We demonstrated that qualitative and quantitative changes occur when the traditional approximation is relaxed: (i) turbulent structures are aligned along the axis of rotation as predicted by the Taylor-Proudman-Poincaré theorem, (ii) the horizontal component of the rotation vector enhances horizontal mixing, (iii) a horizontal mean circulation is established, and (iv) the inverse stratification together with the horizontal mean circulation leads to an equator-ward heat transport as the coldest water at the surface moves northward. These changes have important consequences in all instances where vertical transport of passive or active tracers is considered. The horizontal transport induced by a heat flux of $10^3$ W m$^{-2}$ at a latitude of $\theta = 45^\circ$ is equal to an Ekman transport induced by a a wind field of $U_{10} = 75$ km h$^{-1}$. The Ekman transport is confined to the upper tens of meters, whereas the horizontal transport induced by tilted convection spreads over the upper kilometer. The vertical extent is, however, not very important when vertical velocities due to divergences of the horizontal velocity field are considered. The similarity to the wind stress forcing that leads to horizontal divergence, Ekman pumping and a large scale circulation by Sverdrup balance can be taken further. Gradients in the heat flux at the ocean surface will lead to horizontal divergence, leading to vertical velocities and a large scale circulation by Sverdrup balance. An effect that is completely neglected when the traditional approximation is employed.

For a more thorough discussion we refer the reader to Wirth & Barnier (2008).

### Références

