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ABSTRACT

8 One of the proposed mechanisms to explain the multidecadal variability ob-
9 served in sea surface temperature of the North Atlantic consists of a large-
10 scale low-frequency internal mode spontaneously developing because of the
11 large-scale baroclinic instability of the time-mean circulation. Even though
12 this mode has been extensively studied in terms of the buoyancy variance bud-
13 get, its energetic properties remain poorly known. Here we perform the full
14 mechanical energy budget including available potential energy (APE) and ki-
15 netic energy (KE) of this internal mode and decompose the budget into three
16 frequency bands: mean, low frequency (LF) associated with the large-scale
17 mode and high frequency (HF) associated with mesocale eddy turbulence.
18 This decomposition allows us to diagnose the energy fluxes between the dif-
19 ferent reservoirs and to understand the sources and sinks. Due to the large-
20 scale of the mode, most of its energy is contained in the APE. In our configu-
21 ration, the only source of LF APE is the transfer from mean APE to LF APE
22 that is attributed to the large-scale baroclinic instability. In return the sinks
23 of LF APE are the parameterized diffusion, the flux toward HF APE and to a
24 much lesser extent toward LF KE. The presence of an additional wind-stress
25 component weakens multidecadal oscillations and modifies the energy fluxes
26 between the different energy reservoirs. The KE transfer appears to only have
27 a minor influence on the multidecadal mode compared to the other energy
28 sources involving APE, in all experiments. These results highlight the utility
29 of the full APE/ KE budget.

30 **1. Introduction**

31 The multidecadal large-scale variability of the Sea Surface Temperature (SST) is characterised
32 in the North Atlantic by an anomaly intensified in the subpolar region and by a weaker anomaly
33 of opposite sign south of the equator (Kushnir 1994; Deser et al. 2010; Zhang et al. 2019). This
34 large scale SST variability has been named Atlantic Multidecadal Variability (AMV, Kushnir 1994;
35 Schlesinger and Ramankutty 1994; Kerr 2000; Sutton et al. 2018). Cool AMV phases occurred
36 in the 1900s-1920s, 1960s-1990s and warm phases occurred in the 1930s-1960s and after 1995.
37 These cool and warm phases have been shown to be associated with several regional climate
38 impacts such as the Sahel Indian summer monsoon rainfall, Atlantic hurricanes frequency, summer
39 climate over western Europe and north America (Zhang et al. 2019), wave climate in the Atlantic
40 and Pacific Ocean (Reguero et al. 2019). Observations moreover show that more heat is released
41 from the North Atlantic ocean to the atmosphere during a positive phase of the AMV (Gulev
42 et al. 2013). Thus understanding what controls the dynamics of this variability and its potential
43 predictability have essential societal and economics implications (Sutton et al. 2018).

44 Several mechanisms have been proposed to explain the origin of the AMV and remain actively
45 debated (see for instance the recent discussion in Clement et al. 2015; Zhang et al. 2016; Clement
46 et al. 2016). Some studies suggest a direct role of the atmosphere either via stochastic heat flux
47 (Hasselmann 1976; Frankignoul and Hasselmann 1977; Clement et al. 2015) or via aerosol emis-
48 sions (Booth et al. 2012), while other studies (e.g. Sévellec and Fedorov 2013; Arzel et al. 2018)
49 suggest a role for oceanic processes linked with the internal variability of the Atlantic Merid-
50 ional Overturning Circulation (AMOC). In this work we focus on improving our knowledge of the
51 physics of internal ocean modes which are one of the possible explanations for the AMV. At low
52 resolution, internal interdecadal variability arises in rectangular flat bottomed single hemispheric

53 basin forced by prescribed surface heat fluxes (Greatbatch and Zhang 1995; Huck et al. 1999). This
54 internal variability is due to a large-scale baroclinic instability that gives rise to SST anomalies and
55 to geostrophically-induced Meridional Overturning Circulation variability (Colin de Verdière and
56 Huck 1999; Te Raa and Dijkstra 2002). This mode of variability and its mechanism were also
57 demonstrated to exist in global realistic configuration of an Ocean General Circulation Model
58 (OGCM) (Sévellec and Fedorov 2013), in idealized coupled models (Buckley et al. 2012; Jamet
59 et al. 2016), in climate models (Muir and Fedorov 2017) and in observations (Frankcombe et al.
60 2008). It is also shown to produce maximum SST variance in the region where the AMV signature
61 is observed (Arzel et al. 2018).

62 The mode can be damped in some models (such as in the study of Sévellec and Fedorov (2013))
63 and self-sustained in others (Huck et al. (2015) for instance). The damped or self-sustained na-
64 ture of the mode depends on different parameters such as the topography (Winton 1997), the
65 wind shape and strength, or the vertical and horizontal diffusion (Huck et al. 2001; Arzel et al.
66 2018). In the case of a damped mode, atmospheric stochastic forcing is needed to excite the
67 mode. Frankcombe et al. (2009) showed that the introduction of a North Atlantic Oscillation type
68 stochastic forcing leads to an amplitude of sea surface temperature variability comparable to obser-
69 vations. Arzel et al. (2018) studied the bifurcation structure of the mode in a realistic configuration
70 forced by prescribed surface fluxes and showed that the mode becomes damped for eddy induced
71 diffusivities larger than 600 m s^{-1} .

72 In addition to available potential energy fluxes associated with the large-scale instability mech-
73 anism, ocean mesoscales eddies have been shown to be at the origin of a spatio-temporal inverse
74 cascade of kinetic energy (Arbic et al. 2014; Martin et al. 2020). This latter mechanism has been
75 proposed to be central to the existence of interannual-to-decadal fluctuations of sea level anoma-
76 lies and surface kinetic energy in global-scale eddy simulations (Penduff et al. 2011; Arbic

77 et al. 2014; Sérazin et al. 2015, 2018; Martin et al. 2020) and to influence the AMOC variability
78 (Grégorio et al. 2015; Leroux et al. 2018; Jamet et al. 2019). However, realistic and global-
79 scale eddy resolving simulations of multidecadal variability are still beyond reach because of the
80 long-time integration required to bring the circulation in near equilibrium with the weak interior
81 diffusive vertical fluxes. Therefore most of the studies devoted to this problem are based on simple
82 box-model geometries (e.g., Spall 2008; Huck et al. 2015; Hochet et al. 2020). How the oceanic
83 mesoscale turbulence influences the multidecadal mode that spontaneously develops under pre-
84 scribed surface fluxes has been explored by Huck et al. (2015). These authors show that, in the
85 presence of mesoscale turbulence, the primary mechanism driving multidecadal-scale temperature
86 fluctuations remains the large-scale baroclinic instability mechanism. The presence of a surface
87 restoring boundary condition in Spall (2008) prevents the internal ocean mode from developing so
88 that the wind-driven gyre circulation and subsequent mesoscale instabilities play a major role. The
89 coexistence of the mode described above and mesoscale eddies was shown by Huck et al. (2015)
90 using idealized simulations at eddy-resolving resolution. In such eddying configurations and us-
91 ing a frequency-domain approach, Hochet et al. (2020) have highlighted a non-linear transfer of
92 temperature variance from low to high frequencies: mesoscale eddies are a sink of temperature
93 variance for the low frequency mode. Hence low-frequencies do not arise as the result of the
94 mesoscale eddy field, as in Spall (2008) for instance, but instead draw their energy source from
95 the large-scale stratification. Sévellec et al. (2020) have also shown a similar behavior using moor-
96 ing data in the Southern Ocean, but on shorter timescale. However the use of temperature variance
97 instead of an energetic framework, as in Arbic et al. (2014) or Sérazin et al. (2015, 2018) makes
98 the comparison with results from these studies difficult. Temperature variance is up to a factor
99 equal to the local definition of available potential energy (APE) ϵ_{APE} in a Quasi-Geostrophic (QG)

100 framework:

$$\varepsilon_{APE} \approx \frac{1}{2} \frac{g^2 \rho'^2}{\rho_0 N^2} \quad (1)$$

101 with N the Brunt-Väisälä frequency, g the acceleration of gravity parameter, ρ' the density
102 anomaly and ρ_0 the reference density. However, the internal mode described in the literature
103 cited above occurs in regions where isopycnals outcropping prevents the use of (1). Thus diagnos-
104 tics and budget of temperature variance do not permit identifying the sources and sinks of energy.
105 In this article we seek to obtain the full energy budget (i.e., including both kinetic energy (KE)
106 and available potential energy) for the low frequency mode described for instance in Hochet et al.
107 (2020) and to quantify the energy transfers associated with the time mean flow and mesoscale eddy
108 field. In particular we want to compare the intensity and direction of the conversion between Low
109 Frequency (LF) and High Frequency (HF) KE and between LF and HF APE. Because the only
110 source of KE in the buoyancy forced experiment of Hochet et al. (2020) is the APE/KE conversion
111 terms, we also investigate the effect of a wind stress forcing to add a direct source of KE.

112 The main difficulty in obtaining the energy budget in different frequency bands lies in the APE
113 decomposition. Indeed, contrary to the kinetic energy, the time decomposition of the full APE
114 formula is not straightforward. Scotti and White (2014) circumvented this problem by computing
115 the fluctuating APE as the difference between the APE for the total circulation (i.e., time mean and
116 fluctuations) and the APE for the time mean circulation. This idea has been applied by Zemskova
117 et al. (2015) to an eddy permitting ocean state estimate (from the “Estimating the Circulation and
118 Climate of the Ocean”, Phase II) to decompose the APE and KE budget into time mean and fluc-
119 tuating components. More recently, the same method has been used by Zemskova et al. (2021)
120 to study the influence of several wind intensities over the Southern Ocean on the time mean and
121 fluctuating components of the APE and KE budget. They found that the APE budget is not signif-
122 icantly affected by the surface wind stress and mainly controlled by the surface buoyancy forcing.

123 However we will show that applying this method to our configuration leads to a spurious imprint
124 of the time-independent surface heat flux forcing on the LF and HF APE reservoirs, whereas we
125 would instead expect the energy flux associated with the steady forcing to be entirely imparted
126 to the background stratification. We will thus develop an alternative method that will be used to
127 decompose the APE into mean, low frequency, and high frequency parts.

128 The article is organised as follows: in section 2, we decompose the mean, low- and high-
129 frequencies and derive the budget for APE and KE. In section 3, we give a description of the
130 model configuration used in this study. In section 4, we describe the variability in three numeri-
131 cal simulations with idealized North Atlantic configuration and under prescribed surface heat flux
132 forcing with different wind forcing intensities. In Section 5, we apply the energy budgets described
133 in section 2 on the simulation outputs. In section 6, we conclude and discuss the main findings.

134 2. Theory

135 In this section we derive the APE and KE budgets for the mean, LF and HF circulations. We use
136 a linear equation of state for the density ρ that is only a function of temperature: $\rho = \rho_0(1 - \alpha\theta)$
137 where θ is the temperature, $\alpha = 2 \times 10^{-4} \text{K}^{-1}$ the uniform thermal expansion coefficient and
138 $\rho_0 = 1027.5 \text{kg m}^{-3}$ the reference density (consistently with the ocean model used, see section 3
139 for the full model description). The equation for ρ is then:

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = D + F, \quad (2)$$

140 where $\mathbf{v} = (u, v, w)$ is the 3d velocity with u , v , w the zonal, meridional, and vertical velocities.
141 D and F represent the dissipation and surface forcing of density, respectively, the latter being
142 constant with time and zero below the surface (i.e. no penetrative radiation). The time independent
143 forcing is used here to keep the problem simple, we therefore do not account for slow variation

144 of the forcing linked for instance with climate change. Note that the generalization of the theory
 145 presented below to a time dependent F and to penetrative radiation is straightforward.

146 To obtain a separate budget for the low and high frequency parts of the APE and KE we decom-
 147 pose each field M into mean, low and high frequency parts:

$$M = \overline{M} + M^{LF} + M^{HF}, \quad (3)$$

148 where M^{LF} and M^{HF} are the low and high frequency parts of M , respectively, and \overline{M} the time
 149 mean. The time mean is computed using the following formula:

$$\overline{M} = \frac{1}{T} \int_T M dt \quad (4)$$

150 where T is the time length over which the integral is computed so that: M^{LF} and M^{HF} satisfy
 151 $\overline{M^{HF}} = \overline{M^{LF}} = \overline{M^{LF}M^{HF}} = 0$. To decompose into HF and LF we use a low pass Butterworth filter
 152 (cut-off frequency given in the following section). The low-pass filter is represented by $\widetilde{\cdot}$ so that
 153 M^{HF} satisfies $\widetilde{M^{HF}} = 0$ and $\widetilde{M} = \overline{M} + M^{LF}$.

154 Using this decomposition in frequency bands on Eq. (2) gives the following evolution equations
 155 for the mean, LF and HF of ρ :

$$\frac{\partial \overline{\rho}}{\partial t} = -\overline{\mathbf{v}} \cdot \nabla \overline{\rho} - \overline{\mathbf{v}^{LF} \cdot \nabla \rho^{LF}} - \overline{\mathbf{v}^{HF} \cdot \nabla \rho^{HF}} + \overline{D} + F \quad (5)$$

156

$$\frac{\partial \rho^{LF}}{\partial t} = -\overline{\mathbf{v}} \cdot \nabla \rho^{LF} - \mathbf{v}^{LF} \cdot \nabla (\overline{\rho} + \rho^{LF}) + \overline{\mathbf{v}^{HF} \cdot \nabla \rho^{HF}} - \mathbf{v}^{HF} \cdot \widetilde{\nabla \rho^{HF}} + \overline{\mathbf{v}^{LF} \cdot \nabla \rho^{LF}} + D^{LF} \quad (6)$$

157

$$\frac{\partial \rho^{HF}}{\partial t} = -\overline{\mathbf{v}} \cdot \nabla \rho^{HF} - \mathbf{v}^{LF} \cdot \nabla \rho^{HF} - \mathbf{v}^{HF} \cdot \nabla (\overline{\rho} + \rho^{LF} + \rho^{HF}) + \mathbf{v}^{HF} \cdot \widetilde{\nabla \rho^{HF}} + D^{HF} \quad (7)$$

158 APE is obtained as the difference between Potential Energy (PE) and Background Potential Energy
 159 (BPE). We derive expressions for PE, BPE, and APE in sections a, b and c below.

160 *a. Potential Energy*

161 Multiplying Eq. (5) by zg gives an equation for the mean PE :

$$\frac{\partial gz\bar{\rho}}{\partial t} = -\nabla \cdot (g\bar{\mathbf{v}}z\bar{\rho}) + g\bar{w}\bar{\rho} + \overline{g w^{HF} \rho^{HF}} + \overline{g w^{LF} \rho^{LF}} - \nabla \cdot \left(gz(\overline{\mathbf{v}^{HF} \rho^{HF}} + \overline{\mathbf{v}^{LF} \rho^{LF}}) \right) + gz\bar{D} + gz\bar{F}, \quad (8)$$

162 Integrating Eq. (8) on the volume V of the basin and time averaging results in the following
163 equation:

$$\frac{dPE^{MEAN}}{dt} = \underbrace{\int_V g\bar{w}\bar{\rho} dV}_{-C(PE^{MEAN}, KE^{MEAN})} + \underbrace{\int_V \overline{g w^{LF} \rho^{LF}} dV}_{-C(PE^{MEAN}, PE^{LF})} + \underbrace{\int_V \overline{g w^{HF} \rho^{HF}} dV}_{-C(PE^{MEAN}, PE^{HF})} + \underbrace{\int_V gz\bar{D} dV}_{D_{PE}}, \quad (9)$$

164 Note that the volume integral of the forcing term multiplied by z disappears because $z = 0$ at
165 the surface and \bar{F} is zero below the surface. We note $C(A, B)$ the conversion term from A to B
166 with $C(A, B) = -C(B, A)$, if $C(A, B) > 0$ then $C(A, B)$ acts to increase B i. $C(PE^{MEAN}, KE^{MEAN})$,
167 $C(PE^{MEAN}, PE^{LF})$ and $C(PE^{MEAN}, PE^{HF})$ are respectively the conversion of mean PE to mean
168 KE , of mean PE to LF PE and of mean PE to HF PE . D_{PE} is interpreted as the rate of con-
169 version of internal energy to potential energy (e.g. Hughes et al. 2009). There is no potential
170 energy in anomalies because $\overline{gz\rho^{LF}} = \overline{gz\rho^{HF}} = 0$, therefore $PE^{LF} = PE^{HF} = 0$. However, fol-
171 lowing Zemskova et al. (2015), we attribute the two terms $\int_V \overline{g w^{HF} \rho^{HF}} dV$ and $\int_V \overline{g w^{LF} \rho^{LF}} dV$ to
172 $C(PE^{MEAN}, PE^{HF})$ and $C(PE^{MEAN}, PE^{LF})$ so that the formal budgets for PE^{LF} and PE^{HF} are:

$$\frac{dPE^{LF}}{dt} = -C(PE^{LF}, KE^{LF}) + C(PE^{MEAN}, PE^{LF}) \quad (10)$$

173 and:

$$\frac{dPE^{HF}}{dt} = -C(PE^{HF}, KE^{HF}) + C(PE^{MEAN}, PE^{HF}) \quad (11)$$

174 From which we deduce that: $C(PE^{HF}, KE^{HF}) = C(PE^{MEAN}, PE^{HF})$ and $C(PE^{LF}, KE^{LF}) =$
175 $C(PE^{MEAN}, PE^{LF})$.

176 *b. Background Potential Energy*

177 To obtain an equation for the BPE, we first define the reference level $z_r(\rho, t)$ which is a function
 178 of time and density and corresponds to the depth that ρ would have in the Lorenz state of minimum
 179 potential energy (Lorenz 1955), a state where isopycnal surfaces would be horizontal. Following
 180 Saenz et al. (2015), a simple relationship between ρ and its reference level z_r can be derived using
 181 the result that an adiabatic rearrangement of the fluid parcels conserves the volume:

$$\int_{V(\rho, t)} dV = \int_{z_r}^0 A(z_r) dz_r, \quad (12)$$

182 where $V(\rho, t)$ is the volume of water parcels with density ρ' lower than ρ at time t , $A(z)$ is the
 183 area of the ocean at depth z . With a non-linear equation of state for density that depends on
 184 temperature, salinity and pressure, the procedure to obtain z_r is quite complex and described for
 185 instance in Saenz et al. (2015). However, in this work we use two assumptions that greatly simplify
 186 the calculation of z_r . The first is the assumption of a linear equation of state depending only
 187 on temperature. This implies that z_r is a function of density ρ (which is itself a function of
 188 temperature: $\rho = \rho_0(1 - \alpha\theta)$) and time t : $z_r = Z_r(\rho, t)$. The second assumption is the flat bottom
 189 basin with vertical boundaries so that the basin area is independant of depth, i.e. $A(z) = A$. Using
 190 Eq. (12), z_r is then simply:

$$z_r(\rho, t) = -\frac{V(\rho, t)}{A}, \quad (13)$$

191 Note that with a depth dependent ocean area, the reference depth can easily be obtained by solving
 192 Eq. (12). A schematic illustrating how the reference depth $z_r(\rho, t)$ is obtained using volume $V(\rho)$
 193 is shown in Figure 1. This reference depth can be used to rewrite the density ρ as a function of
 194 z_r such that: $\rho(X, t) = \rho_r(Z_r(X, t), t)$, with $Z_r(X, t) = z_r(\rho(X, t), t)$. On this schematic and in this
 195 article, we call “physical space” the usual space described by $X = (x, y, z)$ and t , and the “reference
 196 space” the space described by the reference depth z_r and t .

197 Figure 2 shows the reference depth z_r as a function of time for different values of temperature
 198 from a simulation that will be presented in a following section. The time variation of the function
 199 $z_r(\rho, t)$ cannot be neglected here because it varies by more than 500 m for the largest densities due
 200 to the presence of the large-scale, low-frequency, mode. In Zemskova et al. (2015) z_r variations
 201 with time are small because their study is not focused on the same timescale as ours: their time
 202 mean is computed over 20 years and their temporal variability is made of inter-annual, seasonal
 203 and shorter timescales while our focus is on decadal to multi-decadal timescales. Note that the
 204 larger variations of reference depth at larger densities can be attributed to the stronger (weaker)
 205 ρ_r gradient at shallower (deeper) reference depth. Time variation of the reference depths due to
 206 seasonal variation and to the presence of eddies was also reported in Zemskova et al. (2015).

207 The background potential energy is defined as follows:

$$BPE = \int_V gZ_r(X, t)\rho(X, t)dV = \int_{-H}^0 Agz'_r\rho_r(z'_r, t)dz'_r, \quad (14)$$

208 where H is the basin depth (so that $AH = V$) and where the last equality is obtained from equation
 209 (12) and describes the calculation of the BPE in the reference space. We now want to obtain
 210 separate budgets for the mean, LF, and HF BPE. In Zemskova et al. (2015) the BPE budget is
 211 computed by first calculating the BPE of the mean flow that they define as:

$$BPE_Z^{\text{MEAN}} = \int_V g\bar{\rho}z_r(\bar{\rho})dV, \quad (15)$$

212 where z_r is the reference depth associated with $\bar{\rho}$. The BPE of anomalies is then defined as the
 213 difference between the total BPE and the mean BPE:

$$BPE_Z^{\text{anomalies}} = \int_V \left(\overline{g\rho z_r(\rho, t)} - g\bar{\rho}z_r(\bar{\rho}) \right) dV, \quad (16)$$

214 Using this method in our simulation leads to part of the time independent heat flux forcing being
 215 attributed to $BPE_Z^{anomalies}$. Indeed, in this framework, the BPE forcing by heat fluxes is:

$$\int_V g \bar{F} \left(\overline{z_r(\rho, t)} - z_r(\bar{\rho}) \right) dV, \quad (17)$$

216 where \bar{F} is the time-independent net heat flux at the surface. Because of the non-linearity of the z_r
 217 function (and its time dependence) we have that:

$$\frac{z_r(\rho_1) + z_r(\rho_2)}{2} \neq z_r\left(\frac{\rho_1 + \rho_2}{2}\right), \quad (18)$$

218 for two different densities $\rho_1 \neq \rho_2$. Thus the term in Eq. (17) is non-zero and the time independent
 219 forcing acts on the anomalies. However an analysis in terms of density variance shows that the
 220 time independent heat flux only acts on time mean density (Hochet et al. 2020). Density anomalies
 221 are found to be forced only by the term $\overline{\rho' \mathbf{u}'} \cdot \nabla \bar{\rho}$ which is usually interpreted as the signature of a
 222 large-scale baroclinic instability (see Colin de Verdière and Huck 1999). The fact that the term in
 223 Eq. (17) is non-zero is thus at odds with this interpretation.

224 To circumvent this problem we develop below an alternative method to separate the BPE in
 225 frequency bands. The readers not interested in the details of the BPE decomposition may skip this
 226 section and refer to figure 3 which gives an intuitive view of the transfer between the different
 227 reservoirs.

228 As in the previous section, ρ is first decomposed into three frequency bands: mean, low-
 229 frequency, and high frequency so that:

$$\rho = \bar{\rho} + \rho^{LF} + \rho^{HF}, \quad (19)$$

230 $\rho(X, t) = \rho_r(z_r, t)$ can trivially be written as:

$$\rho(z_r, t) = -\frac{1}{A} \frac{\partial}{\partial z_r} \int_{V(z_r)} \rho(X, t) dV, \quad (20)$$

231 where $V(z_r)$ is the volume of water with $z'_r > z_r$. This formula simply states that the average of ρ
 232 on z_r surfaces is ρ by definition of $z_r(\rho, t)$. Using Eq. (19) in Eq. (20) gives:

$$\rho = - \underbrace{\frac{1}{A} \frac{\partial}{\partial z_r} \int_{V(z_r)} \bar{\rho} dV}_{\rho_r^{MEAN}(z_r, t)} - \underbrace{\frac{1}{A} \frac{\partial}{\partial z_r} \int_{V(z_r)} \rho^{LF} dV}_{\rho_r^{LF}(z_r, t)} - \underbrace{\frac{1}{A} \frac{\partial}{\partial z_r} \int_{V(z_r)} \rho^{HF} dV}_{\rho_r^{HF}(z_r, t)} \quad (21)$$

233 where ρ_r^{MEAN} , ρ_r^{LF} , and ρ_r^{HF} are the average of ρ^{MEAN} , ρ^{LF} and ρ^{HF} , respectively, on z_r surfaces.
 234 This ensures that ρ_r^{MEAN} , ρ_r^{LF} and ρ_r^{HF} are functions of z_r (and time) and this property will be
 235 useful to obtain an evolution equation for the BPE as will become clear below. The BPE is then
 236 decomposed as follows:

$$BPE = \int_V \overline{\rho g z_r(\rho, t)} dV = \underbrace{\int_V \overline{\rho_r^{MEAN}(z_r, t) g z_r(\rho, t)} dV}_{BPE^{MEAN}} + \underbrace{\int_V \overline{\rho_r^{LF}(z_r, t) g z_r(\rho, t)} dV}_{BPE^{LF}} + \underbrace{\int_V \overline{\rho_r^{HF}(z_r, t) g z_r(\rho, t)} dV}_{BPE^{HF}}, \quad (22)$$

237 where BPE^{MEAN} , BPE^{LF} , and BPE^{HF} are respectively the BPE associated with the mean, LF and
 238 HF densities. The time evolution of the BPE is then the sum of the time evolution of the MEAN,
 239 LF, and HF BPE:

$$\frac{dBPE}{dt} = \frac{dBPE^{MEAN}}{dt} + \frac{dBPE^{LF}}{dt} + \frac{dBPE^{HF}}{dt}, \quad (23)$$

240 The evolution equation for the mean, LF or HF, BPE is:

$$\frac{dBPE^*}{dt} = \int_V g \frac{\partial \rho_r^*}{\partial t} z_r dV + \underbrace{\int_V g \rho_r^* \frac{\partial z_r}{\partial t} dV}_{=0}, \quad (24)$$

241 where * represents either MEAN, LF, or HF. The second term of the r.h.s. is zero as shown
 242 in Winters et al. (1995) and later in Tailleux (2009) because ρ_r^* is constant on z_r surfaces by
 243 construction (see Eq. (21)).

244 The three evolution equations for the mean, LF, and HF density (i.e., Eqs. (5), (6) and (7)) are
 245 averaged on z_r surfaces using the following formula:

$$\frac{\partial \rho_r^*}{\partial t}(z_r, t) = \frac{1}{A} \frac{\partial}{\partial z_r} \left(\int_{V(z_r)} \frac{\partial \rho^*}{\partial t} dV \right), \quad (25)$$

246 where * represents the mean, LF, or HF density. Using Eq (25) with Eqs (5), (6), and (7), and
 247 inserting in formula (24) leads to the following relations for the three BPE budgets:

$$\frac{dBPE^{MEAN}}{dt} = -C(BPE^{MEAN}, BPE^{LF}) - C(BPE^{MEAN}, BPE^{HF}) + F_{BPE^{MEAN}} + D_{BPE^{MEAN}}, \quad (26)$$

$$\frac{dBPE^{LF}}{dt} = -C(BPE^{LF}, BPE^{HF}) + C(BPE^{MEAN}, BPE^{LF}) + D_{BPE^{LF}}, \quad (27)$$

$$\frac{dBPE^{HF}}{dt} = C(BPE^{LF}, BPE^{HF}) + C(BPE^{MEAN}, BPE^{HF}) + D_{BPE^{HF}}, \quad (28)$$

250 where the conversion terms are:

$$C(BPE^{LF}, BPE^{HF}) = g \int_V \overline{z_r(\rho, t) \left(\widetilde{\mathbf{v}^{HF} \cdot \nabla \rho^{HF}} - \overline{\mathbf{v}^{HF} \cdot \nabla \rho^{HF}} \right) - z_r(\rho, t) \overline{\mathbf{v}^{LF} \cdot \nabla \rho^{HF}}} dV, \quad (29)$$

$$C(BPE^{MEAN}, BPE^{LF}) = g \int_V \overline{z_r(\rho, t) \overline{\mathbf{v}^{LF} \cdot \nabla \rho^{LF}} - z_r(\rho, t) \overline{\mathbf{v} \cdot \nabla \rho^{LF}}} dV, \quad (30)$$

$$C(BPE^{MEAN}, BPE^{HF}) = g \int_V \overline{z_r(\rho, t) \overline{\mathbf{v}^{HF} \cdot \nabla \rho^{HF}} - z_r(\rho, t) \overline{\mathbf{v} \cdot \nabla \rho^{HF}}} dV, \quad (31)$$

253 Note that we have used the following relation to obtain the conversion terms formulas:

$$\begin{aligned} \int_V \overline{z_r(\rho, t) \overline{\mathbf{v} \cdot \nabla (\overline{\rho} + \rho^{LF} + \rho^{HF})}} dV = \\ \int_V \overline{z_r(\rho, t) \overline{\mathbf{v}^{HF} \cdot \nabla (\overline{\rho} + \rho^{LF} + \rho^{HF})}} dV = \int_V \overline{z_r(\rho, t) \overline{\mathbf{v}^{LF} \cdot \nabla (\overline{\rho} + \rho^{LF} + \rho^{HF})}} dV = 0. \end{aligned} \quad (32)$$

254 The dissipation of BPE (Hughes et al. 2009; Zemskova et al. 2015) for each frequency band * is:

$$D_{BPE^*} = \int_V \overline{g z_r(\rho, t) D^*} dV, \quad (33)$$

255 and the forcing of the mean BPE is:

$$F_{BPE^{MEAN}} = \int_V \overline{g z_r(\rho, t) \overline{F}} dV \quad (34)$$

256 The advantage of this approach compared to that of Zemskova et al. (2015) is that the time inde-
 257 pendent heat flux forcing is entirely contained in BPE^{MEAN} and that we have explicit equations
 258 for the densities associated with BPE^{LF} and BPE^{HF} .

259 In the three MITgcm configurations described in the following section 3, the diffusive pro-
 260 cesses increase the mean BPE ($D_{BPE^*} > 0$) and the surface heat flux forcing acts to decrease it
 261 ($F_{BPE^{MEAN}} < 0$). Because the APE varies in opposition to the BPE the above mentioned forcing
 262 and dissipation have respectively an increasing and decreasing impact on the APE. The sign of the
 263 forcing and dissipation of BPE is consistent with results from previous studies using BPE (Hughes
 264 et al. 2009; Zemskova et al. 2015).

265 *c. Available potential energy*

266 The mean APE budget is obtained as the difference between the mean PE (Eq. (9)) and the mean
 267 BPE (Eq. (26)) budgets:

$$\begin{aligned}
 \frac{dAPE^{MEAN}}{dt} &= \frac{dPE^{MEAN}}{dt} - \frac{dBPE^{MEAN}}{dt} = \\
 &= \underbrace{-C(APE^{MEAN}, APE^{LF})}_{=-C(PE^{MEAN}, PE^{LF}) + C(BPE^{MEAN}, BPE^{LF})} - \underbrace{-C(APE^{MEAN}, APE^{HF})}_{=-C(PE^{MEAN}, PE^{HF}) + C(BPE^{MEAN}, BPE^{HF})} \\
 &= \underbrace{-C(APE^{MEAN}, KE^{MEAN})}_{=-C(PE^{MEAN}, KE^{MEAN})} + \underbrace{D_{APE^{MEAN}}}_{=D_{PE^{MEAN}} - D_{BPE^{MEAN}}} + \underbrace{F_{APE^{MEAN}}}_{=-F_{BPE^{MEAN}}} \quad (35)
 \end{aligned}$$

268 The conversion, dissipation and forcing terms of PE and BPE are derived in the two previous
 269 sections. Because $APE^{MEAN} = PE^{MEAN} - BPE^{MEAN}$, the evolution terms of BPE^{MEAN} appear
 270 in the APE^{MEAN} budget with a minus sign. $D_{APE^{MEAN}}$ can then either be seen as the dissipation
 271 of mean APE or as the conversion between mean APE and mean BPE due to the time mean
 272 diffusive flux. Note that $D_{PE^{MEAN}}$ does not explicitly appear in the BPE budget but it can be argued
 273 (see Hughes et al. 2009) that it contributes to the BPE budget and is thus added here as part of

274 $D_{APE^{MEAN}}$. Similarly, $F_{APE^{MEAN}}$ can be seen as the forcing of mean APE or as the conversion of
 275 (mean) BPE to APE due to surface heat flux.

276 Because there is no potential energy in anomalies, APE in anomalies is only made of BPE. The
 277 LF APE budget is then:

$$\begin{aligned} \frac{dAPE^{LF}}{dt} &= \frac{dPE^{LF}}{dt} - \frac{dBPE^{LF}}{dt} = \\ & \underbrace{-C(APE^{LF}, KE^{LF})}_{=-C(PE^{LF}, KE^{LF})} + \underbrace{C(APE^{MEAN}, APE^{LF})}_{=C(PE^{MEAN}, PE^{LF}) - C(BPE^{MEAN}, BPE^{LF})} - \underbrace{C(APE^{LF}, APE^{HF})}_{=C(BPE^{LF}, BPE^{HF})} + \underbrace{D_{APE^{LF}}}_{=-D_{BPE^{LF}}}. \end{aligned} \quad (36)$$

278 Similarly, the HF APE budget is:

$$\begin{aligned} \frac{dAPE^{HF}}{dt} &= \frac{dPE^{HF}}{dt} - \frac{dBPE^{HF}}{dt} = \\ & \underbrace{-C(APE^{HF}, KE^{HF})}_{=-C(PE^{HF}, KE^{HF})} + \underbrace{C(APE^{MEAN}, APE^{HF})}_{=C(PE^{MEAN}, PE^{HF}) - C(BPE^{MEAN}, BPE^{HF})} + \underbrace{C(APE^{LF}, APE^{HF})}_{=-C(BPE^{LF}, BPE^{HF})} + \underbrace{D_{APE^{HF}}}_{=-D_{BPE^{HF}}}. \end{aligned} \quad (37)$$

279 *d. Kinetic Energy*

280 In this subsection, the budgets for the total and low frequency kinetic energy are derived. The
 281 horizontal momentum equations are:

$$\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} + V_u + F_u \quad (38)$$

$$\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} + V_v \quad (39)$$

282 where p is the pressure, V_u and V_v are the viscous term in respectively the zonal and meridional
 283 direction, F_u is the zonal, time-independent forcing (we assume no meridional forcing) and f is the
 284 Coriolis parameter. Time averaging Eqs. (38) and (39), multiplying by $\rho_0 \bar{u}$ and $\rho_0 \bar{v}$ and summing

285 give an equation for the local kinetic energy of the time mean flow:

$$\begin{aligned} \frac{\rho_0}{2} \bar{\mathbf{v}} \cdot \nabla (\bar{u}^2 + \bar{v}^2) = & -\rho_0 \bar{u} \nabla \cdot (\overline{\mathbf{v}^{\text{HF}} u^{\text{HF}} + \mathbf{v}^{\text{LF}} u^{\text{LF}}}) - \rho_0 \bar{v} \nabla \cdot (\overline{\mathbf{v}^{\text{HF}} v^{\text{HF}} + \mathbf{v}^{\text{LF}} v^{\text{LF}}}) \\ & - \nabla \bar{p} - g \bar{\rho} \bar{w} + \rho_0 \bar{u} \bar{V}_u + \rho_0 \bar{v} \bar{V}_v + \rho_0 \bar{u} \bar{F}_u \end{aligned} \quad (40)$$

286 The Coriolis term does not play any part in the kinetic energy budget because the Coriolis accel-
 287 eration is normal to the velocity and there is no contribution from the vertical velocity because
 288 the hydrostatic approximation is used and implies no vertical acceleration (Gregory and Tailleux
 289 2011). Integrating (40) over the entire volume gives the budget for KE^{MEAN} :

$$\begin{aligned} \frac{dKE^{\text{MEAN}}}{dt} = & C(\text{APE}^{\text{MEAN}}, KE^{\text{MEAN}}) \\ & - C(KE^{\text{MEAN}}, KE^{\text{LF}}) - C(KE^{\text{MEAN}}, KE^{\text{HF}}) + D_{KE^{\text{MEAN}}} + F_{KE^{\text{MEAN}}} \end{aligned} \quad (41)$$

290 where:

$$C(\text{APE}^{\text{MEAN}}, KE^{\text{MEAN}}) = - \int_V g \bar{\rho} \bar{w} dV \quad (42)$$

$$C(KE^{\text{MEAN}}, KE^{\text{LF}}) = \int_V \overline{\rho_0 \bar{u} \nabla (\mathbf{v}^{\text{LF}} u^{\text{LF}}) + \rho_0 \bar{v} \nabla (\mathbf{v}^{\text{LF}} v^{\text{LF}})} dV \quad (43)$$

$$C(KE^{\text{MEAN}}, KE^{\text{HF}}) = \int_V \overline{\rho_0 \bar{u} \nabla (\mathbf{v}^{\text{HF}} u^{\text{HF}}) + \rho_0 \bar{v} \nabla (\mathbf{v}^{\text{HF}} v^{\text{HF}})} dV \quad (44)$$

$$D_{KE^{\text{MEAN}}} = \int_V \rho_0 \bar{u} \bar{V}_u + \rho_0 \bar{v} \bar{V}_v dV \quad (45)$$

$$F_{KE^{\text{MEAN}}} = \int_V \rho_0 \bar{u} \bar{F}_u dV \quad (46)$$

295 Proceeding similarly for the LF and HF, we obtain the following budgets for KE^{LF} and KE^{HF} :

$$\frac{dKE^{\text{LF}}}{dt} = C(\text{APE}^{\text{LF}}, KE^{\text{LF}}) + C(KE^{\text{MEAN}}, KE^{\text{LF}}) - C(KE^{\text{LF}}, KE^{\text{HF}}) + D_{KE^{\text{LF}}} \quad (47)$$

$$\frac{dKE^{\text{HF}}}{dt} = C(\text{APE}^{\text{HF}}, KE^{\text{HF}}) + C(KE^{\text{MEAN}}, KE^{\text{HF}}) + C(KE^{\text{LF}}, KE^{\text{HF}}) + D_{KE^{\text{HF}}} \quad (48)$$

$$C(\text{APE}^{\text{LF}}, KE^{\text{LF}}) = - \int_V \overline{g \rho^{\text{LF}} w^{\text{LF}}} dV \quad (49)$$

297

$$C(APE^{HF}, KE^{HF}) = - \int_V g \overline{\rho^{HF} w^{HF}} dV \quad (50)$$

298

$$C(KE^{LF}, KE^{HF}) = - \int_V \overline{\rho_0 u^{HF} \nabla(\mathbf{u}^{HF} u^{LF}) + \rho_0 v^{HF} \nabla(\mathbf{u}^{HF} u^{LF})} dV \quad (51)$$

299

$$D_{KE^{LF}} = \int_V \overline{\rho_0 u^{LF} V_u^{LF} + \rho_0 v^{LF} V_v^{LF}} dV \quad (52)$$

$$D_{KE^{HF}} = \int_V \overline{\rho_0 u^{HF} V_u^{HF} + \rho_0 v^{HF} V_v^{HF}} dV \quad (53)$$

300 *e. Practical calculation of the APE/KE budget using model outputs*

301 In Table 1 we describe how each term of the KE and APE budget mapped in figure 3 is computed
 302 using results from the previous subsections. Due to the very long timescales of the LF variability
 303 (~ 50 years) it would require too much storage to resolve the HF terms. However we show
 304 below that the HF budgets can be obtained as the residual of the well-resolved LF budgets. As
 305 explained above, APE forcing and dissipation can also be seen as conversion between BPE and
 306 APE reservoirs. This is shown in figure 3 by the addition of a BPE reservoir that exchanges energy
 307 with the three APE reservoirs.

308 The total conversion from APE to KE can be obtained from the time mean advection of the
 309 temperature which is an output of the model:

$$C(APE, KE) = \int_V g z \nabla \bar{\rho} dV = \underbrace{\int_V g \nabla(z \bar{\rho}) dV}_{=0} - \int_V g \bar{w} \bar{\rho} dV \quad (54)$$

310 $C(APE^{HF}, KE^{HF})$ can then be deduced from the knowledge of $C(APE^{LF}, KE^{HF})$ and
 311 $C(APE^{MEAN}, KE^{MEAN})$:

$$C(APE^{HF}, KE^{HF}) = C(APE, KE) - C(APE^{LF}, KE^{HF}) - C(APE^{MEAN}, KE^{MEAN}). \quad (55)$$

312 $D_{KE^{HF}}$ is obtained from the total KE budget:

$$D_{KE^{HF}} = -C(APE, KE) - F_{KE^{MEAN}} - D_{KE^{MEAN}} - D_{KE^{LF}}. \quad (56)$$

313 $D_{APE^{HF}}$ is obtained using the total APE budget:

$$D_{APE^{HF}} = C(APE, KE) - F_{APE^{MEAN}} - D_{APE^{MEAN}} - D_{APE^{LF}}. \quad (57)$$

314 Similarly, $C(KE^{LF}, KE^{HF})$ is obtained from the KE^{LF} budget, $C(KE^{MEAN}, KE^{HF})$ from the
 315 KE^{MEAN} budget, $C(APE^{LF}, APE^{HF})$ from the APE^{LF} budget, and $C(APE^{MEAN}, APE^{HF})$ from
 316 the APE^{MEAN} budget.

317 **3. Model and configuration**

318 We use the MITgcm (Marshall et al. 1997) in a rectangular flat-bottom basin with a Carte-
 319 sian geometry on a β -plane centered at 40°N . The zonal and meridional extents are respectively
 320 $L_x = 5000\text{km}$ and $L_y = 4500\text{km}$, and the Southern boundary is located 2000km north of the equa-
 321 tor. An eddy-permitting horizontal resolution of 20km is used in both directions. This resolution
 322 is sufficient in Huck et al. (2015) to capture the main characteristics of the effect of eddy turbu-
 323 lence on low-frequency variability. The depth is $H = 4500\text{m}$, there are 40 levels on the vertical
 324 with grid spacing increasing from 10m at the surface to 400m at the bottom.

325 The ocean is forced by constant heat flux at the surface, decreasing linearly with latitude from
 326 50W m^{-2} at $y = 0\text{km}$ to -50W m^{-2} at $y = 4500\text{km}$, similar to Huck et al. (2015). Static instabil-
 327 ity is removed by strong vertical mixing of the water column. We use biharmonic horizontal eddy
 328 diffusivity with a uniform value of $10^{11}\text{m}^4\text{s}^{-1}$ and Leith implicit viscosity. The vertical viscosity
 329 is $\nu_v = 10^{-3}\text{m}^2\text{s}^{-1}$. In this single hemisphere configuration, the strength of the Meridional Over-
 330 turning Circulation (MOC) is a strong function of the vertical diffusivity K_v , in agreement with the
 331 $K_v^{1/2}$ geostrophic scaling (Huang and Chou 1994). Here, we choose to use $K_v = 2 \times 10^{-4}\text{m}^2\text{s}^{-1}$
 332 corresponding to a MOC strength close to 10 Sv . Because the primary objective of this study is
 333 to establish and understand the full energy budget of the low-frequency mode in the configuration

334 used in previous published articles (Huck et al. 1999, 2001, 2015; Hochet et al. 2020) our main ex-
 335 periment does not have wind forcing. However we also perform two additional experiments with
 336 increasing wind forcing intensity to study the effect of a direct KE source on the energy budget.
 337 The zonal wind stress used in the two wind forcing experiments varies with latitude according to
 338 the following formula:

$$\tau_x(y) = \tau_0 \left(\frac{1}{4} \cos \left(\frac{y\pi}{L_y} \right) - \cos \left(\frac{2\pi y}{L_y} \right) \right), \quad (58)$$

339 where τ_0 is the wind stress amplitude. The meridional wind stress is zero. We chose to use a non-
 340 symmetric zonal wind stress as it seems important to achieve a generic dynamical behavior of the
 341 double-gyre circulation (Berloff and McWilliams 1999). The three experiments use $\tau_0 = 0 \text{ N m}^{-2}$
 342 (no wind forcing), $\tau_0 = 0.05 \text{ N m}^{-2}$ (intermediate wind) and $\tau_0 = 0.1 \text{ N m}^{-2}$ (climatological wind)
 343 (Fig. 4). All three experiments are initialized with a state of rest, the spin-up time is then 500
 344 years and the model is run for another 400 years to produce outputs to compute the diagnostics
 345 presented below.

346 4. Time mean circulation and variability

347 In the following section we describe the time mean circulation as well as the low and high
 348 frequency variability obtained for the range of surface wind-stress forcing amplitudes mentioned
 349 above. Hochet et al. (2020) used exactly the same model parameters and configuration as the
 350 present study with zero wind-stress forcing. The turbulent transfer of temperature variance in their
 351 study was shown to act as a source of temperature variance for frequencies higher than 1/3.5 years
 352 and a sink for smaller frequencies. We thus define the limit between low and high frequencies as
 353 being 3.5 yr. Although it is possible that this limit is altered by the surface wind stress that we
 354 use in the two other experiments, we keep the same definition of 1/3.5 years in all experiments
 355 to be able to compare the three configurations. We thus associate LF with multi-decadal, decadal

356 and part of the inter-annual variability and HF with part of the inter-annual and eddy turbulence
357 induced variability.

358 *a. No wind forcing* $\tau_0 = 0 \text{ N m}^{-2}$

359 Note that the simulation used in this subsection (i.e. without wind forcing) is the same as that
360 described in Hochet et al. (2020), and the description of the LF variability is reproduced below. In
361 the absence of wind-stress forcing, LF variability spontaneously develops with a significant and
362 narrow peak frequency of $1/53 \text{ yr}^{-1}$ (Fig. 5). A detailed description of the variability developing
363 in very similar geometries can be found for instance in Huck et al. (1999) and Huck et al. (2015).
364 Here we will only give a short description of its main characteristics. Following Hochet et al.
365 (2020), we use Complex Empirical Orthogonal Function (CEOF) to describe the LF variability of
366 the three dimensional temperature field. The CEOF are calculated using 50-day average outputs
367 on a 400-year long simulation. The 400 year is chosen to obtain a statistical equilibrium of the
368 solution. Similar to the widely used empirical orthogonal function, CEOF are the eigenvectors
369 of the complex covariance matrix of a complex temperature anomaly which is calculated using
370 the Hilbert transform of the detrended temperature anomaly (Von Storch and Zwiers 2001). The
371 leading CEOF contains 60% of the temperature variance (Fig. 6). The temperature anomaly
372 associated to a CEOF can then be reconstructed using the following formula:

$$\theta_{\text{CEOF}}(x, y, z, t) = \text{PC}_{re}(t)\text{CEOF}_{re}(x, y, z) + \text{PC}_{im}(t)\text{CEOF}_{im}(x, y, z) \quad (59)$$

373 where *re* and *im* stand for the real and imaginary parts respectively and PC is the principal com-
374 ponent of the corresponding CEOF. The APE is shown along with the real and imaginary part of
375 the PC (Fig. 6). The phase of the leading CEOF is chosen to match that of the APE time variation.
376 The APE is very well correlated with the real part of the PC and shows that APE multidecadal vari-

377 ations are linked with the SST pattern shown on the upper panel of figure 6. The low-frequency
378 variability takes the form of a large-scale temperature anomaly, located mainly in the northwestern
379 half and in the upper 500 m of the basin with SST anomaly larger than 3 K at some locations (Fig.
380 6). The successive positions of the positive and negative temperature anomalies as shown on figure
381 6, i.e. a negative center located at latitudes around 2500-3000km and longitude 2000km (opposite
382 of the imaginary part), followed by a negative center for latitudes between 3000 and 3500 km,
383 longitude around 2000 km (real part), then a negative center around latitude 4000 km, longitude
384 1500km (imaginary part), then a negative center in the north western corner (opposite of the real
385 part), indicate north-westward propagation of the temperature anomalies.

386 The sea surface height (SSH) varies together with the temperature anomalies of the leading
387 CEOF (left column of Fig. 7). The amplitude of SSH anomalies (15-20 cm) is maximum along
388 the western boundary current and in its eastward extension in the northern-half of the basin. These
389 values compare well with altimetric observations (Stammer 1997). The time mean of the vertical
390 integral of the LF APE and of the LF density variance are shown in figure 7. The largest values for
391 the LF APE are located along the northern boundary. Equation (22) shows that large values of LF
392 APE (=LF BPE) are associated with deep reference depth and thus outcropping of dense waters.
393 The location of LF APE contrasts with the location of the largest values of LF density variance
394 which are located in the northern part of the basin interior (Fig. 7). The differences between these
395 two quantities further demonstrate that the APE cannot be approximated by the density variance
396 in these configurations in contrast with QG theory.

397 *b. Intermediate wind forcing $\tau_0 = 0.05 \text{ N m}^{-2}$*

398 When wind-stress forcing is present the temperature variability is significantly reduced com-
399 pared to the previous case and consists of a broad band of low-frequency signals with a peak

400 frequency of about $1/22 \text{ yr}^{-1}$ (Fig. 5). Huck et al. (2001) explained this effect of the wind forcing
 401 on the low frequency variability by the damping effect of the Ekman pumping on the large-scale
 402 anomalies. To explain this effect, Huck et al. (2001) assumed that the following formula describes
 403 the effect of wind stress on the temperature anomalies (their equation (14)):

$$\frac{\partial \theta'}{\partial t} = -W_E \frac{\partial \theta'}{\partial z} \quad (60)$$

404 where θ' is the temperature anomaly, and W_E the Ekman pumping. Then if θ' is further assumed
 405 to have an exponential profile with depth, $\theta' \propto \exp(-W_E k t)$ with k of the order of 500 m^{-1} . The
 406 temperature anomaly decreases where W_E is positive which is in the Northern half of the basin in
 407 our configuration.

408 The surface signature of the leading CEOF of temperature variability, explaining 28% of the
 409 spatially integrated variance, shows that the variability now occurs predominantly along the eastern
 410 boundary and along a narrow latitudinal band extending across the width of the basin just south
 411 of the intergyre boundary (Fig. 8). The large-scale anomaly emanates from the eastern boundary
 412 and propagates to the west along the mean temperature contours. SSH variability is no longer
 413 collocated with SST variations, as was the case with zero wind-stress forcing, but instead mostly
 414 occurs along a region centered about the intergyre (at $y = 2000 \text{ km}$) along the western boundary
 415 (Fig. 7, second row). The LF density variance is now almost entirely located on the eastern
 416 boundary as also shown by the CEOF (Fig. 8). The LF APE is in the northeastern corner of the
 417 basin in the region where dense waters outcrop.

418 *c. Climatological wind forcing $\tau_0 = 0.1 \text{ N m}^{-2}$*

419 Increasing the amplitude of the zonal surface wind-stress forcing up to realistic values has
 420 the effect of further decreasing (increasing) the temperature variance on interdecadal (monthly

421 to interannual) timescales compared to the case with $\tau_0 = 0.05 \text{ N m}^{-2}$. Indeed the volume av-
422 eraged LF (HF) temperature spectrum (defined as frequencies lower than $\frac{2\pi}{3.5\text{years}}$) is weaker for
423 $\tau_0 = 0.05 \text{ N m}^{-2}$ than for $\tau_0 = 0.1 \text{ N m}^{-2}$ (Fig. 5).

424 The leading CEOF of temperature variability now represents only 7% of the spatially-integrated
425 variance and is mostly apparent south of the intergyre boundary (Fig. 9). This pattern of variability
426 differs from the previous case with $\tau_0 = 0.05 \text{ N m}^{-2}$ (Fig. 8) for which SST variability was also
427 present along the eastern boundary. SSH variability however shares the same pattern and amplitude
428 as that obtained for $\tau_0 = 0.05 \text{ N m}^{-2}$ with enhanced variability along the western boundary current.
429 The LF APE is now located in the northwestern part of the basin with a much smaller amplitude
430 than in the two previous cases.

431 5. Energy budget

432 We now describe the mean, LF, and HF KE, APE and BPE budgets for the three experiments
433 described above, summarized in figures 10,12, and 13 following the schematic given in figure 3.
434 Table 2 gathers the transfer values obtained for the three experiments.

435 a. *No-wind forcing* $\tau_0 = 0 \text{ N m}^{-2}$

436 In the absence of surface wind-stress there is no external source for the KE reservoir and the main
437 energy pathways are located within the APE part of the budget (Fig. 10). Among the 119 GW of
438 conversion between BPE and APE due to surface heat flux 32 GW is converted into KE (mainly
439 at HF) where it is dissipated by viscous forces. The remaining 87 GW are mainly transferred from
440 HF APE to BPE because of dissipation (53 GW from APE HF to BPE, 16GW from LF APE to
441 BPE and 18 GW from mean APE to BPE).

442 Despite the differences between BPE and density variance shown in figure 7, Hochet et al. (2020)
 443 found a similar pathway for the temperature variance. The surface heat flux is the only source of
 444 mean temperature variance and LF temperature variance is forced through a transfer of temperature
 445 variance from the mean flow, this transfer is interpreted as the result of baroclinic instability of the
 446 mean flow. The conversion from mean APE to LF APE is 49GW which is the largest conversion
 447 term in this experiment. The main sink of LF APE is the conversion to HF APE (29 GW), whereas
 448 the dissipation removes 16GW. Because the direction of the LF/HF APE transfer is from LF to HF
 449 and because LF are associated with large scales and HF with mesoscale eddies, we deduce that
 450 mesoscale eddy turbulence is a sink of energy for the low-frequency variability which is one of the
 451 main result of this study. It confirms previous findings of Arbic et al. (2014) who demonstrated the
 452 existence of a direct temporal APE cascade along with the inverse temporal KE cascade under QG
 453 approximation. Conversion between kinetic energy reservoirs is small compared to conversion
 454 between APE reservoirs. There is however a substantial energy transfer (20 GW) between HF
 455 APE and HF KE. This input of HF KE is balanced by the sink linked with viscous terms. The
 456 only source of mean KE is the conversion of mean APE to mean KE (8GW) and the conversion
 457 between LF and HF KE is negligible. The ratio of LF KE to LF APE is of 0.3% showing that the
 458 low frequency variability is predominantly found in APE in this simulation.

459 The two left columns of Fig. 11 show the spatial pattern of $C(APE^{MEAN}, APE^{HF})$,
 460 $C(APE^{MEAN}, APE^{LF})$ integrated vertically over the water column. Strong positive values of
 461 these two terms are generally located close to the northern boundary, where the convection is the
 462 strongest. Positive values of $C(APE^{MEAN}, APE^{HF})$ are located in the eastern part of the northern
 463 boundary and follows closely the values of mean APE forcing (last column of Fig. 11). Negative
 464 values are located in the southern part of the basin interior. The mean APE forcing term is large
 465 in regions where the reference level is the deepest i.e. where dense waters outcrop at the surface.

466 $C(APE^{MEAN}, APE^{LF})$ (Fig. 11, middle-left column) has its largest values close to the northern and
 467 eastern boundary. The vertical integral of the term $-\overline{\mathbf{u}^{LF} \rho^{LF}} \cdot \nabla \rho$ which represents the transfer
 468 from the mean to the LF density variance (see Colin de Verdière and Huck 1999, for instance)
 469 is shown on the third column of figure 11 and is very different from $C(APE^{MEAN}, APE^{LF})$. This
 470 could be expected from the difference between the variance of the LF density and the LF APE
 471 already shown in figure 7: the transfer from mean APE to LF APE occurs in the convection region
 472 where dense waters outcrop whereas the LF density variance transfer occurs in the basin interior.
 473 $C(APE^{LF}, APE^{HF})$ is not shown but follows closely the variation of $C(APE^{MEAN}, APE^{LF})$.

474 *b. Intermediate wind forcing $\tau_0 = 0.05 \text{ N m}^{-2}$*

475 With an intermediate zonal wind stress at the surface, the LF variability becomes weaker and
 476 shifts to mid-latitudes as explained in section 4 b. The energy budget (Fig. 12) shows a decrease
 477 of the energy fluxes from mean APE and LF APE. The transfer of BPE to mean APE is the same
 478 as that obtained for the no-wind experiment (119GW). Part of this mean APE energy input is
 479 transferred back to BPE by parametrized diffusive flux (30GW) whereas most of it is converted
 480 into HF APE (61GW). The conversion of LF APE to HF APE is approximately half the value
 481 obtained for the no-wind case (14GW against 29GW). The total transfer (i.e. KE+APE) is still
 482 directed from LF toward HF. There is now a small direct forcing of mean KE (7GW) which adds
 483 to the conversion from mean APE to mean KE (5GW) to create a source of 12GW of Mean KE.
 484 8GW is directly dissipated by viscous forces, the remaining is mainly transferred to HF KE. The
 485 conversion between LF KE and HF KE is negligible.

486 The ratio of LF KE and LF APE has increased compared with the no wind simulation (the ratio is
 487 now 1%), however LF KE remains negligible compared to LF APE. We explain the predominance
 488 of APE over KE in the LF by the larger scales found at these frequencies. It is indeed known

489 from QG theory that the ratio KE over APE decreases with larger scales (Vallis 2017). Using the
 490 ECCO2 ocean state estimate, Zemskova et al. (2015) found that there is approximately 10 times
 491 more APE than KE in anomalies but do not discriminate between frequency bands which make
 492 the comparison with our results difficult.

493 With wind forcing, the $C(APE^{MEAN}, APE^{HF})$ conversion increases by 17GW (Table 2), its pos-
 494 itive values follow the northern boundary of the basin (Fig. 11) and the mean APE forcing (last
 495 column of Fig. 11). The largest values of $C(APE^{MEAN}, APE^{LF})$ are now almost entirely located
 496 in the northeastern corner where the SST anomalies seem to originate from.

497 *c. Climatological wind forcing $\tau_0 = 0.1 \text{ N m}^{-2}$*

498 In this experiment the zonal wind forcing is twice as strong as in the previous experiment, close
 499 to the climatological amplitude. The wind stress adds 34GW to the mean KE reservoir and 15 GW
 500 is directly dissipated by viscous forces (Fig. 13). The conversion from mean APE to mean KE is
 501 now negative (i.e. from mean KE to mean APE) which is in line with what is calculated in OGCM
 502 (e.g. Toggweiler and Samuels 1998; Gnanadesikan et al. 2005; Gregory and Tailleux 2011). The
 503 remaining 8GW are all converted to HF KE. The conversion of BPE to mean APE due to heat flux
 504 has slightly increased compared to the two other experiments (128GW). The BPE to mean APE
 505 conversion formula $-g \int_V \overline{z_r(\rho, t) \overline{F}} dV$ shows that the value of this conversion mainly depends on
 506 the position of the deepest reference depths and thus on the circulation in the Northern half of the
 507 basin. An explanation for the (small) increase in BPE to mean APE conversion could thus be that
 508 only the climatological wind substantially modifies the circulation in this region. The conversion
 509 from mean APE to HF APE is now much larger than the conversion from mean APE to LF APE
 510 (84GW vs 13GW), 42 GW is directly dissipated. The energy in the LF KE reservoir has increased
 511 compared to the two other simulation and now represents 10% of LF APE.

512 With this realistic amplitude of wind forcing, positive values of the conversion
513 $C(APE^{MEAN}, APE^{HF})$ continue to extend along the northern boundary (Fig. 11 bottom line) and
514 is particularly intense in the northwestern corner. $C(APE^{MEAN}, APE^{LF})$ intensity is weaker than
515 before and almost entirely located in the northwestern corner.

516 Sohail et al. (2018) and Zemskova et al. (2021) report that the input of KE obtained by increas-
517 ing the wind stress over the Southern ocean results mainly in an increase of the KE dissipation
518 term, rather than an increase in APE dissipation. These results are in line with what is found in
519 our experiments: the total dissipation of KE ($D_{KE^{MEAN}} + D_{KE^{LF}} + D_{KE^{HF}}$) has increased by 65%
520 between the no-wind and climatological wind experiment and by 24% for the total dissipation of
521 APE ($D_{APE^{MEAN}} + D_{APE^{LF}} + D_{APE^{HF}}$): most of the additional mean KE forcing is directly dissi-
522 pated by KE total dissipation.

523 **6. Conclusion**

524 In this article we have derived the mechanical energy budget for the large-scale, internally gener-
525 ated, low-frequency ocean mode that was studied extensively in previous works (Colin de Verdière
526 and Huck 1999; Huck et al. 1999; Huck and Vallis 2001; Huck et al. 2001, 2015; Arzel et al. 2018;
527 Hochet et al. 2020). The mechanical energy budget is decomposed into mean, low-frequency, and
528 high-frequency parts to study the effect of the large-scale baroclinic instability of the mean circu-
529 lation and the effect of the eddy field on the LF mode. One of the main achievements of this work
530 is the new way of decomposing the background potential energy into frequency bands that allows
531 us to correctly attribute the source and sink terms associated with each reservoir.

532 The energy budget of the no wind experiment shows that the energy of this multidecadal mode
533 is mostly contained in the LF APE rather than in the LF KE because of its large-scale. Using
534 ECCO v2, Zemskova et al. (2015) also found that the energy is mostly contained in the fluctuating

535 APE rather than in the fluctuating KE, but with a smaller ratio of approximately 10 (compared to
536 a LF APE over LF KE ratio of approximately 300 in our no wind experiment). This difference is
537 probably due to our use of a single hemispheric basin and thus to the omission of the strong wind
538 forcing over the Southern ocean. The source term for the LF APE is the conversion from the mean
539 APE to the LF APE. In agreement with Hochet et al. (2020) where the budget was made in terms
540 of temperature variance, the sink term of the mode is attributed to the parametrized diffusion (\sim
541 36 %) and to the transfer of APE to higher-frequencies linked with mesoscale eddy turbulence (\sim
542 64 %). In the experiment with no wind forcing, the only source of kinetic energy is the conversion
543 from APE at all frequencies. The transfer of KE between high and low-frequencies which is
544 shown to be an important source of low frequency variability in other experiments (Arbic et al.
545 2014; Sérazin et al. 2018) appears negligible in our configuration compared to all the other energy
546 fluxes. Recognising that this might be due to the absence of any direct source of kinetic energy we
547 performed two other experiments with a time independent zonal wind forcing at the surface that
548 drives the classical wind-driven double gyres.

549 With the addition of a wind forcing at the surface, a source term for the mean kinetic energy
550 appears. The structure of the LF mode is modified with intermediate wind strength and almost
551 disappears with climatological wind. With increasing wind forcing, the LF APE and all its asso-
552 ciated conversion terms decrease. Indeed there is a decrease in the values of the conversion terms
553 from mean APE to LF APE and of LF APE to BPE (linked with diffusive flux), of LF APE to
554 HF APE and of LF APE to LF KE for the sink terms. Meanwhile, the energy in the HF APE
555 increases as well as the conversion from mean APE to HF APE. This larger conversion is balanced
556 mostly by a larger conversion of HF APE to BPE due to diffusive flux. The energy contained in
557 the mean APE increases as well as the conversion from BPE to mean APE due to heat flux. This
558 increase is balanced by a larger conversion to BPE due to diffusive flux and by a larger conversion

559 to HF APE. The conversion of mean APE to mean KE becomes negative which is in agreement
560 with what is usually calculated in more realistic models such as in Zemskova et al. (2015). The
561 conversion between LF KE and HF KE remains negligible or very small compared to other con-
562 versions, nonetheless, it is directed from HF to LF for the climatological wind experiment which
563 is in agreement with the temporal inverse KE cascade found in Arbic et al. (2014).

564 For all wind stress intensities studied here, the energy and conversion terms remain mainly con-
565 tained in the APE. There is however a non-negligible transfer of APE to KE at HF where it is
566 dissipated by viscous forces. The fact that most of the transfers of energy occur between the dif-
567 ferent APE and BPE reservoirs rather than between PE or KE reservoirs outline the importance of
568 the APE budget to study large-scale and low frequency variability. In contrast, Arbic et al. (2014);
569 Sérazin et al. (2015, 2018) found a predominant role for the temporal inverse KE cascade, but we
570 attribute this difference for the most part to our focus on multidecadal variability as compared to
571 their focus on shorter inter-annual variability. The transfer from LF APE to HF APE demonstrates
572 the damping role of the mesoscale eddy turbulence for the large scale variability, even for realistic
573 wind intensities. This transfer of APE from LF to HF is very similar to the QG APE direct tempo-
574 ral cascade of APE that has been observed (along with the inverse temporal cascade of KE) in the
575 idealized simulation of Arbic et al. (2014). Temperature variance budget gives similar pathway
576 for the sources and sinks of the low frequency mode (Hochet et al. 2020). However, we have shown
577 that the locations of the APE and of its associated transfers differ significantly from that of the den-
578 sity variance. Large values of APE are indeed linked with the outcropping of dense waters at the
579 surface due to convection that occurs along the northern boundary in our configuration, whereas
580 there is no significant link between large values of density variance and convective regions. The
581 APE budget is therefore more accurate in identifying regions where energy conversions are the
582 most important.

583 This study is limited by several approximations, the first being the representation of the North
584 Atlantic ocean with a simplified geometry and a flat bottom. In particular, the wind forcing over
585 the Southern Ocean is thought to play an important role in setting the mean stratification, in par-
586 ticular the stratification in the North Atlantic (e.g. Nikurashin and Vallis 2011, 2012) and might
587 therefore influence the dynamic of the low frequency mode. Moreover, using energy budget of
588 a global ocean–sea ice model Hogg et al. (2017) have shown that wind change over the South-
589 ern Ocean leads to change of APE and stratification in the North Atlantic. However, it has been
590 shown in Arzel et al. (2007) that the addition of a re-entrant channel representing the Antarctic
591 Circumpolar Current acts to reduce the low frequency variability in the Southern Hemisphere but
592 does not suppress the internal variability in the Northern Hemisphere. Moreover, the physical
593 mechanism giving rise to the internal mode studied in this single hemispheric configuration is also
594 found to give rise to low-frequency variability in realistic ocean-only configuration (e.g. Sévellec
595 and Fedorov 2013; Arzel et al. 2018; Arzel and Huck 2020) and in ocean-atmosphere coupled
596 configuration (e.g. Ortega et al. 2015; Gastineau et al. 2018). The omission of salinity and the use
597 of a linear equation of state for density certainly has an influence on the APE budget. Indeed, it
598 is known that non-linearities of the equation of state are in general not negligible (e.g. Klocker
599 and McDougall 2010; Nycander et al. 2015). Nonetheless, it has been shown in a realistic setup
600 (Sévellec and Fedorov 2013) that the mode is largely controlled by temperature variation in the
601 upper ocean. Our study as well as previous studies of the internal mode (e.g. Sévellec and Fe-
602 dorov 2013; Huck et al. 2015; Arzel et al. 2018) assume that external forcing is either constant or
603 made of natural variability such as the North Atlantic Oscillation (Frankcombe et al. 2009; Arzel
604 and Huck 2020). This assumption is helpful to understand the physics and the mechanisms of the
605 mode. However, with its multi-decadal variability, the internal mode might be affected by anthro-

606 pogenic forcing which act on the same time scales and modifies the characteristics of the ocean
607 stratification (Levitus et al. 2012).

608 The eddy-permitting resolution of 20 km used here is not sufficient to entirely resolve the eddy
609 field, however similar experiments at 10 km with no wind forcing were conducted in Huck et al.
610 (2015) and no qualitative differences were found. Lastly we set the limit between LF and HF to be
611 3.5 years based on the results from Hochet et al. (2020) that showed that in the same configuration
612 without wind forcing, non-linear transfers of temperature variance are a source (sink) term for
613 periods longer (shorter) than 3.5 years. To be able to compare the three configurations studied in
614 this article we kept this limit fixed, however, with the addition of a wind forcing at the surface, we
615 expect a change in this limit and possibly a modification of the LF/HF transfers. The study of the
616 dependence of this limit on external parameters is left to future work, as well as the implementation
617 of this full energy budget in realistic eddy-resolving models.

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792 **LIST OF TABLES**

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794 **Table 2.** Transfer values for all terms in the energy budget for the no wind ($\tau_0 =$
795 0Nm^{-2}), intermediate wind ($\tau_0 = 0.05\text{Nm}^{-2}$) and climatological wind ($\tau_0 =$
796 0.1Nm^{-2}) experiments. 43

KE and APE Dissipation	
$D_{KE^{LF}}$	$\rho_0 \int_V (\overline{u^{LF} v_u^{LF}} + \overline{v^{LF} v_v^{LF}}) dV$
$D_{KE^{MEAN}}$	$\rho_0 \int_V (\overline{u v_u} + \overline{v v_v}) dV$
$D_{KE^{HF}}$	$-F_{KE^{MEAN}} - C(APE, KE) - D_{KE^{LF}} - D_{KE^{MEAN}}$
$D_{APE^{LF}}$	$-\int_V g z_r(\rho, t) \overline{D^{LF}} dV$
$D_{APE^{MEAN}}$	$\int_V g z \overline{D} dV - \int_V g z_r(\rho, t) \overline{D} dV$
$D_{APE^{HF}}$	$-F_{APE^{MEAN}} - D_{APE^{LF}} - D_{APE^{MEAN}} + C(APE, KE)$
KE and APE Forcing	
$F_{KE^{MEAN}}$	$\rho_0 \int_V \overline{u F_u} dV$
$F_{APE^{MEAN}}$	$-\int_V g z_r(\rho) \overline{F} dV$
Conversion between reservoirs	
$C(KE^{LF}, KE^{HF})$	$C(APE^{LF}, KE^{LF}) + C(KE^{MEAN}, KE^{LF}) + D_{KE^{LF}}$
$C(KE^{MEAN}, KE^{LF})$	$\rho_0 \int_V (\overline{u \nabla \cdot (\mathbf{v}^{LF} u^{LF})} + \overline{v \nabla \cdot (\mathbf{v}^{LF} v^{LF})}) dV$
$C(KE^{MEAN}, KE^{HF})$	$F_{KE^{MEAN}} + C(APE^{MEAN}, KE^{MEAN}) + D_{KE^{MEAN}} - C(KE^{MEAN}, KE^{LF})$
$C(APE^{LF}, APE^{HF})$	$-C(APE^{LF}, KE^{LF}) + C(APE^{MEAN}, APE^{LF}) + D_{APE^{LF}}$
$C(APE^{MEAN}, APE^{LF})$	$-\int_V g \overline{\rho^{LF} w^{LF}} dV - g \int_V z_r(\rho, t) \overline{\mathbf{v}^{LF} \cdot \nabla \rho^{LF}} - \overline{z_r(\rho, t) \mathbf{v} \cdot \nabla \rho^{LF}} dV$
$C(APE^{MEAN}, APE^{HF})$	$-C(APE^{MEAN}, APE^{LF}) - C(APE^{MEAN}, KE^{MEAN}) + D_{APE^{MEAN}} + F_{APE^{MEAN}}$
$C(APE^{MEAN}, KE^{MEAN})$	$-\int_V g \overline{\rho w} dV$
$C(APE^{LF}, KE^{LF})$	$-\int_V g \overline{\rho^{LF} w^{LF}} dV$
$C(APE^{HF}, KE^{HF})$	$C(APE, KE) - C(APE^{MEAN}, KE^{LF}) - C(APE^{MEAN}, KE^{MEAN})$
$C(APE, KE)$	$\int_V g z \overline{\nabla \cdot \mathbf{v}} dV$

TABLE 1. List of all terms in the energy budget shown in figure 3

	$\tau_0 = 0 \text{ N m}^{-2}$	$\tau_0 = 0.05 \text{ N m}^{-2}$	$\tau_0 = 0.1 \text{ N m}^{-2}$
KE and APE Dissipation			
$D_{KE^{LF}}$	-5 GW	-3 GW	-2 GW
$D_{KE^{MEAN}}$	-5 GW	-8 GW	-15 GW
$D_{KE^{HF}}$	-22 GW	-22 GW	-36 GW
$D_{APE^{LF}}$	-16 GW	-7 GW	-5 GW
$D_{APE^{MEAN}}$	-18 GW	-30 GW	-42 GW
$D_{APE^{HF}}$	-53 GW	-56 GW	-62 GW
KE and APE Forcing			
$F_{KE^{MEAN}}$	0 GW	7 GW	34 GW
$F_{APE^{MEAN}}$	119 GW	119 GW	128 GW
Conversion between reservoirs			
$C(KE^{LF}, KE^{HF})$	0 GW	0 GW	-1 GW
$C(KE^{MEAN}, KE^{LF})$	1 GW	1 GW	0 GW
$C(KE^{MEAN}, KE^{HF})$	2 GW	3 GW	8 GW
$C(APE^{LF}, APE^{HF})$	29 GW	14 GW	7 GW
$C(APE^{MEAN}, APE^{LF})$	49 GW	23 GW	13 GW
$C(APE^{MEAN}, APE^{HF})$	44 GW	61 GW	84 GW
$C(APE^{MEAN}, KE^{MEAN})$	8 GW	5 GW	-11 GW
$C(APE^{LF}, KE^{LF})$	4 GW	2 GW	1 GW
$C(APE^{HF}, KE^{HF})$	20 GW	19 GW	29 GW
$C(APE, KE)$	32 GW	26 GW	19 GW

797 TABLE 2. Transfer values for all terms in the energy budget for the no wind ($\tau_0 = 0 \text{ N m}^{-2}$), intermediate wind
798 $\tau_0 = (0.05 \text{ N m}^{-2})$ and climatological wind ($\tau_0 = 0.1 \text{ N m}^{-2}$) experiments.

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818 LF/HF 3.5 year separation. 50

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821 imaginary part of the SST. Middle left: CEOF real part along the meridional section (longi-
822 tude = 800 km) shown by a red line on the top left and right panels, middle right: imaginary
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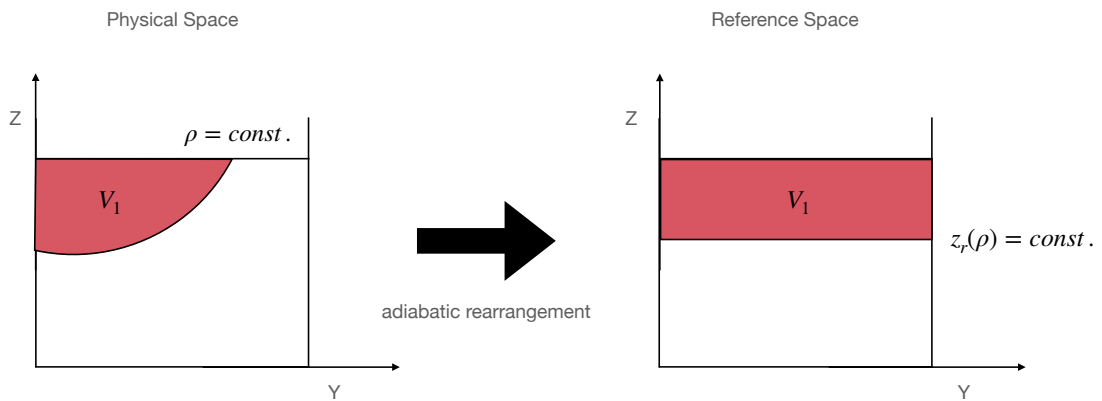
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839 by surface heat fluxes is shown by green arrows, the conversion of APE to BPE because of
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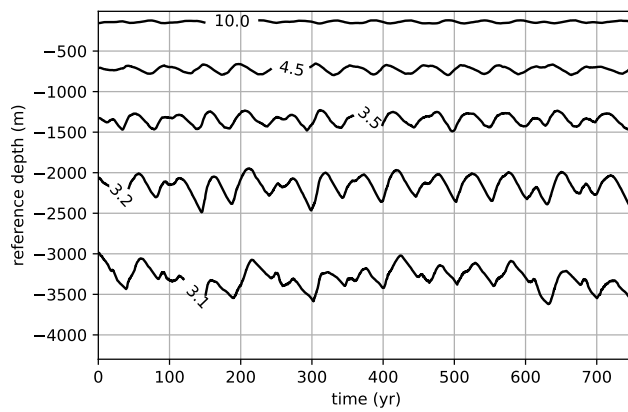
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 845 $C(APE^{MEAN}, APE^{HF})$, mean APE and LF APE $C(APE^{MEAN}, APE^{LF})$ (second column).
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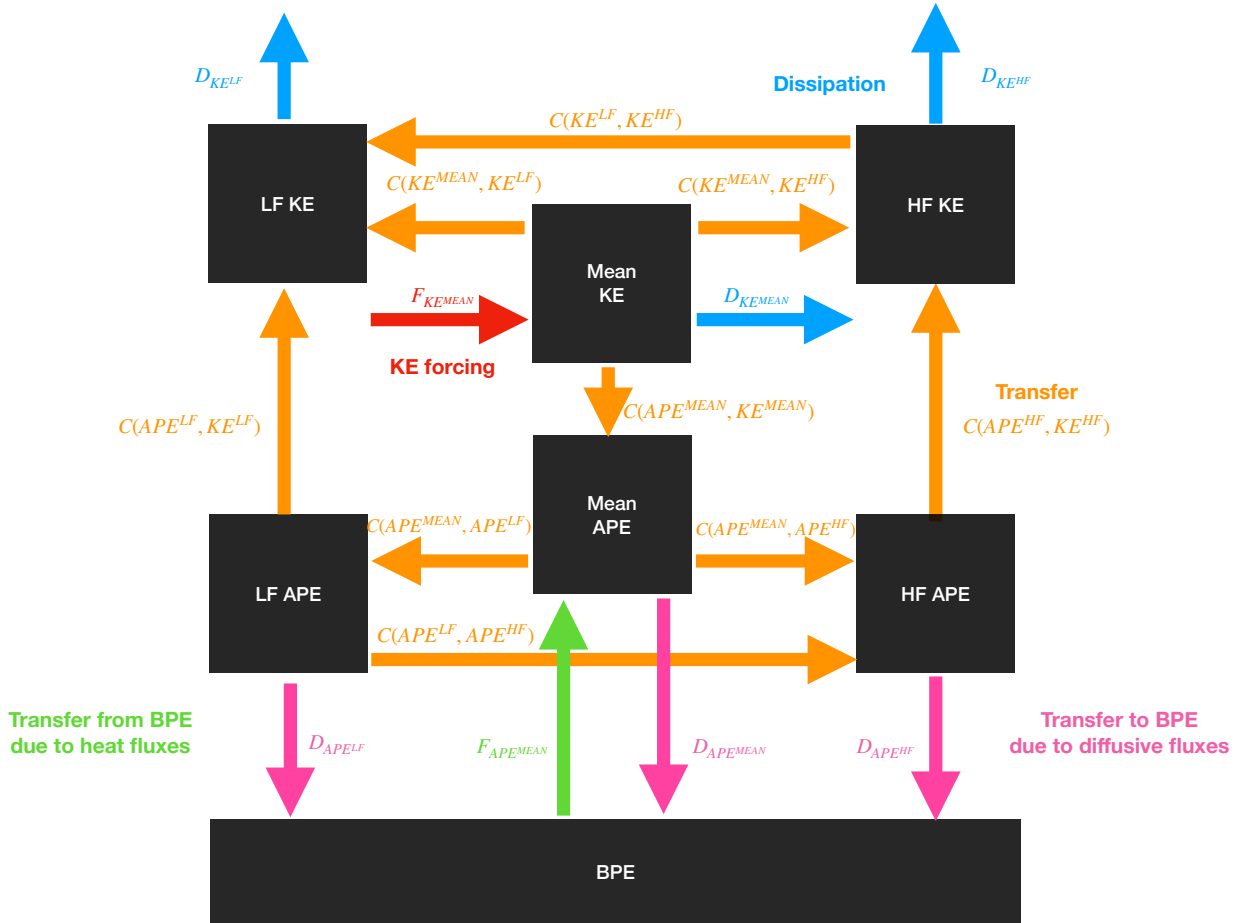
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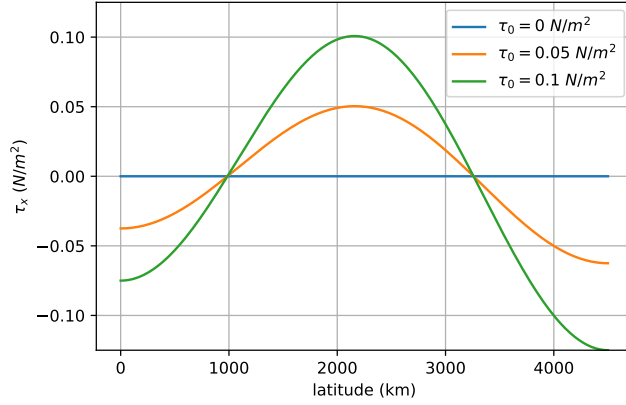
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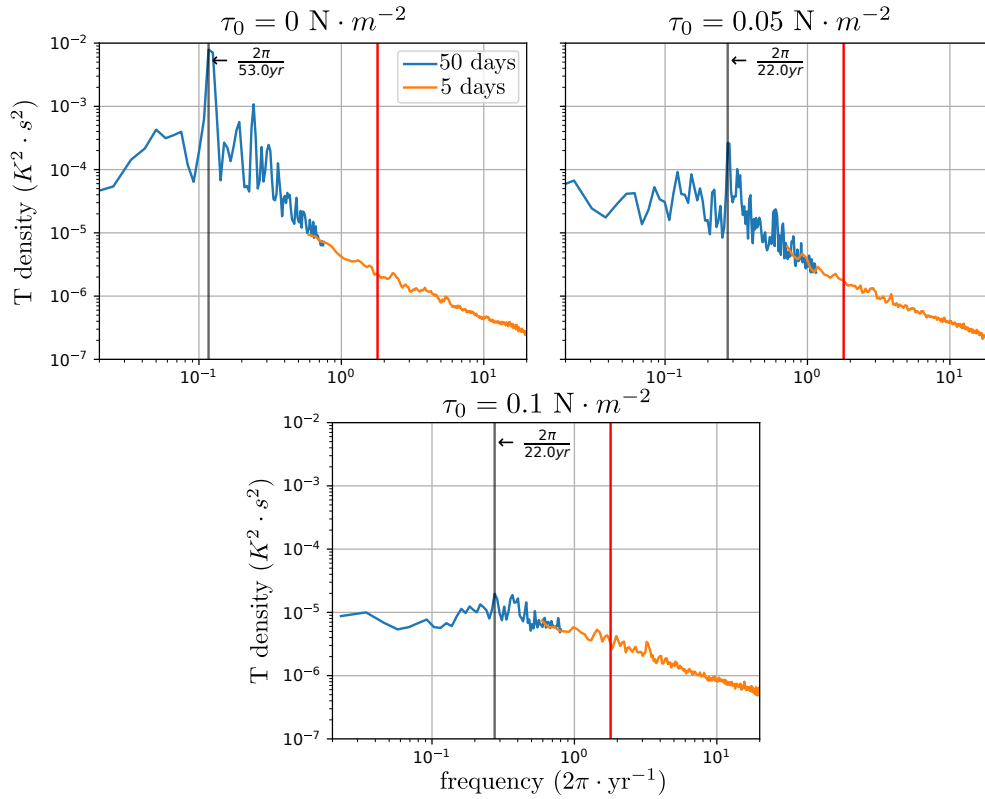
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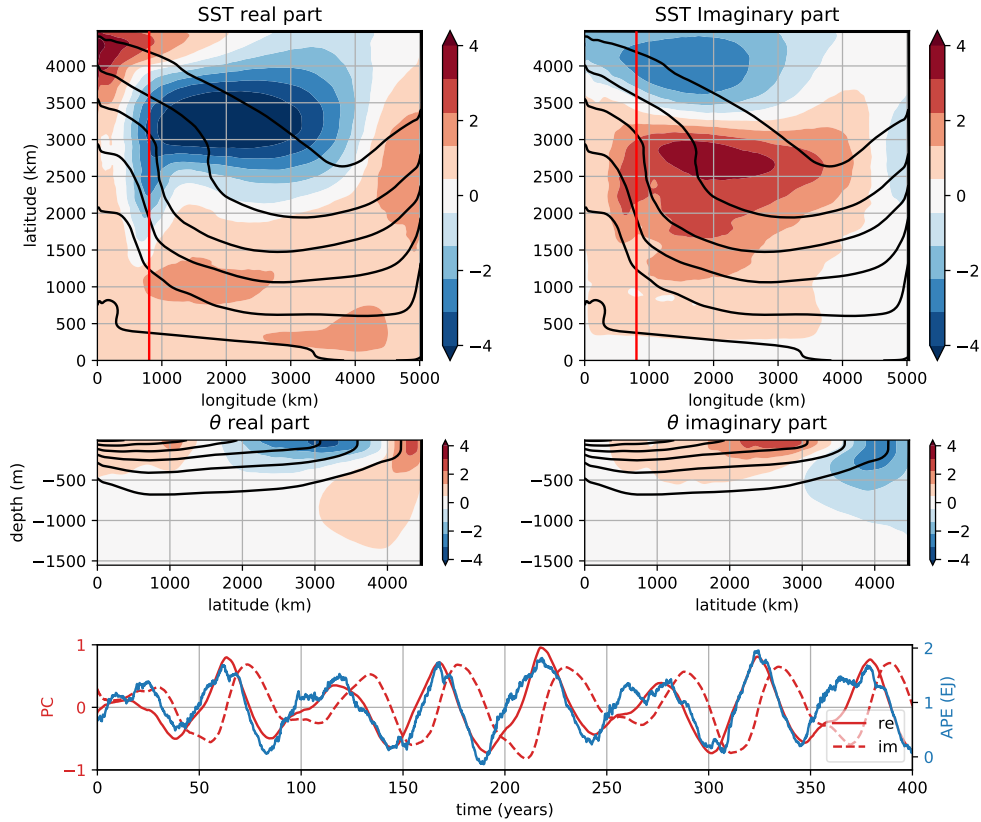
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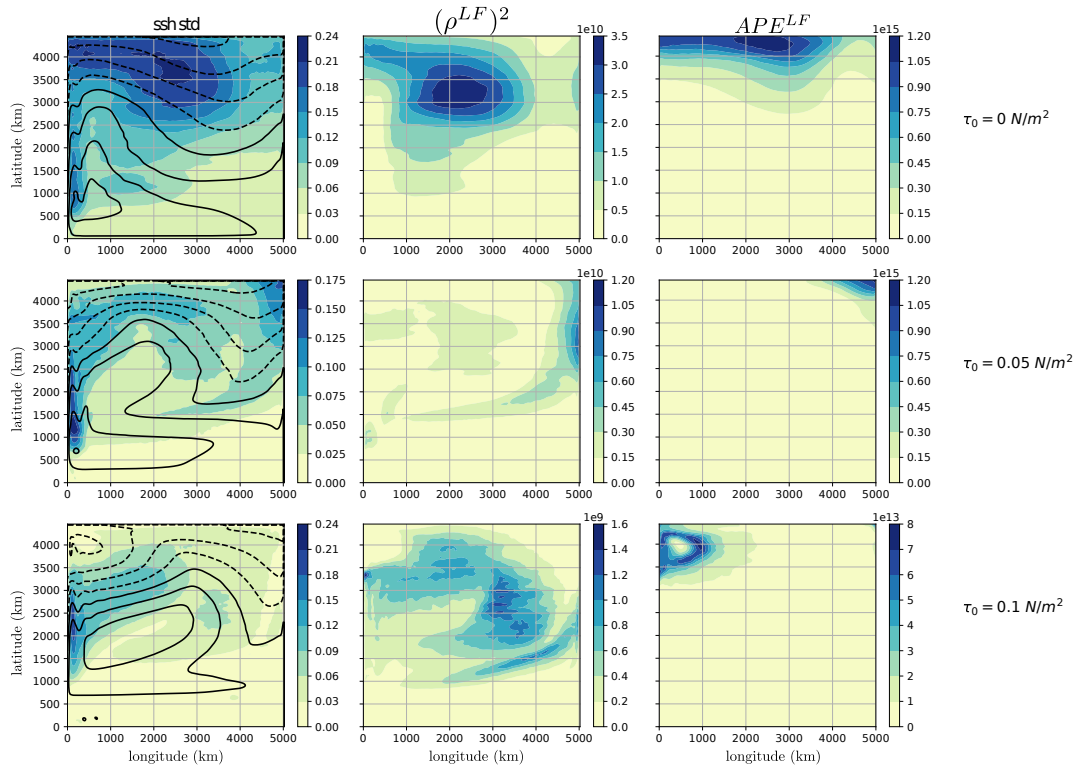
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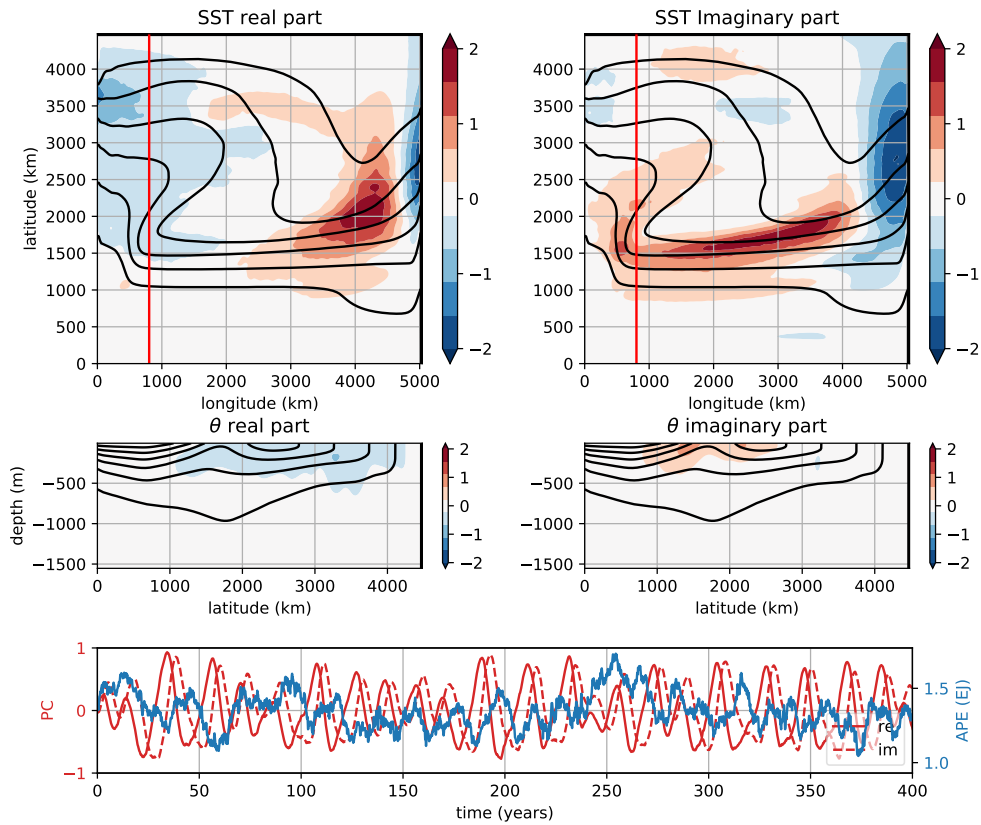
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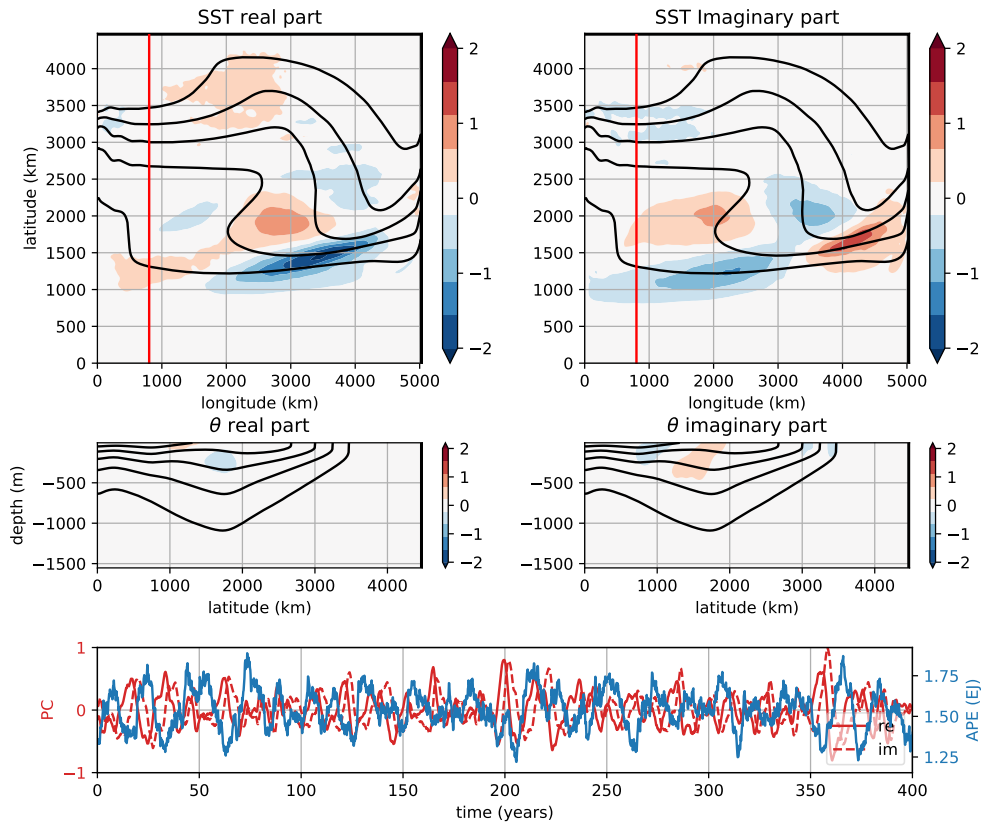
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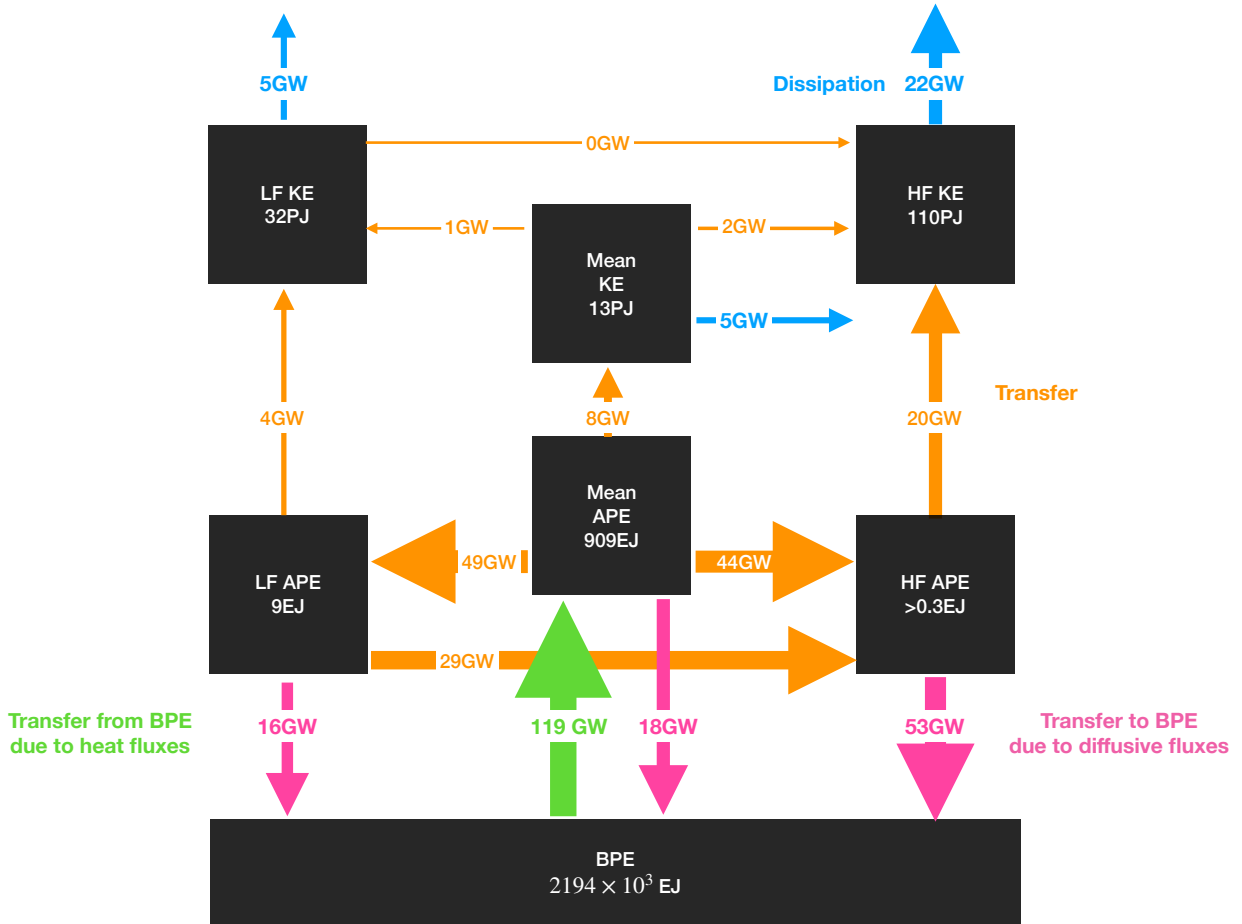
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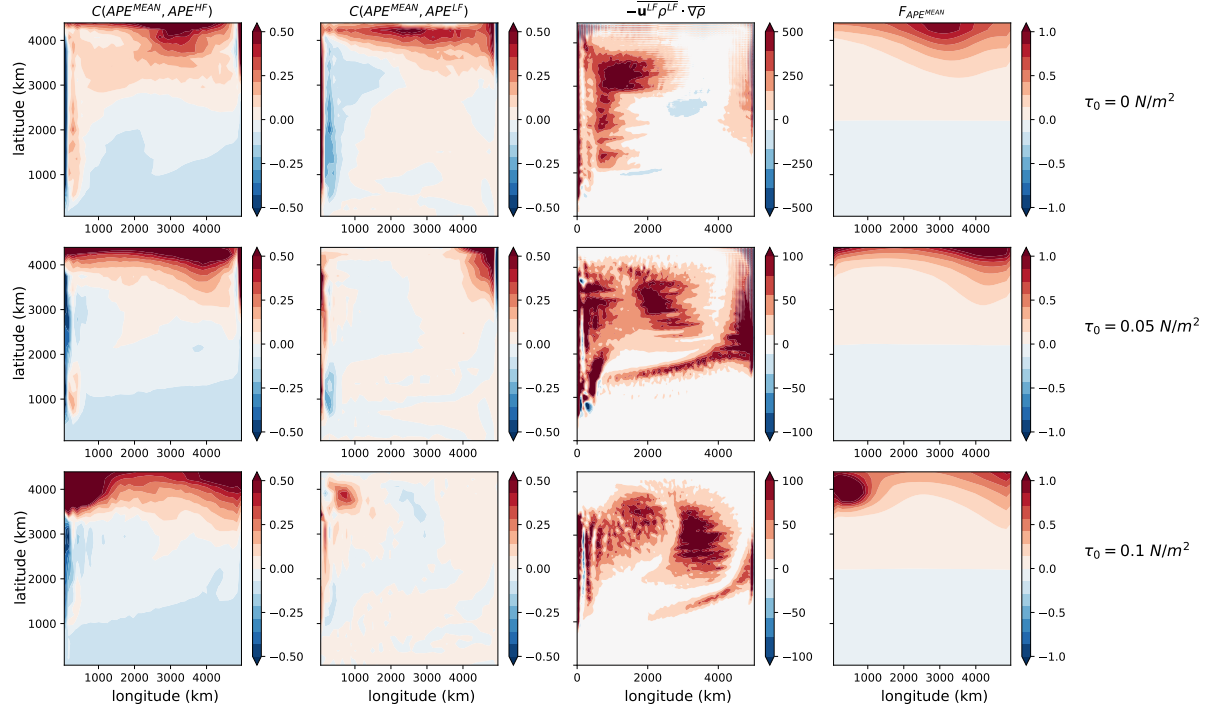
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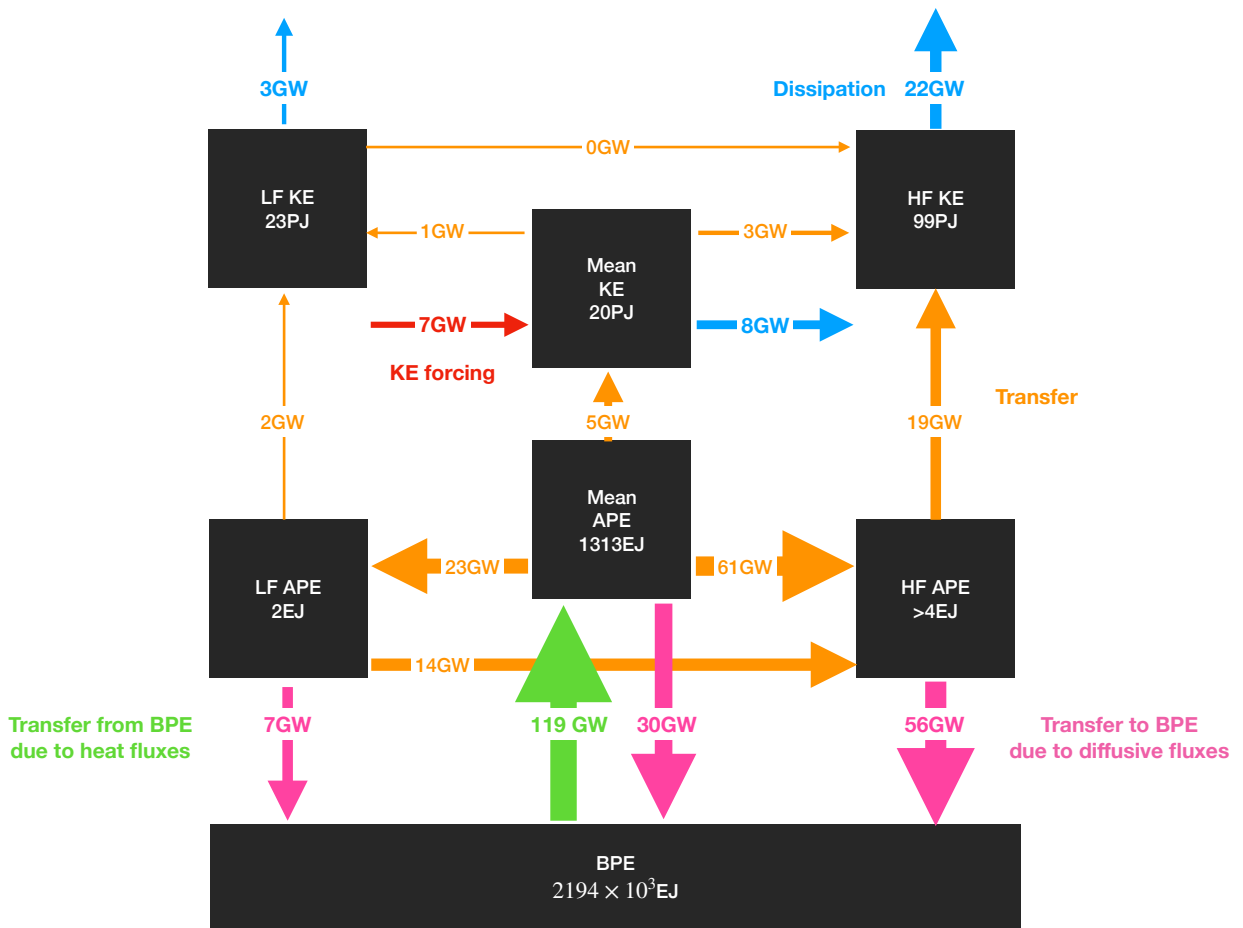
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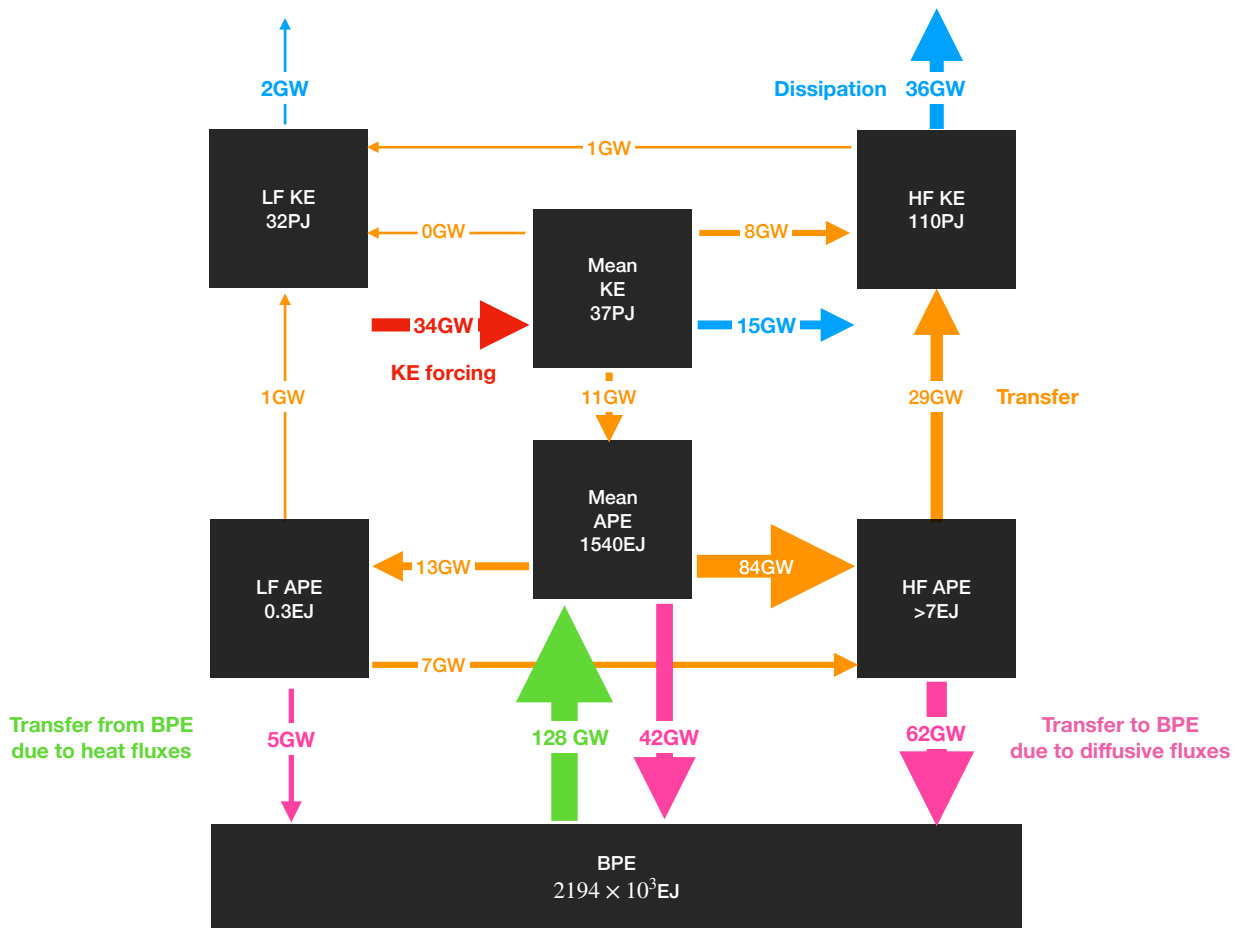
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